Stochastic collapse

Based on arXiv:1910.10000

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Introduction

Three unsolved problems

- What is the origin of dark energy?
- What is the origin of dark matter?
- Did Inflation really occur?

Inflation solves some cosmological problems

- Horizon problem: small causally connected region inflates to large region containing our Universe;
- Flatness problem: $K/a^2 \to$ small, $\Omega \to 1$;

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BONUS

Density perturbations that give rise to large scale structure are generated by inflation.

Basic idea

From Einstein equations, the scale factor satisfies

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3w)\rho \quad \text{where} \quad w = \frac{P}{\rho} \ . \tag{1}$$

If $w < -1/3$, we have an accelerated expansion

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The amount of inflation is quantified by the number of e-folds

$$N = \log\left(\frac{a}{a_i}\right) \ . \tag{3}$$

We require $N \gtrsim 60$ to solve flatness and horizon problems!

Rolling models of inflation

- Equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0$$

- Flat region
  - $V(\phi)$ almost constant
  - $\rho_{\rm vac}$ dominates energy density
  - $a \approx a_i e^{Ht}$
- Decay of ϕ
  - Particle production
  - Reheating

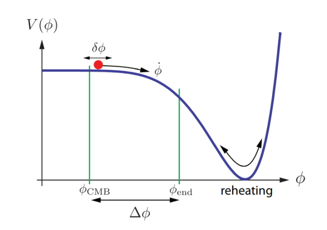

Figure 1: Example of an inflaton potential. **TASI Lectures on Inflation** [arXiv:0907.5424].

Inflationary models and Observation

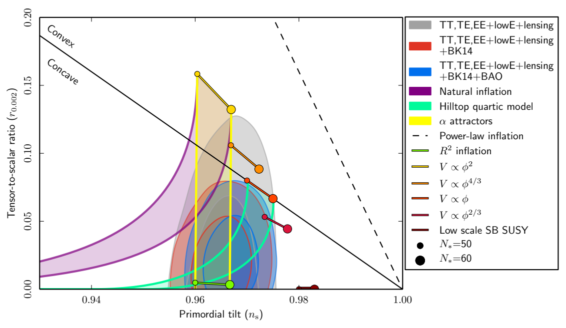

Figure 2: Marginalized joint 68% and 95% CL regions for n_s and r at $k = 0.002\mathrm{Mpc}^{-1}$ from Planck alone and in combination with BK14 or BK14 plus BAO data, compared to the theoretical predictions of selected inflationary models. **Planck 2018 results. X. Constraints on inflation** [arXiv:1807.06211].

Inflation

- slowly-rolling, self-interacting scalar field $\implies$ initial spectrum primordial perturbations due to vacuum fluctuations.
- Accelerated expansion $\implies$ small scales vacuum fluctuations swept up to large scales.
- Slowly varying expansion rate $\implies$ almost scale invariant spectrum.

To summarise

Inflation has some conceptual problems. Among them, we highlight

- The singularity problem, since even if inflation can be achieved by a scalar field coupled to Einstein gravity, this inflationary universe is past incomplete (**Hawking** and **Penrose**, 1970; **Borde** and **Vilekin**, 1994);
- The difficulty of constructing sufficiently flat potentials for inflation (or quasi de Sitter) solutions in string theory or supergravity (**Obied** et al., 2018, **Agrawal** et al., 2018);
- The trans-Planckian problem for fluctuations, if inflation lasts longer than the minimal amount of time necessary to solve the initial condition problems of Cosmology, then the wavelengths of cosmological scales originate at sub-Planckian values (**Martin** and **Brandenberger**, 2001);

Collapsing models

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Alternative scenarios with primordial perturbations from quantum vacuum fluctuations in a collapse phase preceding the Big Bang

- Non-stiff collapse: $P < \rho$ with $V > 0$ (including scale-invariant collapse);
- Pre-Big Bang collapse: $P = \rho$ with $V = 0$ (blue tilted);
- Ekpyrotic collapse: $P \gg \rho$ with $V < 0$ (ultra-stiff fast-roll collapse);

FRW Collapse

$$L = \sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)\right] \quad \text{and} \quad ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \ .$$

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Scalar field with energy density and pressure

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)\;, \quad P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)\;, \tag{4}$$

Equation of State

$$P = w\rho \; . \tag{5}$$

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We choose $V(\varphi) = V_0 e^{-\kappa\lambda\varphi} \implies$ scaling solution with

$$a \propto |t|^p \quad \text{where} \quad p = \frac{2}{\lambda^2} \quad \text{and} \quad \lambda^2 = 3(1+w)\ . \tag{6}$$

Dynamical System

Considering the dimensionless variables

$$x = \frac{\kappa\dot{\varphi}}{\sqrt{6}H} , \quad y = \frac{\kappa\sqrt{\pm V}}{\sqrt{3}H} , \quad (7)$$

The Friedmann constraint becomes

$$x^2 \pm y^2 = 1 , \quad (8)$$

and the dynamics is described by

$$x' = -3x(1-x^2) \pm \lambda\sqrt{3/2}y^2 , \quad (9)$$

$$y' = xy(3x - \lambda\sqrt{3/2}) , \quad (10)$$

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Critical points

$$(A_\pm)\; x_{A_\pm} = +1, -1 \, , \quad y_A = 0 \, ; \quad (11)$$

$$(B)\; x_B = \frac{\lambda}{\sqrt{6}} \, , \quad y_B = \sqrt{1 - \frac{\lambda^2}{6}} \, ; \quad (12)$$

the solution (B) exists for $\pm(6 - \lambda^2) > 0$.

- $\lambda^2 < 6$: flat positive potential
- $\lambda^2 > 6$: steep negative potential

Linear perturbations around the critical point (B) provide us

$$x' = \left(\frac{1}{p} - 3\right)(x - x_B)\ . \tag{13}$$

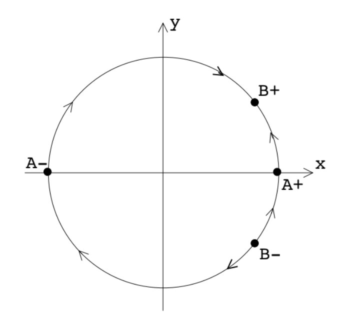

Figure 3: Phase-space for flat positive potentials, $\lambda^2 < 6$. Arrows indicate evolution in cosmic time, t. **Cosmology with positive and negative exponential potentials** [arXiv:0206085v1].

In summary:

- Expanding universe ($N \to +\infty$):
  - The scaling solution exists and is stable for a positive, flat potential $p > 1/3$ (including inflation, $p > 1$).
  - The scaling solution exists but is unstable for a negative, steep potential $p < 1/3$.

$1D$ Phase-space

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- Expanding universe ($N \to +\infty$):
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  - ◇ The scaling solution exists but is unstable for a negative, steep potential $p < 1/3$.
- Contracting universe ($N \to -\infty$):
  - ◇ The scaling solution exists and is stable for a negative steep potential $p < 1/3$ (including ekpyrosis, $p \ll 1$).
  - ◇ The scaling solution exists but is unstable for a positive flat potential $p > 1/3$ (including matter collapse, $p \simeq 2/3$).

Stochastic formalism applied to collapsing models

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By introducing a time-dependent cut-off scale (the so-called coarse-graining scale)

$$k_\sigma = \sigma a H \ , \tag{15}$$

the noises associated to the small wavelength part are described by their two-points correlation matrix $\Xi_{f,g}$, with each entry is given by

$$\Xi_{f,g} = \langle 0|\xi_f \xi_g|0\rangle = \frac{1}{6\pi^2}\frac{dk_\sigma^3}{dN} f_k(N) g_k^\star(N) \ . \tag{16}$$

Field perturbations

During accelerated expansion or collapse

- $|aH|$ increases $\rightarrow$ modes that start on sub-Hubble scales $(k^2 > a^2H^2)$ are stretched up to super-Hubble scales $(k^2 < a^2H^2)$.

Result

Quantum vacuum fluctuations $k^2/a^2 \gg H^2$ at early times[1] $\rightarrow$ well-defined predictions for the power spectrum of perturbations on super-Hubble scales in an expanding cosmology, or in a collapsing cosmology.

[1] which means $\delta\varphi \simeq \frac{e^{-ikt/a}}{a\sqrt{2k}}$ for $k^2/a^2 \gg H^2$.

Field perturbations

The characteristics of the inflation and collapse models for different values of p are summarised in table 1.

Power-law inflation	Collapse
$H > 0$	$H < 0$
$\dot{a} > 0$, $\ddot{a} > 0$	$\dot{a} < 0$, $\ddot{a} < 0$
$p > 1$	$0 < p < 1$

Table 1: Comparing the quantities H, $\dot{a}$, $\ddot{a}$ and p for power-law inflation and collapse. Although $\dot{a}$ is negative in the collapse case, its magnitude $|\dot{a}|$ is increasing. $p < 0$ is not allowed since this requires $\rho_\varphi + P_\varphi < 0$.

Power-law collapse

For a power-law cosmology

$$a \propto |t|^p \,, \tag{17}$$

the Mukhanov-Sasaki equation becomes

$$\frac{d^2 v}{d\eta^2} + \left(k^2 - \frac{\nu^2 - 1/4}{\eta^2}\right) v = 0 \,, \quad \text{where} \quad \nu = \frac{3}{2} + \frac{1}{p-1} \,. \tag{18}$$

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The growing mode solution of quantum fluctuations on large scales (late times) for $k\eta \to 0$

$$\delta\varphi_k = \frac{i}{a}\sqrt{\frac{1}{4\pi k}} \frac{\Gamma(|\nu|)2^{|\nu|}}{|k\eta|^{|\nu|-1/2}} \ . \tag{19}$$

Power-law collapse

It gives a spectrum of field perturbations on super-Hubble scales as $\eta \to 0$

$$\mathcal{P}_{\delta\varphi} = \left[\frac{\Gamma(|\nu|) 2^{|\nu|}}{(\nu - 1/2) 2^{3/2} \Gamma(3/2)} \right]^2 \left(\frac{H}{2\pi} \right)^2 |k\eta|^{3-2|\nu|} \,. \qquad (20)$$

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Thus a power-law collapse gives rise to a power-law spectrum for field fluctuations on super-Hubble scales with spectral tilt

$$\Delta n_{\delta\varphi} = \frac{d \ln \mathcal{P}_{\delta\varphi}}{d \ln k} = 3 - 2|\nu| \, . \tag{21}$$

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$\Delta n_{\delta\varphi} = 0$ for

- Slow-roll inflation ($w = -1$ and $\nu = 3/2$);
- Pressureless collapse ($w = 0$ and $\nu = -3/2$);

Perturbed equation of state w

By perturbing the dimensionless variable x

$$\delta x = \frac{\kappa}{\sqrt{6}} \frac{1}{H} \left(\delta\dot{\varphi} - A\dot{\varphi} - \frac{\dot{\varphi}}{H} \delta H \right) \, , \tag{22}$$

we obtain the correlation matrix of the noise at the critical point (B)

$$\Xi_{x,x}(N) = \frac{\Gamma^2(|\nu|)\nu^2 2^{2|\nu|+4}}{(12\pi)^3 \sigma^{2|\nu|-3}} \left(\frac{2}{2\nu - 1} \right)^{2|\nu|+4} (|\nu| - \nu)^2 \kappa^2 H^2(N) \, . \tag{23}$$

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The noise term always vanishes for $\nu > 0$, hence the scalar field perturbations at leading-order on large scales correspond to **adiabatic** perturbations. This includes the power-law inflation ($\nu = 3/2$) and the ekpyrotic ($\nu = 1/2$) scenarios.

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Solutions for the quantum noise

The stochastic version around the classical critical point $x = x_B$ is

$$\bar{x}' = m(\bar{x} - x_B) + \hat{\xi}_x \quad \text{with} \quad m = \frac{\lambda^2 - 6}{2} \,, \tag{24}$$

and its variance is

$$\begin{aligned} \sigma_x^2(N) = \left\langle (\bar{x}(N) - x_B)^2 \right\rangle &= \sigma_{x,cl}^2(N) + \sigma_{x,qu}^2(N) \\ &= \sigma_x^2(N_\star) e^{2m(N-N_\star)} + \int_{N_\star}^{N} dS \; e^{2m(N-S)} \Xi_{x,x}(S) \end{aligned} \tag{25}$$

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Growth rate of classical and quantum perturbations

The quantum variance decays with time if we have

$$\frac{3 + 2\nu}{\nu - 1/2} > 0 \, . \tag{26}$$

This is the case if either $\nu > 1/2$ or $\nu < -3/2$.

Solutions for the quantum noise

- Classical perturbations grow faster than the quantum noise if $\nu < -3/2$, and the quantum noise grows faster if $\nu > -3/2$;

Also, the condition (26) provides a shift in the spectrum (**Zeldovich** and **Novikov**, 1983; **Wands**, 1999; **Finelli** and **Brandenberger** 2002)

$$n_{\mathrm{s}} = 1 + \frac{12w}{1+3w}\ , \tag{27}$$

and w is related to ν.

Solutions for the quantum noise

When $\nu = -3/2 - \epsilon$, where ϵ is a small positive parameter, $w < 0$ and the spectrum becomes redder.

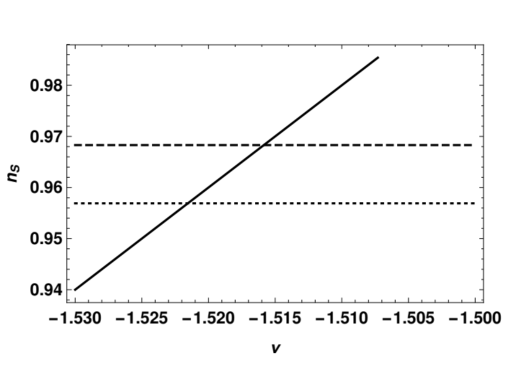

Figure 4: Evolution for $n_{\rm s}$ as function of ν. The horizontal dotted lines enclose the 68% confidence level of the values of $n_{\rm s}$ measured by Planck collaboration 2018.

Maximum lifetime of the collapse phase at the fixed point

If $\sigma^2_{x,qu} = 1 \implies$ does quantum noise change the dynamics?

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Pressureless collapse ($\nu = -3/2$)

$$|H(N)| \approx \sqrt{\frac{134}{N_\star - N}} M_{pl} \,. \tag{28}$$

Drives away from fixed point before the Planck scale if $(N_\star - N) > 134$.

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Radiation-dominated collapse ($\nu = -1/2$)

$$|H(N)| \approx \frac{13}{\sigma} M_{pl} \; . \tag{29}$$

Cannot escape fixed point since $\sigma < 1$.

Maximum lifetime of the collapse phase at the fixed point

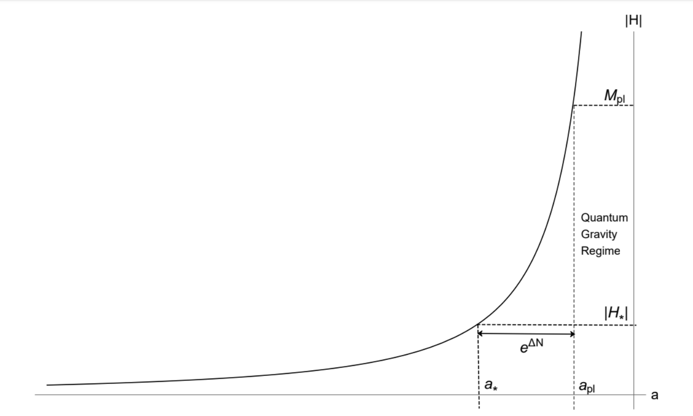

Figure 5: Evolution of the Hubble rate. To get a sensible deviation from the fixed point we start in, the initial scale must be set at low energy.

Final considerations

Summary

Inflation / Ekpyrotic collapse ($\nu > 0$)	Pressureless collapse ($\nu < 0$)
$\delta x = 0$ (adiabatic perturbation)	$\delta x \neq 0$ (non-adiabatic perturbation)

- Inflation and Ekpyrotic collapse are both classical and quantum stable;
- Pressureless collapse is quantum unstable;
- For $\nu = -3/2$, we found the quantum diffusion takes us away from the critical point if we start the collapse from very low energy scales and if it lasts more than 134 e-folds.

To do

Effects of gauge corrections in a collapsing universe

The gauge issue by using the stochastic formalism beyond the usual slow-roll approximation has been discussed by **Pattison et al.** [arXiv:1905.06300] and we intend to extend this idea to a collapsing universe.

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The possibility of a bounce

Since in this work we have used the number of e-folds, which is a monotonic time parameter, our analysis is limited to a collapsing universe. Hence, to modelise a bounce, we need to change our time variable. In doing so, we would be able to introduce stochastic fluctuations of the geometry leading to a bounce.

International Emerging Action

- CNRS funded

- Goal: initiate and strengthen collaborative work between two groups of researchers, one in France and one abroad

Paris ⟷ **Portsmouth**

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Obrigada!