# Black Hole Microscopy: <br> Conformal Field Theory at the bottom of the THROAT OF THE HORIZON 

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February 5, 2021

## What are we talking about?

- The free $\mathcal{N}=(4,4)$ SCFT with target space $\left(T^{4}\right)^{N} / S_{N}$ is a very interesting object:
- It has been the subject of important recent developments in the explicit construction of $\mathrm{AdS}_{3} / \mathrm{CFT}_{2}$ holography ([ ] ] and others)
- It famously holds the microstates accounting for the entropy of black holes in string theory [
- It is related to the D1-D5 system, the simplest example of a black hole in string theory
- Goals:
- Present some new results concerning the deformed theory [
- along with a (utterly) schematic overview of how the SCFT is related to black holes


## With results published in

- Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Dynamics of R-neutral Ramond fields in the D1-D5 SCFT.
- Lima, A.A., Sotkov, G., Stanishkov, M. (2020).

Renormalization of Twisted Ramond Fields in D1-D5 SCFT 2 .

- Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Correlation functions of composite Ramond fields in deformed D1-D5 orbifold SCFT ${ }_{2}$. Physical Review D, 102.
- Lima, A.A., Sotkov, G., Stanishkov, M. (2020). Microstate renormalization in deformed D1-D5 SCFT. Physics Letters B, 808, 135630.


## Outline

A black hole is a black hole

How to build a quantum black hole

The free CFT

The deformed CFT

What is a black hole？
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1. A black hole is a black hole



## $S_{\mathrm{BH}}=\frac{\text { Area }}{4 \mathrm{G}_{\mathrm{N}}}$



What are hairless black holes made of？

## 2. How to build a quantum black hole

- A quantum black hole needs quantum gravity i.e. String Theory.
- In low-energy limit, String Theory becomes SUGRA
- Ingredients:

$$
\begin{array}{rll}
\text { NS-NS (Type IIA and IIB): } & G_{\mu \nu}, & \Phi, \quad B^{(2)} \\
\text { R-R (Type IIA): } & C^{(1)}, & C^{(3)} \\
\text { R-R (Type IIB): } & C^{(0)}, & C^{(2)}, \quad C^{(4)}
\end{array}
$$

- Field equations are complicated, but there are techniques for building supersymmetric solutions
- Type IIB SUGRA
- $N_{1}$ D1 branes electrically coupled to $C^{(2)}$
- $N_{5}$ D5 branes magnetically coupled to $C^{(2)}$
- Array D1 along circle $S^{1}$ with radius $R$
- Array D5 along torus $T^{5}=T^{4} \times S^{1}$
- Wrap!

| 10 dimensions: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D1 | - | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | - | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ |
| D5 | - | $\cdot$ | $\cdot$ | $\cdot$ | $\cdot$ | - | - | - | - | - |

- means "extended"; • means "pointlike"
the compact dimensions of
D1-D5



## A black hole (rather a black ring) with two charges:

$$
\begin{aligned}
e^{\frac{1}{2} \Phi} d s_{E}^{2}= & \frac{1}{\sqrt{H_{1} H_{5}}}\left(-d t^{2}+d y^{2}\right)+\sqrt{H_{1} H_{5}}\left(d r^{2}+r^{2} d \Omega_{3}^{2}\right) \\
& +\sqrt{\frac{H_{1}}{H_{5}}} \sqrt{\operatorname{Vol}\left(T^{4}\right)} d s^{2}\left(T^{4}\right) \\
e^{\Phi}= & \sqrt{H_{1} / H_{5}} \\
F^{(3)}= & d C^{(2)}, \quad F_{r t y}^{(3)}=\partial_{r}\left(\frac{1}{H_{1}}\right), \quad F_{\theta \phi \varphi}^{(3)}=2 Q_{5} \sin ^{2} \theta \sin \phi
\end{aligned}
$$

where $y$ parameterizes $S^{1}$, and

$$
H_{1}=1+\frac{Q_{1}}{r^{2}}, \quad H_{5}=1+\frac{Q_{5}}{r^{2}}
$$

are harmonic functions on the transverse space

Geometry:

$$
\begin{aligned}
e^{\frac{1}{2} \Phi} d s_{E}^{2}= & \frac{1}{\sqrt{H_{1} H_{5}}}\left(-d t^{2}+d y^{2}\right)+\sqrt{H_{1} H_{5}}\left(d r^{2}+r^{2} d \Omega_{3}^{2}\right) \\
& +\sqrt{\frac{H_{1}}{H_{5}}} \sqrt{\operatorname{Vol}\left(T^{4}\right)} d s^{2}\left(T^{4}\right) \\
H_{1}= & 1+\frac{Q_{1}}{r^{2}}, \quad H_{5}=1+\frac{Q_{5}}{r^{2}}
\end{aligned}
$$

Asymptotically: $\quad r \rightarrow \infty \quad$ geometry is $\quad M^{4,1} \times S^{1} \times T^{4}$

Horizon: $\quad r=0, \quad g^{r r}=1 / \sqrt{H_{1} H_{5}}=0$
Near horizon: $\quad H_{1}=\frac{Q_{1}}{r^{2}}, \quad H_{5}=\frac{Q_{5}}{r^{2}}$

## Near-horizon geometry:

with $z=\sqrt{Q_{1} Q_{5}} / r$

$$
\begin{aligned}
e^{\frac{1}{2} \Phi} d s_{E}^{2}= & \frac{\sqrt{Q_{1} Q_{5}}}{z^{2}}\left(-d t^{2}+d z^{2}+d y^{2}\right) \\
& +\underbrace{\sqrt{\text { AdS }_{3} \text { with radius }\left(Q_{1} Q_{5}\right)^{\frac{1}{4}}}}_{S^{3}} \\
& +\sqrt{\frac{Q_{1} Q_{5}}{Q_{5}} \operatorname{Vol}\left(T^{4}\right)} d s^{2}\left(T^{4}\right) \\
e^{\Phi}= & \sqrt{Q_{1} / Q_{5}} \quad \text { (frozen) }
\end{aligned}
$$

Horizon: $r \rightarrow 0$ hence $z \rightarrow \infty$ so AdS boundary

Mink ${ }_{5}$


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## OBS:

Gravity description is good if radius is large and string coupling small:

$$
N=N_{1} N_{5} \gg \frac{1}{g_{s}} \sim \frac{N_{5}}{N_{1}} \gg 1
$$

## So we need large $N$

We have found the geometry

$$
\operatorname{AdS}_{3} \times S^{3} \times T^{4}
$$

This is dual to

$$
\mathrm{CFT}_{2} \quad \text { with } \quad \mathrm{SO}(4)_{E} \quad \text { R-symmetry }
$$

and

$$
\mathrm{SO}(4)_{I} \quad \text { "internal" symmetry }
$$

The central charge is determined by AdS:

$$
c=\frac{3 L_{\mathrm{AdS}_{3}}}{2 G_{N}^{(3)}}=6 N, \quad N=N_{1} N_{5}
$$

The correct CFT is supersymmetric, with 8 supersymmetries, and it is an orbifold defined on the target space $\left(T^{4}\right)^{N} / S_{N}$

3．The free CFT


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## The $\mathcal{N}=(4,4)$ SCFT

Four real bosons and four real fermions: all free Gather fields into complex bosons and complex fermions

$$
X^{\dot{A} A}(z, \bar{z}), \quad \psi^{\alpha \dot{A}}(z), \quad \tilde{\psi}^{\dot{\alpha} \dot{A}}(\bar{z})
$$

$a=1,2,3, \quad$ triplet of $\mathrm{SU}(2)_{L} ; \quad \dot{a}=\dot{1}, \dot{2}, \dot{3}, \quad$ triplet of $\mathrm{SU}(2)_{R}$ $\alpha=+,-, \quad$ doublet of $\operatorname{SU}(2)_{L} \quad \dot{\alpha}=\dot{+}, \dot{-}, \quad$ doublet of $\operatorname{SU}(2)_{R}$ $A=1,2, \quad$ doublet of $\mathrm{SU}(2)_{1} ; \quad \dot{A}=\dot{1}, \dot{2}, \quad$ doublet of $\mathrm{SU}(2)_{2}$

Bosonize fermions:

$$
\psi^{\alpha \dot{1}}(z)=\left[\begin{array}{l}
e^{-i \phi_{2}(z)} \\
e^{-i \phi_{1}(z)}
\end{array}\right], \quad \psi^{\alpha \dot{2}}(z)=\left[\begin{array}{c}
e^{i \phi_{2}(z)} \\
-e^{i \phi_{1}(z)}
\end{array}\right]
$$

Central charge: $c=6$

## Currents

Stress-tensor:

$$
T(z)=\frac{1}{4} \epsilon_{\dot{A} \dot{B}} \epsilon_{A B} \partial X^{\dot{A} A} \partial X^{\dot{B} B}+\frac{1}{4} \epsilon_{\dot{A} \dot{B}} \epsilon_{\alpha \beta} \psi^{\alpha \dot{A}} \partial \psi^{\beta \dot{B}}
$$

Four holomorphic super-currents:

$$
G^{\alpha A}(z)=\epsilon_{\dot{A} \dot{B}} \psi^{\alpha \dot{A}} \partial X^{\dot{B} A}
$$

R-current:

$$
J^{3}(z)=\frac{1}{2} i\left[\partial \phi_{1}(z)-\partial \phi_{2}(z)\right]
$$

Internal $\mathrm{SU}(2)_{2}$ :

$$
\mathfrak{J}^{3}(z)=\frac{1}{2} i\left[\partial \phi_{1}(z)+\partial \phi_{2}(z)\right]
$$

Quantum numbers: $h, j^{3}, \mathfrak{j}^{3}$

## OPERATOR ALGEBRA:

$$
\begin{aligned}
T\left(z^{\prime}\right) T(z)= & \frac{c / 2}{\left(z^{\prime}-z\right)^{4}}+\frac{2 T(z)}{\left(z^{\prime}-z\right)^{2}}+\frac{\partial T(z)}{z^{\prime}-z}+\cdots \\
J^{a}\left(z^{\prime}\right) J^{b}(z)= & \frac{c}{12} \frac{\delta^{a b}}{\left(z^{\prime}-z\right)^{2}}+\frac{i \epsilon^{a b}{ }_{c} J^{c}(z)}{z^{\prime}-z}+\cdots \\
G^{\alpha A}\left(z^{\prime}\right) G^{\beta A}(z)= & -\frac{c}{3} \frac{\epsilon^{A B} \epsilon^{\alpha \beta}}{\left(z^{\prime}-z\right)^{3}}-\epsilon^{A B} \epsilon^{\alpha \beta} \frac{T(z)}{z-z^{\prime}} \\
& +\epsilon^{A B} \epsilon^{\beta \gamma}\left[\sigma^{* a}\right]^{\alpha}{ }_{\gamma}\left[\frac{2 J^{a}(z)}{\left(z^{\prime}-z\right)^{2}}+\frac{\partial J(z)}{z^{\prime}-z}\right]+\cdots \\
J^{a}(z) G^{\alpha A}\left(z^{\prime}\right)= & \frac{1}{2}\left[\sigma^{* a}\right]^{\alpha}{ }_{\gamma} \frac{G^{\gamma A}(z)}{z^{\prime}-z}+\cdots \\
T\left(z^{\prime}\right) G^{\alpha A}(z)= & \frac{3}{2} \frac{G^{\alpha A}(z)}{\left(z^{\prime}-z\right)^{2}}+\frac{\partial G^{\alpha A}(z)}{z^{\prime}-z}+\cdots \\
T\left(z^{\prime}\right) J^{a}(z)= & \frac{J^{a}(z)}{\left(z^{\prime}-z\right)^{2}}+\frac{\partial J^{a}(z)}{z^{\prime}-z}+\cdots
\end{aligned}
$$

The correct CFT is the orbifold $\left(T^{4}\right)^{N} / S_{N}$; make $N$ copies:

$$
\left(T^{4}\right)^{N}=T^{4} \otimes \cdots \otimes T^{4}
$$

Copy index: $I=1, \cdots, N$

$$
X_{I}^{\dot{A} A}(z, \bar{z}), \quad \psi_{I}^{\alpha \dot{A}}(z), \quad \phi_{I}^{a}(z)
$$

Total currents:

$$
\begin{aligned}
T(z) & =\sum_{I=1}^{N}\left[\frac{1}{4} \epsilon_{\dot{A} \dot{B}} \epsilon_{A B} \partial X_{I}^{\dot{A} A} \partial X_{I}^{\dot{B} B}+\frac{1}{4} \epsilon_{\dot{A} \dot{B}} \epsilon_{\alpha \beta} \psi_{I}^{\alpha \dot{A}} \partial \psi_{I}^{\beta \dot{B}}\right] \\
J^{3}(z) & =\frac{1}{2} i \sum_{I=1}^{N}\left[\partial \phi_{I}^{1}(z)-\partial \phi_{I}^{2}(z)\right]
\end{aligned}
$$

etc. Central charge of orbifold: $c=6 N$
... Now identify permutations

$$
000100
$$

## What changes when we orbifold?

Answer: New possible boundary conditions permuting the copies.

## How to implement?

Answer: 'Twist fields' $\sigma_{g}(z), g \in S_{N}$
Going around a twist:

$$
X_{I}^{i}\left(e^{2 \pi i} z, e^{-2 \pi i} \bar{z}\right) \sigma_{g}(z, \bar{z})=X_{g(I)}^{i}(z, \bar{z}) \sigma_{g}(z, \bar{z})
$$

Interpretation in the $(t, y)$ plane:
The twist field $\sigma_{(n)}$ joins $n$ "strings" into a single $n$-wound string



## We have found a CFT dual to the AdS throat

We can compute its degeneracy of states $\Omega$, and we find that

$$
\log \Omega=S_{B H}
$$

This is the famous result of [Strominger and Vafa, 1996]

## 4. The deformed CFT

The free orbifold is dual to a very singular gravitational solution
To go towards the SUGRA description, we need an interacting CFT
[
(States corresponding to the black hole are states which do not renormalize under this deformation)

Deformed theory:

$$
S_{\mathrm{int}}=S_{\mathrm{free}}+\lambda \int d^{2} z O_{[2]}^{(\mathrm{int})}(z, \bar{z})
$$

defined by the MARGINAL interaction operator

$$
O_{[2]}^{(\text {int })}(z, \bar{z})=\epsilon_{A B} G_{-\frac{1}{2}}^{-A} \tilde{G}_{-\frac{1}{2}}^{-B} O_{[2]}^{(0,0)}(z, \bar{z})
$$

The effect of the deformation on an operator $\mathscr{A}_{[n]}$ can be found from the corrections to the two-point function

$$
\left\langle\mathscr{A}_{[n]}^{\dagger}\left(z_{1}, \bar{z}_{1}\right) \mathscr{A}_{[n]}\left(z_{2}, \bar{z}_{2}\right)\right\rangle=\frac{C}{\left|z_{1}-z_{2}\right|^{2 \Delta}}
$$

Change in the conformal dimension

$$
\Delta(\lambda)=\Delta+\lambda \delta \Delta+\frac{1}{2} \lambda^{2} \delta^{2} \Delta+\cdots
$$

where

$$
\begin{aligned}
\delta \Delta & \sim\left\langle\mathscr{A}_{[n]}^{\dagger}(\infty) O_{[2]}^{(\text {int })}(1, \overline{1}) \mathscr{A}_{[n]}(0)\right\rangle \\
\delta^{2} \Delta & \sim \frac{\lambda^{2}}{2} \int d^{2} z_{2} \int d^{2} z_{3}\left\langle\mathscr{A}_{[n]}^{\dagger}\left(z_{1}, \bar{z}_{1}\right) O_{[2]}^{(\mathrm{int})}\left(z_{2}, \bar{z}_{2}\right) O_{[2]}^{(\mathrm{int})}\left(z_{3}, \bar{z}_{3}\right) \mathscr{A}_{[n]}\left(z_{4}, \bar{z}_{4}\right)\right\rangle
\end{aligned}
$$

The four-point function

$$
\left\langle\mathscr{A}_{[n]}^{\dagger}(\infty) O_{[2]}^{(\mathrm{int})}(1, \overline{1}) O_{[2]}^{(\mathrm{int})}(u, \bar{u}) \mathscr{A}_{[n]}(0)\right\rangle
$$

is complicated: TWISTS!
Move $u$ around the other points and fields are permuted:

> Very non-trivial boundary conditions!

What to do?

For

$$
\left\langle\sigma_{\left[n_{1}\right]}(\infty) \sigma_{\left[n_{2}\right]}(1, \overline{1}) \sigma_{\left[n_{3}\right]}(u, \bar{u}) \sigma_{\left[n_{4}\right]}(0)\right\rangle
$$

USE A COVERING SURFACE [Lunin and Mathur, 2001] with coordinates $(t, \bar{t})$
$z=0$
$\downarrow$
$z=u$
$z=\infty$
$\in \quad S_{\text {base }}^{2}$

I
$t=0 \quad t=t_{1}$
$t=x \quad t=\infty$
$\in S_{\text {cover }}^{2}$
such that

$$
\begin{array}{ll}
z(t) \approx b_{1} t^{n_{1}} & \text { as } z \rightarrow 0 \\
z(t) \approx 1+b_{2}\left(t-t_{1}\right)^{n_{2}} & \text { as } z \rightarrow 1 \\
z(t) \approx u+b_{3}(t-x)^{n_{3}} & \text { as } z \rightarrow u \\
z(t) \approx b_{4} t^{n_{4}} & \text { as } z \rightarrow \infty
\end{array}
$$

... plus other images of $\infty$
E.g. for two-point functions $\left\langle\sigma_{2} \sigma_{2}\right\rangle$
the covering surface looks like


## Covering Surface:

Its genus and the number of its ramification points determine the large- $N$ scaling of the corresponding correlation function

$$
\begin{aligned}
\left\langle\mathscr { O } _ { [ n _ { 1 } ] } ^ { 1 } ( z _ { 1 } , \overline { z } _ { 1 } ) \mathscr { O } _ { [ n _ { 2 } ] } ^ { 2 } \left( z_{2},\right.\right. & \left.\left.\bar{z}_{2}\right) \cdots \mathscr{O}_{\left[n_{Q}\right]}^{Q}\left(z_{Q}, \bar{z}_{Q}\right)\right\rangle \\
& \sim N^{\mathbf{s}-\frac{1}{2} \sum_{r=1}^{Q} n_{r}}\left(1+N^{-1}+\cdots\right) \\
& =N^{1-\mathbf{g}-\frac{1}{2} Q}\left(1+N^{-1}+\cdots\right)
\end{aligned}
$$

Here $\mathbf{s}$ is the number of distinct copies entering the permutations, and

$$
\mathbf{g}=\frac{1}{2} \sum_{r=1}^{Q}\left(n_{r}-1\right)-\mathbf{s}+1
$$

(Riemann-Hurwitz Formula)

We have therefore the following program:

1. Choose a relevant operator in the CFT
2. Compute the four-point function with $O_{[2]}^{(\text {int })}$ (Not trivial! Must use covering surface and other paraphernalia)
3. Obtain information about dynamics: structure constants, fusion rules, etc.
4. Integrate the four-point function
(Also not trivial! There are regularization issues and complex-contour gymnastics)
5. Obtain the correction to conformal dimensions

We have done this for Ramond ground states of the $n$-twisted sector:

$$
R_{[n]}^{ \pm} \equiv \frac{1}{\mathscr{S}_{n}(N)} \sum_{h \in S_{N}} \exp \left( \pm \frac{i}{2 n} \sum_{I=1}^{n}\left[\phi_{1, h(I)}-\phi_{2, h(I)}\right]\right) \sigma_{h^{-1}(n) h}(z)
$$

(R-charged doublet of $\operatorname{SU}(2)_{E}$, neutral under internal $\left.\mathrm{SU}(2)_{2}\right)$

$$
\begin{aligned}
R_{[n]}^{\dot{1}} & \equiv \frac{1}{\mathscr{S}_{n}(N)} \sum_{h \in S_{N}} \exp \left(-\frac{i}{2 n} \sum_{I=1}^{n}\left[\phi_{1, h(I)}+\phi_{2, h(I)}\right]\right) \sigma_{h^{-1}(n) h}(z) \\
R_{[n]}^{\dot{2}} & \equiv \frac{1}{\mathscr{S}_{n}(N)} \sum_{h \in S_{N}} \exp \left(+\frac{i}{2 n} \sum_{I=1}^{n}\left[\phi_{1, h(I)}+\phi_{2, h(I)}\right]\right) \sigma_{h^{-1}(n) h}(z)
\end{aligned}
$$

(R-neutral doublet of internal $\mathrm{SU}(2)_{2}$ )

Compute the functions

$$
\begin{aligned}
& \left\langle R_{[n]}^{-}(\infty, \bar{\infty}) O_{[2]}^{(\mathrm{int})}(1, \overline{1}) O_{[2]}^{(\mathrm{int})}(u, \bar{u}) R_{[n]}^{+}(0, \overline{0})\right\rangle \\
& \left\langle R_{[n]}^{\dot{2}}(\infty, \bar{\infty}) O_{[2]}^{(\mathrm{int})}(1, \overline{1}) O_{[2]}^{(\mathrm{int})}(u, \bar{u}) R_{[n]}^{\dot{1}}(0, \overline{0})\right\rangle
\end{aligned}
$$

Genus-zero covering surface:

$$
z(t)=\left(\frac{t}{t_{1}}\right)^{n}\left(\frac{t-t_{0}}{t_{1}-t_{0}}\right)\left(\frac{t_{1}-t_{\infty}}{t-t_{\infty}}\right)
$$

Roaming point on covering is $x$, its image on base is $u$; map $z(x)=u$ is

$$
u(x)=\frac{x^{n-1}(x+n)^{n+1}}{(x-1)^{n+1}(x+n-1)^{n-1}}
$$



Figure: Correction to dimension - R-charged fields


Figure: Correction to dimension - R-neutral fields

We have also analyzed double-cycle operators

$$
\begin{array}{ll}
\llbracket R_{\left[n_{1}\right]}^{ \pm} R_{\left[n_{2}\right]}^{ \pm} \rrbracket(z, \bar{z}), & \llbracket R_{\left[n_{1}\right]}^{ \pm} R_{\left[n_{2}\right]}^{\mp} \rrbracket(z, \bar{z}), \\
\llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{\mathrm{i}} \rrbracket(z, \bar{z}), & \llbracket R_{\left[n_{1}\right]}^{\dot{2}} R_{\left[n_{2}\right]}^{\dot{2}} \rrbracket(z, \bar{z}), \\
\llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{\dot{2}} \rrbracket(z, \bar{z})
\end{array}
$$

Twisted sector corresponding to conjugacy class $\left(n_{1}\right)\left(n_{2}\right)(1)^{N-n_{1}-n_{2}}$
Genus-zero covering surface:

$$
z(t)=\left(\frac{t}{t_{1}}\right)^{n_{1}}\left(\frac{t-t_{0}}{t_{1}-t_{0}}\right)^{n_{2}}\left(\frac{t_{1}-t_{\infty}}{t-t_{\infty}}\right)^{n_{2}}
$$

Roaming map:

$$
u(x)=\left(\frac{x+\frac{n_{1}}{n_{2}}}{x-1}\right)^{n_{1}+n_{2}}\left(\frac{x}{x-1+\frac{n_{1}}{n_{2}}}\right)^{n_{1}-n_{2}}
$$

## Example:

$$
\begin{aligned}
\left\langle\llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{+} \dagger^{\dagger}\right. & \left.(\infty, \bar{\infty}) O_{[2]}^{(\mathrm{int})}(1, \overline{1}) O_{[2]}^{(\mathrm{int})}(u, \bar{u}) \llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{+} \rrbracket(0, \overline{0})\right\rangle \\
& =G(u, \bar{u}) \\
& =\sum_{\mathfrak{a}}\left|G\left(x_{\mathfrak{a}}(u)\right)\right|^{2}
\end{aligned}
$$

where

$$
\begin{aligned}
& G(x)=C \frac{x^{1-n_{1}+n_{2}}(x-1)^{1+n_{1}+n_{2}}( }{}\left(x+\frac{n_{1}}{n_{2}}\right)^{1-n_{1}-n_{2}}\left(x-1+\frac{n_{1}}{n_{2}}\right)^{1+n_{1}-n_{2}} \\
&\left(x+\frac{n_{1}-n_{2}}{2 n_{2}}\right)^{4} \\
& \times\left[(x-1)\left(x-1+\frac{n_{1}}{n_{2}}\right)+x\left(x+\frac{n_{1}}{n_{2}}\right)\right]
\end{aligned}
$$

and $x_{\mathfrak{a}}(u)$ are inverses of roaming map

$$
u(x)=\left(\frac{x+\frac{n_{1}}{n_{2}}}{x-1}\right)^{n_{1}+n_{2}}\left(\frac{x}{x-1+\frac{n_{1}}{n_{2}}}\right)^{n_{1}-n_{2}}
$$

## Results:

- Fusion rules...

$$
\begin{aligned}
& {\left[O_{[2]}^{(\text {int })}\right] \times\left[\llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{+} \rrbracket\right], \quad\left[O_{[2]}^{(\text {int })}\right] \times\left[\llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{\dot{2}} \rrbracket\right],} \\
& {\left[O_{[2]}^{\text {(int })}\right] \times\left[\llbracket R_{\left[n_{1}\right]}^{\mathrm{i}} R_{\left[n_{2}\right]}^{\mathrm{i}} \rrbracket\right]}
\end{aligned}
$$

... and structure constants; e.g.

$$
\left\langle\left(R_{m_{1}}^{\mathrm{i}} R_{m_{2}}^{+}\right)^{\dagger} \sigma_{3}\left(R_{m_{1}}^{\mathrm{i}} R_{m_{2}}^{+}\right)\right\rangle=2^{-\frac{25}{3}} 3^{-\frac{16}{3}}\left(m_{1}^{2}-m_{2}^{2}\right)^{\frac{2}{3}} m_{1}^{-\frac{4}{3}} m_{2}^{-\frac{4}{3}}
$$

## - Selection rule:

Composite fields renormalize for $2<n_{1}+n_{2}<N$ BUT
Fields protected if

$$
n_{1}+n_{2}=N
$$

Consequence:

$$
\begin{aligned}
& \text { The most general Ramond field: } \\
& \prod_{i}\left(R_{\left(n_{i}\right)}^{\left(\zeta_{i}\right)}\right)^{q_{i}}, \text { with } \sum_{i} n_{i} q_{i}=N \\
& \text { is protected }
\end{aligned}
$$

Proof involves showing that the function

$$
\left\langle\prod_{i}\left(R_{\left[n_{i}\right]}^{\left(\zeta_{i}\right) \dagger}\right)^{q_{i}}(\infty, \bar{\infty}) O_{[2]}^{(\mathrm{int})}(1, \overline{1}) O_{[2]}^{(\mathrm{int})}(u, \bar{u}) \prod_{i}\left(R_{\left[n_{i}\right]}^{\left(\zeta_{i}\right)}\right)^{q_{i}}(0, \overline{0})\right\rangle
$$

factorizes into the protected double-cycle functions [Lima et al., 2021] (in preparation)

This result was known based on the relation of the field to a BPS NS chiral, but we can give an explicit proof!

## 5. What is a black hole?

Unitarity problem:
Semi-classically, the black hole radiates as a black body
After the black hole evaporates, where does its information go?

Information must be encoded in the radiation somehow... but the horizon is empty!

## termal

## radiation



## MATHUR'S THEOREM:

## Recovery of information requires CHANGES OF ORDER ONE AT THE HORIZON SCALE

[Mathur, 2009]
Fuzzball Proposal:
Black holes are superposition of horizonless 'microstate geometries' [Lunin and Mathur, 2002]

Construction of microstate geometries is a very active area
[Lunin and Mathur, 2002, Skenderis and Taylor, 2008, Bena et al., 2011, Giusto et al., 2013, Giusto and Russo, 2014, Bena et al., 2015, Bena et al., 2016, Warner, 2019]

## Black hole geometry

Microstate geometry

singularity
fuzzball geometry

Fim

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