Dark Energy

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Outline

• Lecture 1 Introduction to Dark Energy
• Lecture 2 Testing Dark Energy, 1
• Lecture 3 Testing Dark Energy, 2
• Lecture 4 Further probes of DE
• Lecture 5 Euclid
Some collaborators

Luciano Casarini, Adalto Gomes, Valerio Marra, Miguel Quartin, Rogerio Rosenfeld, Ioav Waga, ...

Texts

- **Dodelson**, Modern Cosmology
- **Euclid Theory WG**, Cosmology and Fundamental Physics with the Euclid Satellite, arXiv 1206.1225 +1606.00180
Cosmology Executive Summary

Dark matter 27%
Baryons 5%
Massive neutrinos: 0.1%
Photons: 0.01%
Spatial curvature: very close to 0

Something else: \( \approx 70\% \)
Historical perspective, circa 350 BCE

- Gravity is always attractive: how to avoid that the sky falls on our head?
- Aristotle’s answer: quintessence
Historical perspective, circa 1700

- Gravity is always attractive: how to avoid that the stars fall on our head?
- Newton’s answer: initial conditions

Historical perspective, circa 1900

- Gravity is always attractive: how to avoid that the Universe fall on our head?
- Einstein’s answer: to avoid collapse (to make the universe stable) it is necessary to introduce a form of repulsive gravity, by modifying the equations of General Relativity.
Einstein’s equations

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

Einstein 1917

Time view

We know so little about the evolution of the universe!

\[ \Omega \]

radiation

matter

BBN

CMB

DE
Why now?

The coincidence problem

$\rho$ $\rho$ $\rho$

Beyond the cosmological constant

$p = -\rho$

$\dot{\rho} = -3H(p + \rho)$
From observations to theory
- What we really observe in cosmology is light from sources and from backgrounds.
- How do we connect these observables to cosmological quantities like $\rho_m, \rho_\gamma, k, a(t), H_0$ etc?
- First, define
  $$\Omega_M = \frac{8\pi \rho_0}{3H_0^2}, \quad \Omega_\Lambda = \frac{8\pi \rho_\Lambda}{3H_0^2}, \quad \Omega_k = \frac{8\pi k}{3H_0^2}$$
  and note that
  $$1 = \Omega_M + \Omega_\Lambda + \Omega_k$$
  so rewrite Friedman equation as ($a_0 = 1$)
  $$H^2 = H_0^2(\Omega_m a^{-3} + \Omega_\Lambda a^3 + \Omega_k a^{-2})$$
- Then, generalize it to several components:
  $$H^2 = H_0^2(\Omega_m a^{-3(1+w_m)} + \Omega_\Lambda a^{3(1+w_\Lambda)} + ...)$$
  $$= H_0^2 \sum_i \Omega_i a^{-3(1+w_i)} = H_0^2 E(a)^2$$

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<table>
<thead>
<tr>
<th>name</th>
<th>density</th>
<th>EOS w</th>
</tr>
</thead>
<tbody>
<tr>
<td>baryons</td>
<td>0.05</td>
<td>≈ 0</td>
</tr>
<tr>
<td>CDM</td>
<td>0.27</td>
<td>≈ 0</td>
</tr>
<tr>
<td>radiation</td>
<td>0.0001</td>
<td>1/3</td>
</tr>
<tr>
<td>Massive neutrinos</td>
<td>&lt;0.05</td>
<td>≈ 0</td>
</tr>
<tr>
<td>Cosm. const.</td>
<td>0.68</td>
<td>-1</td>
</tr>
<tr>
<td>curvature</td>
<td>&lt;0.01</td>
<td>-1/3</td>
</tr>
<tr>
<td>Other ?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

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Warning 1

- Not everything we insert in our equations is necessarily an observable!
- Consider only two components, pressureless uncoupled matter and “something else”.
- Then the EOS of this “something else” is related to the expansion $E(z)$ as
  \[ w_{de} = \frac{(1 + z)(E^2)' - 3E^2}{3[E^2 - \Omega_{m0}(1 + z)^3]} \]
- Even a perfect knowledge of $E(z)$ is not sufficient to obtain $w(z)$: one needs also $\Omega_{m0}$
- But all you get from distance indicators is $E(z)$, not $\Omega_{m0}$
- Conclusion: either you “know” $\Omega_{m0}$ and obtain $w(z)$ or you “know” $w(z)$ and obtain $\Omega_{m0}$

Example

LCDM with different $\Omega$

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Warning 2

- We only directly test gravity within the solar system, at the present time, and with “baryons”

![Diagram showing gravitational interaction](image1)

[On Space and Time, Edited by Shahn Majid]

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Testing Gravity

![Diagram showing experimental setup](image2)

Figure 2: Schematic of the masses and the cantilevers. Geometry in the $x-y$ plane is to scale; the $z$-separation and the thicknesses of the cantilever and shield are not to scale.

Smullin et al. 2004 (SLAC)
Testing Gravity

\[ \frac{GM}{r} \rightarrow \frac{GM}{r} (1 + \beta e^{-r/\lambda}) \]

The fourfold way out from local gravity

\[ \Psi = -\frac{MG^*}{r} (1 + \beta e^{-m_{\phi} r}) \]

\[ m_{\phi}, \beta \] depend on:
- time
- space
- local density
- species

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Lighthouses in the dark

$L^2 = \frac{L}{4\pi f}$

Supernovae Ia \( m - M = 5 \log d_L + 25 \)

- This hypothesis can be tested and calibrated through a local sample whose distance we know by other means.

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Then, we compare $m_{\text{obs}}(z)$ with

$$m_{\text{theor}}(z) = M + 25 + \log d(z; \Omega, \Omega, ..)$$

\[ d_L = r(z)(1+z) \]
\[ ds^2 = 0 \quad \Rightarrow \quad r(z) \]
\[ d_L(z) = \frac{1+z}{H_0 \sqrt{-\Omega_{k0}}} \sinh(\frac{H_0 \sqrt{-\Omega_{k0}}}{H(z)} \int \frac{dz}{H(z)}) \]
\[ H(z) = H_0 (\Omega_{m0} a^{-3} + \Omega_\Lambda + \Omega_{k0} a^{-2})^{1/2} \]
Basic property 1

Local Hubble law

\[ r(z) = \frac{z}{H_0} \]

Global Hubble law

\[ r(z) = \int \frac{dz}{H(z)} \]

If \( H(z) \) in the past is smaller (i.e. acceleration), then \( r(z) \) is larger:
larger distances (for a given redshift) make dimmer supernovae

Basic property 2

Curves of constant luminosity distance

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Hubble diagram

Properties of Dark Energy
Properties of Dark Energy

- Isotropy
- Abundance
- Slow evolution
- Weak clustering

Observations:
- Isotropy
- Large abundance
- Slow evolution
- Weak clustering

Theory:
- Scalar field?
- $\Omega_{DE} \approx \Omega_m$
- $w_{eff} \approx -1$
- $c_s \approx 1$

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The magic of Lagrangians

\[ \int dx^4 \sqrt{-g} \left[ \mathcal{R} + L_{\text{matter}} \right] \]

variation

\[ R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = 8 \pi T_{\mu \nu} \]

The two laws of Lagrangian-cooking:

a) form a scalar: Eqs are covariant

b) No explicit functions of coords: Eqs are conserved

Random example:

\[ \int dx^4 \sqrt{-g} \left[ f(\phi) \mathcal{R} + R_{\mu \nu} \phi^{\prime \prime} + V(\phi) + R_{\mu \nu \rho \sigma} \phi^{\prime \prime} \phi^{\prime \prime} + \cdots + L_{\text{matter}} \right] \]

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The magic of Fourier space

Fourier space

\[ \phi(x, y, z, t) \rightarrow \phi(t) \exp(i \vec{k} \cdot \vec{x}) \]

therefore

\[ \frac{\partial}{\partial x} \phi(x, y, z, t) \rightarrow \phi(t) i k_x \exp(i \vec{k} \cdot \vec{x}) \]

\[ \nabla \phi(x, y, z, t) \rightarrow \phi(t) i \vec{k} \exp(i \vec{k} \cdot \vec{x}) \]

\[ \nabla \nabla \phi(x, y, z, t) \rightarrow - \phi(t) k^2 \exp(i \vec{k} \cdot \vec{x}) \]

Very useful for linear equations because then you can drop the space part!
The past ten years of DE research

\[ \int d^4x \sqrt{-g} \left[ R + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + V(\phi) + L_{\text{matter}} \right] \]

\[ \int d^4x \sqrt{-g} \left[ f(\phi)R + \frac{1}{2} \phi_{,\mu} \phi^{,\mu} + V(\phi) + L_{\text{matter}} \right] \]

\[ \int d^4x \sqrt{-g} \left[ f(\phi)R + K \left( \frac{1}{2} \phi_{,\mu} \phi^{,\mu} \right) + V(\phi) + L_{\text{matter}} \right] \]

\[ \int d^4x \sqrt{-g} \left[ f(\phi, \frac{1}{2} \phi_{,\mu} \phi^{,\mu}) R + G_{\mu\nu} \phi^{,\mu} \phi^{,\nu} + K \left( \frac{1}{2} \phi_{,\mu} \phi^{,\mu} \right) + V(\phi) + L_{\text{matter}} \right] \]

Cosmological constant, Dark energy $w=\text{const}$, Dark energy $w=w(z)$, quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

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The Horndeski Lagrangian

The most general 4D scalar field theory with second order equations of motion

\[ \int d^4x \sqrt{-g} \left[ \sum L_i + L_{\text{matter}} \right] \]

\[ C_i = \left( \partial_\mu \partial^\mu \phi \right)^2 - \left( \nabla_\mu \phi \cdot \nabla^\mu \phi \right) \]

\[ L_i = C_i \frac{1}{2} \left( \partial_\mu \partial^\mu \phi \right) \]

\[ C_i = \frac{1}{2} \left( \partial_\mu \partial^\mu \phi \right) \]

First found by Horndeski in 1975

✓ rediscovered by Deffayet et al. in 2011
✓ no ghosts, no classical instabilities
✓ it modifies gravity!
✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc
A riddle for you all

The most general 4D scalar field theory with second order equations of motion

\[ \int d^4x \sqrt{-g} \left[ \sum_i L_i + L_{\text{matter}} \right] \]

\( L_0 = K(\phi, X) \),
\( L_3 = -G_3(\phi, X) \Sigma \phi \),
\( L_4 = \frac{G_4(\phi, X)}{2} R + G_{4, X} \left[ (\nabla^2 \phi)^2 - (\nabla \phi \nabla \phi) (\nabla^2 \phi) \right] \),
\( L_5 = G_5(\phi, X) G_{\mu \nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{2} G_{4, X} \left[ (\nabla^2 \phi)^2 - 3 (\nabla \phi \nabla \phi) (\nabla^2 \phi) + 2 (\nabla^\mu \nabla^\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right] \).

It turns out that

\[ \sum_i L_i = p(\phi, X) \]

Why?

The Ostrogradski theorem

Higher-than-second order equations of motion are unstable

\[ L = L(q, \dot{q}, \ddot{q}) \]

\[ \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} = 0 \]

The Hamiltonian contains a term linear in a canonical momentum

\[ H = P Q_0 + P_i a(Q_0, Q_i, P_i) - L(Q_0, Q_i, a) \]

Energy states are unbounded from below!

Woodard astro-ph/0601672
The Ostrogradski theorem: proof

\[ L = L(q, \dot{q}, \ddot{q}) \]

\[ \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} = 0 \]

\[ Q_1 = q, \quad P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}} \]

\[ Q_2 = \dot{q}, \quad P_2 = \frac{\partial L}{\partial \ddot{q}} \quad \text{← this has to be invertible} \]

\[ \dot{q} = a(Q_1, Q_2, P_2) \]

\[ H(Q_1, Q_2, P_1, P_2) = \sum P_i q^{(i)} - L = P_1 Q_1 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, a) \]

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Caveats...

- The dangerous term in the Hamiltonian can actually be absent in special degenerate cases
- The instability can manifest itself in very long time scales
- The proof assumes locality
The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

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The Great Horndeski Hunt

Let us assume we have only
1) a perturbed FRW metric
2) pressureless matter
3) the Horndeski field

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Standard rulers

\[ D(z) = \frac{R}{\theta} = \frac{(1+z)^{-1}}{H_0 \sqrt{-\Omega_k}} \sinh(H_0 \sqrt{-\Omega_k} \int \frac{dz}{H(z)}} \]

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Standard rulers

\[ H(z) = \frac{dz}{R} \]

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BAO ruler

Then we can measure $H(z)$ and

$$D(z) = \frac{1}{H_0\sqrt{-\Omega_k_{0}}} \sinh(H_0\sqrt{-\Omega_k_{0}}\int \frac{dz}{H(z)})$$

and therefore we can reconstruct the full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1 - \frac{\Omega_k_{0}r^2}{4}\right)}(dx^2 + dy^2 + dz^2)$$
Two free functions

The most general linear, scalar metric

\[ ds^2 = a^2[(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)] \]

• Poisson’s equation

\[ \nabla^2\Psi = 4\pi G \rho_m \delta_m \]

• anisotropic stress

\[ 1 = -\frac{\Psi}{\Phi} \]

Warning: all the perturbation variables in this talk are root mean squares!

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### Modified Gravity at the linear level

<table>
<thead>
<tr>
<th>Type</th>
<th>Equation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>standard gravity</strong></td>
<td>( Y(k, a) = 1 ) &lt;br&gt; ( \eta(k, a) = 1 )</td>
<td>Boisseau et al. 2000; Acquaviva et al. 2004</td>
</tr>
<tr>
<td><strong>scalar-tensor models</strong></td>
<td>( Y(a) = \frac{G}{F_{G, a}} \frac{2(F + F''(a))}{2F + 3F''} ) &lt;br&gt; ( \eta(a) = 1 + \frac{\beta}{1 + 2\eta_{fr}} )</td>
<td>Schimd et al. 2004; L.A., Kunz &amp; Sapone 2007</td>
</tr>
<tr>
<td><strong>f(R)</strong></td>
<td>( Y(a) = \frac{G}{F_{G, a}} \frac{1 + 4n}{1 + 3n} \frac{\eta^2}{\eta^2 + \frac{m}{a^2}} ) &lt;br&gt; ( \eta(a) = 1 + \frac{1 - \frac{2}{3\beta}}{1 + 2\eta_{fr}} )</td>
<td>Bean et al. 2006; Hu et al. 2006; Tsujikawa 2007</td>
</tr>
<tr>
<td><strong>DGP</strong></td>
<td>( Y(a) = 1 - \frac{1}{3\beta} ); ( \beta = 1 + 2Hr_{\text{w}} ) &lt;br&gt; ( \eta(a) = 1 + \frac{2}{3\beta - 1} )</td>
<td>Lue et al. 2004; Koyama et al. 2006</td>
</tr>
<tr>
<td><strong>massive bi-gravity</strong></td>
<td>( Y(a) = \ldots ) &lt;br&gt; ( \eta(a) = \ldots )</td>
<td>F. Koonin and L.A. 2014, Y. Akrami et al. 2014</td>
</tr>
</tbody>
</table>

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### In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

\[ \eta(k, a) = h_2 \left( 1 + k^2 h_4 \right) \]
\[ Y(k, a) = h_1 \left( 1 + k^2 h_5 \right) \]

\( k = \) wavenumber <br> \( h_i = \) time-dependent functions


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Modified Gravity at the linear level

\[ h_1 = \frac{a_0}{a_1} + \frac{c_0^2}{a_1}, \quad h_2 = \frac{a_0}{a_1} + c_1^2, \]
\[ h_3 = \frac{D}{a_1^2}, \quad h_4 = \frac{D}{a_1^2} + \frac{D}{a_1^2} + \frac{D}{a_1^2} + \frac{D}{a_1^2} + \frac{D}{a_1^2} + \frac{D}{a_1^2}, \]
\[ w_1 = 1 + 2(G_2 - 2G_{12}) + XG_{12} - \phi XG_{12}. \]
\[ w_2 = 2(4G_{12} - 2G_{12}) + 2H(1 + 4G_{12} + 2XG_{12} - G_{12} - XG_{12}), \]
\[ w_3 = 3X(K_1 + 2XK_{12} - 2G_{12} - 2G_{12}) + 18(1 + 4G_{12} + XG_{12} - 18H(1 + 4G_{12} + 2XG_{12})), \]
\[ w_4 = 2(G_2 - 2G_{12}) + XG_{12} - \phi XG_{12}. \]


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Yukawa Potential

\[ \eta(k, a) = h_1 \left( 1 + k^2 h_2 \right) \]
\[ Y(k, a) = h_1 \left( 1 + k^2 h_2 \right) \]
\[ k^2 \Psi = 4\pi G Y(a, k) \rho_m(k) \]
\[ k^2 (\Phi + \Psi) = 8\pi G Y(a, k) \rho_m(k) \]
\[ \Psi = -\frac{GM}{r} h_1 (1 + \frac{h_2 - h_3}{h_2}) e^{-c_s r}, \]
\[ \Phi = -\frac{GM}{r} h_1 (1 + \frac{h_2 - h_3}{h_2}) e^{-c_s r}. \]


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Quasi-static approximation

\[ \phi_\phi'' - V' = Q \rho \]
\[ \phi \to (\delta \phi)e^{ikx} \]
\[ -\delta \ddot{\phi} - c_s^2 \delta \phi - V'' \delta \phi = Q(\delta \rho) \]
\[ -\delta \phi'' - (c_s/k \alpha H)^2 \delta \phi - (V''/\alpha^2 H^2) \delta \phi = (Q/\alpha^2 H^2)(\delta \rho) \]

\[ c_s^2 k^2 \gg \alpha^2 H^2 \]

From a wave equation:

To a “Poisson” equation:

\[ E_{\phi \phi} \equiv D_1 \ddot{\Phi} + D_2 \dddot{\phi} + D_3 \dddot{\phi} + D_4 \dot{\phi} + D_5 \dot{\phi} + D_6 \dddot{\psi} + D_7 \dddot{\psi} + \frac{k^2}{\alpha^2} \chi \]
\[ + \left( D_8 \frac{k^2}{\alpha^2} + D_9 \right) \Phi + \left( D_8 \frac{k^2}{\alpha^2} - M^2 \right) \delta \phi + \left( D_8 \frac{k^2}{\alpha^2} + D_9 \right) \Psi + D_1 \frac{k^2}{\alpha^2} \chi = 0, \]

To a “Poisson” equation:

\[ D_7 \frac{k^2}{\alpha^2} \Phi + \left( D_8 \frac{k^2}{\alpha^2} - M^2 \right) \delta \phi + A_6 \frac{k^2}{\alpha^2} \Psi \simeq 0, \]
The magic of 2\textsuperscript{nd} order perturbed Lagrangians

An elegant way to derive the perturbation equations is to write down the perturbed Lagrangian at second order: when you differentiate it, you get first order perturbation equations!

For instance standard EH Lagrangian gives in Minkowski

\[ ds^2 = -(1 + 2\Phi)dt^2 + 2B_{ij}dx^i dx^j + ((1 + 2\Phi)\delta_{ij} + 2E_{ij})dx^i dx^j \]

\[ S_\Phi = \frac{1}{2} \int d^4x \sqrt{-g} \left[ -8B_{ij} \Phi_i^j + 4 \Phi_i \Phi_j - 4 \Phi_i \Box \Phi^j + 2 \Phi_i^2 - 6 \left( \Phi^i \right)^2 \right] \]

Definition

Degree of freedom: a field with two time derivatives in the 2\textsuperscript{nd} order Lagrangian after taking into account all the constraints

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The magic of 2\textsuperscript{nd} order perturbed Lagrangians

Perturbed Horndeski Lagrangian at second order

\[ S^{(3)} = \int d^4x \sqrt{-g} \left[ 2\kappa \Phi - \omega_3 \Phi + \sum \rho_i (1 + w_i) \left( \frac{\partial^2 \Phi}{\partial x^i \partial x^j} \right)^2 + \left( \frac{1}{2} \sum \frac{(1 + w_i)\rho_i}{w_i} + \frac{w_i}{3} \right) \Phi^2 + \frac{w_3}{a^3} (\partial \Phi)^2 - 3w_1 \Phi^2 \right. \\
+ \left( 3w_2 \Phi - 2w_1 \frac{\partial^2 \Phi}{\partial x^i \partial x^j} - \sum \rho_i (1 + w_i) (w_i - 3Hw_{3w}) \right) \Phi + \sum \frac{\rho_i (1 + w_i)}{2w_i} \left( \frac{\partial \Phi}{\partial x^i} \right)^2 + \left( \frac{w_i}{2w_i} - \frac{w_2}{a^2} (\partial \Phi)^2 \right) \right. \\
+ 3 \Phi \sum \rho_i (1 + w_i) (w_i - 3Hw_{3w}) + \frac{3}{2} \sum (1 + w_i) \rho_i \Phi \right] 

\]

De Felice & Tsujikawa 2011

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The magic of 2\textsuperscript{nd} order perturbed Lagrangians

Classifying theories by their dof

Standard gravity: no scalar degree of freedom, two tensor dof (beside matter)

Horndeski: one scalar dof, two tensor dof (beside matter)

Massive grav: one scalar dof, 2+2 tensor dof (beside matter)

Bimetric models: one scalar dof, 2 vector dof, 2+2 tensor dof (beside matter)

2\textsuperscript{nd} order pert. Lagrangian shows explicitly the dofs

After several simplifications:

\[ S_2 = \int dt d^3x a^3 \left[ Q_S \left( \dot{\zeta}^2 - \frac{c_S^2}{a^2} (\partial_i \zeta) \right) + Q_T \left( h_{ij}^2 - \frac{c_T^2}{a^2} (\partial_k h_{ij})^2 \right) \right] , \]

De Felice & Tsujikawa 2011, Bellini & Sawicki 2014

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The magic of 2\textsuperscript{nd} order perturbed Lagrangians

2\textsuperscript{nd} order pert. Lagrangian shows explicitly the dofs

\[ S_2 = \int dt d^3x a^3 \left[ Q_S \left( \dot{\zeta}^2 - \frac{c_S^2}{a^2} (\partial_i \zeta) \right) + Q_T \left( h_{ij}^2 - \frac{c_T^2}{a^2} (\partial_k h_{ij})^2 \right) \right] , \]

scalar \hspace{1cm} tensor

…and the minimal number of free functions: two for each dof (plus a function for the background, i.e. 5 functions for Horndeski)

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The magic of 2nd order perturbed Lagrangians

2nd order pert. Lagrangian also clarifies the stability conditions

\[ S_2 = \int dt d^3 x d^3 \left[ \mathcal{L}_S \left( \frac{\partial^2}{a^2} (\partial_i \zeta) \right) + \mathcal{L}_T \left( \frac{\partial^2}{a^2} (\partial_k h_{ij})^2 \right) \right]. \]

Equation of motion in Fourier space

\[ \ddot{\zeta} - c^2_{s,T} k^2 \zeta = 0 \]

Positive squared sound speeds

\[ c^2_{s,T} \geq 0 \]

Positive kinetic terms

\[ Q_{s,T} \geq 0 \]

---

Parametrizations of the 5 free functions

<table>
<thead>
<tr>
<th>Variable Translations</th>
<th>( M_s^2 )</th>
<th>( M_s^2 H \alpha_M )</th>
<th>( M_s^2 H \alpha_K )</th>
<th>( M_s^2 H \alpha_B )</th>
<th>( M_s^2 \nu_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amendola et al. [10]</td>
<td>( w_1 )</td>
<td>( w_1 )</td>
<td>( \frac{1}{2} w_1 + 6 H w_2 - 6 H w_1 )</td>
<td>(-2H w_1 + w_4 - w_1)</td>
<td></td>
</tr>
<tr>
<td>Boussofield et al. [35, 37]</td>
<td>( m_0^2 \Omega + M_f^2 )</td>
<td>( m_0^2 \Omega + M_f^2 )</td>
<td>( 3 \sigma + 4 M_f^2 )</td>
<td>(-M_f^2 - m_0^2 \Omega )</td>
<td>(-M_f^2 )</td>
</tr>
<tr>
<td>De Felice et al. [89]</td>
<td>( \bar{G}_r )</td>
<td>( \bar{G}_r )</td>
<td>( 2\sigma + 12 H \beta - 6 H \bar{\rho} \bar{G}_r )</td>
<td>(-6 \bar{\rho} + 2H \bar{G}_r )</td>
<td>( \bar{F}_r - \bar{Q}_r )</td>
</tr>
<tr>
<td>Galbato et al. [34, 36]</td>
<td>( M_f^2 \Omega + 2m_0^2 )</td>
<td>( M_f^2 \Omega + 2m_0^2 )</td>
<td>( 2 \sigma + 4 M_f^2 )</td>
<td>(-m_0^2 \Omega - M_f^2 \bar{f} )</td>
<td>(-2 m_0^2 \bar{f} )</td>
</tr>
<tr>
<td>Piazza et al. [38]</td>
<td>( M^2 (1 + \epsilon_4) )</td>
<td>( M^2 (1 + \epsilon_4) )</td>
<td>( 2 M^2 (\bar{C} + 2 \epsilon_4 \bar{\rho}) )</td>
<td>(-M^2 (\epsilon_3 + \rho) )</td>
<td>(-M^2 \epsilon_4 )</td>
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</tbody>
</table>

Bellini & Sawicki 2014
Bellini-Sawicki parametrization

<table>
<thead>
<tr>
<th>Model Class</th>
<th>$\alpha_K$</th>
<th>$\alpha_B$</th>
<th>$\alpha_M$</th>
<th>$\alpha_T$</th>
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<tbody>
<tr>
<td>$\Lambda$ CDM</td>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>cuscuton ($w_X \neq -1$)</td>
<td>0</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>quintessence</td>
<td>(1 - $\Omega_0$)(1 + $w_X$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>k-essence/perfect fluid</td>
<td>(1 - $\Omega_0$)(1 + $w_X$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>kinetic gravity bracing</td>
<td>$m \frac{\delta c_s^2}{H}$</td>
<td>$m \frac{\delta c_s^2}{H}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>galileon cosmology</td>
<td>$-\frac{3\Omega_0^2 H^2}{2M}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BDK</td>
<td>$\frac{\delta K}{H}$</td>
<td>$-\eta M^2$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>metric $f(R)$</td>
<td>0</td>
<td>$-\Omega_M$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MSG/Palatini $f(R)$</td>
<td>$-\frac{3\Omega_0^2 H^2}{2M}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$f$(Gauss-Bonnet)</td>
<td>0</td>
<td>$-\frac{3\Omega_0^2 H^2}{2M}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Beyond Horndeski, I

Look now at the matter equations of motion

$$\int dx^i \sqrt{-g} \left[ \sum L_i + L_{\text{matter}} \right]$$

Standard sub-horizon matter equations

$$\begin{align*}
\theta &= i k \chi^j \\
\delta &= -\theta \\
\dot{\delta} &= -\mathcal{H} \theta + k^2 \Psi \\
k^2 \Phi &= 4 \pi G \alpha^2 \rho \delta
\end{align*}$$

Modifying gravity

$$k^2 \Phi = 4 \pi G \alpha^2 \rho \delta \eta$$

Modifying continuity equation

$$\Delta = 4 \alpha^2 \mathcal{H} \frac{M^2}{\rho \Omega_0^2} k^2 \Phi$$

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Gleyzes et al 2014; Lombrisier et al 2015
Beyond Horndeski, II

Or, we can add several Horndeski fields!

\[ \int d^4x \sqrt{-g} \left[ L_{H1} + L_{H2} + ... + L_{\text{matter}} \right] \]

\[ Y \equiv A_1 \frac{1 + A_2 k^2 + A_3 k^4}{1 + A_4 k^2 + A_5 k^4}, \]

\[ \eta \equiv -\frac{\Phi}{\Psi} = B_1 \frac{1 + B_2 k^2 + B_3 k^4}{1 + B_4 k^2 + B_5 k^4}. \]

A. Silvestri et al. 2013
T. Baker et al. 2013

Beyond Horndeski, III

- Torsion
- Non-metricity
- Palatini
- Vectors
- Tensors

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Beyond Horndeski, IV

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field
\[ \int d^4x \sqrt{g} R_g + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} (\eta^{\mu\alpha} \eta^{\nu\beta} - \eta^{\mu\nu} \eta^{\alpha\beta}) \]

The three deadly sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for \( m \to 0 \) (vDVZ disc.)
- It violates diffeomorphism invariance
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)

---

Ghost-free Bigravity

- The first problem was partially solved by Vainshtein (1972): there exists a radius below which the linear theory cannot be applied;
  - For the Sun, this radius is larger than the solar system!
- The second and third problems can be solved introducing a second metric

\[ S = -\frac{M_0^2}{2} \int d^4x \sqrt{-\text{det} g} R(g) - \frac{M_f^2}{2} \int d^4x \sqrt{-\text{det} f} R(f) \]
\[ + \ m^2 M_0^2 \int d^4x \sqrt{-\text{det} g} \sum_{n=0}^{4} \beta_n c_n(\sqrt{g^{\alpha\beta} f_{\alpha\beta}}) + \int d^4x \sqrt{-\text{det} g} L_m(g, \Phi) \]

The only ghost-free local non-linear massive gravity theory!  

deRham, Gabadadze, Tolley 2010  
Hassan & Rosen, 2011
Observing $\eta$

$$\eta(k, a) = -\frac{\Phi}{\Psi}$$

Reconstruction of the metric

$$ds^2 = a^2 [(1 + 2\Psi)dt^2 - (1 + 2\Phi)(dx^2 + dy^2 + dz^2)]$$

massive particles respond to $\Psi$

$$\delta^\gamma + \left(1 + \frac{H'}{H}\right)\delta' = \nabla^2 \Psi$$

$$\delta = \frac{\delta \rho}{\rho}$$

massless particles respond to $\Phi \cdot \Psi$

$$\alpha = \int \nabla_{\text{perp}} (\Psi - \Phi) dz$$
Correlation function and Power spectrum

\[ dN_{ab} = \langle dN_a dN_b \rangle = \rho_0^2 dV_a dV_b (1 + \xi(r_{ab})) \]

Average number of pairs = 1

Average number of pairs = 4

\[ P(k) = \int \xi(r) e^{ikr} dV \]

Reality check

\[ \delta = \frac{\rho(x) - \rho_h}{\rho_h} \] Density fluctuation in space

\[ \left\langle \delta_k^2 \right\rangle = P(k, z) \]

\[ P_{\text{matter}}(k, z) \]

Matter power spectrum

\[ b^2(k, z) P_{\text{matter}}(k, z) \]

Galaxy power spectrum

\[ (1 + \frac{f(k, z)}{b(k, z)}) \cos^2 \theta b^2(k, z) P_{\text{matter}}(k, z) \]

\[ f = \frac{\delta \log \Delta}{\delta \log a} \]
Peculiar velocities

\[ r = \frac{v}{H_0} \]

Real space

Redshift space

\[ \beta = \frac{\delta'}{\delta b} \] Kaiser 1987

\[ P_z = (1 + \beta \mu^2) P_r \]

redshift distortion parameter

Guzzo et al.
Deconstructing the galaxy power spectrum

\[ P_{\text{galaxy}}(k, z, \cos \theta) = \left(1 + \frac{f(k, z)}{b(k, z)} \cos^2 \theta \right)^2 b^2(k, z)P_{\text{matter}}(k, z) \]

Redshift distortion

\[ \delta_{\text{gal}}(k, z, \mu) = Gb\sigma_8(1 + \frac{f}{b}\mu^2)\delta(k) \]

Line of sight angle

\[ f = \frac{d \log \delta}{d \log a} \]

Galaxy clustering

Present mass power spectrum

Growth function

Galaxy bias

normalization

Three linear observables: A, R, L

Amplitude \( A \)

\[ \mu=0 \]

Redshift distortion \( R \)

\[ \mu=1 \]

Lensing \( L \)

\[ k^2 \Phi_{\text{lens}} = k^2(\Psi - \Phi) = -\frac{3}{2} \sum G\Omega_m \sigma_8 \delta_{m,0}^2(k) \equiv L \]

\[ \Sigma = Y(1 + \eta) \]
The only model-independent ratios

Redshift distortion/Amplitude
\[ P_1 = \frac{R}{A} = \frac{f}{b} \]

Lensing/Redshift distortion
\[ P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y (1 + \eta)}{f} \]

Redshift distortion rate
\[ P_3 = \frac{R'}{R} = \frac{f'}{f} + f \]

Expansion rate
\[ E = \frac{H}{H_0} \]

How to combine them to test the theory?

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Summarizing...

Matter conservation equation independent of gravity theory
\[ \delta''_{mm} + (1 + \frac{\dot{H}}{H})\delta'_{mm} = -k^2 \Psi \]

Observables
\[ P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y (1 + \eta)}{f} \quad P_3 = \frac{R'}{R} = \frac{f'}{f} + f \quad E = \frac{H}{H_0} \]

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The anisotropic stress is directly observable

A unique combination of model independent observables

\[ \frac{3P_2(1+z)^3}{2E^2(P_3 + 2 + \frac{E'}{E})} - 1 = \eta \]

Observables  Theory

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Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

\[ \frac{3P_2(1+z)^3}{2E^2(P_3 + 2 + \frac{E'}{E})} - 1 = \eta = h_2 \left( \frac{1 + k^2h_4}{1 + k^2h_5} \right) \]

Observables  Theory

L. Amendola, Vitória, Brasil 2016

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439
1305.0008
Horndeski Lagrangian: not too big to fail

\[ g(z,k) \equiv \frac{(REa^2)'}{LEa^2} \]

\[ 2g_{,k}g_{,kk} - 3(g_{,kk})^2 = 0 \]

If this relation is falsified, the Horndeski theory is rejected*

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy
15,000 square degrees
70 million redshifts, 2 billion images
Median redshift \( z = 1 \)
PSF FWHM \( \sim 0.18'' \)
>1000 peoples, >10 countries

arXiv Red Book 1110.3193
Results...

\[ \eta(k,a) = H_\frac{1 + k^2 H_a^2}{1 + k^2 H_a^5} \]

Model 1: \( \eta \) constant for all \( z, k \)

Error on \( \eta \) around 2%

Model 2: \( \eta \) varies in \( z \)

Error on \( \eta \)\n
\[ \eta = \frac{1}{2} \text{ for } f(R)! \]

Table X. Fiducial values and errors for the parameters \( P_1, P_2, P_3, E/E \) and \( \eta \) for every bin. The last bin has been omitted since \( R \) is not defined there.

| \( z \) | \( P_1 \) | \( \Delta P_1 \) | \( \Delta P_1(\%) \) | \( P_2 \) | \( \Delta P_2 \) | \( \Delta P_2(\%) \) | \( P_3 \) | \( \Delta P_3 \) | \( \Delta P_3(\%) \) | \( (E/E) \) | \( \Delta E/E \) | \( \Delta E/E(\%) \) | \( \eta \) | \( \Delta \eta \) | \( \Delta \eta(\%) \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.6 | 0.766 | 0.012 | 1.6 | 0.729 | 0.013 | 1.8 | 0.134 | 0.13 | 99 | -0.920 | 0.022 | 2.4 | 1 | 0.11 | 11 |
| 0.8 | 0.819 | 0.010 | 1.2 | 0.682 | 0.011 | 1.6 | 0.317 | 0.12 | 38 | -1.04 | 0.086 | 4.4 | 1 | 0.091 | 9.1 |
| 1.0 | 0.859 | 0.0093 | 1.1 | 0.650 | 0.011 | 1.7 | 0.460 | 0.12 | 26 | -1.13 | 0.099 | 8.7 | 1 | 0.090 | 9.0 |
| 1.2 | 0.888 | 0.0092 | 1.0 | 0.628 | 0.014 | 2.3 | 0.569 | 0.13 | 23 | -1.21 | 0.12 | 10 | 1 | 0.097 | 9.7 |
| 1.4 | 0.911 | 0.010 | 1.1 | 0.613 | 0.020 | 3.3 | 0.654 | 0.11 | 16 | -1.26 | 0.09 | 7.1 | 1 | 0.073 | 7.3 |

In collab. with Laura Taddei and Matteo Martinelli

Observing Y

\[ \nabla^2 \Psi = 4\pi G Y(k,a) \rho_m \delta_m \]

In collab. with Laura Taddei and Matteo Martinelli

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Observing $Y$

Excluded in the Lab

$$\Psi(r) = -\frac{GM}{r}(1 + \alpha e^{-r/\lambda})$$

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Observing $Y$

Modified-gravity sub-horizon matter equations

$$\theta = ik_j v^j$$

$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$

$$k^2 \Phi = 4\pi G a^2 \rho \eta$$

continuity Euler Poisson

This can be written as a single equation:

$$\delta'' + (1 + \frac{H'}{H})\delta' + \frac{3}{2}\Omega_m Y\delta = 0$$

prime is $d/d\log(a)$

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Observing $Y$

$$\delta'' + \left(2 + \frac{H'}{H}\right)\delta' + \frac{3}{2} \Omega_m Y \delta = 0$$

$$f = \frac{d \log \delta}{d \log a}$$

Growth rate

$$f \sigma_s(z) = \sigma_8 \frac{\delta'}{\delta_0}$$

By measuring the growth of fluctuations we can find $Y$!

Two problems

$$\delta'' + \left(2 + \frac{H'}{H}\right)\delta' + \frac{3}{2} \Omega_m \delta Y = 0$$

Changing initial conditions the evolution can change considerably:

marginalize over initial conditions

$$\delta_{gal}(k, z, 1) = G \sigma_8 f \delta_{m,0}(k) = R$$

The observable is $R$ not $f\sigma(z)$:

marginalize over an overall factor
Observing Y

$$\left(2 + \frac{2}{H^2}\right) \delta' + \frac{3}{2} \Omega_m \delta Y = 0$$

Changing initial conditions the evolution can change considerably.

Free parameters:
$$\Omega_m, w_0, Y, \sigma, \alpha = \frac{\delta'}{\delta}$$

current SN, growth data

Planck best fit

Table I data

M. Martinelli, L. Taddei

Observing Y: current constraints

Free parameters:
$$\Omega_m, w_0, w_a, Y, \sigma, \alpha = \frac{\delta'}{\delta}$$

current SN, growth data
Observing Y: Euclid forecast

WL, SN, growth data

Free parameters:
\[ \Omega_m, \Omega_k, \alpha = \frac{\delta}{\delta_m} \]

Standard case \( Y = \alpha = 1 \)

General mod grav case

\[
\begin{align*}
\Omega_m & , \Omega_k , \alpha = \frac{\delta}{\delta_m} \\
\Omega_m & , \Omega_k , \alpha = \frac{\delta}{\delta_m} \\
\end{align*}
\]

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Observing Y: Euclid forecast

WL, SN, growth data

\[
\begin{align*}
P/P_{max} & \\
\end{align*}
\]

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**Method 3: Cosmological exclusion plot**

\[ \dot{Y} = \Omega_{m0} Y \]

**Yukawa Potential**

\[ Y(k, a) = \frac{h}{1 + k^2 h_i} \]  
Momentum space

\[ \nabla^2 \Psi = 4\pi G Y(k, a) \rho_m \delta_m \]

\[ \Psi = -\frac{GM}{r} h_z \left(1 + \frac{h_z - h_s}{h_s} e^{-r/\sqrt{2}} \right) = -\frac{GM}{r} (1 + Qe^{-mr}) \]  
Real space
Cosmological exclusion plot

\[ \Psi(r) = -\frac{GM}{r} \left( 1 + Qe^{-r/\lambda} \right) \]


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Modified Gravity and clusters

Collab. with S. Borgani, B. Sartoris, L. Pizzuti and the CLASH-VLT team

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Modified Gravity and clusters

- Clusters are the largest gravitationally bound objects
- Their gravitational potential can be mapped with galaxy dynamics, X-ray intracluster gas emission, weak and strong lensing, SZ effect
- They are a bridge between linear and strongly non linear scales

MACS 1206
z=0.44

X-ray emission

Clusters contain a hot intracluster medium (10^7 K) that emits in the X-ray; if the gas is in equilibrium between its own pressure and gravity then

$$\nabla P_{\text{gas}} = -\rho_{\text{gas}} \nabla \Phi_N,$$

further, assuming spherical symmetry and ideal gas

$$M(r) = -\frac{r}{G \mu m_p} \left( \frac{d \ln \rho_{\text{gas}}}{d \ln r} + \frac{d \ln T}{d \ln r} \right).$$
As before, lensing and galaxy dynamics are governed by different combinations of potentials

\[ k^2 \Psi = 4 \pi G Y(k) \rho_m(k) \]
\[ k^2 (\Phi + \Psi) = 8 \pi G Y(k) [1 + \eta(k)] \rho_m(k) \]

So if we replace

\[ \eta = h_2 \left( \frac{1 + k^2 h_4}{1 + k^2 h_5} \right), \quad Y = h_1 \left( \frac{1 + k^2 h_6}{1 + k^2 h_7} \right) \]

and move to real space, we obtain for \( \eta(r) = -\frac{\Phi(r)}{Y(r)} \)

\[ \eta(r) = h_2 \frac{\int_{r_0}^{r} \frac{dr'}{r'^3} [M(r') + 2 \hat{Q}^2 M_{mg}(r')] \int_{r_0}^{r} \frac{dr''}{r''^3} [M(r'') + 2 \hat{Q}^2 M_{mg}(r'')] \right)}{1 + 2 \hat{Q}^2 \Sigma(r)} \]

Now if we assume a NFW profile

\[ \rho(r) = \frac{\rho_0}{r \left(1 + \frac{r}{R_s}\right)^2} \]

we obtain

\[ \Sigma(r) = \frac{2}{\pi} \int_{0}^{\infty} \frac{W(k R_s)}{m^2 + k^2} \left[ \frac{\sin(k r_0)}{r_0} - \frac{\sin(k r)}{r} \right] dk \]

By combining lensing and galaxy dynamics we can measure the parameters \( h_2, Q, \hat{Q}, m, R_s \)
Galaxy dynamics obey the Jeans equation (spherical symmetry, virialized system)

\[ \frac{d\nu \sigma_r^2}{dr} + 2\beta \frac{\nu \sigma_r^2}{r} = -\frac{d\Psi}{dr} \]

- Variance radial velocity
- Velocity anisotropy parameter
- Density of sources

\[ \beta = \infty \]

 Purely tangential orbits

\[ \beta = 0 \]

 Purely radial orbits
Similarly, the lensing effective mass is determined by the distortion of background galaxies:

\[ M_{2D(< \theta_i)} = \pi(D_i \theta)^2 \Sigma_{\text{crit}} \kappa_{\text{min}} + 2\pi D_i^2 \Sigma_{\text{crit}} \int_{\theta_{\text{min}}}^{\theta_i} d \ln \theta \theta^2 \kappa(\theta). \]

---

**Single cluster MACS1206**

\[ \eta(1.96 \, Mpc) = 1.01^{+0.34}_{-0.28} \, (\text{stat}) \pm 0.34 \, (\text{syst}), \]

*Fixing all parameters except \( r_s \) and \( \eta \)!*

---

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In a $f(R)$ theory we have

$$\eta = \frac{1 + 2\mathcal{Q}^2 \Sigma(r)}{1 + 2\mathcal{Q}^2 \Sigma(r)}$$

with

$$\Sigma(r) \rightarrow 1 \text{ for } r \rightarrow 0$$

$$\Sigma(r) \rightarrow 0 \text{ for } r \rightarrow \infty$$

So $\eta$ goes from 0.5 to 1

So we can “exclude” $m > (2 \text{ Mpc})^{-1}$…
Modified Gravity and Grav. Waves

Gravitational waves in GR

General perturbation
\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

redefinition
\[ \bar{h}^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h \]

Gauge freedom
\[ x^{\bar{\alpha}} = x^{\alpha} + \xi^{\alpha}(x^\beta) \]

Gauge choice
\[ \bar{h}^{\mu\nu}_{\rho\sigma} = 0 \]

Linearized Einstein equations
\[ \Box \bar{h}^{\alpha\beta} = -16\pi T^{\alpha\beta} \]

In FRW
\[ \ddot{h} + 3H\dot{h} + k^2 h = 0 \]
Gravitational waves in GR

General solution in vacuum

\[ h_{\alpha\beta}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{ik_{\alpha}x^{\alpha}} \]

Erik Ellgren, Tomas Samuelson

Ligo collaboration
B- and E-modes

B-modes are excited only by GW and lensing!

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GW in modified gravity

GW equation in GR
\[ \ddot{h} + 3H\dot{h} + k^2 h = 0 \]

GW equation in modified gravity
\[ \ddot{h} + 3H(1 + \alpha_M)\dot{h} + (1 + \alpha_T)k^2 h = 0 \]

Bellini & Sawicki 2013
I. Saltas, I. Sawicki, L.A., M. Kunz 1406.7139

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It turns out that if \( \eta \neq 1 \) then the GW equation is modified. CMB B-polarization can be a tool to detect modified gravity!

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BICEP collab. 2014
### Gravitational wave speed

Gravitational wave speed is a fundamental aspect of cosmology, crucial for understanding the evolution of the universe. The speed of gravitational waves is given by:

\[ c_T^2 = 1 + \alpha_T \]

where \( \alpha_T \) is a parameter that can vary depending on the model.

#### CT

- **CDM, \( \Omega_{0.05} = 0 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \)**
- **CDM, \( \Omega_{0.05} = 0 \), \( \chi = 1.7 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \), \( \chi = 1 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \), \( \chi = 0.5 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \), \( \chi = 0.3 \)**

#### GW speed and lensing

GW speed and lensing are closely related, with lensing providing insights into the structure of the universe and the propagation of gravitational waves.

- **CDM lensing**
- **CDM, \( \Omega_{0.05} = 0 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \), \( \chi = 0.2 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \), \( \chi = 1.5 \)**
- **CDM, \( \Omega_{0.05} = 0.2 \), \( \chi = 2.2 \)**


See also Ravetti, Silvestri and Zhou, 2014
Constraints on GW speed

The proposed satellite LiteBIRD plans to measure $r$ at the reionization peak to 0.001!

GW speed at reionization

L. Amendola, Vitória, Brasil 2016
Simultaneous CMB constraints on all modified gravity parameters

With M. Zumalacarregui, V. Pettorino, I. Sawicki, J. Lesgourgues

\[ G^* = G(1 + e^{-m\phi}) \]

\( m_\phi, \alpha \) depend on time, depend on space, depend on local density, depend on species
Screening mechanisms

\[ G^* = G(1 + ae^{-m\rho}) \]

\[ m = m(\phi) \]

The field \( \phi \) obeys a Poisson equation

\[ \nabla^2 \phi + m^2 \phi = \alpha^{1/2} \rho \delta \]

So the solution is something like

\[ \phi = \phi(\rho \delta) = \phi(\text{local density}) \]

A density-dependent range!

---

Screening mechanisms

(mass increases with the local density)

\[ \text{range} = m^{-1} \]

Small density, small mass

Long range

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Screening mechanisms

(range = $m^{-1}$)

Large density, large mass

Short range

Problem of non-linearity: screening effects mix linear and non-linear scales

same density contrast
different physics
Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy
15,000 square degrees
70 million redshifts, 2 billion images
Median redshift \( z = 1 \)
PSF FWHM \( \sim 0.18'' \)
>1000 peoples, >10 countries

arXiv Red Book 1110.3193