Super-renormalizable models of quantum gravity

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Three choices for Quantum Gravity (QG)

The existence of fundamental Planck units ($M_P \sim 10^{19}\ GeV$) indicates new fundamental physics at this very high energy scale. How to interpret this result of dimensional analysis?

General classification of possible approaches two Quantum Gravity (QG). Three distinct groups:

- **Quantize both gravity and matter fields. This is, definitely, the most fundamental possible approach.**

- **Quantize only matter fields on classical curved background (semiclassical approach).**

- **Quantize something else. E.g., in case of (super)string theory both matter and gravity are induced.**

Which approach is "better"?
Indeed, they have something in common.

Semiclassical approach: background gravity

The vacuum effective action includes contributions of fields Φ,

$$e^{i\Gamma(g_{\mu\nu})} \,=\, e^{iS_{vac}(g_{\mu\nu})}\,\int d\Phi\, e^{iS_m(\Phi,\,g_{\mu\nu})}\,.$$

The vacuum action of renormalizable QFT in curved space is

$$S_{vac} = S_{EH} + S_{HD}, \quad \textbf{where} \quad S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}\,,$$

$$S_{HD} = \int d^4x\sqrt{-g}\,\{a_1 C^2 + a_2 E + a_3 \Box R + a_4 R^2\}\,,$$

$$\textbf{where} \quad C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2$$

Without HD terms in the vacuum sector there is no consistency (renormalizability, running etc) in the semiclassical gravity.

Quantum Gravity (QG)

starts from some covariant action of gravity,

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu})\,.$$

$\mathcal{L}(g_{\mu\nu})$ **can be of GR,** $\mathcal{L}(g_{\mu\nu}) = -\kappa^{-2}(R + 2\Lambda)$ **or some other action, including with finite or infinite number of derivatives, that is local or nonlocal. Let us stress that we never give up the requirement of covariance.**

Gauge transformation $x'^{\mu} = x^{\mu} + \xi^{\mu}$. **The metric transforms as**

$$\delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\nabla_{\mu}\xi_{\nu} - \nabla_{\nu}\xi_{\mu} = R_{\mu\nu\,,\,\alpha}\,\xi^{\alpha}.$$

In the case of covariant theory

$$\frac{\delta S}{\delta g_{\mu\nu}}\;\cdot\; R_{\mu\nu\,,\,\alpha}\,\xi^{\alpha} = 0.$$

Covariant renormalizability

Let us start from the Faddeev-Popov approach. In the framework of background field method one has to shift

$$g_{\mu\nu}(x) \;\longrightarrow\; \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)$$

and define $S_{FP} \,=\, S(\bar{g}+h) + S_{gh}(\phi,\bar{g}) + S_{gf}(\phi,\bar{g}),$

with additional gauge fixing and ghost actions. Then

$$Z(J,\bar{g}) \,=\, \int d\phi\, \exp\Big\{\frac{i}{\hbar}\big[S_{FP}(\phi,\bar{g}) + J\phi\big]\Big\} \,=\, \exp\Big\{\frac{i}{\hbar}W(J,\bar{g})\Big\}.$$

There is a formal proof [*P.M. Lavrov & I.Sh., arXiv:1902.04687.*] **of that the effective action which results from this definition is covariant,**

$$\frac{\delta\Gamma(\bar{g})}{\delta\bar{g}_{\mu\nu}} \,\cdot\, R_{\mu\nu\,,\,\alpha}\,\xi^\alpha = 0,$$

with the same generators of diffeomorphism transformations as in the classical theory. This is called covariant renormalizability, which is <u>not</u> the same as multiplicative renormalizability!

Power counting in QG

General definition:

$$D + d = \sum_{l_{int}} (4 - r_l) - 4n + 4 + \sum_{\nu} K_\nu \quad \textbf{with} \quad l_{int} = p + n - 1$$

As the first example consider quantum GR.

$$S_{EH} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R, \qquad \textbf{Power counting: } D + d = 2 + 2p.$$

At the 1-loop level we can expect the divergences like

$$\mathcal{O}(R^2_{\dots}) = R^2_{\mu\nu\alpha\beta}\,, \quad R^2_{\mu\nu}\,, \quad R^2\,.$$

t'Hooft and Veltman; Deser and van Nieuwenhuisen, (1974); ...

At 2-loop level we have [*M.H. Goroff, A. Sagnotti, NPB (1986).*]

$$\mathcal{O}(R^3_{\dots}) = R_{\mu\nu} \Box R^{\mu\nu}\,, \dots R^3\,, \quad R_{\mu\nu} R^{\mu}_{\alpha} R^{\alpha\nu}\,, \quad R_{\mu\nu\alpha\beta} R^{\mu\nu}{}_{\rho\sigma} R^{\mu\nu\rho\sigma}\,.$$

The last structure doesn't vanish on-shell, hence the theory is not renormalizable.

The most natural choice is four derivative model, because we need four derivatives anyway for quantum matter field.

Already known action: $S_{gravity} = S_{EH} + S_{HD}$

where S_{HD} **includes square of the Weyl tensor and** R

$$S_{HD} = -\int d^4x\sqrt{-g}\left\{\frac{1}{2\lambda}\,C^2 + \frac{\omega}{3\lambda}\,R^2 + \text{ surface terms}\right\},$$

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + R^2/3\,,$$

Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ **and we have** K_4, K_2, K_0 **vertices.**

The superficial degree of divergence

$$D + d = 4 - 2K_2 - 4K_0.$$

Dimensions of counterterms are 4, 2, 0.
This theory is definitely renormalizable.

K. Stelle, Phys. Rev. D (1977).

However there is a price to pay: massive ghosts

$$G_{\text{spin}-2}(k) \sim \frac{1}{m^2} \left(\frac{1}{k^2} - \frac{1}{k^2 + m^2} \right), \quad m \propto M_P\,.$$

The tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and a huge mass.

Particle with negative energy means instability of vacuum state.

Even Minkowski space is not protected from spontaneous creation of massive ghost and many gravitons from vacuum.

Different sides of the HDQG problems with massive ghosts:

- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** *(M.V. Ostrogradsky, 1850).*

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** *(M.J.G. Veltman, 1963).*

- **Ghost produce violation of unitarity of the S -matrix.**

One can include more than four derivatives,

$$S = S_{EH} + \sum_{n=0}^{N} \int d^4x\sqrt{-g}\Big\{\omega_n^C C_{\mu\nu\alpha\beta}\Box^n C_{\mu\nu\alpha\beta} + \omega_n^R R\Box^n R\Big\} + \mathcal{O}\big(R^3_{\dots}\big).$$

Simple analysis shows that this theory is superrenormalizable, BUT massive ghost-like states are still present.

For the real poles case:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2+m_1^2} + \frac{A_2}{k^2+m_2^2} + \cdots + \frac{A_{N+1}}{k^2+m_{N+1}^2}\,.$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$,
the signs of the corresponding terms alternate: $A_j \cdot A_{j+1} < 0$.

M. Asorey, J.-L. Lopez & I. Sh., IJMPhA (1997), hep-th/9610006.

Exact β-functions in QG

In the superrenormalizable QG one can derive exact RG equations by working at the one-loop level !

M. Asorey, J.-L. Lopez & I. Sh., IJMPhA (1997), hep-th/9610006.

$$\beta_\Lambda = \mu \frac{d\rho_\Lambda}{d\mu} = \frac{1}{(4\pi)^2} \left(\frac{5\omega_{N-2,C}}{\omega_{N,C}} + \frac{\omega_{N-2,R}}{\omega_{N,R}} - \frac{5\omega_{N-1,C}^2}{2\omega_{N,C}^2} - \frac{\omega_{N-1,R}^2}{2\omega_{N,R}^2} \right).$$

L. Modesto, L. Rachwal & I.Sh., arXiv:1704.03988, EJPC (2018).

$$\beta_G = \mu \frac{d}{d\mu} \left(-\frac{1}{16\pi G} \right) = -\frac{1}{6(4\pi)^2} \left(\frac{5\omega_{N-1,C}}{\omega_{N,C}} + \frac{\omega_{N-1,R}}{\omega_{N,R}} \right).$$

Different from four-derivative quantum gravity these β-functions do not depend on the choice of a gauge-fixing condition.

And for $N \geq 3$ **they are exact.**

Two sides of higher derivatives in QG.

The consistent theory which is supposed to work at arbitrary energy scale can not be constructed without at least fourth derivatives.

If the higher derivative terms are included, then the tree-level spectrum includes massless graviton and massive spin-2 “ghost” with negative kinetic energy and huge mass.

If we do not include the higher derivative terms into classical action, they will emerge with infinite coefficients and (most relevant) with logarithmically running parameters. In any case, the unphysical ghosts come back.

No way to live with ghosts and no way to live without ghosts.

Still we can live, so there must be some explanation, of course.

Ghost-free HD models of gravity

Consider an example of ghost-free HD model of QG.

- **In the (super)string theory, the object of quantization is a kind of non-linear sigma-model in two space-time dimensions.**

Both metric and matter fields are induced, implying unification of all fundamental forces.

The σ-model approach is close to QFT in curved space,

$$S_{str} = \int d^2\sigma\sqrt{g}\,\Big\{\frac{1}{2\alpha'}g^{\mu\nu}G_{ij}(X)\partial_\mu X^i\partial_\nu X^j$$

$$+\frac{1}{\alpha'}\,\frac{\varepsilon^{\mu\nu}}{\sqrt{g}}\,A_{ij}(X)\partial_\mu X^i\partial_\nu X^j + B(X)R + T(X)\Big\}\,, \qquad i,j = 1,2,...,D\,.$$

The Polyakov approach: conditions of anomaly cancellation order by order in α'. Critical dimensions:

D=26 **for bosonic string,** D=10 **for superstrings.**

At the first order in α' the effective equations give GR !

E.S. Fradkin & A. Tseytlin (1985);
C. Callan, D. Friedan, E. Martinec, M. Perry, (1985).

- **Metric reparametrization remove ghosts at all orders in α' .**

In the torsionless case the effective action can be written as

$$S_M = \frac{2}{\kappa^2} \int d^D x \sqrt{G}\, e^{-2\phi} \Big\{ - R + 4\,(\partial\phi)^2$$

$$+\alpha' \,\big(a_1 R_{\lambda\mu\nu\rho} R^{\lambda\mu\nu\rho} + a_2 R_{\mu\nu} R^{\mu\nu} + a_3 R^2\big)\Big\} + \;...$$

In order to remove ghosts one performs reparametrization of the background metric $G_{\mu\nu}$

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha'\,(x_1\, R_{\mu\nu} + x_2\, R\, G_{\mu\nu}) + ...$$

where $x_{1,2,...}$ are specially tuned parameters.

B. Zweibach, S. Deser & A.N. Redlich, ... A. Tseytlin (1985-1987).

Ghost-killing reparametrization doesn't affect string S-matrix,

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha' \left(x_1 \, R_{\mu\nu} + x_2 \, R \, G_{\mu\nu} \right) + ...$$

At the same time, Zweibach reparametrization is ambiguous and this actually produce ambiguous physical solutions.

A. Maroto & I.Sh., PLB, hep-th/9706179.

- **Even more subtle point is that the effectively working ghost-killing transformation must be absolutely precise!**

Any infinitesimal change produce a ghost with a huge mass. Moreover, smaller violation of fine-tuning leads to a greater mass of the ghost, hence (according to a "standard wisdom") smaller violation of fine-tuning produce greater gravitational instability.

At low energies we know that the quantum effects are described by QFT, not string theory. Hence, string theory is ghost-free and unitary only if it completely controls QFT, even in the deep IR.

An alternative to Zweibach transformation

In the non-local theory

$$S = -\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,\Big\{R + \,G_{\mu\nu}\,\frac{a(\Box)-1}{\Box}\,R^{\mu\nu}\Big\}, \quad a(\Box) = e^{-\Box/m^2}\,.$$

A. Tseytlin, PLB, hep-th/9509050.

In this and similar theories propagator of metric perturbations has a single massless pole, corresponding to gravitons.

With this choice there are no ghosts!

The idea is to use Zweibach-like transformation, but arrive at the non-local theory which is non-polynomial in derivatives, instead of “killing” all higher derivatives that one can kill.

One more ambiguity in the (super)string theory.

There was a proposal to use the same kind of non-local models to construct superrenormalizable and unitary models of QG.

E.T. Tomboulis, hep-th/9702146; PRD (2015), arXiv:1507.00981.

...

L. Modesto, L. Rachwal, NPB (2014), arXiv:1407.8036.

The propagator is defined by the terms bilinear in curvature's,

$$S = \int_x \Big\{ -\frac{1}{\kappa^2}\,R + \frac{1}{2}\,C_{\mu\nu\alpha\beta}\,\Phi(\Box)\,C^{\mu\nu\alpha\beta} + \frac{1}{2}\,R\,\Psi(\Box)\,R \Big\}\,.$$

The equation for defining the poles:

$$p^2\left[1 \,+\, \kappa^2 p^2 \Phi(-p^2)\right] \,=\, p^2\, e^{\alpha p^2} \,=\, 0.$$

In this particular case there is only a massless pole corresponding to gravitons. But unfortunately, it is impossible to preserve the ghost-free structure at the quantum level.

I.Sh., PLB, arXive:1502.00106.

Typically there are infinitely many poles on the complex plane.

Complex ghosts and Lee-Wick unitarity in QG

Starting from *Tomboulis (1977)* **and** *Salam and Strathdee (1978)* **the main hope in the "minimal" fourth-derivative QG was that the real ghost pole splits into a couple of complex conjugate poles under the effect of quantum corrections.**

One-loop effects, large-*N* approximation and lattice-based considerations indicated an optimistic picture, but unfortunately all of them are not conclusive, as shown by *Johnston (1988)*.

However, for six- or more- derivative theory of QG, one can just start from the theory which has only complex massive poles.

L. Modesto, and I.Sh. PLB (2016), arXiv:1512.07600.

It turns out that such a theory is unitary and, moreover, this property may probably hold even at the quantum level.

Quantum consistency

There is yet another difficulty of non-local gravity, which is possibly shared by other e.g. polynomial models.

In the recent paper

M. Asorey, L. Rachwal, I.Sh., Galaxies - 2018; arXiv:1802.01036

it was shown that within the non-local models of exponential type the reflection positivity condition is not satisfied.

The Euclidean 2-point function $S_2(x, y)$ should satisfy Osterwalder-Schrader reflection positivity property

$$\int \theta f(x) S_2(x, y) f(y) \geqslant 0.$$

For the non-local gravity this is not true.

This means that this theory has unphysical modes regardless of the absence of massive pole in the tree-level propagator.

The main issue is stability

Certainly, the unitarity of the S- matrix is not the unique condition of consistency of the quantum gravity theory.

The most important feature is the stability of physically relevant solutions of classical general relativity in the presence of higher derivatives and massive ghosts.

The problem is well explored for the cosmological backgrounds.
Gravitational waves on de Sitter space (energy $\ll M_p$):

A. A. Starobinsky, Let. Astr. Journn. (in Russian) (1983).

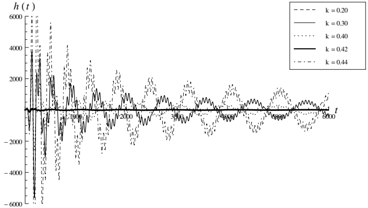
S. Hawking, T. Hertog, and H.S. Real, PRD (2001).
J. Fabris, A. Pelinson and I.Sh., NPB (2001).

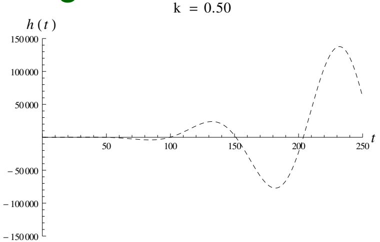
J. Fabris, A. Pelinson, F. Salles and I.Sh., JCAP, arXiv:1112.5202.

More general FRW-backgrounds:

F. Salles and I.Sh., PRD, arXiv:1401.4583.

More general cosmological backgrounds





Example: radiation-dominated Universe. There are no growing modes until the frequency k achieves the value ≈ 0.5 in Planck units. Starting from this value, we observe instability as an effect of massive ghost.

The anomaly-induced quantum correction is $\mathcal{O}(R^3_{....})$. Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical $\mathcal{O}(R^2_{....})$ - terms.

Massive ghosts are present only in the vacuum state. We just do not observe them “alive” until the energy scale M_P .

What can we do Planck or greater frequencies?

The simplest possible equation is for the fourth-derivative gravity without quantum (semiclassical) corrections,

$$\frac{1}{3}\overset{....}{h} + 2H\dddot{h} + \Big(H^2 + \frac{M_{\rm P}^2}{32\pi a_1}\Big)\ddot{h} + \frac{1}{6}\frac{\nabla^4 h}{a^4} - \frac{2}{3}\frac{\nabla^2 \ddot{h}}{a^2} - \frac{2H}{3}\frac{\nabla^2 \dot{h}}{a^2}$$
$$-\Big(H\dot{H} + \ddot{H} + 6H^3 - \frac{3M_{\rm P}^2 H}{32\pi a_1}\Big)\dot{h} - \Big[\frac{M_{\rm P}^2}{32\pi a_1} - \frac{4}{3}\left(\dot{H} + 2H^2\right)\Big]\frac{\nabla^2 h}{a^2}$$
$$-\Big[24\dot{H}H^2 + 12\dot{H}^2 + 16H\ddot{H} + \frac{8}{3}\dddot{H} - \frac{M_{\rm P}^2}{16\pi a_1}\left(2\dot{H} + 3H^2\right)\Big]h = 0.$$

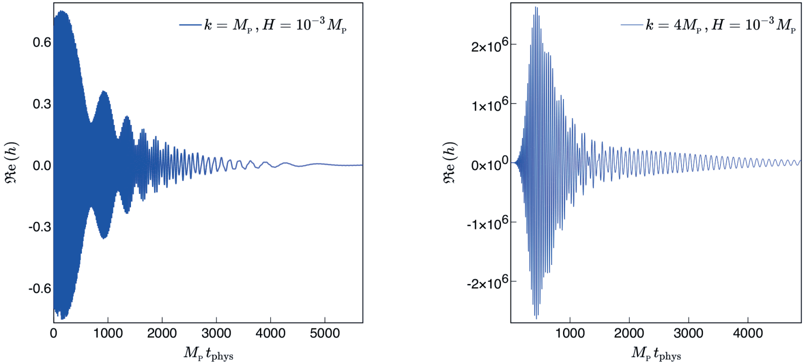
It is easy to note that the space derivatives ∇ and hence the wave vector $\vec{k}$ enter this equation only in the combination

$$\vec{q} = \frac{\vec{k}}{a(t)}.$$

When universe expands, each frequency becomes smaller!

Filipe de O. Salles, Patrick Peter, I.Sh., On the ghost-induced instability on de Sitter background. PRD (2018), arXiv:1801.00063

The qualitative conclusion is perfectly well supported by numerical analysis, including the case when the semiclassical corrections are taken into account.



The growth of the waves really stops at some point. At least in the cosmological setting this may be a solution of the problem.

General qualitative situation.

1) We know there is no way to have semiclassical or quantum gravity without higher derivatives.

2) Higher derivatives mean ghosts and instabilities. But in the closed system the problem can be solved because there is no energy to provide a global and total explosion of ghost or even tachyonic ghost modes (Lee-Wick approach).

G. Dvali, S. Folkerts, C. Germani, PRD (2011), arXiv:1006.0984;
G. Dvali and C. Gomez, Fortschr. Phys. (2013), arXiv:1112.3359.

May be there is some general unknown principle which forbids Planck-scale concentration of gravitons.

3) Then this restriction can be violated only for the Planck-scale background, which “opens” the phase space of quantum states and enables the production of instabilities. But after that the expansion of the universe reduce the frequencies and the instabilities do stabilize.

Conclusions

- **The construction of QG theory which is not restricted to the IR region, is not possible without higher derivative terms.**

- **Most important: higher derivative terms are needed for a consistent formulation of semiclassical theory.**

- **Including more than four derivatives provides theoretical advantages: superrenormalizable QG and well-defined renormalization group flow, free from gauge-fixing ambiguities.**

- **Lee-Wick type unitarity of the S- matrix for the gravitational field takes place in case of complex massive poles.**

- **The solution of the issue of ghosts will certainly require new ideas or new insights. One can say that we are now we are now waiting for and expecting new elements of this great puzzle.**