



# Induced vacuum currents in topologically nontrivial locally AdS spacetimes

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# Outline

- Anti-de Sitter spacetime as a background geometry in QFT
- Geometry and Field
- Current density in AdS spacetime with toroidal compact dimensions
- Current density in the presence of a brane
- Conclusions

# Publications

- E. R. Bezerra de Mello, A. A. Saharian, V. Vardanyan, Phys. Lett. B **741**, 155 (2015)
- S. Bellucci, A.A. Saharian, V. Vardanyan, JHEP 11(2015)092
- S. Bellucci, A. A. Saharian, V. Vardanyan, Phys. Rev. D **93**, 084011 (2016)
- S. Bellucci, A. A. Saharian, V. Vardanyan, Phys. Rev. D **96**, 065025 (2017)
- S. Bellucci, A. A. Saharian, D. H. Simonyan, V. Vardanyan, Phys. Rev. D **98**, 085020 (2018)

# Aim

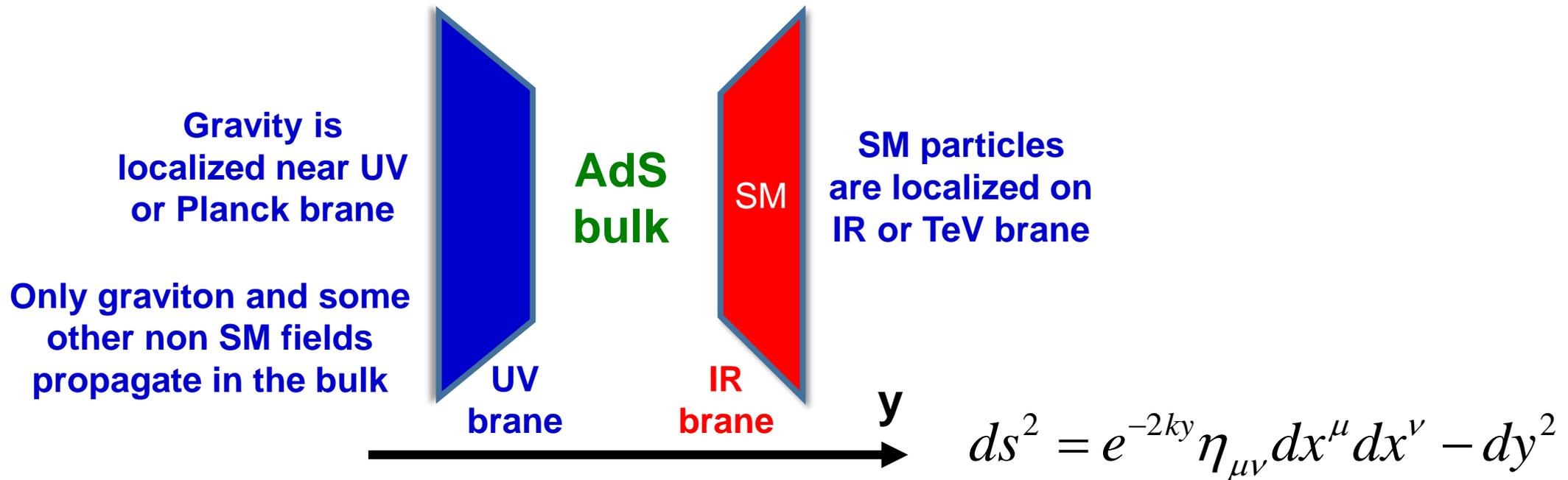
- We aim to consider combined effects of **topology** and **gravity** on the properties of quantum vacuum
- Gravitational field is considered as a **classical curved background**
- **Back-reaction** of quantum effects is described by Einstein equations with the expectation value of the energy-momentum tensor for quantum fields in the right-hand side
- This hybrid but very useful scheme is an important intermediate step to the development of **quantum gravity**
- Among the most interesting effects in this field are the **particle production** and the **vacuum polarization** by strong gravitational fields
- As a background geometry we consider **AdS spacetime**

# Importance of AdS in QFT on curved backgrounds

- Because of the **high symmetry**, numerous problems are **exactly solvable** on AdS bulk and this may shed light on the influence of a classical gravitational field on the quantum matter in more general geometries
- **Questions of principal nature** related to the quantization of fields propagating on curved backgrounds
- AdS spacetime generically arises as a **ground state** in extended supergravity and in string theories
- **AdS/Conformal Field Theory correspondence**: Relates string theories or supergravity in the bulk of AdS with a conformal field theory living on its boundary
- **Braneworld models**: Provide a solution to the hierarchy problem between the gravitational and electroweak scales
- Braneworlds naturally appear in **string/M-theory** context and provide a novel setting for discussing phenomenological and cosmological issues related to extra dimensions

# Randall-Sundrum-type braneworlds

- Original **Randall-Sundrum model** (RS1) offers a solution to the **hierarchy problem** by postulating 5D AdS spacetime bounded by two (3+1)-dimensional branes



- Hierarchy problem between the gravitational and electroweak scales is solved for

*$k \cdot \text{distance between branes} = 40$*

# Geometry

- (D+1)-dimensional AdS spacetime

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{-2y/a} \eta_{ik} dx^i dx^k - dy^2 \quad \eta_{ik} = \text{diag}(1, -1, \dots, -1)$$

- New coordinate  $z = ae^{y/a}, 0 \leq z < \infty$

$$ds^2 = (a/z)^2 (\eta_{ik} dx^i dx^k - dz^2)$$

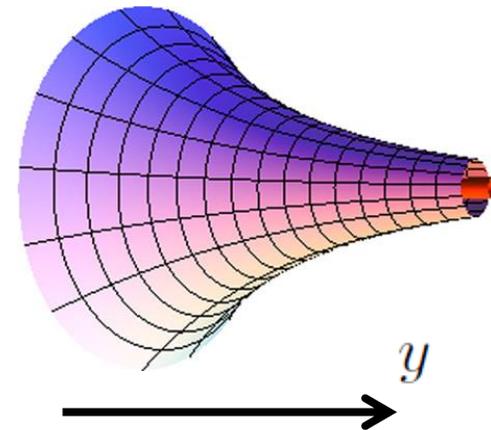
- Topology  $R^p \times (S^1)^q, q + p = D - 1$   
 $q$ -dimensional **torus**

- Cartesian coordinates along **uncompactified** and **compactified** dimensions

$$\mathbf{x}_q \downarrow = (x^{p+1}, \dots, x^{D-1}) \quad \mathbf{x}_p \downarrow = (x^1, \dots, x^p)$$



$L_l \rightarrow$  length of the  $l$ -th compact dimension



# Fields

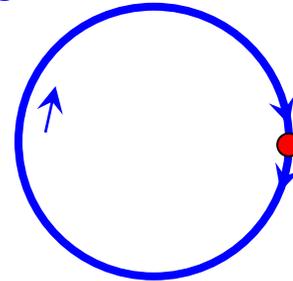
- Charged **scalar field** with general curvature coupling

$$(g^{\mu\nu} D_\mu D_\nu + m^2 + \xi R) \varphi(x) = 0, \quad D_\mu = \nabla_\mu + ieA_\mu$$

*External classical gauge field*

- In models with nontrivial topology one need also to specify the **periodicity conditions** obeyed by the field operator along compact dimensions

$$\varphi(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l} \varphi(t, \mathbf{x}_p, \mathbf{x}_q)$$

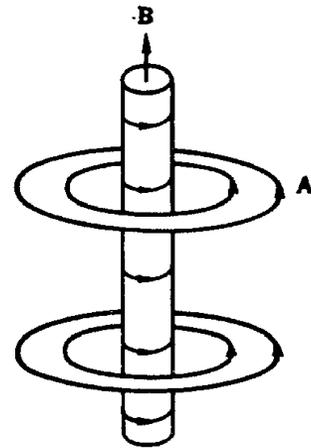
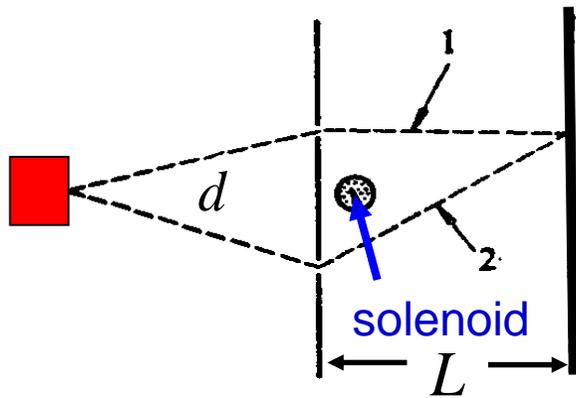


- Special cases:
  - **Untwisted fields**  $\alpha_l = 0$
  - **Twisted fields**  $\alpha_l = \pi$

- We assume that the gauge field is **constant**:  $A_\mu = \text{const}$

- Though the corresponding **field strength vanishes**, the nontrivial topology gives rise to **Aharonov-Bohm-like effects**

# Aharonov-Bohm effect



inside ↷

$$A_r = A_z = 0, \quad A_\phi = Br/2, \quad B_r = B_\phi = 0, \quad B_z = B,$$

$$A_r = A_z = 0, \quad A_\phi = BR^2/2r, \quad B_r = B_\phi = B_z = 0,$$

↶ outside

Phase of the wave function is changed

$$\alpha \rightarrow \alpha - (e/\hbar)\vec{A} \cdot \vec{r}$$

Phase change along the trajectory

$$\Delta\alpha = -(e/\hbar) \int_{\text{trajectory}} \vec{A} \cdot d\vec{r}$$

Change of the phase difference

$$\Delta\delta = \Delta\alpha_1 - \Delta\alpha_2 = (e/\hbar) \oint_{2-1} \vec{A} \cdot d\vec{r} = (e/\hbar) \int_{2-1} \text{rot}\vec{A} \cdot d\vec{S} = (e/\hbar)\Phi \leftarrow \text{magnetic flux}$$

Interference pattern is shifted by  $\Delta x = L\lambda\Delta\delta/d$

First experimental confirmation of the Aharonov-Bohm effect

Chambers R. G. *Phys. Rev. Letters* **5**, 3 (1960)

# Current density

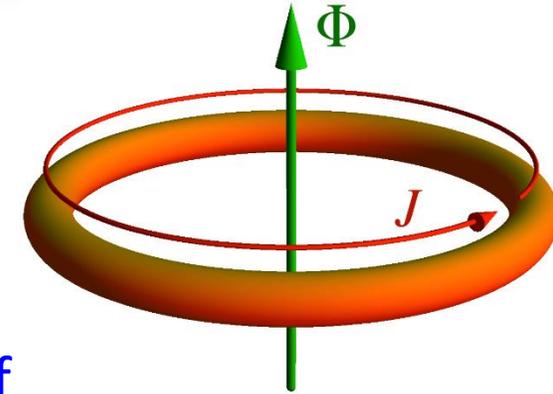
- We are interested in the effects of non-trivial topology and gravity on the **vacuum expectation value** (VEV) of the current

$$j_{\mu}(x) = ie[\varphi^{+}(x)D_{\mu}\varphi(x) - (D_{\mu}\varphi^{+}(x))\varphi(x)]$$

- This VEV is among the most important quantities that characterize the **properties of the quantum vacuum**
- Although the corresponding operator is **local**, due to the **global nature of the vacuum**, the VEV carries important information about the **global properties** of the background space-time
- Current acts as the **source** in the **Maxwell equations** and therefore plays an important role in modeling a **self-consistent dynamics** involving the electromagnetic field

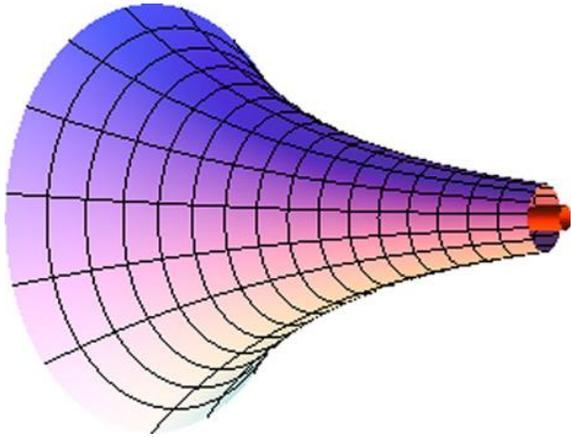
# Analog from condensed matter physics: Persistent currents

- Persistent currents in metallic rings are predicted in *M. Büttiker, Y. Imry, R. Landauer, Phys. Lett. A 96, 7 (1983)*
- Existence of persistent currents in normal metal rings is a signature of phase coherence in mesoscopic systems and an example of the Aharonov-Bohm effect
- Temperature must be sufficiently low to reduce the probability of inelastic scattering and the circumference of the ring short enough that the phase coherence of the electronic wave functions is preserved around the loop
- Measurements of persistent currents in nanoscale gold and aluminum rings: *A.C. Bleszynski-Jayich et. al., Science 326 (2009); H. Bluhm et. al., Phys. Rev. Lett. 102 (2009)*



# Induced currents in models with compact dimensions

- In the problem under consideration the presence of a **constant gauge field** is equivalent to the **magnetic flux** enclosed by the compact dimension



- **Flux** of the field strength which threads the  $l$ -th compact dimension

$$A_l L_l = \oint dx^l A_l$$

- **Gauge transformation**

$$\varphi = \varphi' e^{-ie\chi}, \quad A_\mu = A'_\mu + \partial_\mu \chi \quad \chi = A_\mu x^\mu$$

- Problem with a constant gauge field is reduced to the problem in the **absence of the gauge field** with the **shifted phases** in the periodicity conditions:

$$\varphi'(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\tilde{\alpha}_l} \varphi'(t, \mathbf{x}_p, \mathbf{x}_q), \quad \tilde{\alpha}_l = \alpha_l + eA_l L_l$$

# Evaluation procedure

- VEV of the current density can be expressed in terms of the **Hadamard function**

$$G^{(1)}(x, x') = \langle 0 | \varphi(x) \varphi^+(x') + \varphi^+(x') \varphi(x) | 0 \rangle \leftarrow \text{Vacuum state}$$

- The corresponding relation

$$\langle 0 | j_\mu(x) | 0 \rangle = \langle j_\mu \rangle = \frac{i}{2} e \lim_{x' \rightarrow x} (\partial_\mu - \partial'_\mu) G^{(1)}(x, x')$$

- **Mode sum** for the Hadamard function

$$G^{(1)}(x, x') = \sum_{\sigma} \sum_{s=\pm} \varphi_{\sigma}^{(s)}(x) \varphi_{\sigma}^{(s)*}(x')$$

$\{\varphi_{\sigma}^{(+)}(x), \varphi_{\sigma}^{(-)}(x)\} \leftarrow$  **Complete set** of normalized positive- and negative-energy **solutions** to the field equation obeying the periodicity conditions

# Vacuum current density

- Charge density vanishes
- Components of the current density along **uncompact dimensions** vanish
- Current density along  **$l$ -th compact dimension**

$$\langle j^l \rangle = \frac{4ea^{-1-D}L_l}{(2\pi)^{(D+1)/2}} \sum_{n_l=1}^{\infty} n_l \sin(\tilde{\alpha}_l n_l) \sum_{\mathbf{n}_{q-1}} \cos(\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1}) q_{\nu-1/2}^{(D+1)/2} (1 + g_{\mathbf{n}_q}^2 / (2z^2))$$

$$\mathbf{n}_{q-1} = (n_{p+1}, \dots, n_{l-1}, n_{l+1}, \dots, n_{D-1}), \quad -\infty < n_i < +\infty, \quad i \neq l$$

$$\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1} = \sum_{i=1, \neq l}^{D-1} \tilde{\alpha}_i n_i, \quad g_{\mathbf{n}_q} = \left( \sum_{i=p+1}^{D-1} n_i^2 L_i^2 \right)^{1/2} \quad \tilde{\alpha}_l = \alpha_l + eA_l L_l$$

$$q_{\alpha}^{\mu}(x) = \frac{e^{-i\pi\mu} Q_{\alpha}^{\mu}(x)}{(x^2 - 1)^{\mu/2}} \quad \leftarrow \text{Associated Legendre function of the second kind}$$

# Current density: Properties

- Current density along the  $l$ -th compact dimension is an **odd periodic function** of the phase  $\tilde{\alpha}_l$  and an **even periodic function** of the phases  $\tilde{\alpha}_i, i \neq l$ , with the period  $2\pi$
- In particular, the current density is a **periodic function** of the **magnetic fluxes** with the period equal to the flux quantum  $2\pi/|e|$
- In the **absence of the gauge field**, the current density along the  $l$ -th compact dimension **vanishes** for **untwisted** and **twisted** fields along that direction
- Charge flux through  $(D-1)$ -dimensional hypersurface  $x^l = \text{const}$   
 $n_l \langle j^l \rangle, n_l = a/z$  ← Normal to the hypersurface
- **Charge flux** depends on the coordinate lengths of the compact dimensions in the form of  
 $L_i/z$  = proper length of the compact dimension, measured in units of the AdS curvature radius  
 $L_{(p)i} = aL_i/z$  ← **proper length** of the compact dimension

# Limiting cases

- Large values of the curvature radius  $a \rightarrow \infty$

$$\langle j^l \rangle \approx \frac{4eL_l m^{(D+1)/2}}{(2\pi)^{(D+1)/2}} \sum_{n_l=1}^{\infty} n_l \sin(\tilde{\alpha}_l n_l) \sum_{\mathbf{n}_{q-1}} \cos(\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1}) \frac{K_{(D+1)/2}(mg_{\mathbf{n}_q})}{g_{\mathbf{n}_q}^{(D+1)/2}}$$

$K_{(D+1)/2}(x)$  ← MacDonald function

Leading term coincides with the current in Minkowski spacetime with toroidally compactified dimensions

- Large values of the proper length compared with the AdS curvature radius:  $L_l/z \gg 1$

- ◆  $\tilde{\alpha}_i = 0, i = p+1, \dots, D-1, i \neq l,$

$$\langle j^l \rangle \approx \frac{4e\Gamma(p/2 + \nu + 2)}{\pi^{p/2+1}\Gamma(\nu + 1)a^{D+1}V_q} \frac{z^{D+2\nu+2}}{L_l^{p+2\nu+2}} \sum_{n_l=1}^{\infty} \frac{\sin(\tilde{\alpha}_l n_l)}{n_l^{p+2\nu+3}}$$

- ◆ At least one of the phases  $\tilde{\alpha}_i, i \neq l,$  is not equal to zero

$$\langle j^l \rangle \approx \frac{2ea^{-1-D}}{\pi^{(p+1)/2}} \frac{\sin(\tilde{\alpha}_l)z^{D+2\nu+2}}{\Gamma(\nu + 1)V_q e^{\beta_{q-1}L_l}} \frac{\beta_{q-1}^{(p+3)/2+\nu}}{(2L_l)^{(p+1)/2+\nu}}, \beta_{q-1} = (\sum_{i=p+1, \neq l}^{D-1} \tilde{\alpha}_i^2 / L_i^2)^{1/2}$$

# Limiting cases

- Small values of the proper length compared with the AdS curvature radius:  $L_l/z \ll 1$

$$\langle j^l \rangle \approx \frac{2e\Gamma((D+1)/2)}{\pi^{(D+1)/2}(a/z)^{D+1}L_l^D} \sum_{n_l=1}^{\infty} \frac{\sin(\tilde{\alpha}_l n_l)}{n_l^D}$$

Coincides with the VEV of the current density for a massless scalar field in  $(D+1)$ -dimensional Minkowski spacetime compactified along the direction  $x^l$

- Near the AdS boundary,  $z \rightarrow 0$

$$\langle j^l \rangle \approx \frac{4eL_l\Gamma(\nu + D/2 + 1)}{\pi^{D/2}\Gamma(\nu + 1)a^{D+1}} z^{D+2\nu+2} \sum_{n_l=1}^{\infty} n_l \sin(\tilde{\alpha}_l n_l) \sum_{\mathbf{n}_{q-1}} \frac{\cos(\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1})}{g_{\mathbf{n}_q}^{D+2\nu+2}}$$

- Near the AdS horizon,  $z \rightarrow \infty$   $\langle j^l \rangle \approx (z/a)^{D+1} \langle j^l \rangle_M$ ,

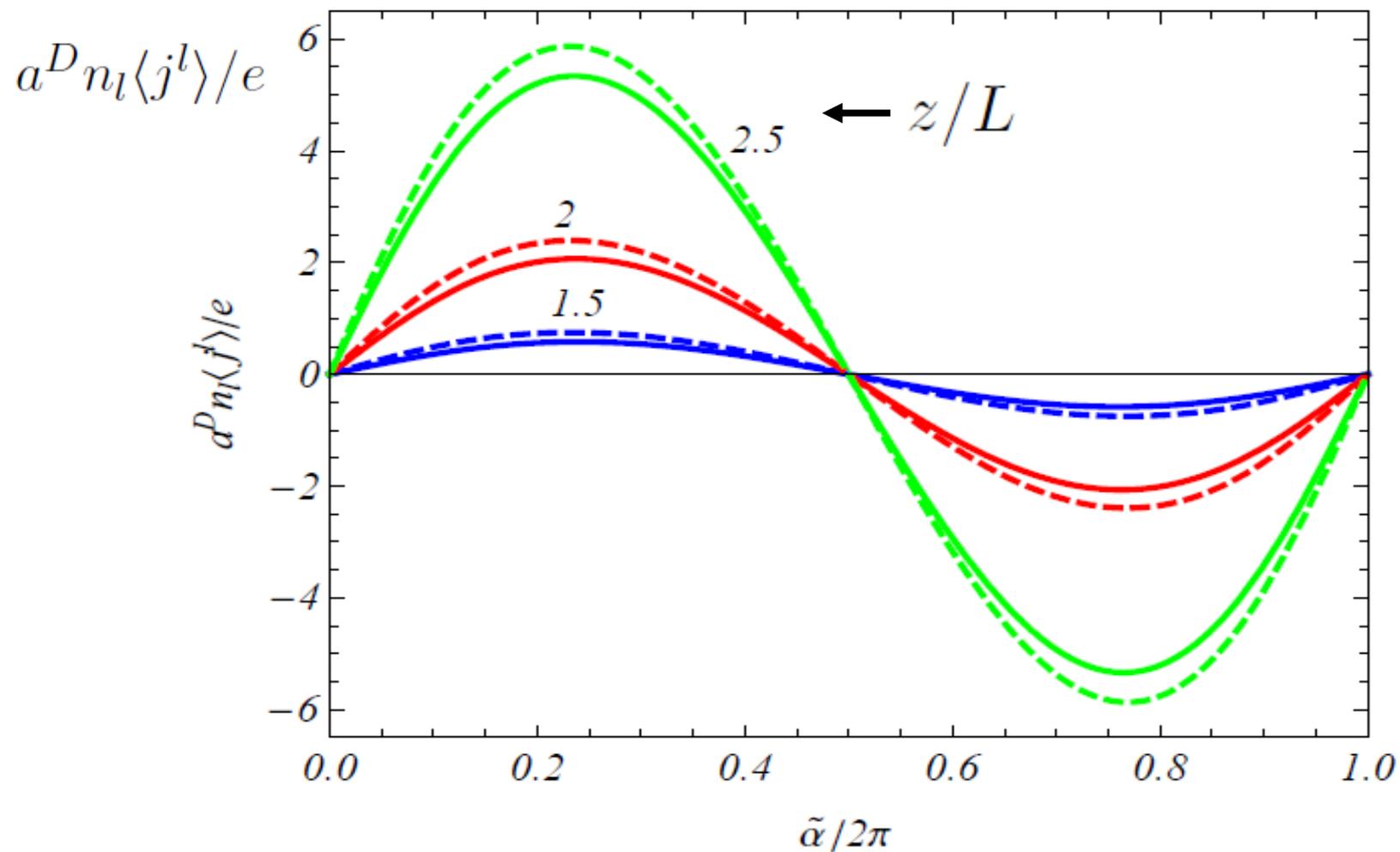
$$\langle j^l \rangle_M = 2eL_l \frac{\Gamma((D+1)/2)}{\pi^{(D+1)/2}} \sum_{n_l=1}^{\infty} n_l \sin(\tilde{\alpha}_l n_l) \sum_{\mathbf{n}_{q-1}} \frac{\cos(\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1})}{g_{\mathbf{n}_q}^{D+1}}$$



Current density in Minkowski spacetime for a massless scalar field

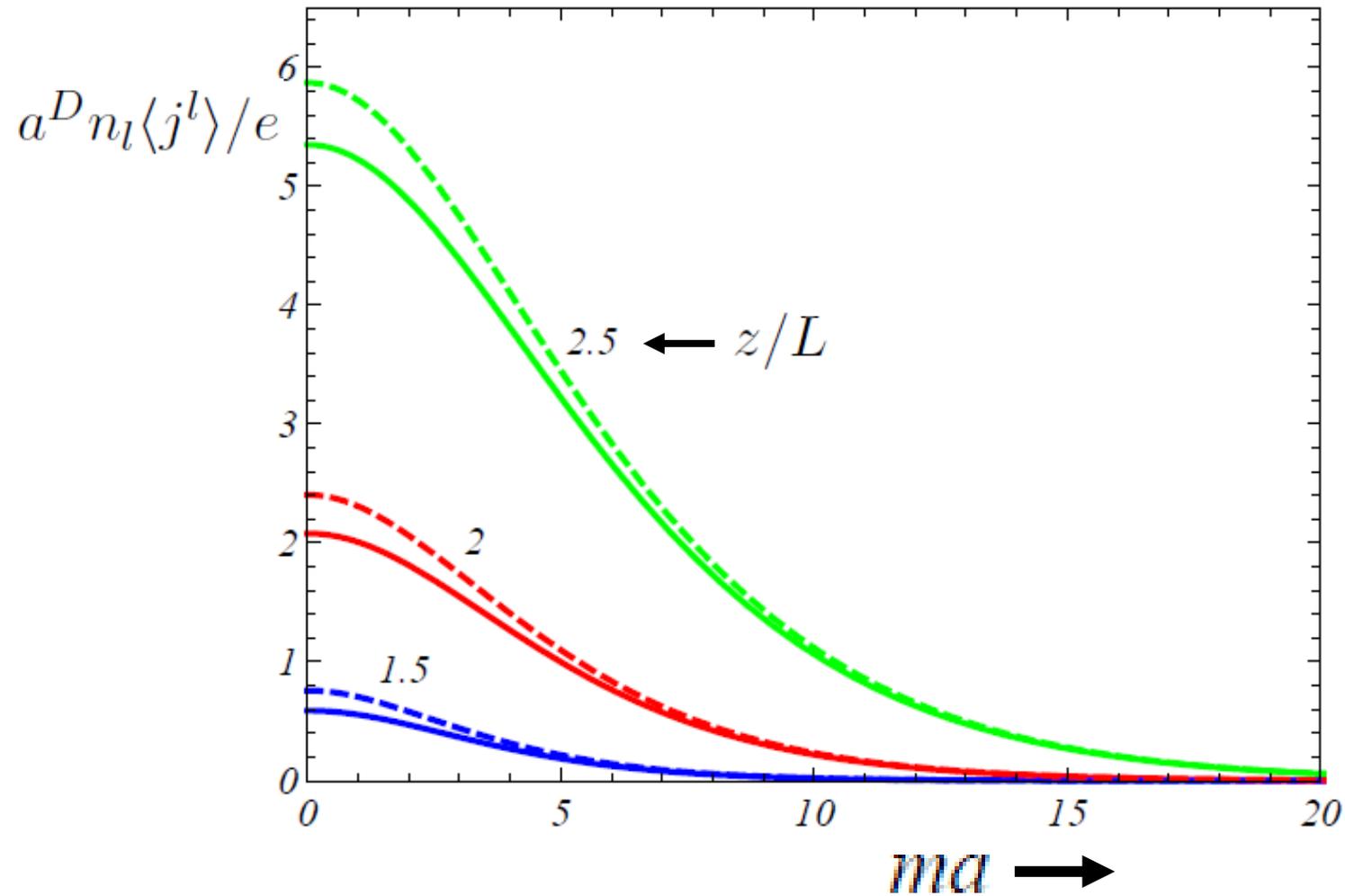
# Numerical example

- D=4 **minimally** (full curves) and **conformally** (dashed curves) coupled fields
- Single compact dimension

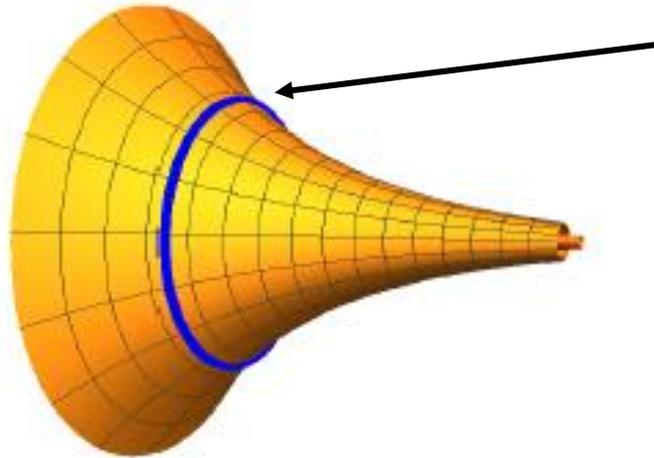


# Numerical example

$$\tilde{\alpha} = \pi/2$$



# Geometry with a brane



Brane at  $y = y_0$

■ Boundary condition on the brane

$$(1 + \beta n^\mu D_\mu) \varphi(x) = 0, \quad y = y_0$$

Constant      Normal to the brane

■ Robin boundary condition

■ Special cases: Dirichlet ( $\beta = 0$ ) and Neumann ( $\beta = \infty$ )

■ There is a region in the space of the parameter  $\beta$  in which the vacuum becomes unstable

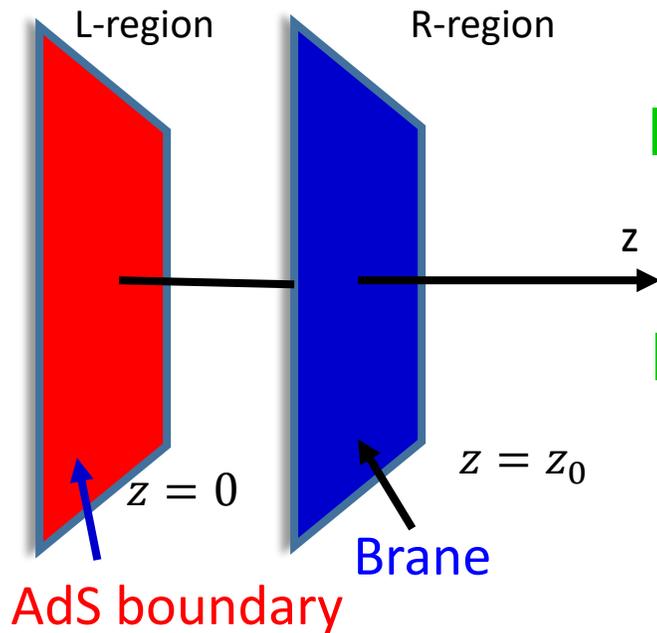
■ Critical value for the Robin coefficient depends on the lengths of the compact dimensions, on the phases in periodicity conditions and on the mass of the field

# Geometry with a brane

- Properties of the vacuum are different in L- and R-regions

L-region  $\rightarrow$  Region between the brane and AdS boundary

R-region  $\rightarrow$  Region between the brane and horizon



- For both L- and R-regions the Hadamard function is decomposed into pure AdS and brane-induced contributions

- Current density along the  $l$ -th compact dimension

$$\langle j^l \rangle = \langle j^l \rangle_0 + \langle j^l \rangle_b, \quad l = p + 1, \dots, D - 1,$$

Pure AdS (in the absence of the brane)

Brane-induced

# Brane-induced current density

## ■ Brane-induced current density in the R-region

$$\langle j^l \rangle_b = -\frac{eC_p z^{D+2}}{2^{p-1} a^{D+1} V_q} \sum_{\mathbf{n}_q} k_l \int_{k_{(q)}}^{\infty} dx x (x^2 - k_{(q)}^2)^{\frac{p-1}{2}} \frac{\bar{I}_\nu(z_0 x)}{\bar{K}_\nu(z_0 x)} K_\nu^2(zx),$$

$I_\nu, K_\nu$  modified Bessel functions  $C_p = \frac{\pi^{-(p+1)/2}}{\Gamma((p+1)/2)}$

$$k_{(q)}^2 = \sum_{l=p+1}^{D-1} k_l^2 = \sum_{l=p+1}^{D-1} (2\pi n_l + \tilde{\alpha}_l)^2 / L_l^2,$$

Barred notation for a given function  $F(x)$

$$\bar{F}(x) = A_0 F(x) + B_0 x F'(x), \quad A_0 = 1 + \frac{D}{2} \delta_y \beta / a, \quad B_0 = \delta_y \beta / a$$

$\delta_y = 1$  in the R-region and  $\delta_y = -1$  in the L-region

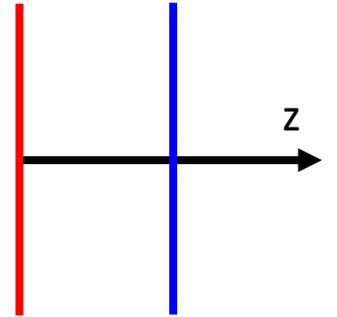
## ■ Brane-induced current density in the L-region by the replacements $I_\nu \rightarrow K_\nu, K_\nu \rightarrow I_\nu$

# Asymptotics of the brane-induced current density in R-region

- At large distances from the brane compared with the AdS curvature radius  $z \gg z_0$

$$\langle j^l \rangle_b \approx -\frac{e z^{D-(p-1)/2} \tilde{\alpha}_l k_{(q)}^{(0)(p-1)/2}}{2^{p+1} \pi^{(p-1)/2} a^{D+1} V_q L_l} \frac{\bar{I}_\nu(z_0 k_{(q)}^{(0)})}{\bar{K}_\nu(z_0 k_{(q)}^{(0)})} e^{-2z k_{(q)}^{(0)}} \quad k_{(q)}^{(0)2} = \sum_{i=p+1}^{D-1} \tilde{\alpha}_i^2 / L_i^2$$

Near the horizon the boundary-free part dominates



- When the location of the brane tends to the AdS boundary,  $z_0 \rightarrow 0$ , the VEV vanishes as  $z_0^{2\nu}$
- An important result is that the VEV of the current density is **finite on the brane**
- For **Dirichlet** boundary condition both the current density and its normal derivative **vanish on the brane**

# Finiteness of the current density on the brane

- Finiteness of the current density is in clear contrast to the behavior of the VEVs for the field squared and the energy-momentum tensor which suffer surface divergences
- Feature that the VEV of the current density is finite on the brane could be argued on the base of general arguments
- In quantum field theory the ultraviolet divergences in the VEVs of physical observables bilinear in the field are determined by the local geometrical characteristics of the bulk and boundary
- On the background of standard AdS geometry with non-compact dimensions the VEV of the current density in the problem under consideration vanishes by the symmetry
- Compactification of the part of spatial dimensions to torus does not change the local bulk and boundary geometries and does not add new divergences compared with the case of trivial topology

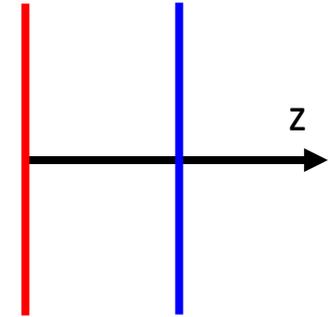
# Asymptotics of the brane-induced current density in L-region

■ On the **AdS boundary** the brane-induced contribution **vanishes** as  $z^{D+2\nu+2}$  ,  $z \rightarrow 0$

■ Near the **AdS boundary** the boundary-free part in the VEV of the current density behaves in a similar way



On the AdS boundary the ratio of the brane-induced and boundary-free contributions tend to a **finite limiting value**



■ For a fixed value of  $z$ , when the brane location tends to the **AdS horizon**, the brane-induced contribution is **exponentially suppressed**

$$\langle j^l \rangle_b \approx \frac{(1 - 2\delta_0 B_0) e \tilde{\alpha}_l z^{D+2} e^{-2z_0 k_{(q)}^{(0)}}}{2^p \pi^{(p-1)/2} a^{D+1} V_q L_l z_0^{(p+1)/2}} k_{(q)}^{(0)(p+1)/2} I_\nu^2(z k_{(q)}^{(0)}) , z_0 \rightarrow \infty$$

# Applications to Randall-Sundrum 1-brane model

- From the results for the R-region one can obtain the current density in  $Z_2$ - symmetric braneworld models of the **Randall-Sundrum** type with a **single brane**
- In the original RS 1-brane model the universe is realized as a  $Z_2$ -symmetric **positive tension brane** in **5D AdS** and the only contribution to the curvature comes from the negative cosmological constant in the bulk
- Most scenarios motivated from string theories predict the presence of other **bulk fields**, such as scalar fields
- In addition, string theories also predict **small compact dimensions** originating from 10D string backgrounds

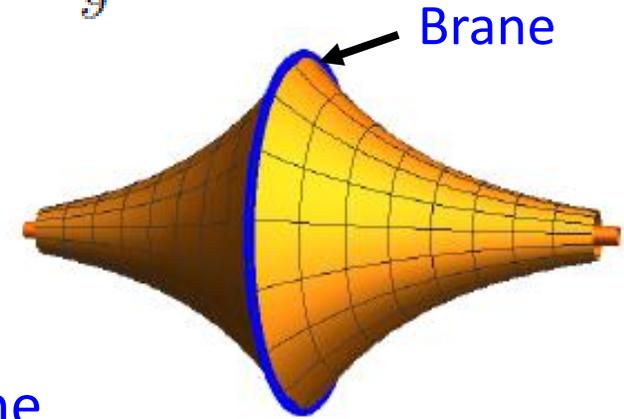
# Generalized RS 1-brane model with compact dimensions

- Background geometry:  $ds^2 = e^{-2|y-y_0|/a} \eta_{ik} dx^i dx^k - dy^2$

Topology  $R^p \times T^q$

- Background geometry contains **two patches**  $y > y_0$  of the AdS glued by the brane and related by the  $Z_2$ -symmetry identification  $y - y_0 \longleftrightarrow y_0 - y$

- Spatial geometry in the case  $D = 2$ , embedded into a 3D Euclidean space



- For fields even under the reflection with respect to the brane (**untwisted scalar field**) the boundary condition is of the **Robin** type with

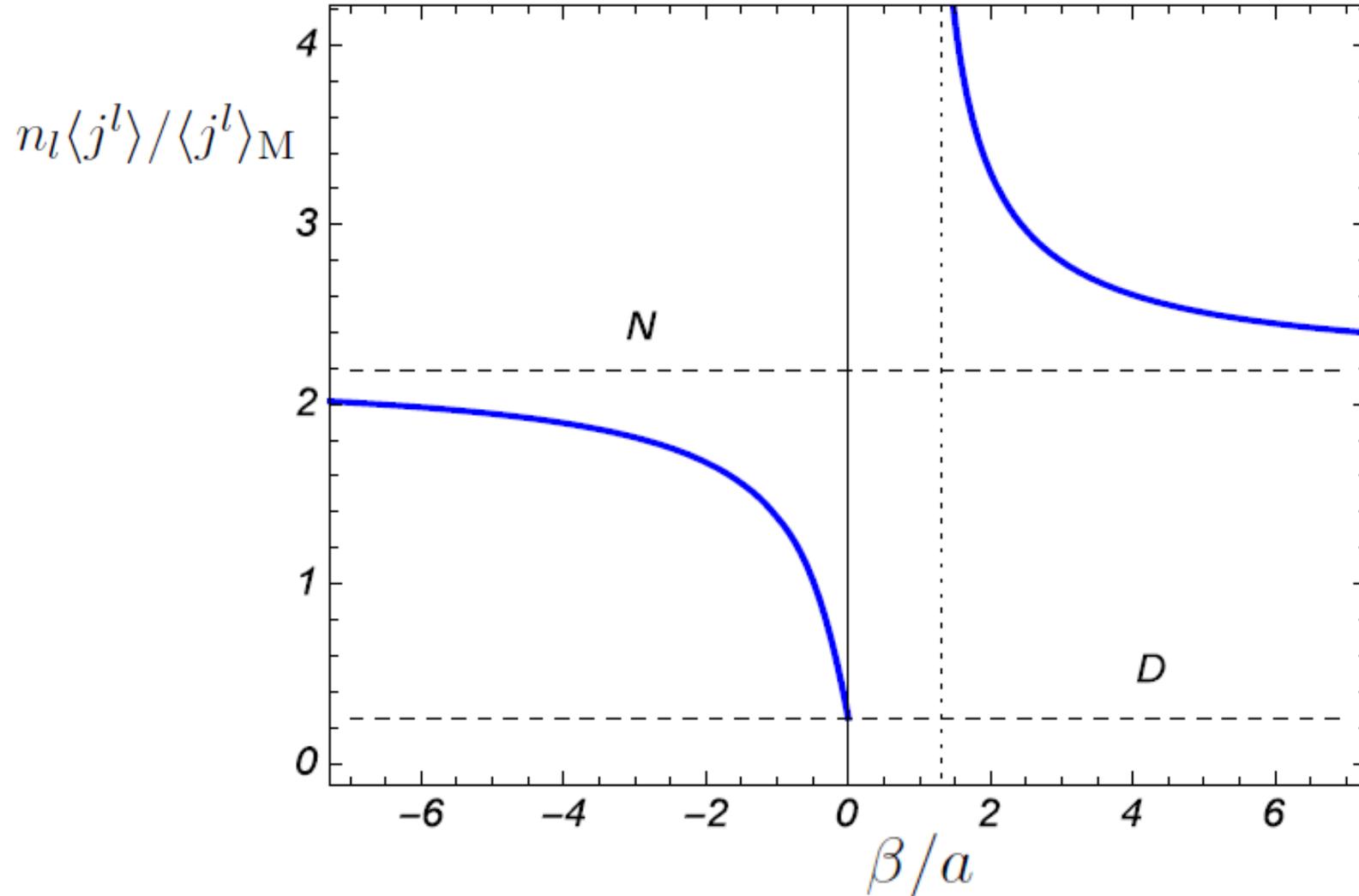
$$\beta = -1/(c + 2D\xi/a) , \text{ with } c \text{ being the } \text{brane mass term}$$

- For fields odd with respect to the reflection (**twisted fields**) the boundary condition is reduced to the **Dirichlet** one

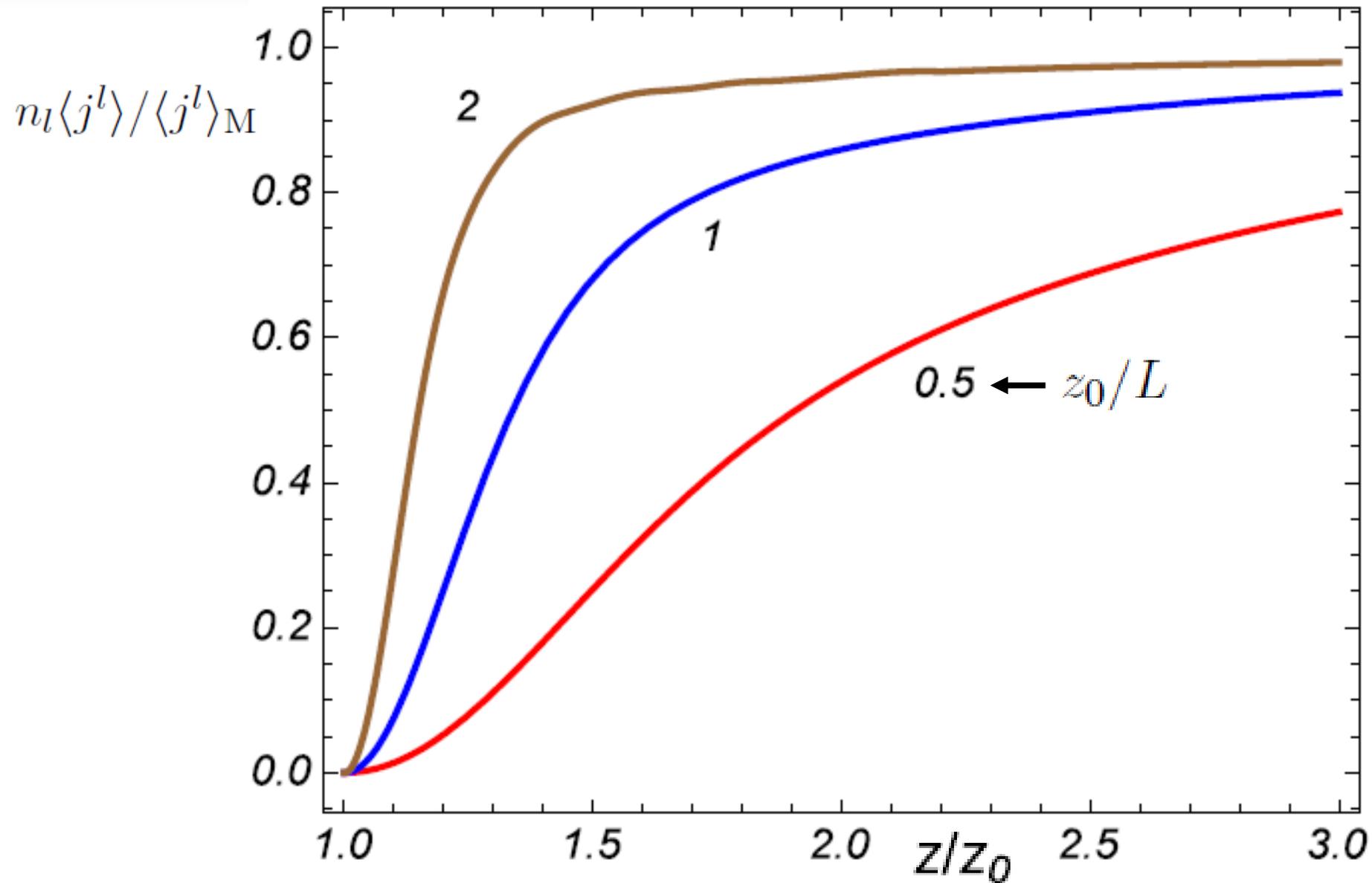
# Vacuum current as a function of the Robin coefficient

$D = 4$  AdS space with a single compact dimension

$$\tilde{\alpha} = \pi/2, z/z_0 = 1.2, z_0/L = 1$$

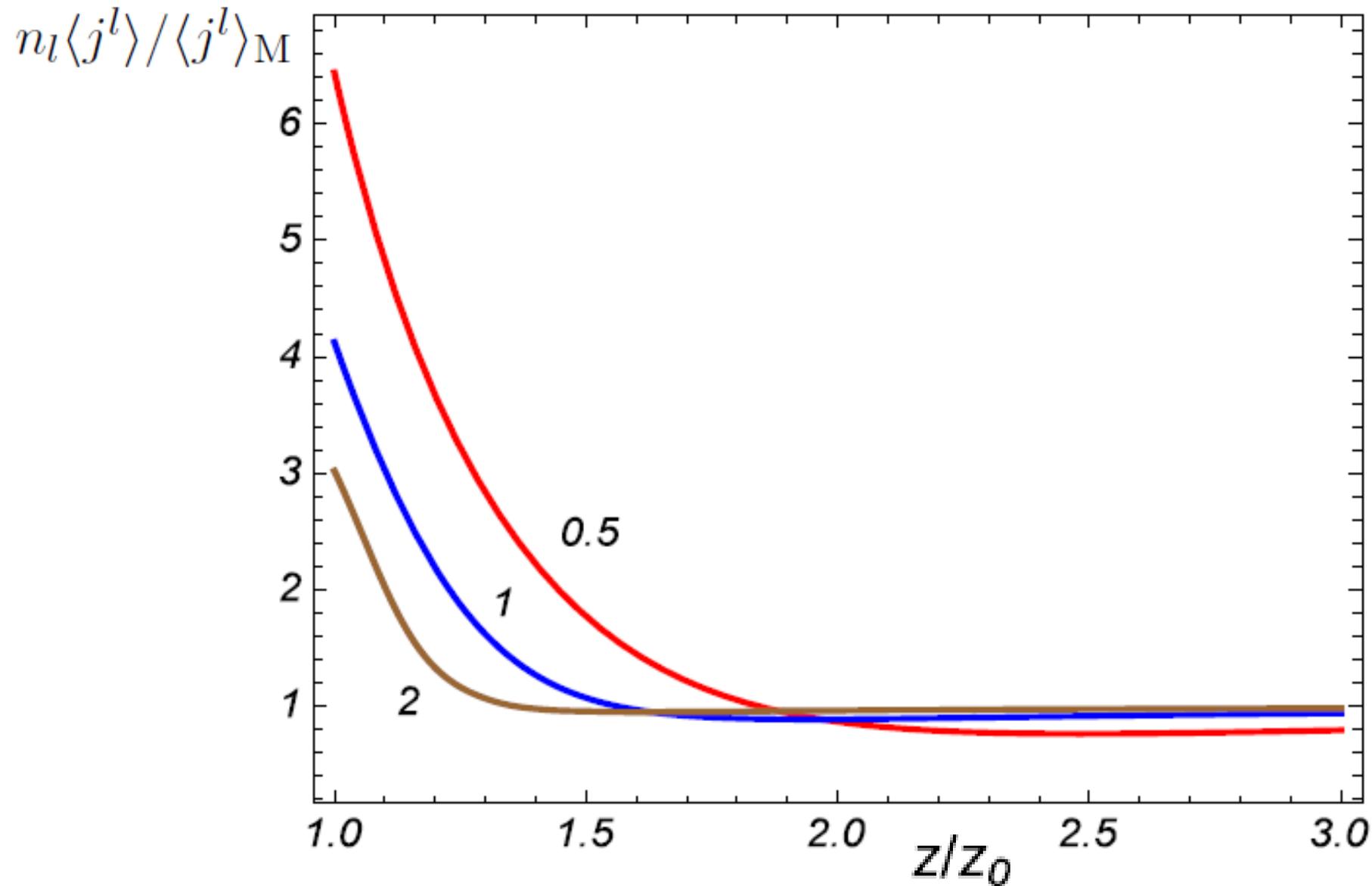


# Vacuum current: Dirichlet BC



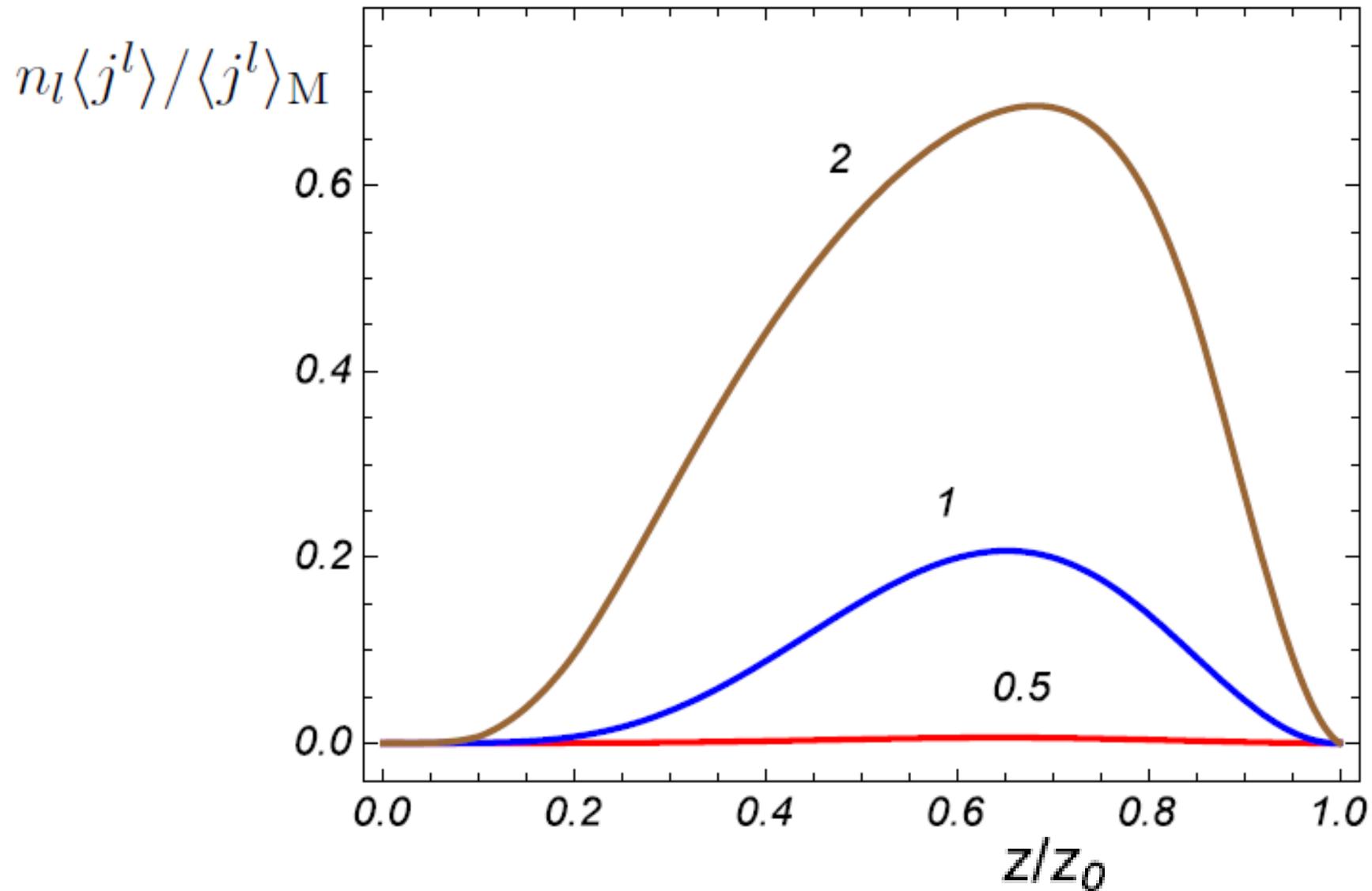
$$\tilde{\alpha} = \pi/2$$

# Vacuum current: Neumann BC



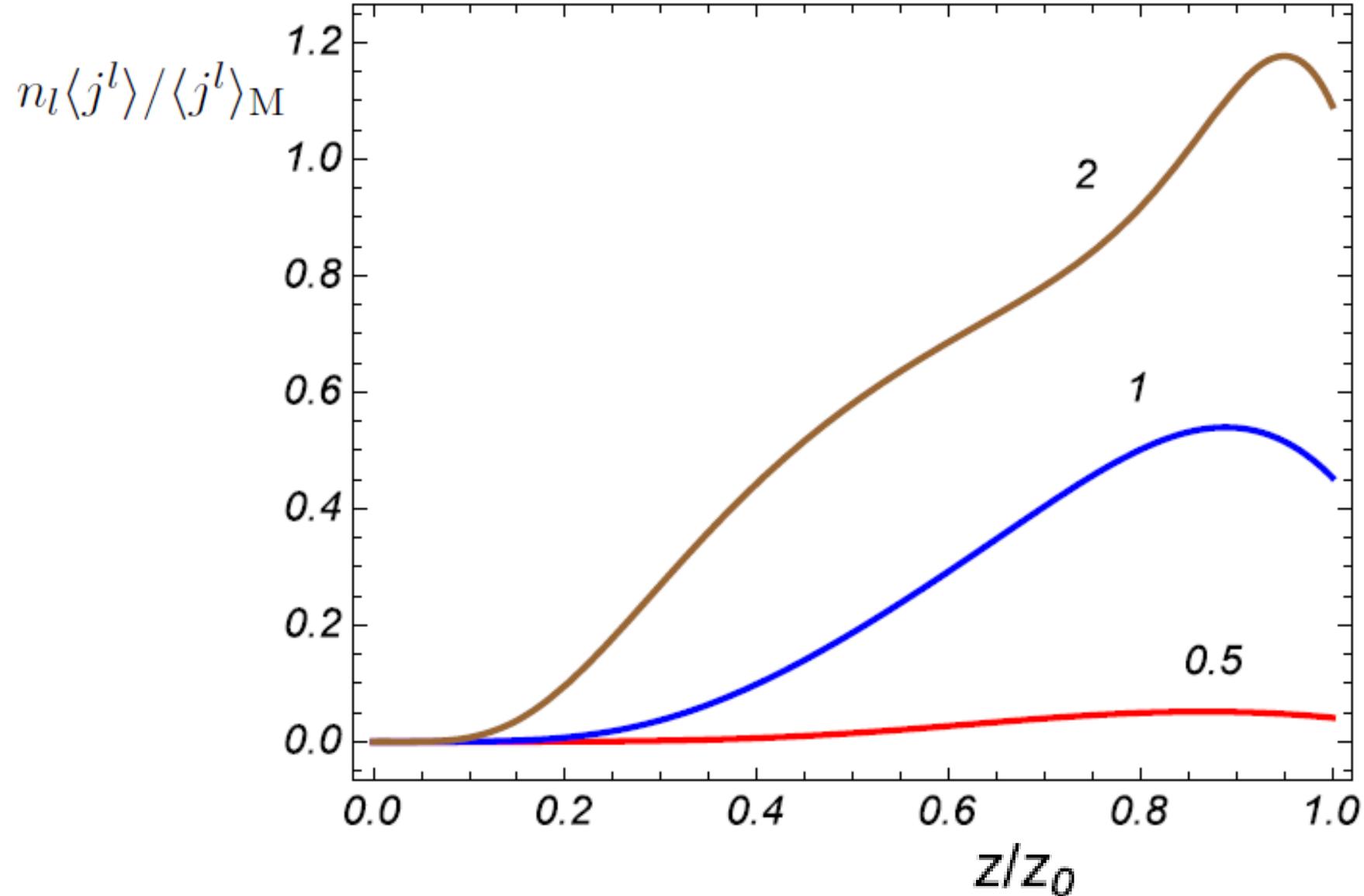
# Vacuum current: Dirichlet BC

Region between the AdS boundary and the brane



# Vacuum current: Neumann BC

Region between the AdS boundary and the brane

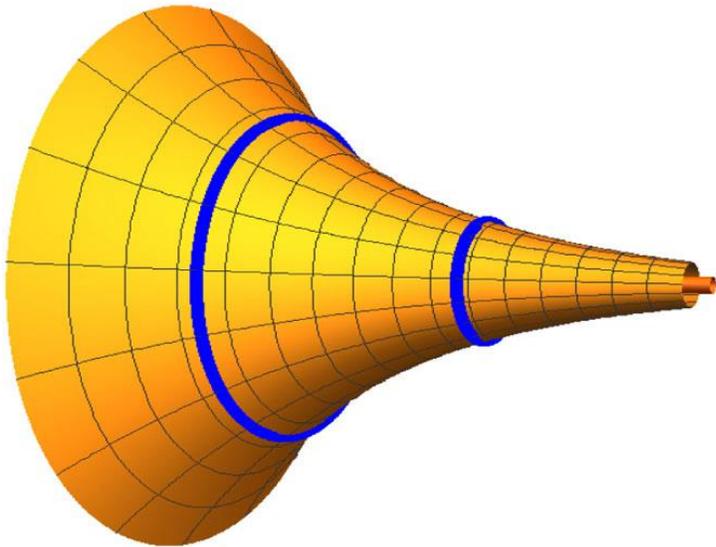


# Conclusions

- VEV of the **current density** for a massive scalar field is investigated in the background of AdS spacetime with spatial topology  $R^p \times (S^1)^q$
- Charge density and the components along the uncompactified dimensions vanish
- Current density along compactified dimensions is a **periodic function of the magnetic flux** with the period of the flux quantum
- Current density vanishes on the **AdS boundary**
- Near the **horizon** the effects induced by the background curvature are small
- In **Kaluza-Klein-type** models the current with the components along compact dimensions is a source of **cosmological magnetic fields**

# Models with two parallel branes

- Vacuum currents in models with two parallel branes on AdS bulk are investigated in *S. Bellucci, A. A. Saharian, and V. Vardanyan, PRD 93, 084011 (2016)*



- Boundary conditions

$$(1 + \beta_j n_j^\mu D_\mu) \varphi(x) = 0, \quad y = y_j,$$

- Current densities in generalized Randall-Sundrum braneworld models with extra compact dimensions

# Vacuum currents for fermion fields

■ Fermionic vacuum currents on AdS bulk with toroidally compactified spatial dimensions are investigated in *S. Bellucci, A. A. Saharian, and V. Vardanyan, PRD 96, 065025 (2017)*

■ Field equation:  $i\gamma^\mu D_\mu \psi - m\psi = 0$ ,  $D_\mu = \partial_\mu + \Gamma_\mu + ieA_\mu$

■ Periodicity conditions:  $\psi(t, x^1, \dots, x^l + L_l, \dots, x^D) = e^{i\alpha_l} \psi(t, x^1, \dots, x^l, \dots, x^D)$

■ In the **Minkowski bulk**, for the system of a fermionic field and  $N/2$  charged scalar fields, with the same masses, charges and the same phases in the quasiperiodicity conditions, the **total current vanishes** as a consequence of the cancellation between the fermionic and scalar contributions

*In the corresponding supersymmetric models no net current appears*

■ In the AdS bulk the influence of the gravitational field on the VEVs for scalar and fermionic fields, in general, is different and there is **no cancellation** between the scalar and fermionic counterparts

Thank you