Growth of struture in interacting vacuum cosmologies

Humberto. A. Borges

Seminário PPGCosmo

30 de julho de 2021

Sumário

Consider a slow-roll inflaton field $\varphi = \varphi_0 + \delta\varphi$ with the average fluctuation $< \varphi >= \varphi_0$ but the variance is not zero $< \delta\varphi^2 >\neq 0$.
To first order

$$\zeta^{(1)} = -\frac{H}{\dot{\varphi}_0}\delta\varphi, \qquad \Rightarrow \qquad P_\zeta(k) = A_\zeta^2\left(\frac{k}{aH}\right)^{n_\zeta - 1}. \tag{1}$$

At second order

$$\zeta^{(2)} = \left(\frac{\dot{H}}{\dot{\varphi}^2} + \frac{H\ddot{\varphi}}{\dot{\varphi}^3}\right)\delta\varphi^2 \qquad \Rightarrow \qquad \zeta^{(2)} = (\epsilon - 2\eta)(\zeta^{(1)})^2. \tag{2}$$

Primordial curvature perturbation

$$\zeta \approx \zeta^{(1)} + \frac{1}{2}\zeta^{(2)} = \zeta^{(1)} + \frac{3}{5}f_{NL}(\zeta^{(1)})^2\,. \tag{3}$$

Bispectrum

$$lim_{k_1\to 0} < \zeta_{k_1}\zeta_{k_2}\zeta_{k3} >\propto n_\zeta - 1.$$

The galaxy's total velocity

$$cz = \mathcal{H}r + \vec{v}_g \cdot \hat{r}. \tag{4}$$

$$\delta(s) \simeq \delta(r) - \frac{1}{aH}\frac{\partial v_g}{\partial r}. \tag{5}$$

The linear continuity equation

$$-\frac{\vec{\nabla} \cdot v_g}{aH} = f\delta, \tag{6}$$

$$f = \Omega_m^{6/11}, \qquad f = \frac{\dot{\delta}}{\mathcal{H}\delta}. \tag{7}$$

We measure $f\sigma_8$.

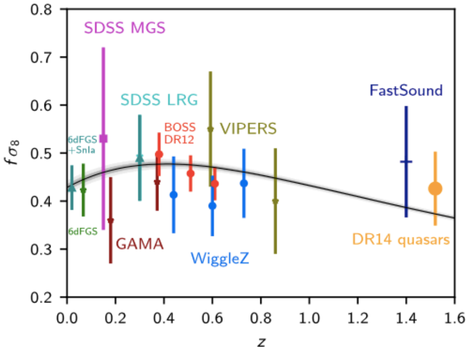


Figura 1: ΛCDM cosmology with $\Omega_m \approx 0.3$

From Planck Collaboration, 2018. arXiv:1807.06209

Dimensionless interaction parameter

$$g \equiv -\frac{a\bar{Q}}{\mathcal{H}\bar{\rho}_{dm}} \,. \tag{8}$$

Model 1

$$\bar{Q} = 3\alpha H\bar{\rho}_{dm}\bar{\rho}_V/(\bar{\rho}_m + \bar{\rho}_V) \quad \Rightarrow \quad g = -3\alpha(1-\Omega_{dm}).$$

The matter density parameter and the Hubble parameter are given by

$$\Omega_{dm}(a) = \frac{\Omega_{dm0}}{\Omega_{dm0} + (1-\Omega_{dm0})a^{3(1+\alpha)}}, \tag{9}$$

$$\mathcal{H}(a) = aH_0\left[1 - \Omega_{dm0} + \frac{\Omega_{dm0}}{a^{3(1+\alpha)}}\right]^{\frac{1}{2(1+\alpha)}}. \tag{10}$$

- Planck 2018+JLA+SHOES prior implies $0.13 < \alpha < 0.27$

M. Benetti, H. A. Borges, C. Pigozzo, S. Carneiro, and J. Alcaniz, arXiv:2102.10123

Model 2

$$\bar{Q} = qH\rho_V \quad \Rightarrow \quad g = -q(1-\Omega_{dm})/\Omega_{dm}.$$

The matter density parameter and the Hubble parameter, given by

$$\mathcal{H}(a) = H_0\sqrt{\frac{3(1-\Omega_{dm0})a^{3+q} + 3\Omega_{dm0} + q}{(3+q)a}}. \tag{11}$$

$$\Omega_{dm}(a) = \frac{3\Omega_{dm0} + q - q(1-\Omega_{dm})a^{3+q}}{3\Omega_{dm0} + q + 3(1-\Omega_{dm0})a^{3+q}}, \tag{12}$$

- Inclusion of the physical prior $q < 0$ in the statistical analysis considering Planck 2018 and SNIa.

R. von Marttens, H.A. Borges, S. Carneiro, J.S. Alcaniz, W. Zimdahl, Eur.Phys.J.C 80 (2020) 12, 1110.

Model 3

$$\bar{Q} = \epsilon H \bar{\rho}_{dm} \quad \Rightarrow \quad g = -\epsilon.$$

The matter density parameter and the Hubble parameter, given by

$$\Omega_{dm}(a) = \frac{(3+\epsilon)\Omega_{dm0} a^{-(3+\epsilon)}}{(3+\epsilon) + 3\Omega_{dm0}(a^{-(3+\epsilon)} - 1)}, \tag{13}$$

$$\mathcal{H}(a) = a\sqrt{\frac{\rho_{dm0}}{3+\epsilon} a^{-(3+\epsilon)} + \frac{\Lambda}{3}}. \tag{14}$$

The energy-momentum tensor of matter plus vacuum is

$$T_{\mu\nu} = \rho_{dm} u_\mu u_\nu - \rho_V g_{\mu\nu} \,. \tag{15}$$

The energy-momentum conservation equations

$$\nabla^\mu T_{(V)\mu\nu} = Q_\nu \,, \tag{16}$$

$$\nabla^\mu T_{(dm)\mu\nu} = -Q_\nu \,, \tag{17}$$

where the energy-momentum transfer is

$$Q_\nu = \nabla_\nu p_V = Q u_\nu . \tag{18}$$

The four velocity in this gauge is $u_\nu = [-a, 0, 0, 0]$.

Comoving-synchronous gauge

$$ds^2 = a^2(\eta)[-d\eta^2 + \gamma_{ij} dx^i dx^j] \,. \tag{19}$$

Deformation tensor and the perturbed scalar expansion is

$$\vartheta^i_j = \frac{1}{2}\gamma^{ik}\gamma'_{jk} \,, \qquad \vartheta = \vartheta^i_i \,. \tag{20}$$

The perturbed Raychaudhuri equation for the expansion, energy continuity equation and energy constraint are

$$\vartheta' + \mathcal{H}\vartheta + \vartheta^i_j\vartheta^j_i + \frac{1}{2}a^2\bar{\rho}_{dm}\delta_{dm} = 0. \tag{21}$$

$$\rho'_{dm} + (3\mathcal{H} + \vartheta)\rho_{dm} = -a\bar{Q} \,. \tag{22}$$

$$\vartheta^2 - \vartheta^i_j\vartheta^j_i + 4\mathcal{H}\vartheta + \mathcal{R} = 2a^2\rho_{dm} \,, \tag{23}$$

The metric and comoving matter density contrast can be expanded up to second order using only scalar quantities as

$$\gamma_{ij} \approx [1 - 2\psi^{(1)} - 2\psi^{(2)}]\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\nabla^2)\chi^{(1)} + (\partial_i\partial_j - \frac{1}{3}\nabla^2)\chi^{(2)}, \quad (24)$$

$$\delta_{dm} \approx \delta_{dm}^{(1)} + \frac{1}{2}\delta_{dm}^{(2)}. \quad (25)$$

Using the $0 - j$ component of the Einstein equations require

$$\mathcal{R}_c' = \left[\psi^{(1)} + \frac{1}{6}\nabla^2\chi^{(1)}\right]' = 0\,. \quad (26)$$

The perturbed Raychaudhuri, energy continuity and energy constraint equations up to first-order are

$$\vartheta'^{(1)} + \mathcal{H}\vartheta^{(1)} + \frac{1}{2}a^2\bar{\rho}_{dm}\delta^{(1)}_{dm} = 0, \tag{27}$$

$$\delta'^{(1)}_{dm} + g\mathcal{H}\delta^{(1)}_{dm} + \vartheta^{(1)} = 0, \tag{28}$$

$$\mathcal{H}\vartheta^{(1)} - \frac{1}{2}a^2\bar{\rho}_{dm}\delta^{(1)}_{dm} + \nabla^2\mathcal{R}_c = 0. \tag{29}$$

Combining the continuity equation (28) with the constraint (29), we find a first integral

$$2\mathcal{H}\delta'^{(1)}_{dm} + \left[a^2\bar{\rho}_{dm} + 2g\mathcal{H}^2\right]\delta^{(1)}_{dm} = 2\nabla^2\mathcal{R}_c\,. \tag{30}$$

$$-\frac{\vartheta^{(1)}}{\mathcal{H}} = f_{rsd}\delta^{(1)}_{dm}\,. \tag{31}$$

$$\delta^{(1)}_{dm}(\eta,\vec{x}) = \left(f_{rsd} + \frac{3\Omega_{dm}}{2}\right)^{-1}\frac{\nabla^2\mathcal{R}_c}{\mathcal{H}^2}\,. \tag{32}$$

$$\psi^{(1)} = \mathcal{R}_c + \frac{1}{3}\nabla^2\mathcal{R}_c\left[\frac{1}{\mathcal{H}^2}\left(f_{rsd} + \frac{3}{2}\Omega_{dm}\right)^{-1} + \int\frac{g}{\mathcal{H}}\left(f_{rsd} + \frac{3}{2}\Omega_{dm}\right)^{-1}d\eta\right]. \tag{33}$$

$$\chi^{(1)} = -2\mathcal{R}_c\left[\frac{1}{\mathcal{H}^2}\left(f_{rsd} + \frac{3}{2}\Omega_{dm}\right)^{-1} + \int\frac{g}{\mathcal{H}}\left(f_{rsd} + \frac{3}{2}\Omega_{dm}\right)^{-1}d\eta\right]. \tag{34}$$

Here $f_{rsd} = f_1 + g$.
The only surviving perturbation is the primordial curvature perturbation $\psi^{(1)} \to \zeta^{(1)}$

$\mathcal{R}_c = -\zeta^{(1)}$ express our initial conditions in terms of gauge-invariant curvature perturbation on uniform-density hypersurfaces.

First-order differential equation for the redshift-space distortion parameter

$$2\mathcal{H}^{-1} f_{rsd}' + (2f_{rsd} + 4 - 3\Omega_{dm} - 2g) f_{rsd} = 3\Omega_{dm}. \tag{35}$$

$$f_{rsd} \approx \Omega_{dm}^{\gamma}. \tag{36}$$

$$\gamma = \frac{6 + 6\alpha}{11 + 6\alpha}, \qquad \gamma = \frac{6 + 2q}{11 + 2q}, \qquad \gamma = \frac{6 + 2\epsilon}{11 + 3\epsilon}. \tag{37}$$

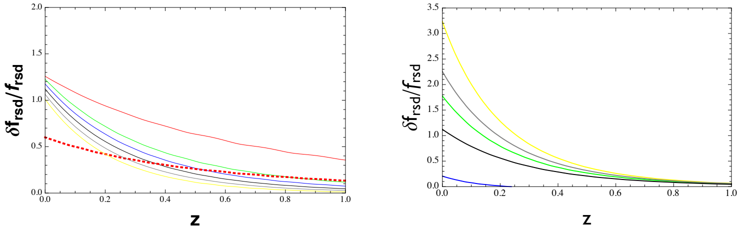
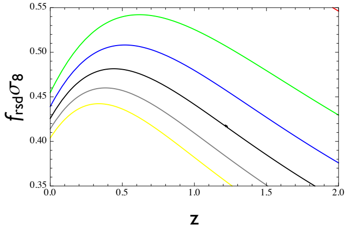


Figura 2: **Left panel**: Plot of the relative percentage difference $\left(\frac{\Omega_{dm}^{\tilde{\gamma}}}{f_1+g} - 1\right) \times 100$ between the analytical formula f_{rsd} and the numerical solution $f_1 + g$ for $-0.5 < \alpha < 0.2$.
Right panel: For the model with $-0.2 < q < 0.2$.



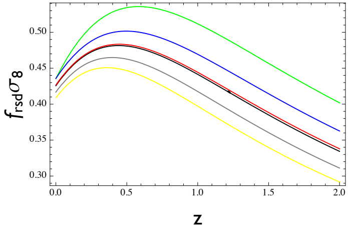


Figura 3: Magnitude of redshift space distortions for dark matter, $f_{rsd}\sigma_8$ versus redshift, z, normalised to $\sigma_8 = 0.83$ at present. **Left panel**: ΛCDM model (black curve) and model 1: $\alpha = -0.2$ (green curve), $\alpha = -0.1$ (blue curve), $\alpha = 0.1$ (grey curve) and $\alpha = 0.2$ (yellow curve) all with $\Omega_{dm0} = 0.3$. **Right panel**: ΛCDM model (black curve) and model 2: $q = -0.2$ (green curve), $q = -0.1$ (blue curve), $q = 0.1$ (grey curve) and $q = 0.2$ (yellow curve). For the model 3 we have plotted for $\epsilon = -0.01$

As we did for the first-order equations, we can obtain a first integral

$$4\mathcal{H}\delta'^{(2)}_{dm} + 2\left[a^2\rho_{dm} + 2g\mathcal{H}^2\right]\delta^{(2)}_{dm} - \mathcal{R}^{(2)} = 2\vartheta^{(1)2} - 2\vartheta^{(1)i}_{\ \ j}\vartheta^{(1)j}_{\ \ i} - 8\mathcal{H}\delta^{(1)}_{dm}\vartheta^{(1)}, \quad (38)$$

where

$$\frac{1}{2}\mathcal{R}^{(2)} = 2\nabla^2\psi^{(2)} + 6\partial^i\psi^{(1)}\partial_i\psi^{(1)} + 16\psi^{(1)}\nabla^2\psi^{(1)} + \mathcal{O}(\nabla^4)\,. \quad (39)$$

The coupled system of these equations is solved by separating the solution

$$\delta^{(2)}_{dm} = \delta^{(2)}_{dmh} + \delta^{(2)}_{dmp}, \qquad \mathcal{R}^{(2)}(\vec{x},\eta) = \mathcal{R}^{(2)}_h(\vec{x}) + \mathcal{R}^{(2)}_p(\vec{x},\eta) \quad (40)$$

$$\delta_{dm}^{(2)} = -\frac{24}{5[2f_{rsd}+3\Omega_{dm}]}\left[\left(f_{NL}+\frac{5}{12}\right)\frac{\partial^i\mathcal{R}_c\partial_i\mathcal{R}_c}{\mathcal{H}^2}+\left(f_{NL}-\frac{5}{3}\right)\frac{\mathcal{R}_c\nabla^2\mathcal{R}_c}{\mathcal{H}^2}\right]+ + \frac{\mathcal{S}(a,\Sigma)}{2(4f_2+3\Omega_{dm}+2g)}\left(\frac{\nabla^2\mathcal{R}_c}{\mathcal{H}^2}\right)^2 . \quad (41)$$

where we introduce the dimensionless shape coefficient

$$\Sigma(\vec{x}) = \frac{\vartheta^i_j\vartheta^j_i}{\vartheta^2} = \frac{\partial^i\partial_j\mathcal{R}_c\partial^j\partial_i\mathcal{R}_c}{(\nabla^2\mathcal{R}_c)^2}, \quad (42)$$

and define the dimensionless source function

$$\mathcal{S}(a,\Sigma) = \frac{2f_{rsd}^2(1-\Sigma)+8f_{rsd}+4(f_{rsd}+\frac{3}{2}\Omega_{dm})(1+\Sigma)}{(f_{rsd}+\frac{3}{2}\Omega_{dm})^2} + +4(1+\Sigma)\mathcal{H}^2\int\frac{g}{a\mathcal{H}^2}\left(f_{rsd}+\frac{3}{2}\Omega_{dm}\right)^{-1}da . \quad (43)$$

Bispectrum

$$< \delta_{k_2}\delta_{k_2}\delta_{k3} >= (2\pi)^3\delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)\times[2F(\vec{k}_1,\vec{k}_1)P_L(k_1)P_L(k_2)+2perm.].$$

Kernel in Fourier space:

$$F_i(k_1,k_2)=\frac{3}{5}\mathcal{H}^2(2f_{rsd}+3\Omega_{dm})\left[\left(f_{NL}+\frac{5}{12}\right)\frac{\vec{k}_1\cdot\vec{k}_2}{k_1^2k_2^2}+\left(f_{NL}-\frac{5}{3}\right)\frac{k_1^2+k_2^2}{2k_1^2k_2^2}\right], \quad (44)$$

and

$$F_n(k_1,k_2)=\frac{f_{rsd}^2+3(2f_{rsd}+\Omega_{dm})}{2(4f_2+3\Omega_{dm}+2g)}+\frac{(2f_{rsd}+3\Omega_{dm}-f_{rsd}^2)}{2(4f_2+3\Omega_{dm}+2g)}\frac{(\vec{k}_1\cdot\vec{k}_2)^2}{k_1^2k_2^2}+\frac{\vec{k}_1\cdot\vec{k}_2(k_1^2+k_2^2)}{2k_1^2k_2^2}$$
$$+\left[\frac{2\mathcal{H}^2}{4f_2+3\Omega_{dm}+2g}\left(1+\frac{\vec{k}_1\cdot\vec{k}_2}{k_1^2k_2^2}\right)+\mathcal{H}^2(2f_{rsd}+3\Omega_{dm})\frac{\vec{k}_1\cdot\vec{k}_2(k_1^2+k_2^2)}{2k_1^2k_2^2}\right]\times$$
$$\times\int\frac{g}{\mathcal{H}(2f_{rsd}+3\Omega_{dm})}d\eta. \quad (45)$$

Humberto. A. Borges and D. Wands Phys.Rev. D101 (2020)

Thanks !