Cosmology with matter diffusion

based on S. Calogero and H. Velten *arXiv:1308.3393, to be submitted to JCAP*

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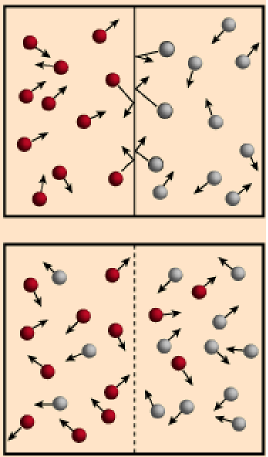
Introducing velocity diffusion

Diffusion is a very known physical process!

- Transported quantities:
- Momentum ⟶ Newton's law of viscosity
- Energy ⟶ Fourier's Law of thermal conduction
- Mass ⟶ Osmosis, Fick's law



- The purpose here is to propose and explore the idea that diffusion may also play a fundamental role in the large scale dynamics of the matter in the universe.

- **The general relativistic theory of diffusion processes is not a standard topic treated in text books**
- **It has received considerable attention only in recent years**
- J. Franchi, Y. Le Jan. Relativistic Diffusions and Schwarzschild Geometry. Comm. Pure Appl. Math., 60 : 187251, 2007;
- Z. Haba. Relativistic diffusion with friction on a pseudoriemannian manifold. Class. Quant. Grav., 27 : 095021, 2010;
- J. Hermann. Diffusion in the general theory of relativity. Phys. Rev. D, 82: 024026, 2010;
- S. Calogero. A kinetic theory of diffusion in general relativity with cosmological scalar field. J. Cosmo. Atrop. Phys. 11 2011, 016.

Kinetic diffusion on flat spacetimes

The kinetic evolution of particles is governed by a partial differential equation on the distribution function f. The particular form of this equation depends on the interaction among the particles. The non-relativistic free-transport equation reads:

$$\partial_t f + p.\nabla_x f = 0 \tag{1}$$

If the particles are interacting either by internal or external forces, a new term has to be added to the right hand side.
For example,

$$\partial_t f + p.\nabla_x f = \sigma \Delta_p f \tag{2}$$

is the kinetic Fokker-Planck equation. Here $\sigma > 0$ is the diffusion constant and Δ_p denotes the Laplace operator.

Kinetic diffusion on flat spacetimes

The relativistic, Lorentz invariant generalization of (2) is

$$p^{\mu}\partial_{x^{\mu}} f = \sigma \Delta_p f \tag{3}$$

The relativistic current density vector and energy-momentum tensor are given by

$$J^{\mu}(t,x) = \int_{R^3} f(t,x,p) p^{\mu} \frac{dp}{p^0} \tag{4}$$

$$T^{\mu\nu}(t,x) = \int_{R^3} f(t,x,p) p^{\mu} p^{\nu} \frac{dp}{p^0} \tag{5}$$

independently of the equation satisfied by the distribution function f.

It is possible to show that (S. Calogero. A kinetic theory of diffusion in general relativity with cosmological scalar field. J. Cosmo. Atrop. Phys. 11/2011, 016).

$$\partial_{x^{\mu}} J^{\mu} = 0, \qquad \partial_{x^{\mu}} T^{\mu\nu} = 3\sigma J^{\nu} \tag{6}$$

Kinetic diffusion on curved spacetimes

The generalization of (3) on the curved spacetime is

$$p^\mu \partial_{x^\mu} f - \Gamma^i_{\mu\nu} p^\mu p^\nu \partial_{p^i} f = \sigma D_p f \tag{7}$$

from which it is also possible to find analogue identities

$$\nabla_\mu J^\mu = 0, \qquad \nabla_\mu T^{\mu\nu} = 3\sigma J^\nu . \tag{8}$$

Now, how to accomodate $\nabla_\mu T^{\mu\nu} \neq 0$ with Einstein equations?

Coupling with Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu} \qquad (9)$$

Bianchi Identitities $\rightarrow \nabla^{\mu} G_{\mu\nu} = 0 \rightarrow \nabla^{\mu} T_{\mu\nu} = 0$.
Let's add an additional cosmological field (ϕ).

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \phi g_{\mu\nu} = T_{\mu\nu} \qquad (10)$$

such that

$$\nabla_{\mu}\phi = 3\sigma J_{\mu} \qquad (11)$$

In the absence of diffusion ϕ is constant! Thus, the field ϕ plays the role of cosmological constant Λ, i.e., DARK ENERGY!!!!!
For $\sigma > 0$ we have a dynamical DE scenario.

A Viable Cosmological Scenario

The matter is said to undergo (velocity) diffusion in a cosmological scalar field ϕ if the following equations hold:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \phi g_{\mu\nu} = T_{\mu\nu}, \tag{12}$$

$$\nabla_\mu T^{\mu\nu} = 3\sigma J^\mu, \tag{13}$$

$$\nabla_\mu J^\mu = 0. \tag{14}$$

The constant $\sigma > 0$ is the diffusion constant. The value 3σ measures the energy transferred from the scalar field to the matter per unit of time due to diffusion. We use units $8\pi G = c = 1$. Taking a covariant divergence of (12), using (13) and the Bianchi identity $\nabla^\mu (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = 0$, we obtain the following evolution equation for ϕ:

$$\nabla_\mu \phi = 3\sigma J_\mu. \tag{15}$$

If the matter undergoing diffusion is a perfect fluid,

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \quad J^\mu = n u^\mu, \tag{16}$$

where ρ is the rest-frame energy density, p the pressure, u^μ the 4-velocity and n the particle number density of the fluid. For a perfect fluid, Eq. (15) reads

$$\nabla_\mu \phi = 3\sigma n u_\mu, \tag{17}$$

while Eq. (14) becomes

$$\nabla_\mu (n u^\mu) = 0. \tag{18a}$$

Projecting (13) into the direction of u^μ and onto the hypersurface orthogonal to u^μ we obtain

$$\nabla_\mu (\rho u^\mu) + p \nabla_\mu u^\mu = \sigma n, \tag{18b}$$

$$(\rho + p) u^\mu \nabla_\mu u^\nu + u^\nu u^\mu \nabla_\mu p + g^{\mu\nu} \nabla_\mu p = 0. \tag{18c}$$

The ϕCDM model

We consider a spacetime with the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad k = 0, \pm 1. \tag{19}$$

Eq. (18b) becomes

$$\dot{\rho} + 3H(\rho + p) = \sigma n_0 a^{-3}, \tag{20}$$

where

$$H = \frac{\dot{a}}{a} \tag{21}$$

is the Hubble function. Eq. (18c) is identically satisfied. The cosmological scalar field equation (17) reduces to

$$\dot{\phi} = -\sigma n_0 a^{-3}. \tag{22}$$

The ϕCDM model

Introducing the dimensionless variables

$$\Omega_m(z) = \frac{\rho(z)}{3H_0^2}, \qquad \Omega_\phi(z) = \frac{\phi(z)}{3H_0^2}, \qquad E(z) = \frac{H(z)}{H_0}$$

and the dimensionless parameters

$$\tilde{\sigma} = \frac{\sigma n_0}{3a_0^3 H_0^3}, \quad K = \frac{k}{H_0^2 a_0^2} \tag{23}$$

we can rewrite (20), (22) as

$$\frac{d\Omega_m(z)}{dz} = \frac{3\gamma\Omega_m(z)}{1+z} - \tilde{\sigma}\frac{(1+z)^2}{E(z)}, \tag{24}$$

$$\frac{d\Omega_\phi(z)}{dz} = \tilde{\sigma}\frac{(1+z)^2}{E(z)}, \tag{25}$$

$$E(z) = \sqrt{\Omega_m(z) + \Omega_\phi(z) - K(1+z)^2}. \tag{26}$$

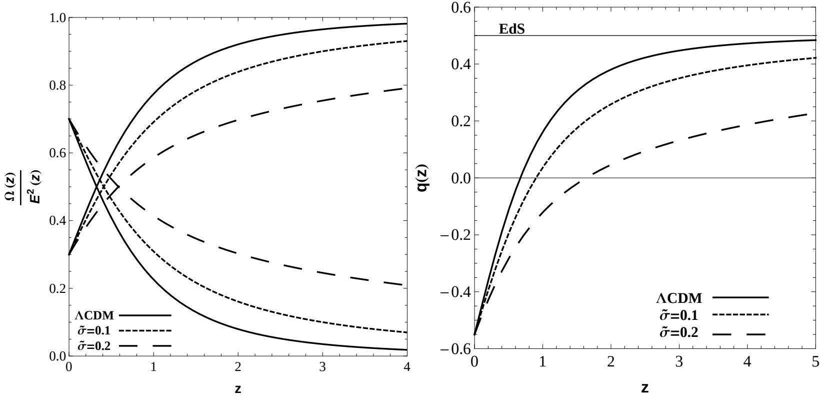


Abbildung: *Left Panel:* Evolution of the fractional density parameters. *Right panel:* Evolution of the deceleration parameter (right). Both assume $\Omega_{m0} = 0.3$. In the right panel, the solid horizontal line at the value $q = 0.5$ describes the Einstein-de-Sitter (EdS) model and solid horizontal line at the value $q = 0$ denotes the transition to the accelerated expansion.

Observational constraints on the magnitude of the cosmic diffusion

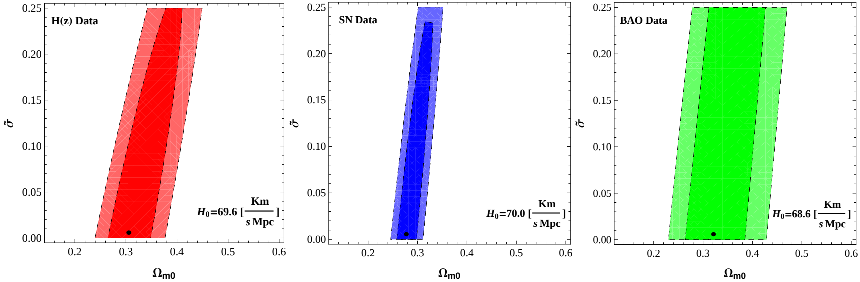


Abbildung: Confidence level contours at 1σ and 2σ for H(z) (Red), Supernovae (Blue), BAO (Green) and data sets.

Cosmological perturbations of spatially flat solutions

Using the conformal Newtonian gauge in the absence of anisotropic stresses, we write the metric in the form

$$g = a(\eta)^2[-(1+2\Psi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j] = \overline{g} + \delta g, \quad (27)$$

where $\Psi = \Psi(\eta, x^1, x^2, x^3)$ is the Newtonian potential,

$$\overline{g} = a(\eta)^2(-d\eta^2 + \delta_{ij}dx^i dx^j)$$

is the spatially flat RW metric in the conformal time η and

$$\delta g = -2a(\eta)^2\Psi(d\eta^2 + \delta_{ij}dx^i dx^j)$$

is the (scalar) metric perturbation.

Next the perturbations of the scalar field and of the fluid variables will be considered. We set

$$\phi = \overline{\phi} + \delta\phi, \quad \rho = \overline{\rho} + \delta\rho, \quad p = \overline{p} + \delta p, \quad n = \overline{n} + \delta n. \qquad (28)$$

As to the four-velocity, we have $u_\mu = \overline{u_\mu} + \delta u_\mu$, where

$$\overline{u_\mu} = -a(t)\delta^0{}_\mu.$$

Since at first order

$$g^{\mu\nu} u_\mu u_\nu = -1 + 2(\Psi + a^{-1}\delta u_0),$$

the requirement that $g^{\mu\nu} u_\mu u_\nu = -1$ hold at the first order entails

$$\delta u_0 = -a\Psi.$$

Moreover it will be shown below that diffusion in a scalar field restricts the velocity perturbations to be of the form $\delta u_i = \partial_{x^i} V$, for some scalar function V. It is convenient to set $V = a(\eta)\theta(\eta, x^1, x^2, x^3)$.

In conclusion

$$u_0 = -a\,(1+\Psi), \quad u_i = a\,\partial_{x^i}\theta \tag{29}$$

and therefore

$$u^0 = a^{-1}(1-\Psi), \quad u^i = a^{-1}\partial_{x^i}\theta. \tag{30}$$

It follows that the Einstein equations (12) at zero order, i.e., $\overline{G^{\mu}_{\ \nu}} + \overline{\phi}\,\delta^{\mu}_{\ \nu} = \overline{T^{\mu}_{\ \nu}}$, read

$$3\frac{\mathcal{H}^2}{a^2} = \overline{\rho} + \overline{\phi}, \quad \mathcal{H}' = -\frac{1}{2}[a^2(\overline{p} - \overline{\phi}) + \mathcal{H}^2], \tag{31}$$

while at first order, i.e., $\delta G^{\mu}_{\nu} = \delta T^{\mu}_{\nu}$, they give

$$\nabla^2\Psi - 3\mathcal{H}(\mathcal{H}\Psi + \Psi') = \frac{1}{2}a^2(\delta\rho + \delta\phi), \tag{32}$$

$$\partial_{x^i}(\mathcal{H}\Psi + \Psi') = \frac{1}{2}a^2(\overline{\rho} + \overline{p})\partial_{x^i}\theta \Rightarrow \mathcal{H}\Psi + \Psi' = \frac{1}{2}a^2(\overline{\rho} + \overline{p})\theta, \tag{33}$$

$$\Psi'' + 3\mathcal{H}\Psi' + (2\mathcal{H}' + \mathcal{H}^2)\Psi = \frac{1}{2}a^2(\delta p - \delta\phi), . \tag{34}$$

Now Eq. (18a) at zero order gives

$$\overline{n}' + 3\mathcal{H}\overline{n} = 0 \Rightarrow \overline{n}(\eta) = \overline{n}_0\, a(\eta)^{-3}, \tag{35}$$

where $\overline{n}_0 = \overline{n}(0)$ and $a(0) = 1$. At first order we obtain

$$\delta n' + 3\mathcal{H}\delta n + \overline{n}\nabla^2\theta - 3\overline{n}\Psi' = 0. \tag{36}$$

The equation (17) for the scalar field gives, at zero order,

$$\overline{\phi}' = -\sigma a\overline{n}, \tag{37}$$

and at first order

$$\delta\phi' = -\sigma a(\delta n + \overline{n}\Psi) \quad \partial_{x^i}\delta\phi = \sigma a\overline{n}\partial_{x^i}\theta \Rightarrow \delta\phi = \sigma a\overline{n}\theta. \tag{38}$$

To close the system we need an equation of state, which we take to be that of a dust fluid. Setting $\overline{p} = \delta p = 0$ and using the second equation in (31) in we obtain

$$\nabla^2\Psi - 3\mathcal{H}(\mathcal{H}\Psi + \Psi') = \frac{a^2}{2}\delta\rho + \frac{1}{2}\sigma\overline{n}_0\theta, \tag{39a}$$

$$\mathcal{H}\Psi + \Psi' = \frac{1}{2}a^2\overline{\rho}\theta, \tag{39b}$$

$$\Psi'' + 3\mathcal{H}\Psi' + a^2\overline{\phi}\,\Psi = -\frac{1}{2}\sigma\overline{n}_0\theta, \tag{39c}$$

where $a, \overline{\rho}, \overline{\phi}, \mathcal{H}$ solve

$$a' = a\mathcal{H}, \quad \mathcal{H}' = \frac{a^2}{2}\overline{\phi} - \frac{\mathcal{H}^2}{2}, \quad \overline{\phi}' = -\frac{\sigma\overline{n}_0}{a^2}, \quad \overline{\rho}' + 3\mathcal{H}\overline{\rho} = \frac{\sigma\overline{n}_0}{a^2}, \tag{40}$$

with initial data $a(0) = 1$, $\mathcal{H}_0 > 0$, $\overline{\rho}_0 > 0$, $\overline{\phi}_0 = 3\mathcal{H}_0^2 - \overline{\rho}_0 > 0$.

Combining the second and the third equation of the system (39) we obtain an equation for Ψ alone:

$$\Psi'' + \left(3\mathcal{H} + \frac{\sigma\overline{n}_0}{a^2\overline{\rho}}\right)\Psi' + \left(a^2\overline{\phi} + \mathcal{H}\frac{\sigma\overline{n}_0}{a^2\overline{\rho}}\right)\Psi = 0. \qquad (41)$$

Evolution of the gravitational potential

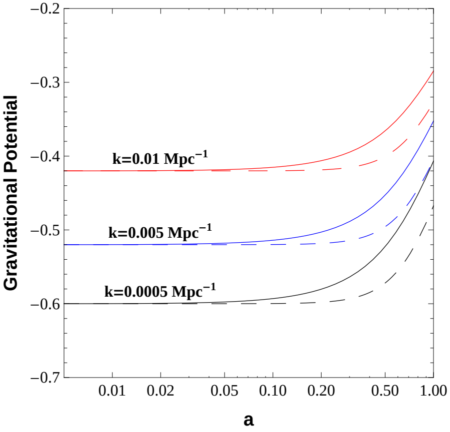


Abbildung: Gravitational potential for three different scales as a function of the scale factor for the ΛCDM (dashed) and the diffusion model with $\tilde{\sigma} = 0.1$ (solid).

The integrated Sachs-Wolfe effect

The temperature anisotropies on the CMB sky are connected to the linear fluctuations of matter via the Sachs-Wolfe effect. While most of such anisotropies were already present at the time of last scattering ($z \sim 1100$), a relevant part of them has been produced thereafter due to the fact that photons traveled through time varying gravitational potential wells. The latter contribution, known as the integrated Sachs-Wolfe effect (ISW), can be computed as the integral of the derivative of the potential fluctuations along the photon trajectory $\hat{\mathbf{n}}$ from $\eta_{\rm lss}$ (conformal time at the last scattering surface or decoupling time) to η_0 (conformal time today). For the ISW only, and for the case where there is no shear stress, we can write the CMB temperature anisotropy as

$$\left(\frac{\Delta T}{T}\right)_{\rm ISW} = 2\int_{\eta_{lss}}^{\eta_0} d\eta \frac{\partial \Psi}{\partial \eta}\left[(\eta_0 - \eta)\,\hat{\mathbf{n}}, \eta\right]. \qquad (42)$$

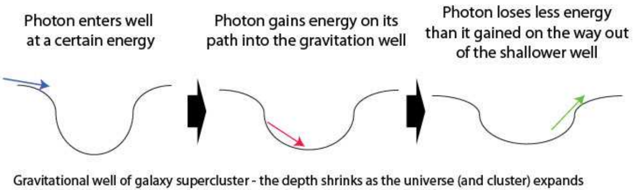


Abbildung: The integrated Sachs-Wolfe effect

With the perturbed equations deduced before we can calculate the ISW signal Eq. (42). A comparison between the ΛCDM model and the diffusion one is performed by calculating relative amplifications (Q) of the ISW effect as

$$Q \equiv \frac{\left(\frac{\Delta T}{T}\right)^{\mathrm{Diff}}_{\mathrm{ISW}}}{\left(\frac{\Delta T}{T}\right)^{\Lambda\mathrm{CDM}}_{\mathrm{ISW}}} - 1. \tag{43}$$

If $Q > 0\,(< 0)$ the diffusion model produces more (less) temperature variation to the CMB photons via the ISW effect than the fiducial ΛCDM model.

Assuming a fiducial ΛCDM model with $\Omega_{m0} = 0.30$ and $h = 0.7$ we calculate the quantity $\left(\frac{\Delta T}{T}\right)^{\Lambda\mathrm{CDM}}_{\mathrm{ISW}}$ using the ΛCDM equations, i.e. the case $\tilde{\sigma} = 0$. Then, we calculate $\left(\frac{\Delta T}{T}\right)^{\mathrm{Diff}}_{\mathrm{ISW}}$ for many different values of the parameters $\tilde{\sigma}$ and Ω_{m0}.

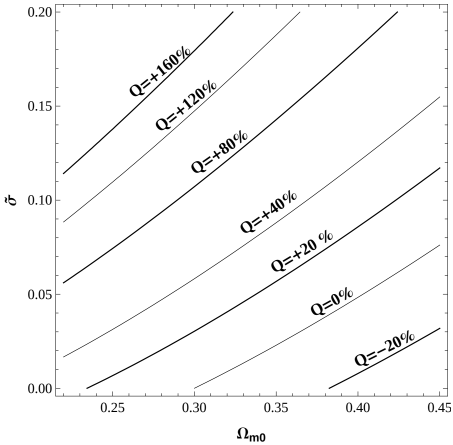


Abbildung: Contours for the parameter Q in the $\tilde{\sigma}$ x Ω_{m0} plane.

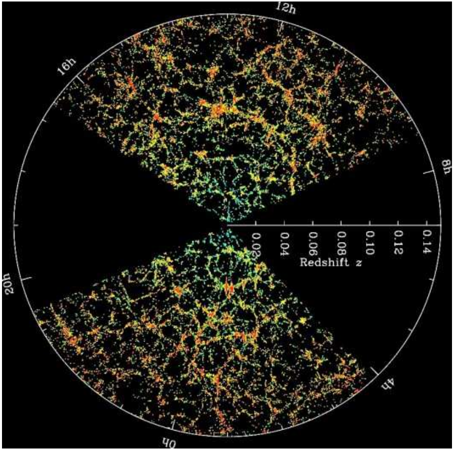


Abbildung: Sloan Digital Sky Survey distribution of galaxies

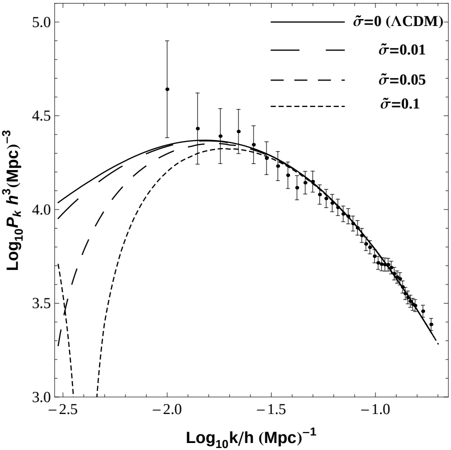


Abbildung: Matter power spectrum. The ΛCDM ($\tilde{\sigma} = 0$) model is shown in the solid line. The data points correspond to the 2dFGRS data. The case of cosmic diffusion is plotted in the dashed lines for different values of the parameter $\tilde{\sigma}$.

The moment of matter-radiation equality

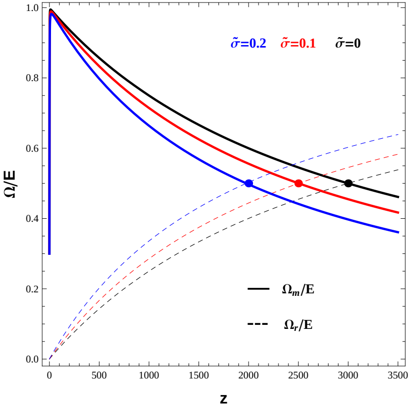


Abbildung: Evolution of the matter and radiation fractional densities. The initial data are $\Omega_{\phi 0} = 0.7$, $\Omega_{m0} = 0.2999$, $\Omega_{r0} = 3 \cdot 10^{-4}$. The Universe becomes radiation dominated at $z \sim 3000$ for $\sigma = 0$ (ΛCDM), at $z \sim 2500$ for $\sigma = 0.1$ and at $z \sim 2000$ for $\sigma = 0.2$.

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- What next? Things to think about...
  How to prove the existence of the cosmic diffusion mechanism?
  Is it possible to reproduce specific interacting scenarios?