BLACK HOLE MICROSCOPY: Conformal Field Theory at the bottom of the throat of the horizon

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What are we talking about?

- The free $\mathcal{N} = (4,4)$ SCFT with target space $(T^4)^N / S_N$ is a very interesting object:
 - It has been the subject of important recent developments in the explicit construction of AdS_3/CFT_2 holography ([Eberhardt et al., 2019] and others)
 - It famously holds the microstates accounting for the entropy of black holes in string theory [Strominger and Vafa, 1996]
 - It is related to the D1-D5 system, the simplest example of a black hole in string theory
- \cdot Goals:
 - Present some new results concerning the deformed theory [Lima et al., 2020s, Lima et al., 2020b, Lima et al., 2020c, Lima et al., 2020d],
 - along with a (*utterly*) schematic overview of how the SCFT is related to black holes

With results published in

· Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Dynamics of R-neutral Ramond fields in the D1-D5 SCFT.

· Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Renormalization of Twisted Ramond Fields in D1-D5 SCFT₂.

· Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Correlation functions of composite Ramond fields in deformed D1-D5 orbifold SCFT₂. *Physical Review D, 102*.

· Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Microstate renormalization in deformed D1-D5 SCFT. *Physics Letters B*, 808, 135630.

Outline

A black hole is a black hole

How to build a quantum black hole

The free CFT

The deformed CFT

What is a black hole?

1. A black hole is a black hole

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What are hairless black holes made of?

2. How to build a quantum black hole

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- A quantum black hole needs quantum gravity i.e. String Theory.
- $\cdot\,$ In low-energy limit, String Theory becomes SUGRA
- \cdot Ingredients:

NS-NS (Type IIA and IIB): $G_{\mu\nu}, \quad \Phi, \quad B^{(2)}$ R-R (Type IIA): $C^{(1)}, \quad C^{(3)}$ R-R (Type IIB): $C^{(0)}, \quad C^{(2)}, \quad C^{(4)}$

• Field equations are complicated, but there are techniques for building supersymmetric solutions

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- $\cdot\,$ Type IIB SUGRA
- N_1 D1 branes electrically coupled to $C^{(2)}$
- + N_5 D5 branes magnetically coupled to $C^{(2)}$
- $\cdot\,$ Array D1 along circle S^1 with radius R
- · Array D5 along torus $T^5 = T^4 \times S^1$
- · Wrap!

– means "extended"; \cdot means "pointlike"

the compact dimensions of

D1-D5



A black hole (rather a black ring) with two charges:

$$e^{\frac{1}{2}\Phi}ds_{E}^{2} = \frac{1}{\sqrt{H_{1}H_{5}}}(-dt^{2} + dy^{2}) + \sqrt{H_{1}H_{5}}(dr^{2} + r^{2}d\Omega_{3}^{2}) + \sqrt{\frac{H_{1}}{H_{5}}}\sqrt{\operatorname{Vol}(T^{4})}ds^{2}(T^{4}) e^{\Phi} = \sqrt{H_{1}/H_{5}} F^{(3)} = dC^{(2)}, \quad F_{rty}^{(3)} = \partial_{r}\left(\frac{1}{H_{1}}\right), \quad F_{\theta\phi\varphi}^{(3)} = 2Q_{5}\sin^{2}\theta\sin\phi$$

where y parameterizes S^1 , and

$$H_1 = 1 + \frac{Q_1}{r^2}, \qquad H_5 = 1 + \frac{Q_5}{r^2}$$

are *harmonic functions* on the transverse space

Geometry:

$$e^{\frac{1}{2}\Phi}ds_E^2 = \frac{1}{\sqrt{H_1H_5}}(-dt^2 + dy^2) + \sqrt{H_1H_5}(dr^2 + r^2d\Omega_3^2) + \sqrt{\frac{H_1}{H_5}}\sqrt{\operatorname{Vol}(T^4)}ds^2(T^4) H_1 = 1 + \frac{Q_1}{r^2}, \qquad H_5 = 1 + \frac{Q_5}{r^2}$$

Asymptotically:
$$r \to \infty$$
 geometry is $M^{4,1} \times S^1 \times T^4$
Horizon: $r = 0$, $g^{rr} = 1/\sqrt{H_1H_5} = 0$
Near horizon: $H_1 = \frac{Q_1}{r^2}$, $H_5 = \frac{Q_5}{r^2}$

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Near-horizon geometry:

with
$$z = \sqrt{Q_1 Q_5}/r$$

 $e^{\frac{1}{2}\Phi} ds_E^2 = \frac{\sqrt{Q_1 Q_5}}{z^2} (-dt^2 + dz^2 + dy^2)$
AdS₃ with radius $(Q_1 Q_5)^{\frac{1}{4}}$
 $+ \sqrt{Q_1 Q_5} d\Omega_3^2$
 $+ \sqrt{\frac{Q_1}{Q_5}} Vol(T^4) ds^2(T^4)$
 $e^{\Phi} = \sqrt{Q_1/Q_5}$ (frozen)

Horizon: $r \to 0$ hence $z \to \infty$ so AdS boundary





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OBS: Gravity description is good if radius is large and string coupling small:

$$N = N_1 N_5 \gg \frac{1}{g_s} \sim \frac{N_5}{N_1} \gg 1$$

So we need large ${\cal N}$

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We have found the geometry

 $\mathrm{AdS}_3 \times S^3 \times T^4$

This is dual to

 CFT_2 with $SO(4)_E$ R-symmetry

and

 $SO(4)_I$ "internal" symmetry

The central charge is determined by AdS:

$$c = \frac{3L_{\text{AdS}_3}}{2G_N^{(3)}} = 6N, \qquad N = N_1 N_5$$

The correct CFT is supersymmetric, with 8 supersymmetries, and it is an **orbifold** defined on the target space $(T^4)^N/S_N$

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3. The free CFT

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The
$$\mathcal{N} = (4, 4)$$
 SCFT

Four real bosons and four real fermions: all free Gather fields into complex bosons and complex fermions

$$X^{\dot{A}A}(z,\bar{z}), \quad \psi^{\alpha \dot{A}}(z), \quad \tilde{\psi}^{\dot{\alpha} \dot{A}}(\bar{z})$$

$$\begin{split} a &= 1, 2, 3, \quad \text{triplet of } \mathrm{SU}(2)_L; \quad \dot{a} &= \dot{1}, \dot{2}, \dot{3}, \quad \text{triplet of } \mathrm{SU}(2)_R \\ \alpha &= +, -, \quad \text{doublet of } \mathrm{SU}(2)_L \quad \dot{\alpha} &= \dot{+}, \dot{-}, \quad \text{doublet of } \mathrm{SU}(2)_R \\ A &= 1, 2, \quad \text{doublet of } \mathrm{SU}(2)_1; \quad \dot{A} &= \dot{1}, \dot{2}, \quad \text{doublet of } \mathrm{SU}(2)_2 \end{split}$$

Bosonize fermions:

$$\psi^{\alpha \dot{1}}(z) = \begin{bmatrix} e^{-i\phi_2(z)} \\ e^{-i\phi_1(z)} \end{bmatrix} , \qquad \psi^{\alpha \dot{2}}(z) = \begin{bmatrix} e^{i\phi_2(z)} \\ -e^{i\phi_1(z)} \end{bmatrix} ,$$

Central charge: c = 6

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Currents

Stress-tensor:

$$T(z) = \frac{1}{4} \epsilon_{\dot{A}\dot{B}} \epsilon_{AB} \partial X^{\dot{A}A} \partial X^{\dot{B}B} + \frac{1}{4} \epsilon_{\dot{A}\dot{B}} \epsilon_{\alpha\beta} \psi^{\alpha\dot{A}} \partial \psi^{\beta\dot{B}}$$

Four holomorphic super-currents:

$$G^{\alpha A}(z) = \epsilon_{\dot{A}\dot{B}}\psi^{\alpha\dot{A}}\partial X^{\dot{B}A}$$

R-current:

$$J^{3}(z) = \frac{1}{2}i[\partial\phi_{1}(z) - \partial\phi_{2}(z)]$$

Internal $SU(2)_2$:

$$\mathfrak{J}^3(z) = \frac{1}{2}i[\partial\phi_1(z) + \partial\phi_2(z)]$$

Quantum numbers: h, j^3, j^3

OPERATOR ALGEBRA:

$$T(z')T(z) = \frac{c/2}{(z'-z)^4} + \frac{2T(z)}{(z'-z)^2} + \frac{\partial T(z)}{z'-z} + \cdots$$

$$J^a(z')J^b(z) = \frac{c}{12}\frac{\delta^{ab}}{(z'-z)^2} + \frac{i\epsilon^{ab}{}_c J^c(z)}{z'-z} + \cdots$$

$$G^{\alpha A}(z')G^{\beta A}(z) = -\frac{c}{3}\frac{\epsilon^{AB}\epsilon^{\alpha\beta}}{(z'-z)^3} - \epsilon^{AB}\epsilon^{\alpha\beta}\frac{T(z)}{z-z'} + \epsilon^{AB}\epsilon^{\beta\gamma}[\sigma^{*a}]^{\alpha}_{\gamma} \left[\frac{2J^a(z)}{(z'-z)^2} + \frac{\partial J(z)}{z'-z}\right] + \cdots$$

$$J^a(z)G^{\alpha A}(z') = \frac{1}{2}[\sigma^{*a}]^{\alpha}_{\gamma}\frac{G^{\gamma A}(z)}{z'-z} + \cdots$$

$$T(z')G^{\alpha A}(z) = \frac{3}{2}\frac{G^{\alpha A}(z)}{(z'-z)^2} + \frac{\partial G^{\alpha A}(z)}{z'-z} + \cdots$$

$$T(z')J^a(z) = \frac{J^a(z)}{(z'-z)^2} + \frac{\partial J^a(z)}{z'-z} + \cdots$$

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The correct CFT is the **orbifold** $(T^4)^N/S_N$; make N copies: $(T^4)^N = T^4 \otimes \cdots \otimes T^4$

Copy index: $I = 1, \cdots, N$

$$X_{\underline{I}}^{\dot{A}A}(z,\bar{z}), \quad \psi_{\underline{I}}^{\alpha\dot{A}}(z), \quad \phi_{\underline{I}}^{a}(z)$$

Total currents:

$$T(z) = \sum_{I=1}^{N} \left[\frac{1}{4} \epsilon_{\dot{A}\dot{B}} \epsilon_{AB} \partial X_{I}^{\dot{A}A} \partial X_{I}^{\dot{B}B} + \frac{1}{4} \epsilon_{\dot{A}\dot{B}} \epsilon_{\alpha\beta} \psi_{I}^{\alpha\dot{A}} \partial \psi_{I}^{\beta\dot{B}} \right]$$
$$J^{3}(z) = \frac{1}{2} i \sum_{I=1}^{N} \left[\partial \phi_{I}^{1}(z) - \partial \phi_{I}^{2}(z) \right]$$

etc. Central charge of orbifold: c = 6N

... Now identify permutations

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What changes when we orbifold?

<u>Answer:</u> New possible boundary conditions permuting the copies.

How to implement? Answer: 'Twist fields' $\sigma_q(z), g \in S_N$

Going around a twist:

$$X^i_{I}(e^{2\pi i}z, e^{-2\pi i}\bar{z})\sigma_g(z, \bar{z}) = X^i_{g(I)}(z, \bar{z})\sigma_g(z, \bar{z}).$$

Interpretation in the (t, y) plane:

The twist field $\sigma_{(n)}$ joins *n* "strings" into a single *n*-wound string





We have found a CFT dual to the AdS throat We can compute its degeneracy of states Ω , and we find that $\log \Omega = S_{BH}$

This is the famous result of [Strominger and Vafa, 1996]

4. The deformed CFT

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The free orbifold is dual to a very singular gravitational solution

To go towards the SUGRA description, we need an interacting CFT

[Larsen and Martinec, 1999, Seiberg and Witten, 1999, David et al., 2002]

(States corresponding to the black hole are states which do not renormalize under this deformation)

Deformed theory:

$$S_{\rm int} = S_{\rm free} + \lambda \int d^2 z \, O^{\rm (int)}_{[2]}(z,\bar{z})$$

defined by the MARGINAL interaction operator

$$O_{[2]}^{(\text{int})}(z,\bar{z}) = \epsilon_{AB} G_{-\frac{1}{2}}^{-A} \tilde{G}_{-\frac{1}{2}}^{-B} O_{[2]}^{(0,0)}(z,\bar{z})$$

The effect of the deformation on an operator $\mathscr{A}_{[n]}$ can be found from the corrections to the two-point function

$$\left\langle \mathscr{A}_{[n]}^{\dagger}(z_1, \bar{z}_1) \mathscr{A}_{[n]}(z_2, \bar{z}_2) \right\rangle = \frac{C}{|z_1 - z_2|^{2\Delta}}$$
fixed by conf. symm

Change in the conformal dimension

$$\Delta(\lambda) = \Delta + \lambda \delta \Delta + \frac{1}{2} \lambda^2 \delta^2 \Delta + \cdots$$

where

$$\delta\Delta \sim \left\langle \mathscr{A}_{[n]}^{\dagger}(\infty)O_{[2]}^{(\text{int})}(1,\bar{1})\mathscr{A}_{[n]}(0) \right\rangle$$

$$\delta^{2}\Delta \sim \frac{\lambda^{2}}{2} \int d^{2}z_{2} \int d^{2}z_{3} \left\langle \mathscr{A}_{[n]}^{\dagger}(z_{1},\bar{z}_{1})O_{[2]}^{(\text{int})}(z_{2},\bar{z}_{2})O_{[2]}^{(\text{int})}(z_{3},\bar{z}_{3})\mathscr{A}_{[n]}(z_{4},\bar{z}_{4}) \right\rangle$$

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The four-point function

$$\left\langle \mathscr{A}^{\dagger}_{[n]}(\infty) O^{(\mathrm{int})}_{[2]}(1,\bar{1}) O^{(\mathrm{int})}_{[2]}(u,\bar{u}) \mathscr{A}_{[n]}(0) \right\rangle$$

is complicated: TWISTS!

Move u around the other points and fields are permuted:

Very non-trivial boundary conditions!

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What to do?

For

$$\left\langle \sigma_{[n_1]}(\infty)\sigma_{[n_2]}(1,\bar{1})\sigma_{[n_3]}(u,\bar{u})\sigma_{[n_4]}(0)\right\rangle$$

<u>USE A COVERING SURFACE</u> [Lunin and Mathur, 2001] with coordinates (t, \bar{t})

such that

$$z(t) \approx b_1 t^{n_1} \qquad \text{as } z \to 0$$

$$z(t) \approx 1 + b_2 (t - t_1)^{n_2} \qquad \text{as } z \to 1$$

$$z(t) \approx u + b_3 (t - x)^{n_3} \qquad \text{as } z \to u$$

$$z(t) \approx b_4 t^{n_4} \qquad \text{as } z \to \infty$$

... plus other images of ∞

E.g. for two-point functions $\langle \sigma_2 \sigma_2 \rangle$

the covering surface looks like





Covering Surface:

Its genus and the number of its ramification points determine the large-N scaling of the corresponding correlation function

$$\left\langle \mathscr{O}^{1}_{[n_{1}]}(z_{1}, \bar{z}_{1}) \mathscr{O}^{2}_{[n_{2}]}(z_{2}, \bar{z}_{2}) \cdots \mathscr{O}^{Q}_{[n_{Q}]}(z_{Q}, \bar{z}_{Q}) \right\rangle$$

$$\sim N^{\mathbf{s} - \frac{1}{2} \sum_{r=1}^{Q} n_{r}} \left(1 + N^{-1} + \cdots \right)$$

$$= N^{1 - \mathbf{g} - \frac{1}{2}Q} \left(1 + N^{-1} + \cdots \right)$$

Here \mathbf{s} is the number of distinct copies entering the permutations, and

$$\mathbf{g} = \frac{1}{2} \sum_{r=1}^{Q} (n_r - 1) - \mathbf{s} + 1$$

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(Riemann-Hurwitz Formula)

We have therefore the following program:

- 1. Choose a relevant operator in the CFT
- Compute the four-point function with O^(int)_[2]
 (Not trivial! Must use covering surface and other paraphernalia)
- 3. Obtain information about dynamics: structure constants, fusion rules, etc.

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- 4. Integrate the four-point function (Also not trivial! There are regularization issues and complex-contour gymnastics)
- 5. Obtain the correction to conformal dimensions

We have done this for Ramond ground states of the n-twisted sector:

$$R_{[n]}^{\pm} \equiv \frac{1}{\mathscr{S}_{n}(N)} \sum_{h \in S_{N}} \exp\left(\pm \frac{i}{2n} \sum_{I=1}^{n} \left[\phi_{1,h(I)} - \phi_{2,h(I)}\right]\right) \sigma_{h^{-1}(n)h}(z)$$

(R-charged doublet of $SU(2)_E$, neutral under internal $SU(2)_2$)

$$\begin{aligned} R_{[n]}^{\mathbf{i}} &\equiv \frac{1}{\mathscr{S}_{n}(N)} \sum_{h \in S_{N}} \exp\left(-\frac{i}{2n} \sum_{I=1}^{n} \left[\phi_{1,h(I)} + \phi_{2,h(I)}\right]\right) \sigma_{h^{-1}(n)h}(z) \\ R_{[n]}^{\mathbf{i}} &\equiv \frac{1}{\mathscr{S}_{n}(N)} \sum_{h \in S_{N}} \exp\left(+\frac{i}{2n} \sum_{I=1}^{n} \left[\phi_{1,h(I)} + \phi_{2,h(I)}\right]\right) \sigma_{h^{-1}(n)h}(z) \end{aligned}$$

(R-neutral doublet of internal $SU(2)_2$)

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Compute the functions

$$\left\langle R^{-}_{[n]}(\infty,\bar{\infty})O^{(\text{int})}_{[2]}(1,\bar{1})O^{(\text{int})}_{[2]}(u,\bar{u})R^{+}_{[n]}(0,\bar{0}) \right\rangle \\ \left\langle R^{\dot{2}}_{[n]}(\infty,\bar{\infty})O^{(\text{int})}_{[2]}(1,\bar{1})O^{(\text{int})}_{[2]}(u,\bar{u})R^{\dot{1}}_{[n]}(0,\bar{0}) \right\rangle$$

Genus-zero covering surface:

$$z(t) = \left(\frac{t}{t_1}\right)^n \left(\frac{t-t_0}{t_1-t_0}\right) \left(\frac{t_1-t_\infty}{t-t_\infty}\right)$$

Roaming point on covering is x, its image on base is u; map z(x) = u is

$$u(x) = \frac{x^{n-1}(x+n)^{n+1}}{(x-1)^{n+1}(x+n-1)^{n-1}}$$

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Figure: Correction to dimension – R-charged fields



Figure: Correction to dimension – R-neutral fields

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We have also analyzed double-cycle operators

$$\begin{split} & [\![R^{\pm}_{[n_1]}R^{\pm}_{[n_2]}]\!](z,\bar{z}), \quad [\![R^{\pm}_{[n_1]}R^{\mp}_{[n_2]}]\!](z,\bar{z}), \\ & [\![R^{i}_{[n_1]}R^{i}_{[n_2]}]\!](z,\bar{z}), \quad [\![R^{\dot{2}}_{[n_1]}R^{\dot{2}}_{[n_2]}]\!](z,\bar{z}), \quad [\![R^{i}_{[n_1]}R^{\dot{2}}_{[n_2]}]\!](z,\bar{z}) \end{split}$$

Twisted sector corresponding to conjugacy class $(n_1)(n_2)(1)^{N-n_1-n_2}$

Genus-zero covering surface:

$$z(t) = \left(\frac{t}{t_1}\right)^{n_1} \left(\frac{t-t_0}{t_1-t_0}\right)^{n_2} \left(\frac{t_1-t_\infty}{t-t_\infty}\right)^{n_2}$$

Roaming map:

$$u(x) = \left(\frac{x + \frac{n_1}{n_2}}{x - 1}\right)^{n_1 + n_2} \left(\frac{x}{x - 1 + \frac{n_1}{n_2}}\right)^{n_1 - n_2}$$

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Example:

$$\begin{split} \left\langle \llbracket R_{[n_1]}^{\mathbf{i}} R_{[n_2]}^+ \rrbracket^{\dagger}(\infty, \bar{\infty}) O_{[2]}^{(\mathrm{int})}(1, \bar{1}) O_{[2]}^{(\mathrm{int})}(u, \bar{u}) \llbracket R_{[n_1]}^{\mathbf{i}} R_{[n_2]}^+ \rrbracket(0, \bar{0}) \right\rangle \\ &= G(u, \bar{u}) \\ &= \sum_{\mathfrak{a}} |G(x_{\mathfrak{a}}(u))|^2 \end{split}$$

where

$$G(x) = C \frac{x^{1-n_1+n_2}(x-1)^{1+n_1+n_2}(x+\frac{n_1}{n_2})^{1-n_1-n_2}(x-1+\frac{n_1}{n_2})^{1+n_1-n_2}}{(x+\frac{n_1-n_2}{2n_2})^4} \times \left[(x-1)(x-1+\frac{n_1}{n_2}) + x(x+\frac{n_1}{n_2}) \right]$$

and $x_{\mathfrak{a}}(u)$ are inverses of roaming map

$$u(x) = \left(\frac{x + \frac{n_1}{n_2}}{x - 1}\right)^{n_1 + n_2} \left(\frac{x}{x - 1 + \frac{n_1}{n_2}}\right)^{n_1 - n_2}$$

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Results:

 \cdot Fusion rules...

$$\begin{split} &[O_{[2]}^{(\text{int})}] \times [\llbracket R_{[n_1]}^{\mathbf{i}} R_{[n_2]}^+ \rrbracket], \quad [O_{[2]}^{(\text{int})}] \times [\llbracket R_{[n_1]}^{\mathbf{i}} R_{[n_2]}^{\mathbf{j}} \rrbracket], \\ &[O_{[2]}^{(\text{int})}] \times [\llbracket R_{[n_1]}^{\mathbf{i}} R_{[n_2]}^{\mathbf{i}} \rrbracket] \end{split}$$

... and structure constants; e.g.

$$\left\langle (R_{m_1}^{\dot{1}}R_{m_2}^+)^{\dagger} \sigma_3 (R_{m_1}^{\dot{1}}R_{m_2}^+) \right\rangle = 2^{-\frac{25}{3}} 3^{-\frac{16}{3}} (m_1^2 - m_2^2)^{\frac{2}{3}} m_1^{-\frac{4}{3}} m_2^{-\frac{4}{3}}$$

 \cdot Selection rule:

Composite fields renormalize for $2 < n_1 + n_2 < N$ BUT

Fields protected if

 $n_1 + n_2 = N$

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Consequence:

The most general Ramond field: $\prod_{i} (R_{(n_i)}^{(\zeta_i)})^{q_i}, \text{ with } \sum_{i} n_i q_i = N$ is protected

Proof involves showing that the function

 $\left\langle \prod_{i} (R_{[n_i]}^{(\zeta_i)\dagger})^{q_i}(\infty,\bar{\infty}) \ O_{[2]}^{(\text{int})}(1,\bar{1}) \ O_{[2]}^{(\text{int})}(u,\bar{u}) \ \prod_{i} (R_{[n_i]}^{(\zeta_i)})^{q_i}(0,\bar{0}) \right\rangle$

factorizes into the protected double-cycle functions [Lima et al., 2021] (in preparation)

> This result was known based on the relation of the field to a BPS NS chiral, but we can give an explicit proof!

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5. What *is* a black hole?

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Unitarity problem:

Semi-classically, the black hole radiates as a black body

After the black hole evaporates, where does its information go?



Information must be encoded in the radiation somehow...

but the horizon is empty!



MATHUR'S THEOREM:

Recovery of information requires CHANGES OF ORDER ONE AT THE HORIZON SCALE

[Mathur, 2009]

<u>Fuzzball Proposal:</u> Black holes are superposition of horizonless 'microstate geometries' Lunin and Mathur, 2002

> Construction of microstate geometries is a very active area [Lunin and Mathur, 2002, Skenderis and Taylor, 2008, Bena et al., 2011, Giusto et al., 2013, Giusto and Russo, 2014, Bena et al., 2015, Bena et al., 2016, Warner, 2019]

Black hole geometry Microstate geometry E horizon cap singularity





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Bena, I., Bobev, N., Giusto, S., Ruef, C., and Warner, N. P. (2011).
An Infinite-Dimensional Family of Black-Hole Microstate Geometries. *JHEP*, 03:022.
[Erratum: JHEP 04, 059 (2011)].

 Bena, I., Giusto, S., Martinec, E. J., Russo, R., Shigemori, M., Turton, D., and Warner, N. P. (2016).
 Smooth horizonless geometries deep inside the black-hole regime.

Phys. Rev. Lett., 117(20):201601.

 Bena, I., Giusto, S., Russo, R., Shigemori, M., and Warner, N. P. (2015).
 Habemus Superstratum! A constructive proof of the existence of superstrata. *JHEP*, 05:110.

David, J. R., Mandal, G., and Wadia, S. R. (2002). Microscopic formulation of black holes in string theory.

500

Phys. Rept., 369:549–686.

- - Eberhardt, L., Gaberdiel, M. R., and Gopakumar, R. (2019). The Worldsheet Dual of the Symmetric Product CET

The Worldsheet Dual of the Symmetric Product CFT. *JHEP*, 04:103.

- Giusto, S., Lunin, O., Mathur, S. D., and Turton, D. (2013).
 D1-D5-P microstates at the cap. JHEP, 02:050.
- Giusto, S. and Russo, R. (2014). Superdescendants of the D1D5 CFT and their dual 3-charge geometries. *JHEP*, 03:007.
- Larsen, F. and Martinec, E. J. (1999).
 U(1) charges and moduli in the D1 D5 system. JHEP, 06:019.
- Lima, A. A., Sotkov, G. M., and Stanishkov, M. (2020a).

Correlation functions of composite Ramond fields in deformed D1-D5 orbifold SCFT₂. *Phys. Rev. D*, 102(10):106004.

- Lima, A. A., Sotkov, G. M., and Stanishkov, M. (2020b). Dynamics of R-neutral Ramond fields in the D1-D5 SCFT.
- Lima, A. A., Sotkov, G. M., and Stanishkov, M. (2020c). Microstate Renormalization in Deformed D1-D5 SCFT. *Phys. Lett. B*, 808:135630.
- Lima, A. A., Sotkov, G. M., and Stanishkov, M. (2020d). Renormalization of Twisted Ramond Fields in D1-D5 SCFT₂.
 - , .

, .

Lima, A. A., Sotkov, G. M., and Stanishkov, M. (2021). On the Dynamics and Protection of Twisted Ramond Ground States in the deformed D1-D5 SCFT. Lunin, O. and Mathur, S. D. (2001). Correlation functions for M**N / S(N) orbifolds. Commun. Math. Phys., 219:399–442.

Lunin, O. and Mathur, S. D. (2002). AdS / CFT duality and the black hole information paradox.

Nucl. Phys. B, 623:342-394.

Mathur, S. D. (2009). The Information paradox: A Pedagogical introduction. *Class. Quant. Grav.*, 26:224001.

(日) (日) (日) (日) (日) (日) (日)

- Seiberg, N. and Witten, E. (1999).
 The D1 / D5 system and singular CFT. JHEP, 04:017.
- Skenderis, K. and Taylor, M. (2008). The fuzzball proposal for black holes. *Phys. Rept.*, 467:117–171.
- Strominger, A. and Vafa, C. (1996).

Microscopic origin of the Bekenstein-Hawking entropy. Phys. Lett. B, 379:99-104.

- Warner, N. P. (2019).

Lectures on Microstate Geometries.