Warm Inflation, Cosmological Fluctuations and Constraints from Planck[1]

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[1] In collaboration with: M. Bastero-Gil (Granada), A. Berera (Edinburgh) and I. G. Moss (Newcastle)

Cosmological Inflation

Standard Big-Bang cosmological model problems:

- Flatness problem ($\Omega_T = 1 \Rightarrow \Omega_T(t_{\rm nucleos.}) - 1 \approx 10^{-16}$)
- Horizon problem (The observable Universe was larger than the particle horizon at LSS)
- Magnetic monopoles, etc

Problems are solved by an early phase of inflation:

$$\ddot{a} > 0, \quad p < -\rho/3$$

(Starobinsky '80; Guth '81; Albrecht, Steinhardt '82; Linde '82)

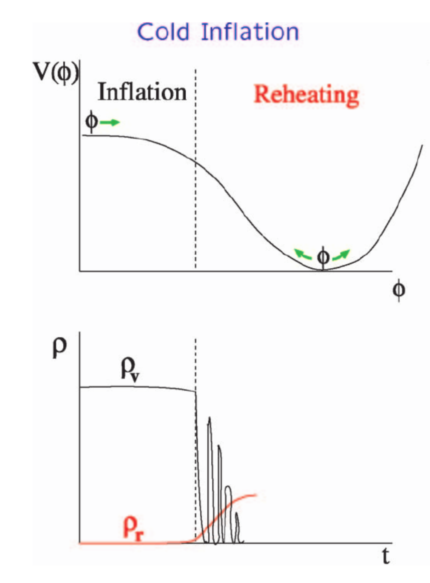
Extra bonus: The standard theory of inflation predicts that the large scale distribution of galaxies can be traced back to quantum vacuum fluctuations of a weakly coupled field during the inflationary era.

Cold Inflation

$$\begin{array}{ccccc} \text{Inflaton} & \Longrightarrow & \text{Radiation (p)reheating} & \Longrightarrow & \text{Matter} \\ \phi & \longrightarrow & \chi, \psi_\chi & \longrightarrow & \text{SM dof} \end{array}$$

$$\text{for example :}\ \ \mathcal{L}_I = -V(\phi) - \frac{1}{2} g_\chi^2 \phi^2 \chi^2 - g_\psi \phi \bar{\psi}_\chi \psi_\chi + \mathcal{L}_I[\chi, \psi_\chi, SM]$$

$\Rightarrow$ density perturbations sourced by inflaton's quantum fluctuations $\Rightarrow$ CMBR amplitude fixes the energy scale for the inflaton potential V: e.g., for $V(\phi) = \lambda\phi^4/4 \Longrightarrow \lambda \sim 10^{-14}$


Cold Inflation
V(ϕ)
Inflation
Reheating
ϕ
ϕ
ϕ
ρ
ρ_v
ρ_r
t

Relevent cosmological parameters for inflation:

- Curvature perturbations: $\Phi = H\delta\phi/\dot{\phi}, \quad \delta\phi(\mathbf{x},t) = \phi(\mathbf{x},t) - \phi(t)$

$$P_R^2 = \frac{k^3}{2\pi^2}\int_{k'} \langle \Phi(k)\Phi(k')\rangle = P_R^2(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}, \quad \text{amplitude} = P_R^2(k_0)$$

- Spectral index:

$$n_s - 1 = \frac{d\ln P_R^2}{dN_e} = \frac{d\ln P_R^2}{d\ln k}$$

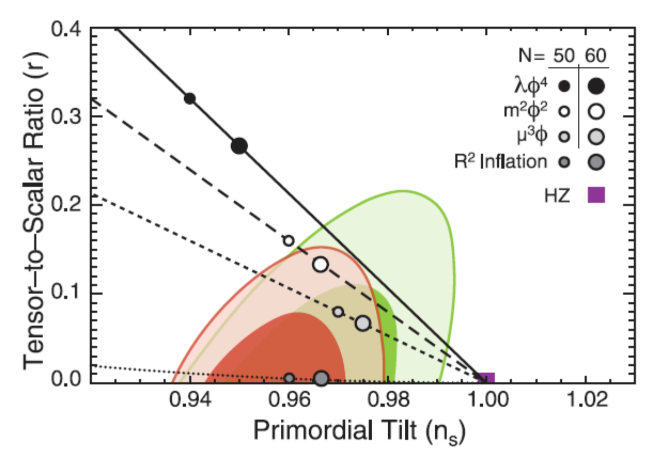
- Tensor to scalar curvature perturbation ratio:

$$r = \frac{P_T^2}{P_R^2}$$

- NonGaussianity:

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$
$$B_\Phi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = f_{NL}\; F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

WMAP 9yrs



0.4
0.3
0.2
0.1
0.0
Tensor–to–Scalar Ratio (r)
0.94
0.96
0.98
1.00
1.02
Primordial Tilt (n_s)
N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
$\mu^3\phi$
R^2 Inflation
HZ

Planck

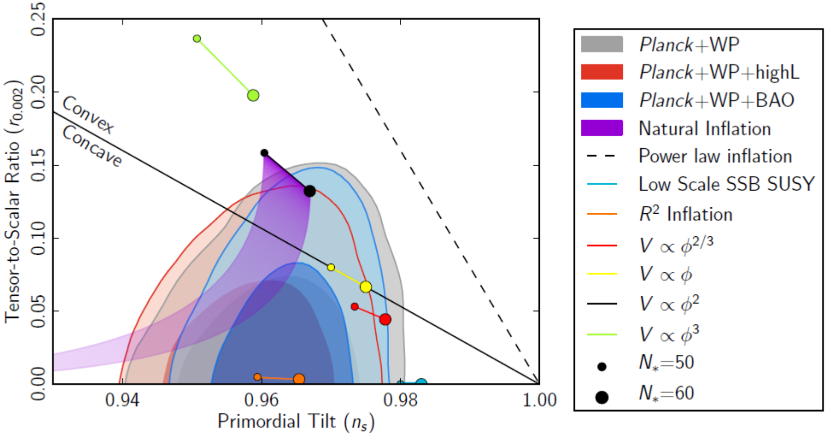


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

- In cold inflaton, interactions with other d.o.f. only important during (p)reheating.
- Interactions of the inflaton with other fields are considered negligible during inflation:

$$\dot{\rho}_r + 4H\rho_r = 0, \quad (H = \dot{a}/a \sim cte)$$

$$\Downarrow$$

The radiation density during inflation redshifts away: $\rho_r \sim 1/a^4$

$$\Downarrow$$

Universe supercools during inflation

But what if the universe did not supercool ?

Warm Inflation[a]:

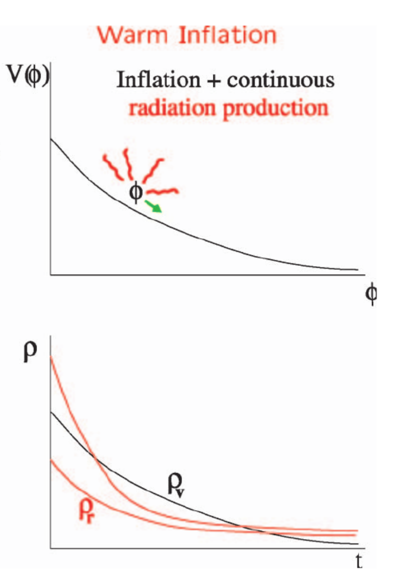
[a] Ian G. Moss PLB154 1985, Yokoyama and Maeda PLB207 1988, A. Berera and L. Z. Fang PRL74 1995, Berera PRL75 1995

$$\text{Inflaton} \implies \text{Decay} \implies \text{Radiation} \implies \text{Matter}$$

Interactions of the inflaton with other d.o.f. are important during inflation, generate dissipation/viscosity terms $\Rightarrow$ small fraction of vacuum energy density can be converted to radiation

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2$$

$\Rightarrow$ The radiation density during inflation stabilises: $\Rightarrow \rho_r \sim \Upsilon\dot{\phi}^2/(4H)$


Warm Inflation
V(ϕ)
Inflation + continuous
radiation production
ϕ
ϕ
ρ
ρv
ρr
t

Warm Inflation

- The production of radiation is associated with a friction term in the inflaton equation,

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + \Upsilon\dot{\phi} + \text{additional terms} = 0,$$

- $\Upsilon\dot{\phi}$ describes how inflaton's interactions with other fields backreact on the inflaton dynamics.
- Effectiveness of warm inflation measured by $Q = \frac{\Upsilon}{3H}$
- Cold or warm inflation: $T \lesssim H$ or $T \gtrsim H$
- adiabatic density fluctuations are sourced by thermal fluctuations: amplitude $\delta\phi^2 \sim H^2 + HT + \Upsilon T$, (WI: $T > H$), curvature perturbations $\zeta = H\delta\phi/\dot{\phi}$. CMB amplitude $\Delta_R^2(k_0) = (2.41 \pm 0.10) \times 10^{-9}$ (WMAP Nine-year Mean) fixes the energy scale V.

Warm Inflation: how to built/get it ?

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For example:

$$V(\phi, \chi) = V(\phi) + \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} g^2 \phi^2 \chi^2$$

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Equation of motion for the (homogeneous) inflaton field (ensemble averaged):

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + g^2\phi\langle\chi^2\rangle = 0,$$

$\langle\chi^2\rangle \equiv$ time dependent thermal average. In perturbation theory:

$$\begin{aligned} g^2\phi\langle\chi^2\rangle &\simeq g^2\phi\langle\chi^2\rangle_0 \\ &- g^4\phi(t)\int d^4x'\theta(t-t')\left[\phi^2(t')-\phi^2(t)\right] i\langle[\chi^2(x),\chi^2(x')]\rangle \\ &+ \mathcal{O}(g^6) \end{aligned}$$

During slow-roll[2]:

$$\begin{aligned} &-g^4\phi(t)\int d^4x'\theta(t-t')\left[\phi^2(t')-\phi^2(t)\right] i\langle[\chi^2(x),\chi^2(x')]\rangle \\ &\simeq 2g^2\phi^2\dot{\phi}\int d^4x'\theta(t-t')(t-t')i\langle[\chi^2(x),\chi^2(x')]\rangle \\ &= \Upsilon(\phi,T)\dot{\phi} \end{aligned}$$

[2] M. Gleiser and ROR, PRD50, 2441 (1994); A. Berera, M. Gleiser and ROR, PRD58, 123508 (1998)

and

$$g^2\phi\langle\chi^2\rangle_0$$

$$= g^2\phi\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[1+2n_{BE}(\omega_k)\right]$$

$$\simeq (T=0\ \mathrm{term})+g^2\phi\frac{T^2}{12}$$

Putting everything together:

$$\ddot{\phi}+3H\dot{\phi}+V_{,\phi}+g^2\phi\frac{T^2}{12}+\Upsilon(\phi,T)\dot{\phi}=0,$$

What about the equation for the radiation energy density ?

From:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + g^2\phi\frac{T^2}{12} + \Upsilon(\phi, T)\dot{\phi} = 0,$$

equation for entropy density:

$$T\dot{s} + 3HTs - \Upsilon(\phi, T)\dot{\phi}^2 = 0$$

and using $s = -\partial V(\phi, T)/\partial T$, $\rho = V(\phi, T) + Ts$, $Ts = 4\rho_r/3$,

$$\left(1 - \frac{V_{,\phi T} T\phi}{4\rho_r}\right)\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2 + \text{extra terms}$$

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$\Rightarrow$ direct coupling to radiation can throw the inflaton away from the inflationary trajectory. Need couplings $\ll 1$, but this spoils thermalization ! (See, however, A. Berera, M. Gleiser and ROR, PRL83, 264 (1999))

Warm Inflation: how to built it[3]

Solution: Decouple the inflaton from the radiation bath:

[3] For a review: Berera, Moss and ROR, Rep. Prog. Phys. **72**, 026901 (2009)

Warm Inflation: how to built it[3]

Solution: Decouple the inflaton from the radiation bath:

If the field(s) coupled to the inflaton is(are) heavy, e.g., $m_\chi \gg T$, then:

$$n_{BE}(\omega_k) \approx e^{-m_\chi/T} \approx 0,$$

and

$$\begin{aligned} g^2\phi\langle\chi^2\rangle_0 &= g^2\phi \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left[1 + 2n_{BE}(\omega_k)\right] \\ &\approx (T = 0 \text{ term}) \end{aligned}$$

$\Rightarrow$ only T=0 corrections (Coleman-Weinberg type of corrections) to the inflaton potential, which can be kept under control.

[3] For a review: Berera, Moss and ROR, Rep. Prog. Phys. **72**, 026901 (2009)

Warm Inflation: how to built it[4]

Working model: the two-stage decay model:

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$$\underset{\text{inflaton}}{\phi} \longrightarrow \underset{\text{heavy field}}{\chi, \psi_\chi} \longrightarrow \underset{\text{light fermions or scalars}}{\psi_\sigma, \sigma}$$

for example : $\mathcal{L}_I = -\frac{\lambda}{4}\phi^4 - \frac{1}{2} g_\chi^2 \phi^2 \chi^2 - g_\psi \phi \bar{\psi}_\chi \psi_\chi - h_\sigma M \chi \sigma^2 - h_\psi \chi \bar{\psi}_\sigma \psi_\sigma$

very same interactions found/needed in (p)reheating in cold inflation !

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- decouple the radiation from the inflaton ($m_\chi, m_{\psi_\chi} \gg T, m_{\sigma,\psi_\sigma} \ll T$)
- couplings between fields of order 0.1
- SUSY to reduce vacuum corrections, e.g., $W = g\Phi X^2 + hXY^2$ where Φ, X, Y are superfields with scalar and fermion components given by (ϕ, ψ_ϕ), (χ, ψ_χ) and (σ, ψ_σ) respectively.

[4] For a review: Berera, Moss and ROR, Rep. Prog. Phys. **72**, 026901 (2009)

Let $\phi \equiv \phi(x,t)$ and average out (integrate over) the other fields and over the quantum modes of the inflaton field. This gives a stochastic (Langevin-like) effective equation of motion for the inflaton[5]

$$\left[\frac{\partial^2}{\partial t^2} + (3H + \Upsilon)\frac{\partial}{\partial t} - e^{-2Ht}\nabla^2\right]\phi_c(\vec{x},t) + \frac{\partial V(\phi_c)}{\partial \phi_c} = \xi_q(\vec{x},t) + \xi_T(\vec{x},t)$$

We have a dissipation (friction) term $\Upsilon\dot{\phi}$ and stochastic thermal ξ_T and quantum ξ_q sources. The sources have a Gaussian distribution with local correlation functions, e.g.,

$$\langle \xi_T(\mathbf{x},t)\xi_T(\mathbf{x}',t')\rangle = a^{-3}\Upsilon T\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

$\Rightarrow$ Eq. for ϕ_c is similar to a Langevin equation with quantum and thermal noise terms (stochastic process w/ Gaussian noises).

[5] ROR and L. A. da Silva, JCAP 03 (2013) 032

The friction and (Markovian) noise terms emerge in an adiabatic approximation [6], $\dot{\phi}/\phi, H, \dot{T}/T < \Gamma_\chi \approx h^2 m_\chi/(8\pi)$.

Dissipation coefficient Υ:
$\phi \to \sigma$, $\psi_\sigma \Rightarrow$ mediated by the excitation of the intermediate (catalyst) fields, χ, ψ_χ

Typically[7] $(c + 2a - 2b = 1)$:

$$\Upsilon = C_\phi \frac{T^c \phi^{2a}}{m_X^{2b}}$$

At low-T $(m_\chi, m_{\psi_\chi} \gg T)$ (T-corrections to $V(\phi)$ are suppressed) $\Rightarrow$ leading friction coefficient is:

$$\Upsilon \sim g^2 h^4 (T^3/m_\chi^2) \approx T^3/\phi^2$$

[6] A. Berera, I. G. Moss and ROR, PRD76, 083520 (2007)

[7] M. Basteiro-Gil, A. Berera, ROR, JCAP 09 (2011) 033

Equation for the fluctuations:

$$\delta\ddot{\varphi}(\vec{k},t) + (3H+\Upsilon)\delta\dot{\varphi}(\vec{k},t) + V_{,\phi\phi}(\phi)\delta\varphi(\vec{k},t) + a^{-2}k^2\delta\varphi(\vec{k},t) = \tilde{\xi_T}(\vec{k},t) + \tilde{\xi_q}(\vec{k},t)$$

General solution found by expressing it in terms of a Green function (analytical).

Using also that the thermal ξ_T and quantum ξ_q noises are uncorrelated (decoupled), they give separated contributions to the power spectrum:

$$P_{\delta\varphi}(z) = \frac{k^3}{2\pi^2}\int \frac{d^3k'}{(2\pi)^3}\langle\delta\varphi(\mathbf{k},z)\delta\varphi(\mathbf{k}',z)\rangle = P^{(\mathrm{th})}_{\delta\varphi}(z) + P^{(\mathrm{qu})}_{\delta\varphi}(z)$$

$z = \frac{k}{aH}$

CMB Observables and Warm Inflation

scalar curvature perturbation spectrum (at horizon crossing, $z = 1$):

$$P^2_{\mathcal{R}} \simeq \left(\frac{H_*}{\dot{\phi}_*}\right)^2 \left(\frac{H_*}{2\pi}\right)^2 \left[1 + 2n_* + \frac{2\pi T_*}{H_*} \frac{\sqrt{3}\, Q_*}{\sqrt{3 + 4\pi Q_*}}\right], \quad Q = \frac{\Upsilon}{3H}$$

tensor-to-scalar ratio ($P^2_t = (2/\pi^2) H^2_*/M^2_P$):

$$r = \frac{P^2_t}{P^2_{\mathcal{R}}} \simeq \frac{16\epsilon_*}{1 + 2n_* + \kappa_*}\ , \quad \kappa_* = \frac{2\pi T_*}{H_*} \frac{\sqrt{3}\, Q_*}{\sqrt{3 + 4\pi Q_*}}$$

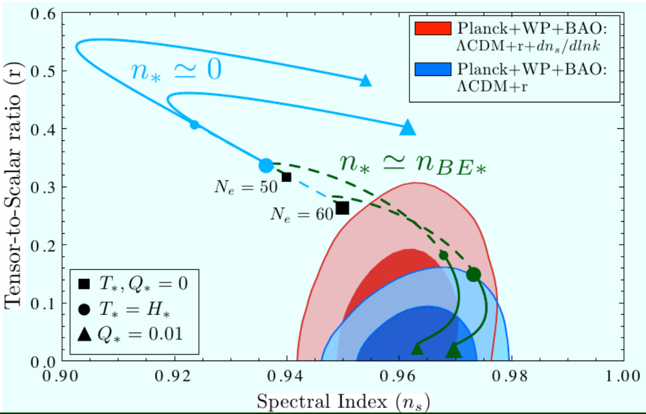
spectral index:

$$n_s - 1 \simeq 2\eta_* - 6\epsilon_* + \frac{2\kappa_*}{1 + \kappa_*} (7\epsilon_* - 4\eta_* + 5\sigma_*)\ ,$$

$$\epsilon = (M^2_P/2)(V'/V)^2 \ll 1 + Q, \quad , \eta = M^2_P V''/V \ll 1 + Q,$$

$$\sigma = M^2_P \Upsilon' V'/(\Upsilon V) < 1 + Q$$

Trajectories in the (n_s, r) plane for $\lambda\phi^4$ potential[8]



black lines: nearly-thermal inflaton, $T_* \gtrsim H_*$
light blue lines: vanishing inflaton occupation numbers ($T_\phi = 0$)
dashed lines: $T_* \lesssim H_*$

[8](S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, ROR and J. G. Rosa, arXiv:1307.5868)

Accounting for the perturbations of the radiation bath: Coupled two-fluid system [9]

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$\Downarrow$

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dissipative fluxes

$\Downarrow$

hydrodynamics: transport coefficients (shear and bulk)

[9] (M. Bastero-Gil, A. Berera, I. G. Moss and ROR, arXiv:1401.1149)

Dissipation in Fluids: Transport Coefficients

(no conserved charges, $\mu_i = 0 \to$ no heat transport)

Relativistic Dissipative Fluid Dynamics

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^{\mu}u^{\nu} + \Delta T^{\mu\nu}$$

where $P = P(T)$ is pressure, $s = dP/dT$ is entropy density, $\epsilon = -P + Ts$ is energy density, and $w = Ts = P + \epsilon$ is enthalpy density.

In the Landau-Lifshitz approach u is the velocity of energy transport.

$$\Delta T^{\mu\nu} = \eta\left(\Delta^{\mu}u^{\nu} + \Delta^{\nu}u^{\mu}\right) + \left(\tfrac{2}{3}\eta - \zeta\right)H^{\mu\nu}\partial_{\rho}u^{\rho}$$

$$H^{\mu\nu} \equiv u^{\mu}u^{\nu} - g^{\mu\nu}, \quad \Delta_{\mu} \equiv \partial_{\mu} - u_{\mu}u^{\beta}\partial_{\beta},$$

$$s^{\mu} = sU^{\mu}$$

$$\partial_{\mu}s^{\mu} = \frac{\eta}{2T}\left(\partial_i u^j + \partial_j u^i - \tfrac{2}{3}\delta^{ij}\partial_k u^k\right)^2 + \frac{\zeta}{T}\left(\partial_k u^k\right)^2$$

Shear vs. Bulk Viscosity

Shear viscosity is relevant for change in shape at constant volume.

Bulk viscosity is relevant for change in volume at constant shape.

Bulk viscosity is zero for point particles and for a radiation equation of state. It is generally small unless internal degrees of freedom (rotation, vibration) can easily be excited in collisions. But this is exactly the case for a resonance gas – expect bulk viscosity to be large near the critical temperature!

Bulk and shear viscosities

From hydrodynamics[10]

$$\eta_s = \frac{4}{15}\rho_r \tau, \quad \eta_b = 4\rho_r \tau \left(\frac{1}{3} - v_s^2\right)^2 .$$

- for conformal field theories ($v_s^2 = 1/3$), dilatation is a symmetry, fluid remains always in equilibrium, also for scale invariant field theories (ideal equation of state, $\omega_r = 1/3$) $\Rightarrow \eta_b = 0$
- Quantum corrections break scale invariance in general $\Rightarrow \eta_b \neq 0$
- modeling the radiation bath with $m_\sigma^2 \sigma^2/2 + \lambda_\sigma \sigma^4/4!$, ($m_\sigma \ll T, \;\; \lambda_\sigma \ll 1$):

$$\eta_b \simeq 8.9 \times 10^{-5} \lambda_\sigma T^3 \ln^2(0.064736\lambda_\sigma), \quad \eta_s \simeq 3.04 \times 10^3 \frac{T^3}{\lambda_\sigma^2} ,$$

- $\eta_b \ll \eta_s$

[10] S. Weinberg, Gravitation and Cosmology, (New York, NY, Wiley, 1972)

inflaton: both dissipation and fluctuation (stochastic noise) as a result of interactions

$\Downarrow$

same for the radiation fluid (analogous to Landau's theory of random fluids):

$\Downarrow$

Random sources and dissipative stresses are introduced via a stress term Π_{ab} in the stress-energy tensor

$$T^{(f)}{}_{ab} = (p^{(f)} + \rho^{(f)})\, u_a{}^{(f)} u_b{}^{(f)} + p^{(f)}\, g_{ab} + \Pi_{ab},$$

$$\Pi_{ij} = -\left(\eta_s \nabla_i u_j{}^{(f)} + \eta_s \nabla_j u_i{}^{(f)} + (\eta_b - \frac{2}{3}\eta_s)\delta_{ij}\nabla_k u^{(f)k}\right) - \Sigma_{ij},$$

fluctuations are generated by a Gaussian noise term Σ_{ab}. The correlation functions of the stochastic noise term Σ_{ij} are assumed to be local and determined by the fluctuation-dissipation relation,

$$\langle \Sigma_{ij}(x,t)\Sigma_{kl}(x',t')\rangle =$$
$$2T\left(\eta_s\delta_{ik}\delta_{jl} + \eta_s\delta_{il}\delta_{jk} + (\eta_b - \frac{2}{3}\eta_s)\delta_{ij}\delta_{kl}\right)\, \delta^{(3)}(x-x')\delta(t-t').$$

Cosmological perturbations

Perturbed spacetime metric:

$$ds^2 = -(1+2\alpha)dt^2 - 2\beta_{,i} dt\, dx^i + a^2 \left(\delta_{ij}(1+2\varphi) + 2\gamma_{,ij}\right) dx^i dx^j,$$

$$\begin{aligned}\chi &= a(\beta + a\dot{\gamma}),\\ \kappa &= 3H\alpha - 3\dot{\varphi} - \nabla^2\chi.\end{aligned}$$

perturbed Einstein equations in gauge-ready form[11]:

$$\begin{aligned}\nabla^2\varphi + H\kappa &= -4\pi G\delta\rho,\\ \kappa + \nabla^2\chi &= -12\pi G(\rho+p)\delta v,\\ \dot{\chi} + H\chi - \alpha - \varphi &= 8\pi G\delta\Pi,\\ \dot{\kappa} + 2H\kappa + \nabla^2\alpha - 3(\rho+p)\alpha &= 4\pi G(\delta\rho + 3\delta p).\end{aligned}$$

[11] J. -c. Hwang, H. Noh, Class. Quant. Grav. 19 (2002) 527

Fluid and scalar perturbations

$$(\delta\dot{\phi} - \alpha\dot{\phi})^{\cdot} + 3H(\delta\dot{\phi} - \alpha\dot{\phi}) - \nabla^2\delta\phi + \Omega_{,\phi\phi}\delta\phi - \kappa\dot{\phi}$$
$$+\delta\Upsilon\dot{\phi} + \Upsilon\delta\dot{\phi} - \alpha\ddot{\phi} = (2\Upsilon T)^{1/2}\xi^{(\phi)}$$

$$\delta\dot{\rho}^{(f)} - \alpha T\dot{s} + 3H(\delta\rho^{(f)} + \delta p^{(f)} - \eta_b\kappa)$$
$$+(Ts - 3H\eta_b)(\nabla^2\delta v^{(f)} - \kappa) + s_{,\phi}\delta q = -\delta Q^{(\phi)},$$

$$a^{-3}\{a^3(Ts - 3H\eta_b)\delta v^{(f)}\}^{\cdot} + \alpha(Ts - 3H\eta_b) + \delta p^{(f)}$$
$$-\eta_b\kappa - \eta'\nabla^2(\delta v^{(f)} + \chi) = -\delta J^{(\phi)} + (2\eta' T)^{1/2}\xi^{(f)}.$$

$$\eta' = \frac{4}{3}\eta_s + \eta_b.$$

energy and momentum transfer terms:

$$\begin{aligned} \delta Q^{(\phi)} &= -\delta\Upsilon\dot{\phi}^2 - 2\Upsilon\dot{\phi}\,(\delta\dot{\phi} - \alpha\dot{\phi}) + (2\Upsilon T)^{1/2}\dot{\phi}\,\xi^{(\phi)} + \boldsymbol{\nabla}\cdot\mathbf{P}, \\ \delta J^{(\phi)} &= \Upsilon\dot{\phi}\,\delta\phi + \nabla^{-2}\boldsymbol{\nabla}\cdot(\dot{\mathbf{P}} + 4H\mathbf{P}), \end{aligned}$$

where $\mathbf{P}$ is a stochastic energy flux added to the stress energy tensor:

$$\mathbf{P} = -C_P(2\Upsilon T)^{1/2}\dot{\phi}\,\nabla^{-2}\boldsymbol{\nabla}\xi^{(\phi)}.$$

The two cases $C_P = 0$ and $C_P = 1$ govern whether the noise source $\xi^{(\phi)}$ appears in the energy flux or in the momentum flux.

Background equations:

$$\begin{aligned} \ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{,\phi} &= 0, \\ \dot{\rho}_r + 4H\left(\rho_r - \frac{9}{4}H\eta_b\right) &= \Upsilon\,\dot{\phi}^2\,, \\ 3H^2 &= 8\pi G\rho\,. \end{aligned}$$

Prolonged inflation requires the slow-roll conditions $|\epsilon_X| \ll 1$, where $\epsilon_X = -d\ln X/Hdt$, and X is any of the background field quantities. At leading order in the slow-roll approximation:

$$\begin{aligned} 3H(1+Q)\dot{\phi} &\simeq -V_{,\phi}, \\ 4\rho_r &\simeq 3Q\dot{\phi}^2 + 9H\eta_b, \\ 3H^2 &\simeq 8\pi GV, \end{aligned}$$

where $Q = \Upsilon/(3H)$.

dissipative parameter, with cubic dependence with the temperature, $c = 3$, $\Upsilon = C_\phi\frac{T^3}{\phi^2}$, and a quartic chaotic model with inflationary potential $V = \lambda\phi^4/4$

Metric perturbations, choose a gauge (e.g. the zero-shear gauge, $\chi = 0$)
From the slow-roll equations, the gauge-invariant curvature perturbation (Lukash variable) Φ:

$$\Phi = -\frac{1}{1+Q}\zeta^{\phi} - \frac{Q}{1+Q}\zeta^{v}\,,$$

where

$$\begin{aligned}\zeta^{\phi} &= -\varphi + H\delta\phi/\dot{\phi}\,,\\ \zeta^{v} &= \varphi + H\delta\rho_r/4\rho_r\,.\end{aligned}$$

At late times, when $z \equiv k/(aH) \to 0$, we have $\Phi = -\zeta^{\phi} = -\zeta^{v}$.
Power spectrum:

$$\langle \zeta^{i}(k,t)\zeta^{i}(k',t)\rangle = P^{i}(k,t)\,(2\pi)^3\delta^{(3)}(k+k')\,,$$

quantum noise term ξ_q accounts for the inflaton thermal and quantum vacuum fluctuations:

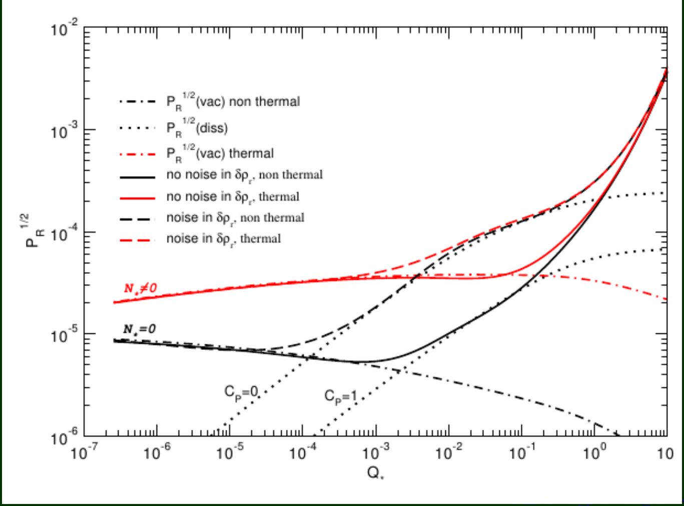
$$H\frac{\sqrt{1+2\mathcal{N}_*}}{\sqrt{2}}\,\xi^{(q)}$$

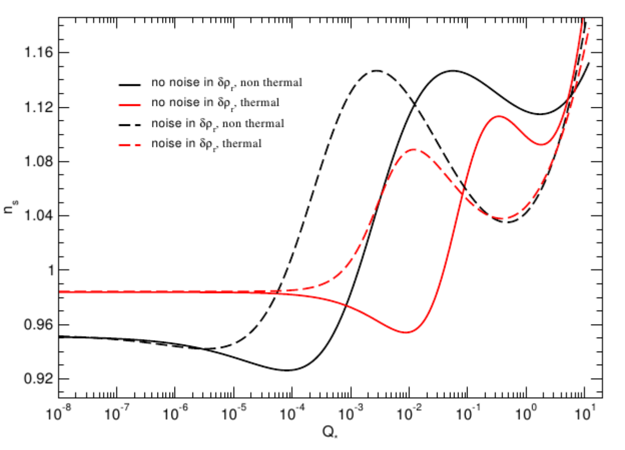
$\xi^{(q)}$ is a zero mean, unit variance Gaussian distribution

Cases considered:

- $\mathcal{N}_* = 0$ for a nonthermal (vacuum) inflaton fluctuation, or
- $\mathcal{N}_* = n_{BE}(k) = 1/(e^{k/(aT)} - 1)$ for a thermal statistical state for the inflaton
- $C_P = 0$, field stochastic $\xi^{(\phi)}$ term in the energy flux
- $C_P = 1$, no field stochastic $\xi^{(\phi)}$ term in the energy flux
- with and without viscosities

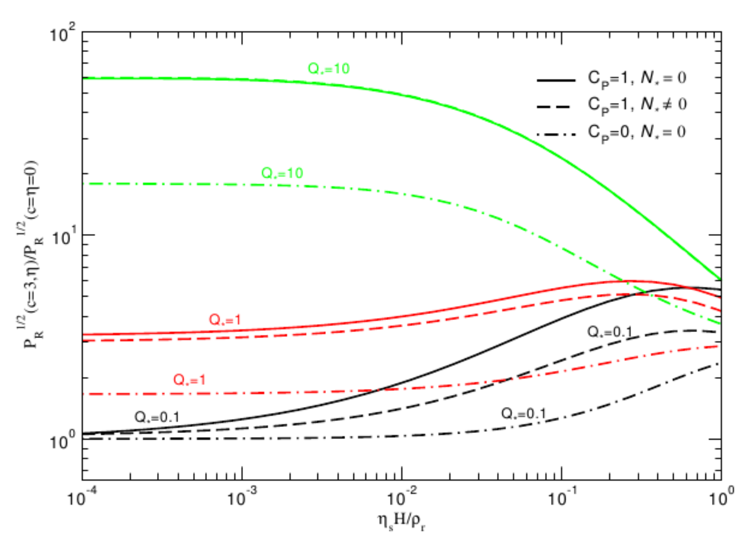
$$P_{\mathcal{R}} = (P_{\mathcal{R},\,diss} + P_{\mathcal{R},\,vac}) = \left(\frac{H_*^2}{2\pi\dot{\phi}_*}\right)^2 \left[\frac{T_*}{H_*}\frac{2\pi Q_*\,(40)^{1-C_P}}{\sqrt{1+4\pi Q_*/3}} + 1 + 2\mathcal{N}_*\right],$$



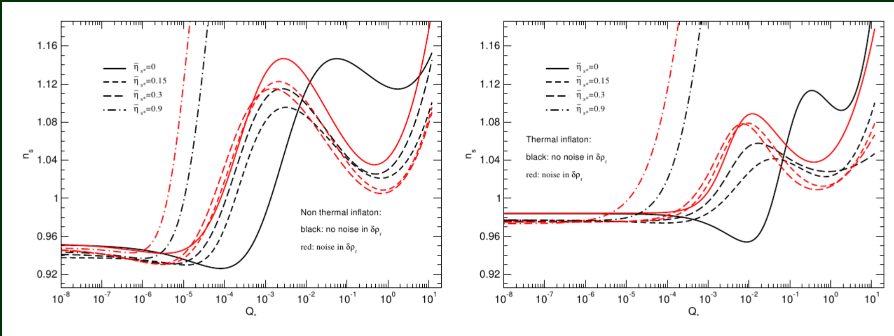


$$n_s - 1 \simeq \frac{1}{1+Q_*}\left[2\eta_* - 6\epsilon_* + \frac{4\mathcal{N}_*}{1+2\mathcal{N}_*+\Delta_{Q*}}(2\epsilon_* - \eta_* + \sigma_*) + \frac{2\Delta_{Q*}}{1+2\mathcal{N}_*+\Delta_{Q*}}(7\epsilon_* - 4\eta_* + 5\sigma_*)\right]$$

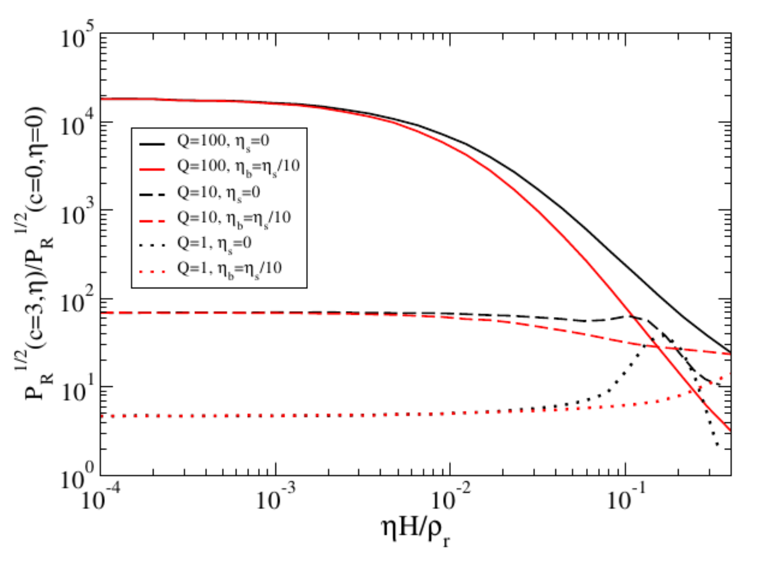
$$\Delta_{Q*} \simeq 2\pi Q_* \frac{T_*}{H_*}(40)^{1-C_P}$$



Q*=10
Q*=10
Q*=1
Q*=1
Q*=0.1
Q*=0.1
Q*=0.1
CP=1, N* = 0
CP=1, N* ≠ 0
CP=0, N* = 0
P_R^1/2(c=3,η)/P_R^1/2(c=η=0)
ηsH/ρr
10^-4
10^-3
10^-2
10^-1
10^0
10^0
10^1
10^2



n_s
Q_*
η̄_{s*}=0
η̄_{s*}=0.15
η̄_{s*}=0.3
η̄_{s*}=0.9
Non thermal inflaton:
black: no noise in δρ_r
red: noise in δρ_r
Thermal inflaton:
black: no noise in δρ_r
red: noise in δρ_r



$P_R^{1/2}(c=3,\eta)/P_R^{1/2}(c=0,\eta=0)$
$\eta H/\rho_r$
10^0
10^1
10^2
10^3
10^4
10^5
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Q=100, η_s=0
Q=100, η_b=η_s/10
Q=10, η_s=0
Q=10, η_b=η_s/10
Q=1, η_s=0
Q=1, η_b=η_s/10

Q_*	10^{-5}			10^{-3}	
$\bar{\eta}_{b*}$	2.2×10^{-5}	0.1	0.188	0.035	0.21
(a)	0.97 ± 0.02	0.980 ± 0.001	1.008 ± 0.003	0.969 ± 0.004	1.89 ± 0.01
(b)	0.98 ± 0.01	0.989 ± 0.008	1.51 ± 0.01	1.03 ± 0.01	2.10 ± 0.01
(c)	0.97 ± 0.02	0.981 ± 0.001	0.999 ± 0.007	0.949 ± 0.006	1.73 ± 0.01
(d)	0.98 ± 0.01	0.99 ± 0.01	1.172 ± 0.009	0.973 ± 0.008	2.05 ± 0.01

TABLE I: The spectral tilt n_s for different values of Q and bulk viscosity parameter $\bar{\eta}_b = \eta_b H/\rho_r$. (a) with radiation noise term ($C_P = 0$), thermal inflaton fluctuations; (b) $C_P = 0$, non-thermal inflaton fluctuations; (c) without the radiation noise term ($C_P = 1$), thermal inflaton fluctuations; (d) $C_P = 1$, non-thermal inflaton fluctuations. These are for $N_* = 50$ and for a pivot scale of $k_0 = 1000H_0$.

Conclusions

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- Warm inflation $\rightarrow$ dissipation mechanisms $\rightarrow$ radiation production during inflation $\rightarrow$ backreaction effect on the power spectrum
- Thermal radiation bath reconciles $\lambda\phi^4$ model with observations without the need of extra parameters (than those already required for reheating in conventional inflation) and completely in the context of renormalizable interactions

[12] (M. Basteiro-Gil, A. Berera, I. Moss, ROR, to appear)

- Decay processes in the radiation bath $\rightarrow$ departures from thermal equilibrium $\rightarrow$ non ideal radiation fluid $\Rightarrow$ viscosities effects + additional stochastic fluctuations (random cosmological fluids)

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- red spectrum heavily constrain dissipation ($Q_* \lesssim 0.01$) and the magnitude for shear and bulk viscosities ($\eta_{s(b)*} H_* / \rho_{r*} \lesssim 0.1$)
- nongaussianities compatible with Planck (Planck team result: $f_{NL}^{\rm warm} = 4 \pm 33$ at 68%CL) is expected to further differentiate WI from CI [12]

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