

Quark Matter in Neutron Stars: from a particle physics perspective

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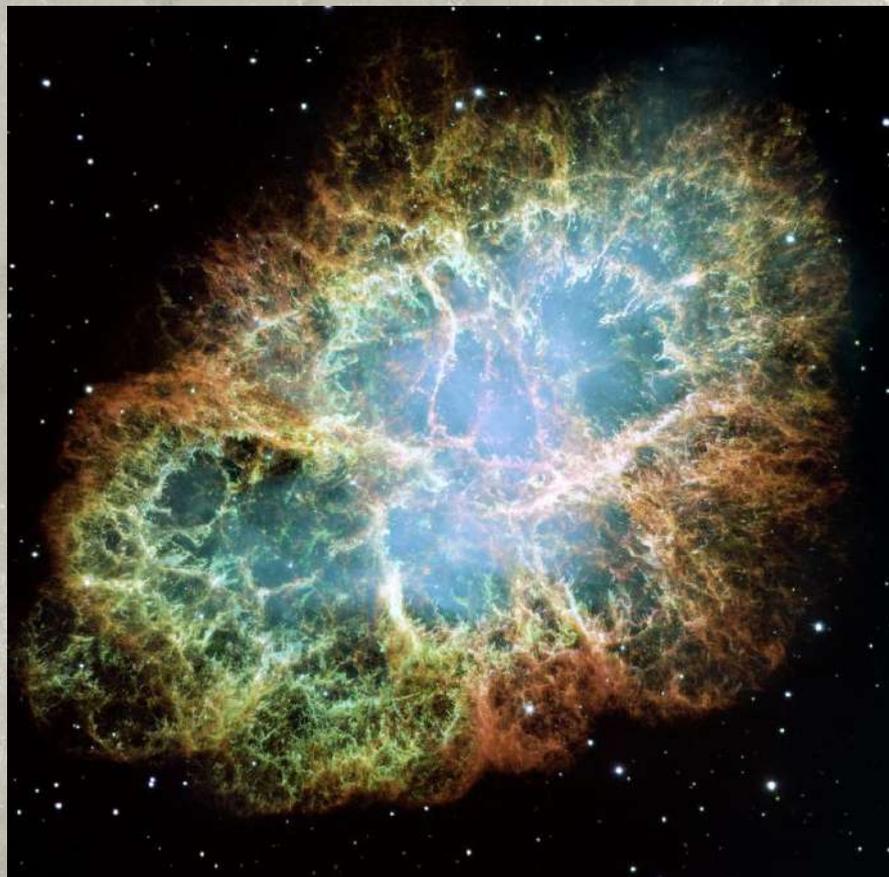
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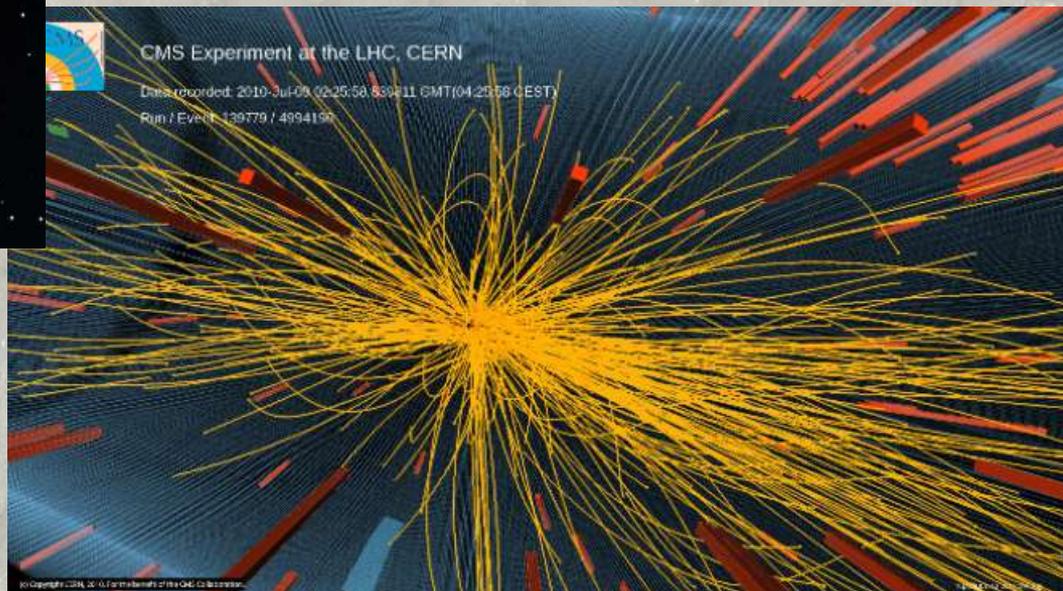
Neutron stars & particle physics ???!



★ A new (nearby) future, one that deeply connects high-energy nuclear & particle physics under extreme conditions to astrophysics of neutron stars!

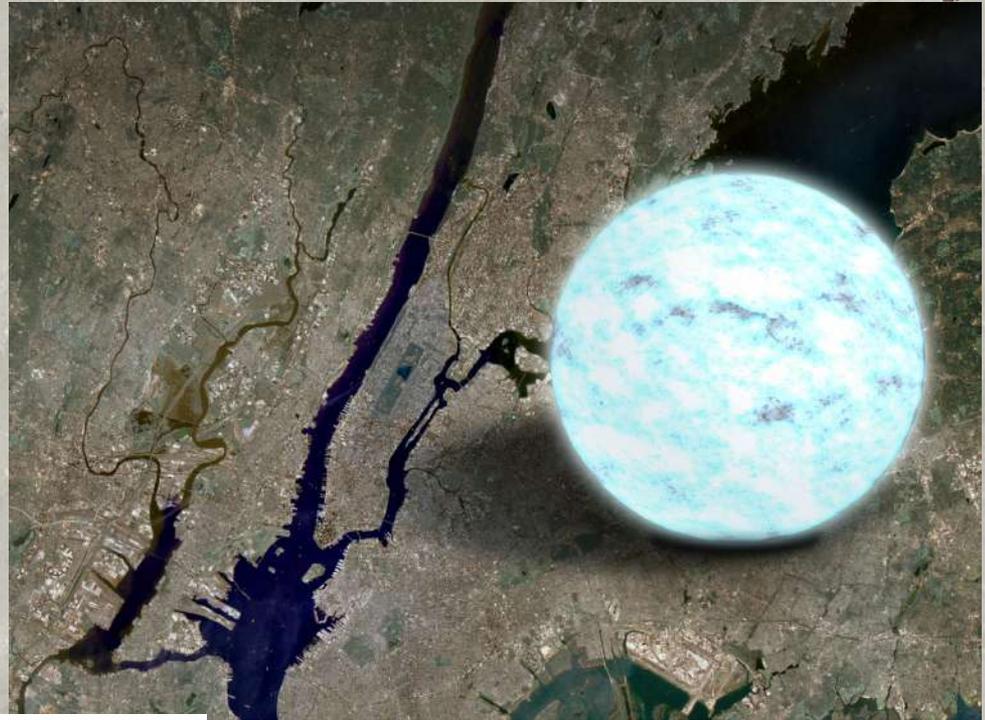


[NASA, ESA, J. Hester and A. Loll (Arizona State University)]

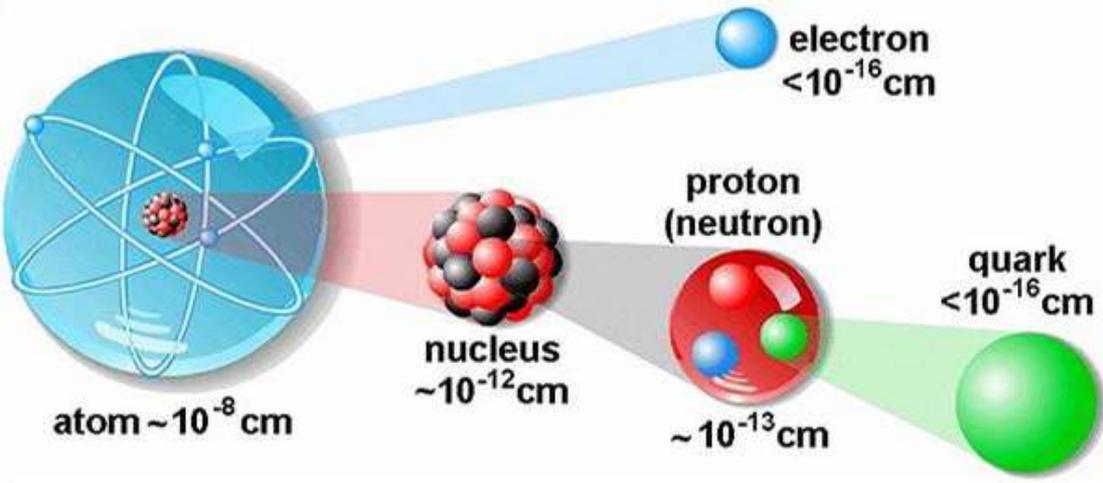




Very different scales,
yet somehow connected?



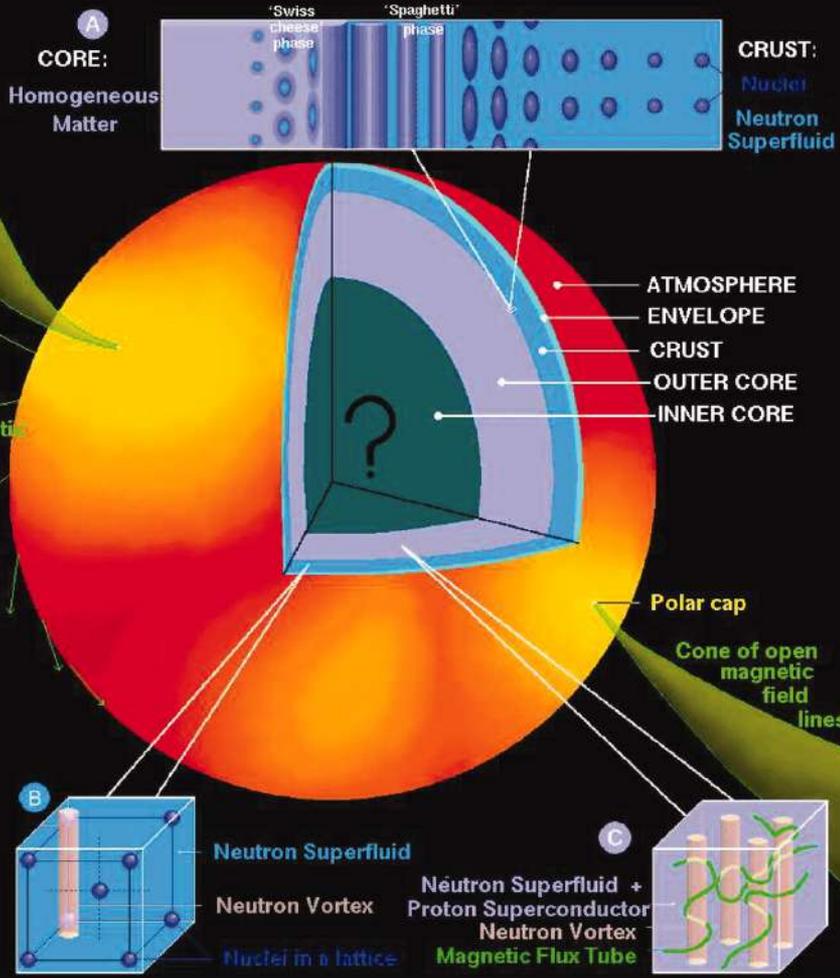
[NASA's Goddard Space Flight Center]



[ETH Zürich]

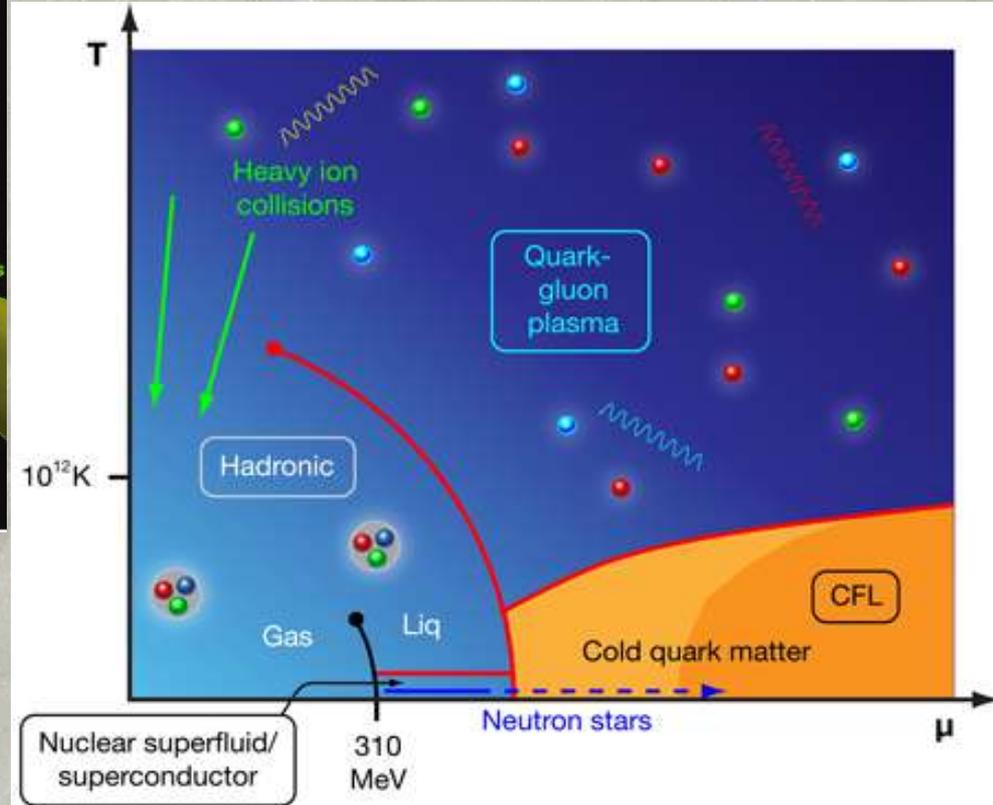


A NEUTRON STAR: SURFACE and INTERIOR



[Lattimer & Prakash,

★ Large domains of the phase diagram of strong interactions not reachable by lab experiments: great challenges!

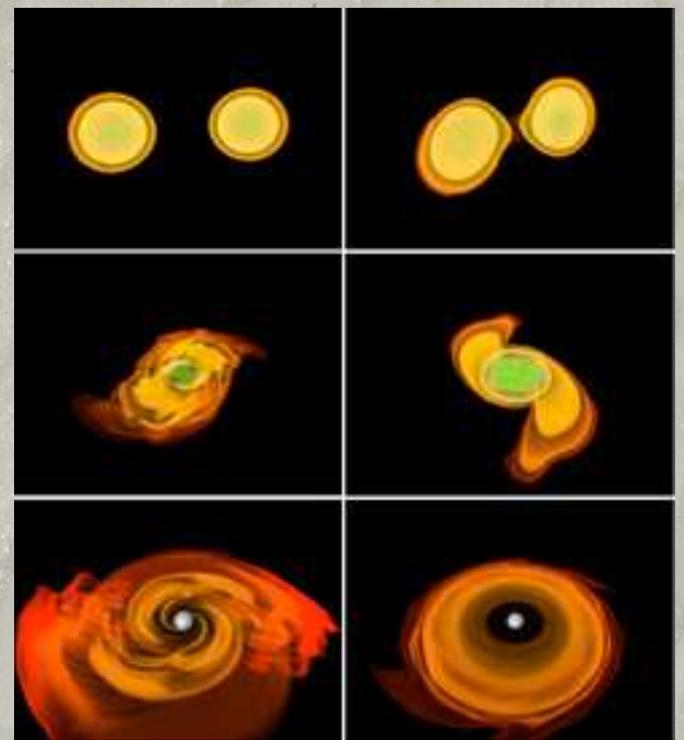


[Alford, 2010]

★ NS provide a new window to the Standard Model, complementary to accelerator experiments.



[NASA]



[Rezzolla et al., 2008]

★ Observations of compact objects have entered a new era after the first direct observation of gravitational waves from BH merging. Higher-precision NS mass and radius measurements are hopefully at the corner.

★ Hence, yes, neutron stars & particle physics! :-)



★ One expects that NS will exhibit new states of matter.

★ The most basic motivation has been expressed F. Wilczek:

The behavior of QCD at large net baryon density (and low temperature) is also of obvious interest. It answers yet another childlike question: **What will happen when you keep squeezing things harder and harder?** [Wilczek, Phys. Today, August 2000]

★ Naturally, a high-energy physicist, especially one that likes QCD, will ask questions such as:

- **Is there quark matter in NS?**
- Can it reach an equilibrium configuration?
- **What is the formation mechanism?**
- Can we make predictions from first principles?

★ To address these questions one needs to deal with the structure of neutron stars.



- ★ There is too much to cover on QM in NS, so one has to make choices – limited by time and bias, of course!
- ★ Mine will focus on the equation of state.
- ★ For a much broader coverage, please see:

EMMI Rapid Reaction Task Force Meeting on “Quark Matter in Compact Star”

Michael Buballa,¹ Veronica Dexheimer,² Alessandro Drago,³ Eduardo Fraga,^{4,5,6} Pawel Haensel,⁷ Igor Mishustin,⁵ Giuseppe Pagliara,³ Jürgen Schaffner-Bielich,⁴ Stefan Schramm,⁵ Armen Sedrakian,⁴ and Fridolin Weber^{8,9}

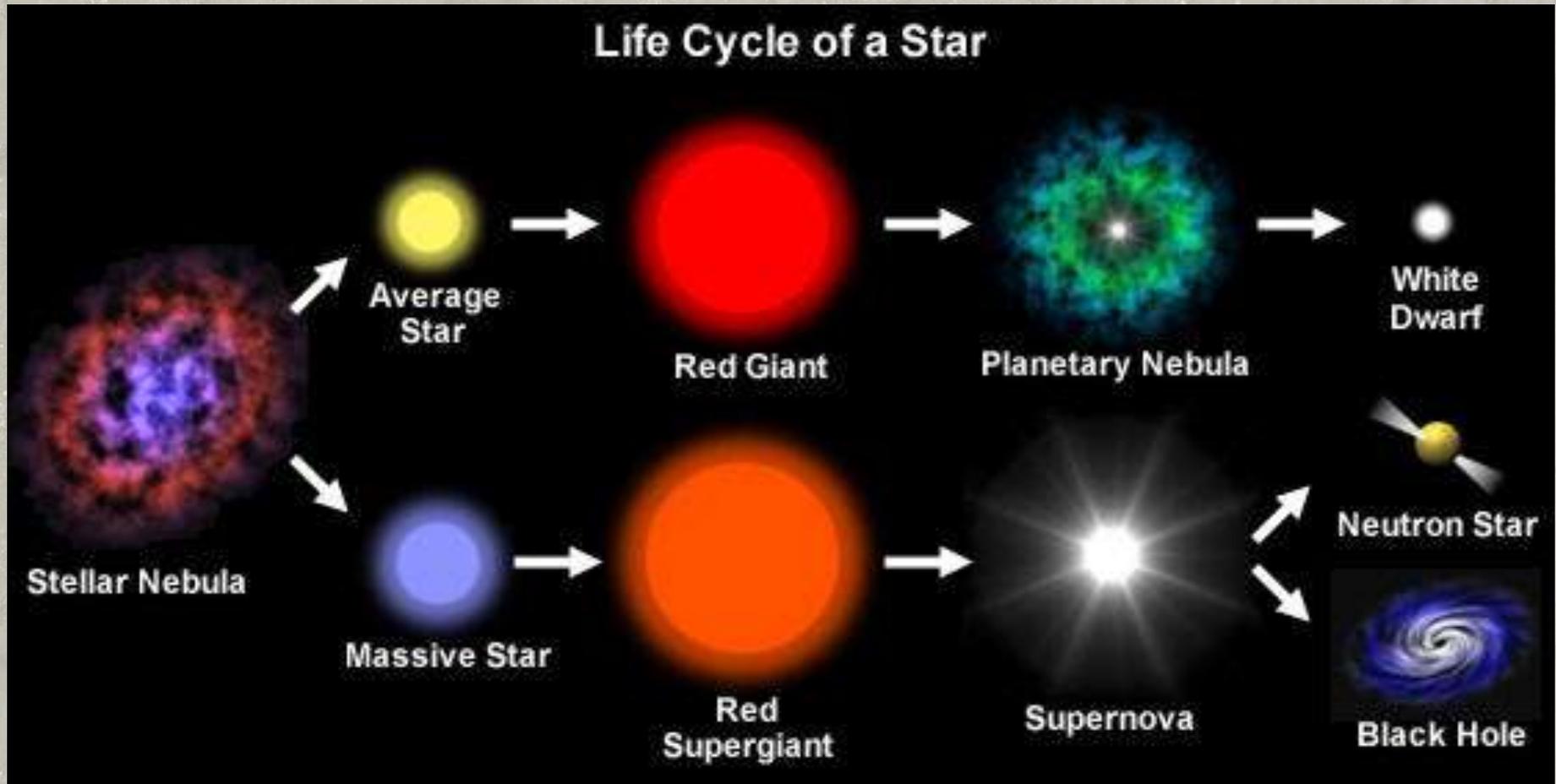
Michael Buballa et al 2014 J. Phys. G: Nucl. Part. Phys. 41 123001



Outline

- ★ What kind of matter do we expect to find inside NS?
- ★ Approaches to compute the equation of state.
- ★ Some serious difficulties.
- ★ The good and bad heritage of the MIT bag model.
- ★ Dense quark matter via QCD, perturbatively.
- ★ Asking questions we can answer using controlled approximations in QCD.
- ★ Final remarks.

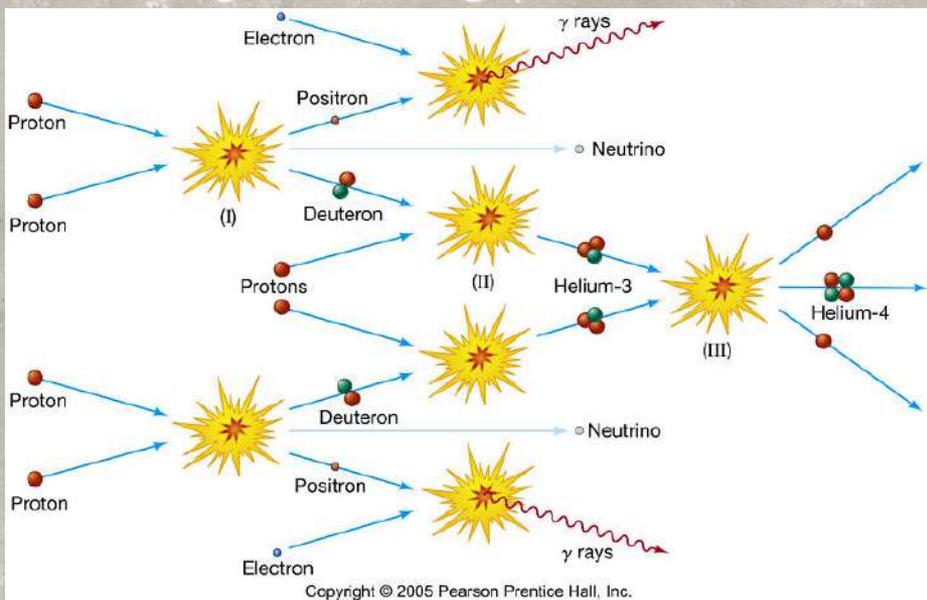
First, a tiny bit on stellar formation and structure...



Inner pressure to equilibrate gravity...



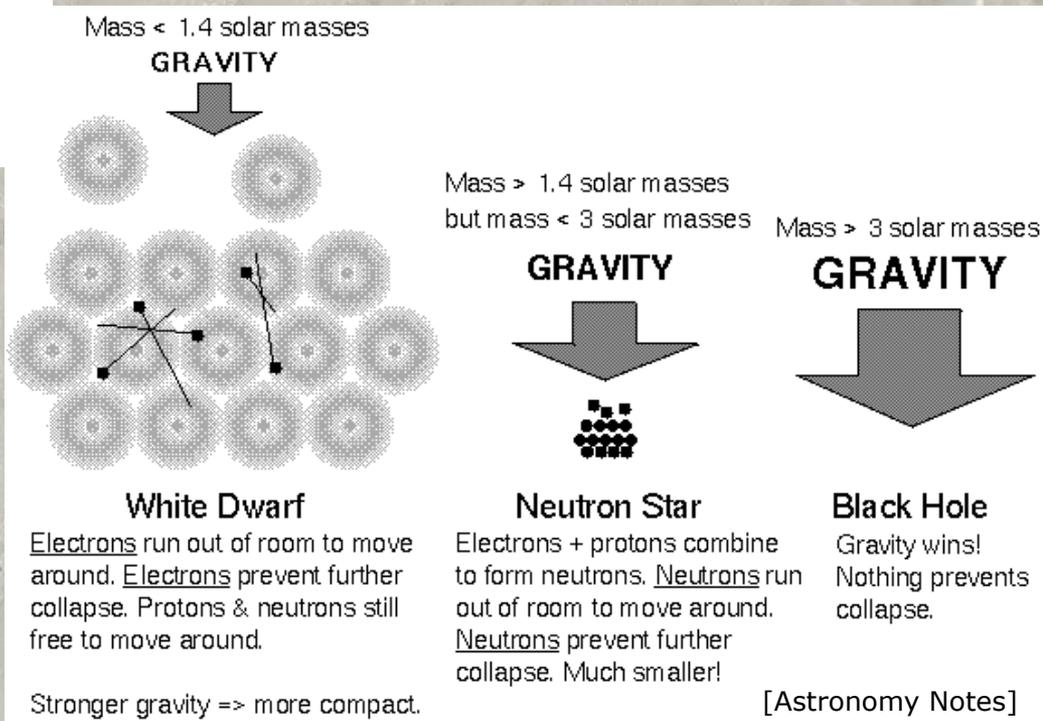
★ Average star (e.g., our Sun)



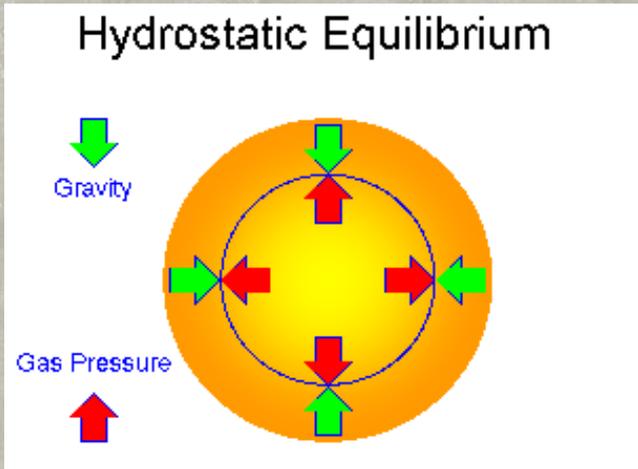
Inner pressure generated by thermonuclear reactions.

★ Compact star

Fermi pressure of degenerate fermion gas equilibrates gravity!



Hydrostatic equilibrium in stars - Newtonian stars:



[Astronomy, Ohio State]



★ Inner pressure generated by thermonuclear reactions.

★ Newton's law + spherical symmetry + hydrostatic equilibrium.

3.2. Limite da deformabilidade

- Para uma distribuição esféricamente simétrica de matéria não-relativística, temos

$p \approx p_0$

$$m(r) = \int_0^r 4\pi r'^2 dr' \rho$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho \quad (2)$$

\vec{F}_in \vec{F}_out

equilíbrio hidrostático

$$\frac{G m(r)}{r^2} dm = [p(r) - p(r+dr)] dA \Rightarrow \left[\frac{dm}{A dr} = \rho \right]$$

$$\Rightarrow \left[\frac{dp}{dr} = - \frac{G m(r) \rho}{r^2} \right]$$

[equações de estrutura (1)(2)]

[Modelos Newtonianos!]

Hydrostatic equilibrium in stars - compact stars:



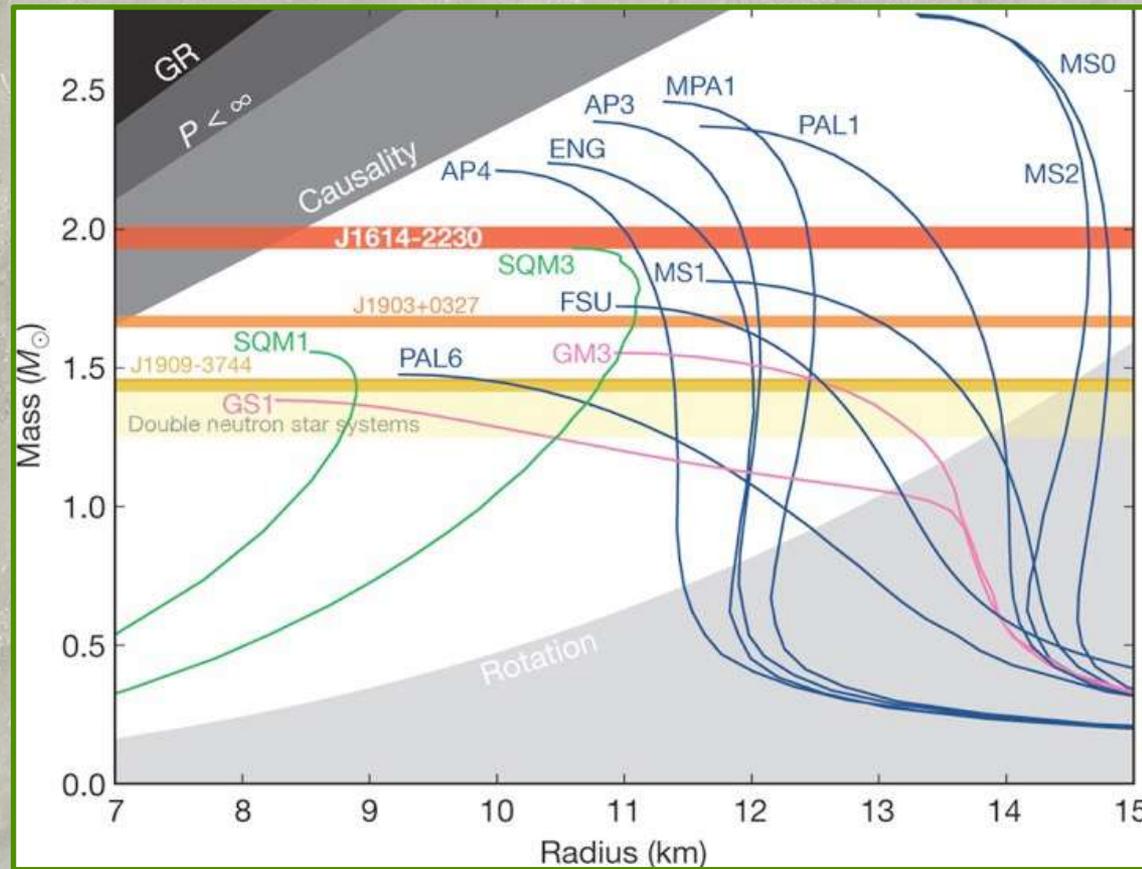
- ★ Tolman–Oppenheimer–Volkov (TOV) equations: Einstein's GR field equations + spherical symmetry + hydrostatic equilibrium.
- ★ In GR, energy density and pressure also contribute to gravity!

$$\frac{dp}{dr} = -\frac{GM(r)\epsilon(r)}{r^2 \left[1 - \frac{2GM(r)}{r}\right]} \left[1 + \frac{p(r)}{\epsilon(r)}\right] \left[1 + \frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$
$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \epsilon(r) ; \quad \mathcal{M}(R) = M$$

- ★ One equation missing in the system: the equation of state!



★ Equation of state: different EoSs define different types of stars (neutron stars, strange stars, hybrid stars).



[Demorest et al (2010)]

★ So, we need the EoS for the kind of matter we expect to find inside neutron stars, which goes way beyond neutrons!



What kind of matter do we expect to find in neutron stars?

- ★ Tough question! So far observations cannot tell much in this respect!
- ★ Some numbers can give a hint, though:

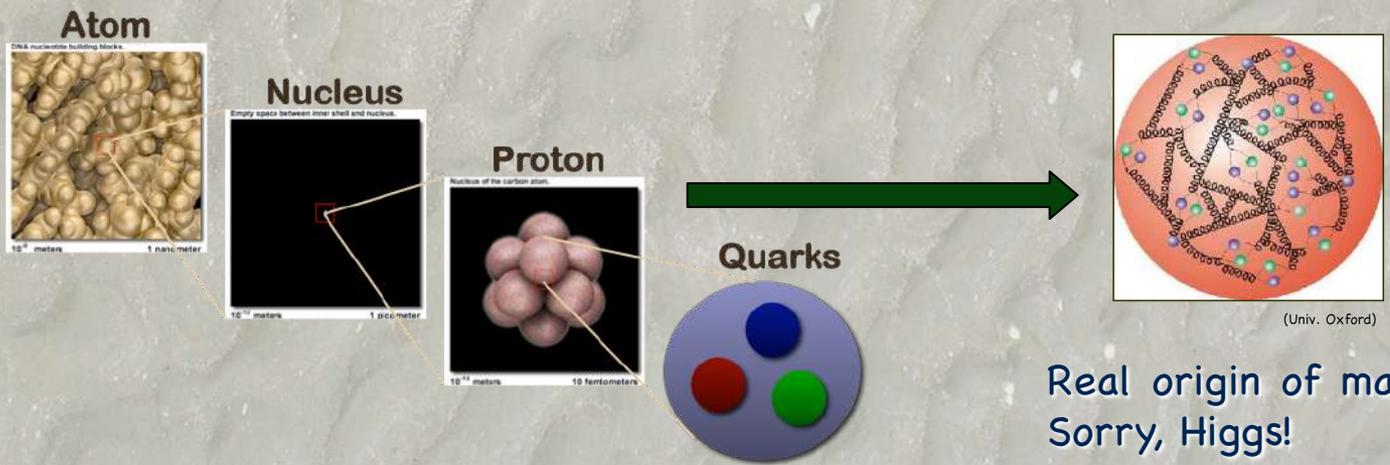
$$n_0 = 3 \times 10^{14} \text{ g/cm}^3 = 0.16 \text{ fm}^{-3}$$

$$n_{\text{core}} \approx (4 - 15) n_0 \quad [\langle n_{\text{Earth}} \rangle \approx 5.5 \text{ g/cm}^3]$$

$$M \approx 1 - 2 \text{ solar masses} \quad [M_S \approx 2 \times 10^{30} \text{ kg} \approx 10^{57} \text{ GeV}]$$

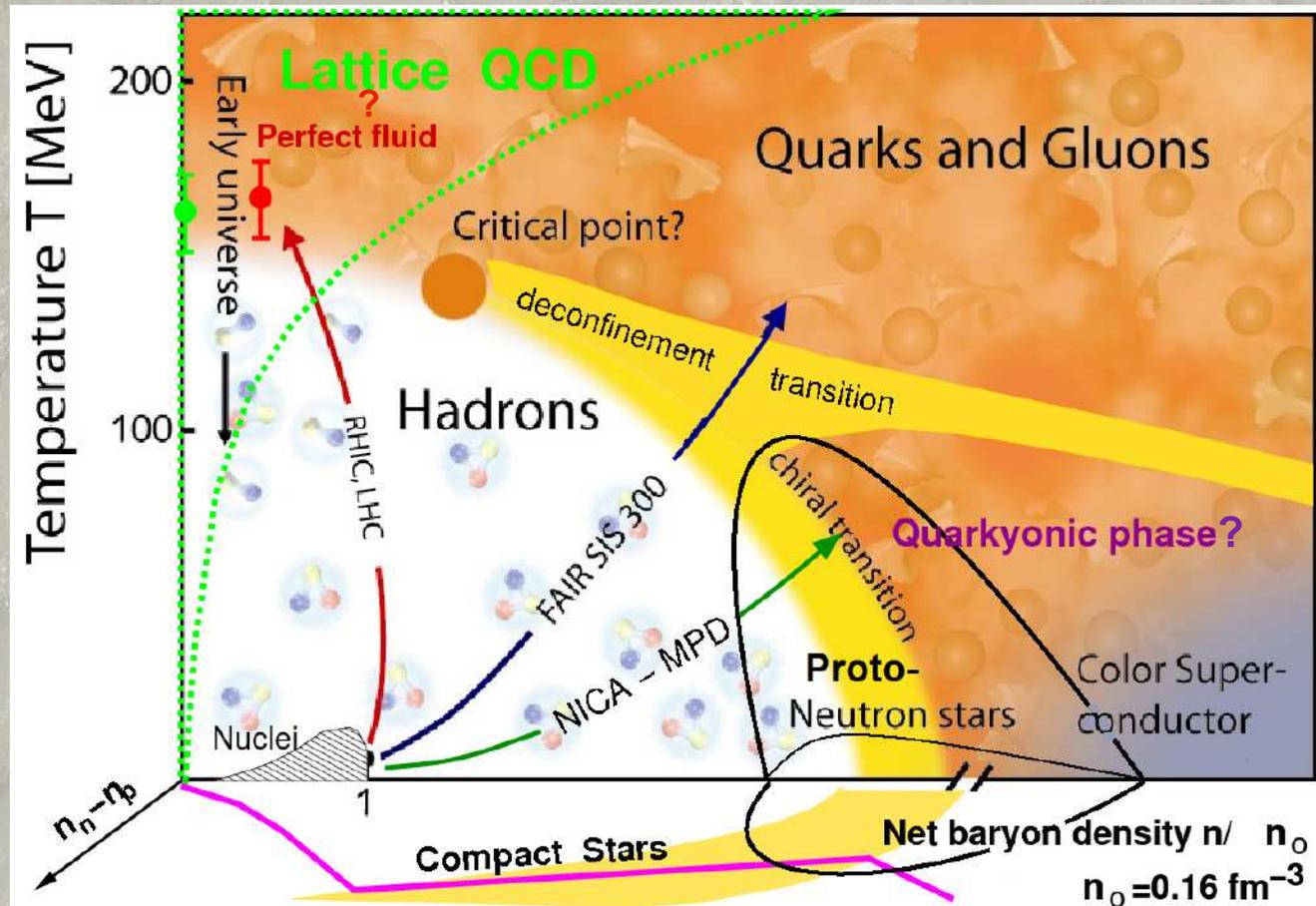
$$R \approx 6 - 16 \text{ Km} \quad [R_S \approx 7 \times 10^5 \text{ km}]$$

➔ QCD at "high" density + General Relativity





Then, neutron stars provide a unique window to strong interactions under extreme conditions!



We need the EoS in this region !!



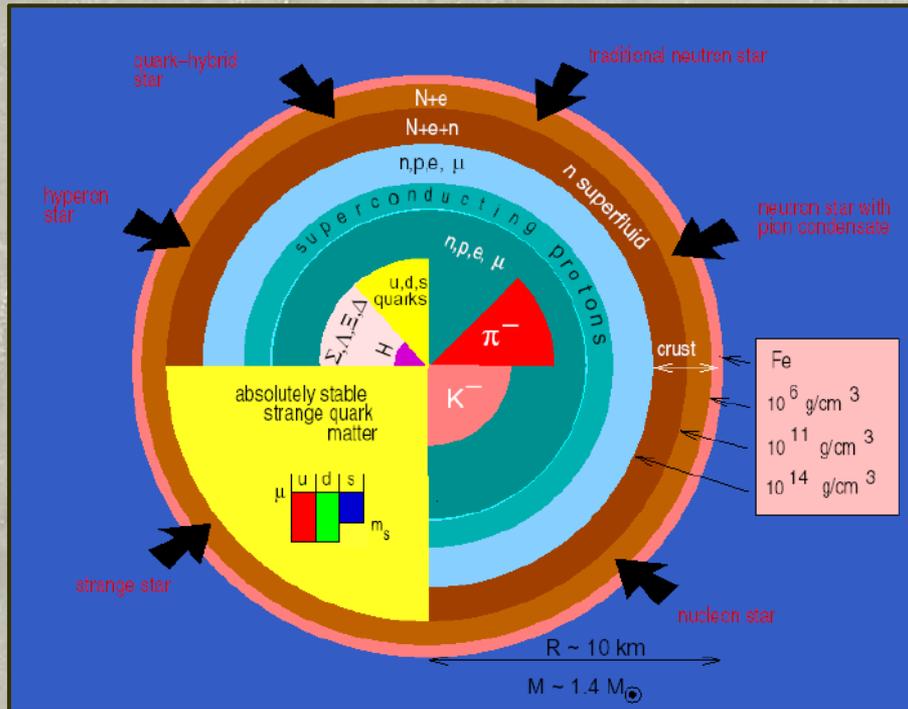
★ So, the kind of matter we expect inside NS is strongly interacting matter (QCD matter) under extreme conditions.

★ This brings several exciting possibilities!

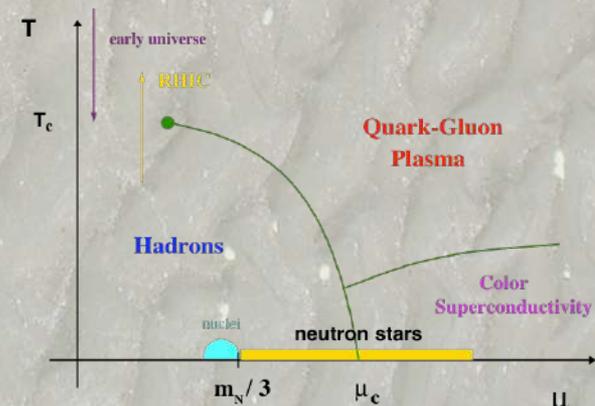
- New phases, condensates, color superconductivity, etc in the core.

- Deconfinement and chiral symmetry partial restoration (might affect SN explosion mechanism via EoS).

- Strange matter (strange stars?).



[F. Weber, 2000]



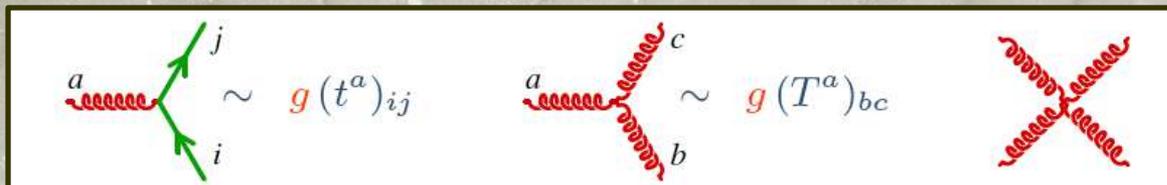


Theory (QCD)

The theory is, in principle, given:

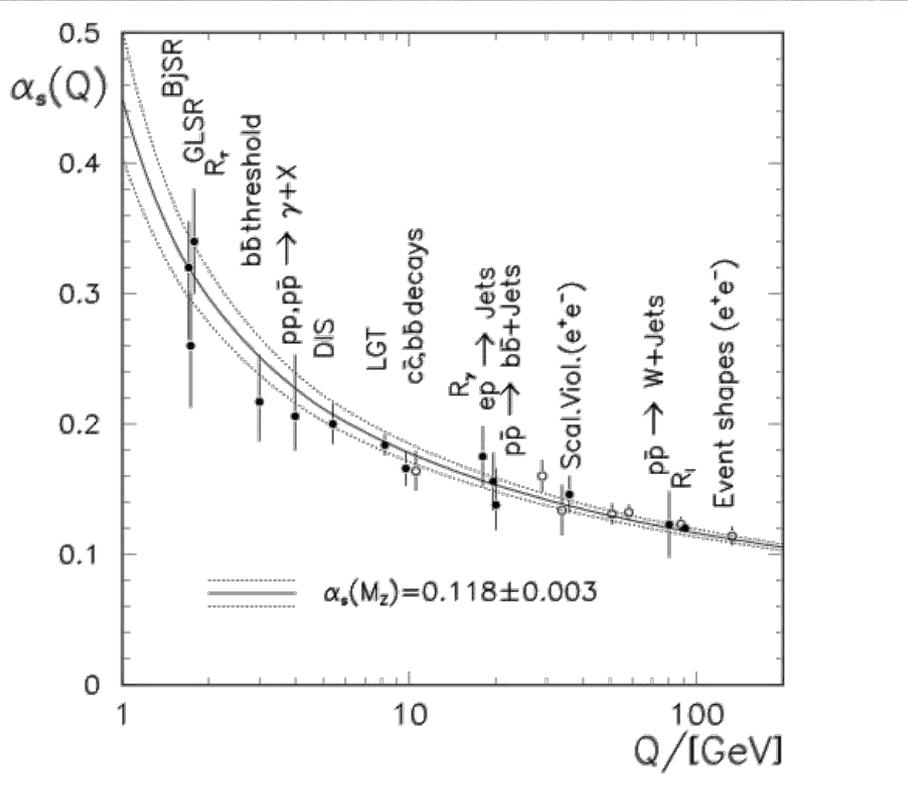
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \sum_f \bar{\psi}_f (i\gamma^\mu \partial_\mu - m_f - ig\gamma^\mu A_\mu^a \tau^a) \psi_f$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$



Then: compute the thermodynamic potential and then the EoS!
Well, it's not so simple...

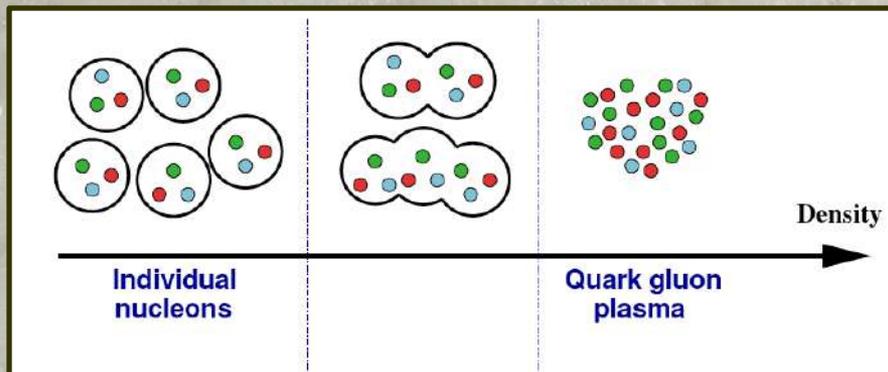
Asymptotic freedom x the vacuum of QCD



- Matter becomes simpler at very high densities (baryon chemical potential μ as energy scale), but very complicated in the opposite limit...

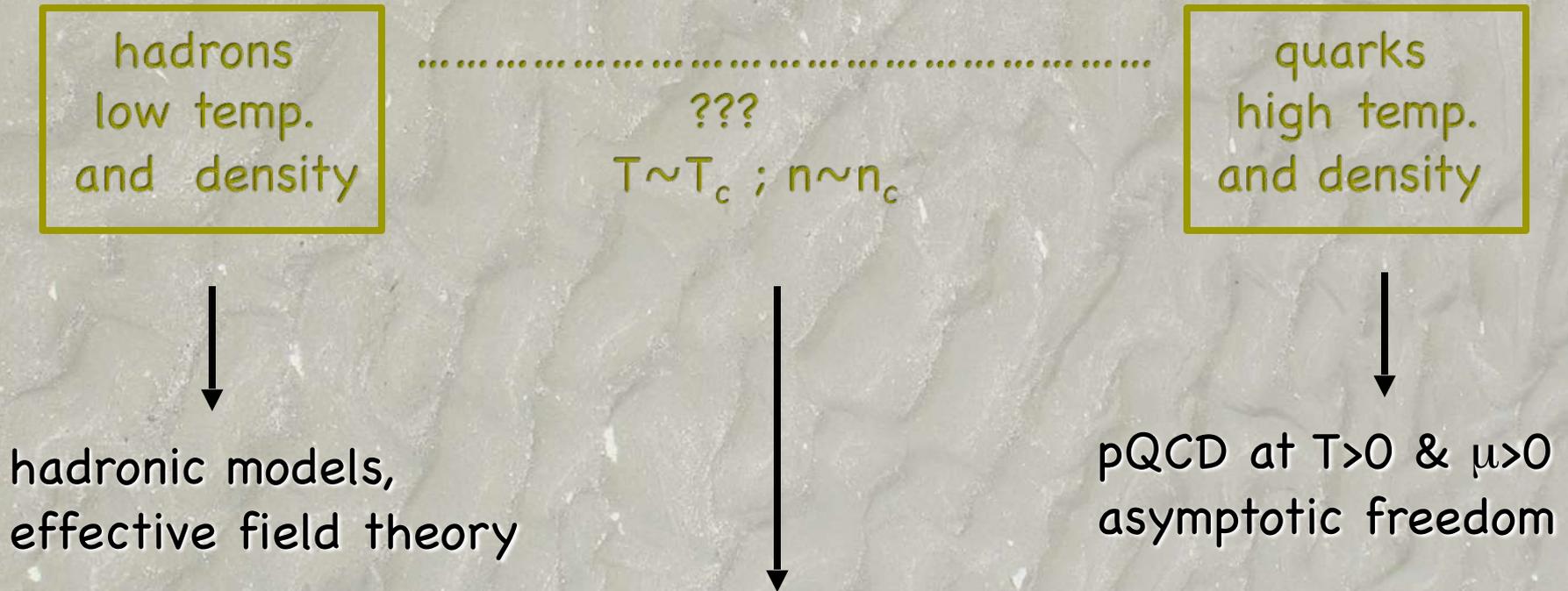
- μ is not high enough in the interesting cases...

- In-medium pQCD needs optimization to produce sensible results.





Equation of state - naïve field map



where all the things that matter happen...

there is no appropriate formalism yet...



Ok, let's not desperate... There are several possibilities...

Some popular examples:

- Very intelligent and sophisticated brute force: **lattice QCD**
- Intensive use of symmetries: **effective field theory models**
- Redefining degrees of freedom: **quasiparticle models**
- "Moving down" from high-energy pQCD
- "Moving up" from hadronic low-energy (nuclear) models



Some serious difficulties:

★ Differently from the hot and dilute case, the cold and dense physical setup suffers from the Sign Problem* -> no Lattice QCD guidance...

★ We are left with the following possibilities, then:

- moving up from effective field theory (EFT)
- models (NJL, LSM, etc and extensions)
- moving down from pQCD & imposing constraints
- the MIT bag model (a long story...)

★ A few (important) things I will not cover in this talk:

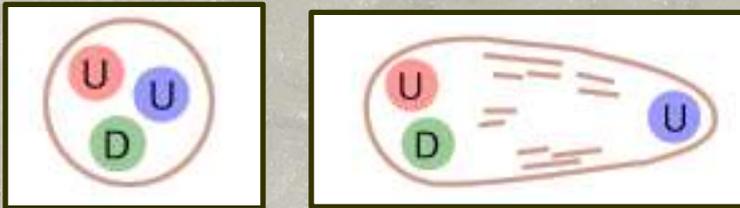
- superfluidity & color superconductivity
- cooling of NS with a QM core
- signals of QM in core-collapse NS
- twin stars
- rotation
- nucleation & burning in QM formation in NS
- magnetic fields in NS

*Sign Problem: in QCD with $\mu \neq 0$, the effective S_{eucl} is complex, so that one does not have a well-defined weight factor any more... Monte Carlo calculations hindered...



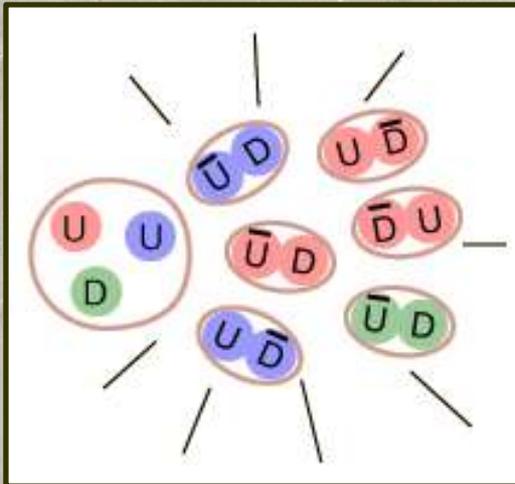
★ Let us start with a discussion of an important but also in a way “bad start”: the MIT bag model, perhaps the most popular approach to quark matter in NS.

★ The MIT bag model (70's) is something REALLY simple:



Asymptotic freedom + confinement in the simplest and crudest fashion: bubbles (bags) of perturbative vacuum in a confining medium.

+ eventual corrections $\sim \alpha_s$



- Asymptotic freedom: free quarks and gluons inside color singlet bags
- Confinement: vector current vanishes on the boundary

★ This makes it easy to use and dangerous...



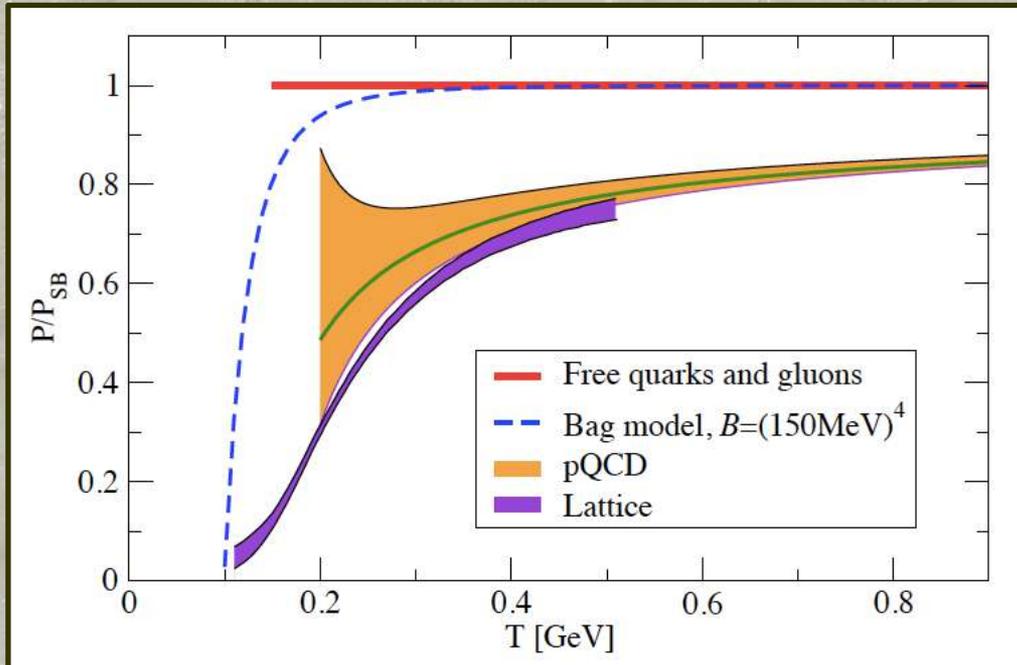
For instance, at finite temperature the pressure has the form:

$$p = \left(\nu_b + \frac{7}{4} \nu_f \right) \frac{\pi^2 T^4}{90} - B$$

★ B: phenomenological constant to be adjusted (for instance, originally to hadron masses) – very unconstrained...

★ Counting of Stefan-Boltzmann degrees of freedom.

Comparison to lattice QCD and pQCD:



[ESF, Kurkela & Vuorinen (2013)]

★ Gives false impression of precision and accuracy. In fact, very crude!

★ Comparison to lattice shows that it misses: the nature of the phase transition, relevant scales (quantitative), general behavior of the EoS, ...

[1 GeV ~ 10¹³ K ~ 10⁶ T_{sun} (core)]



Cold quark matter via pQCD, also an old story...

Freedman & McLerran, 1977-1978

Baluni, 1978 ; Toimela, 1980's

Kajantie et al, 2001 ; Peshier et al, 1999-2003

Blaizot, Iancu & Rebhan, 1999-2003

ESF, Pisarski & Schaffner-Bielich, 2001

Andersen & Strickland, 2002

Rebhan & Romatschke, 2003

Vuorinen, 2004-2007



Quark stars & hybrid stars
with large masses from pQCD
(15 years ago...)

[sorry for unavoidable omissions...]

Including the strange quark mass:

Freedman & McLerran, 1977-1978; Baluni, 1978
(considered irrelevant for over 20 years...)

ESF and Romatschke, 2005

Kurkela, Romatschke & Vuorinen, 2010



NLO, NNLO: vacuum diagrams & ring
resummation (the state of the art)

New T & μ state of the art:

Cool quark matter

Kurkela & Vuorinen, 2016



Higher order cold QM:

Gorda et al, 2018, 2021

ESF, Kurkela & Vuorinen (2014, 2016)

Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)

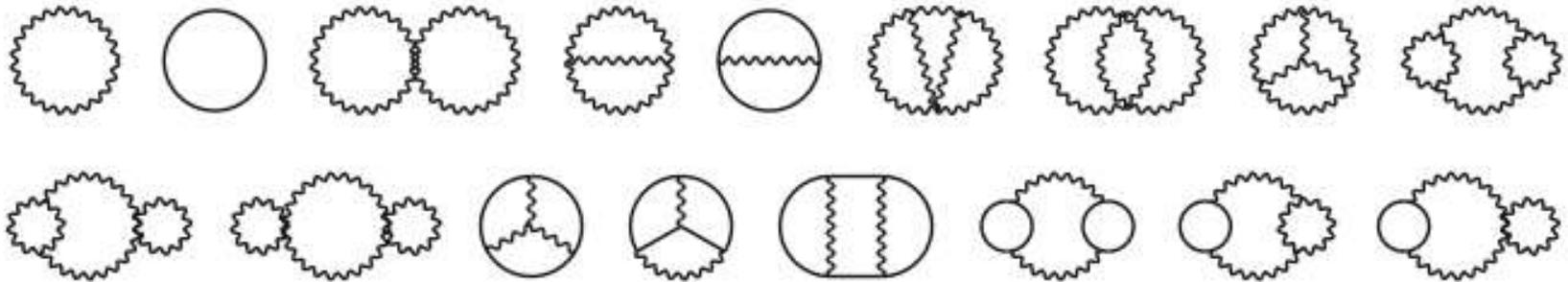


Very high densities:

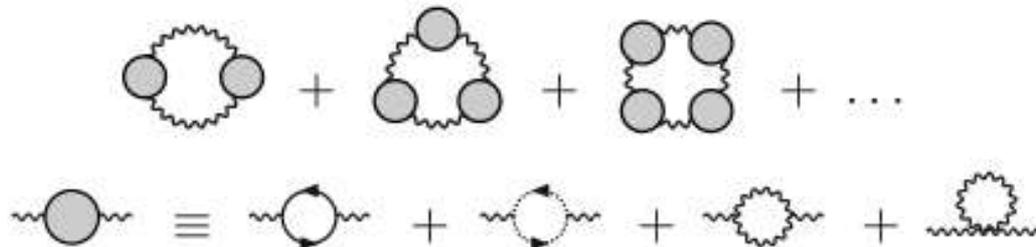
Pressure from pQCD (diagrammatically speaking...)

(Vuorinen, 2004)

Up to three loops (g^4) the diagrams to be evaluated are (without ghosts)



Also at low T the first IR problems (for the pressure) appear at 3-loop order: on top of 2PI graphs must resum

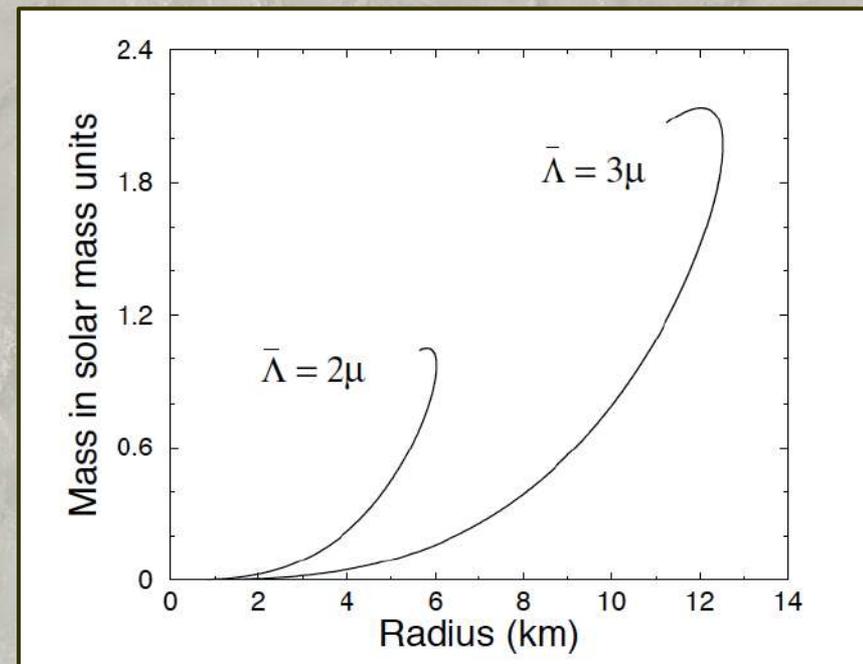
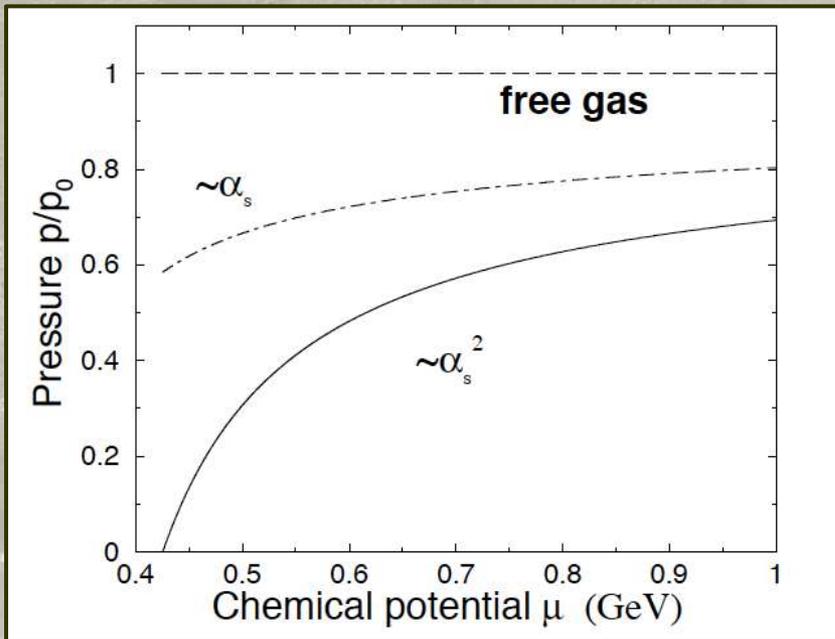
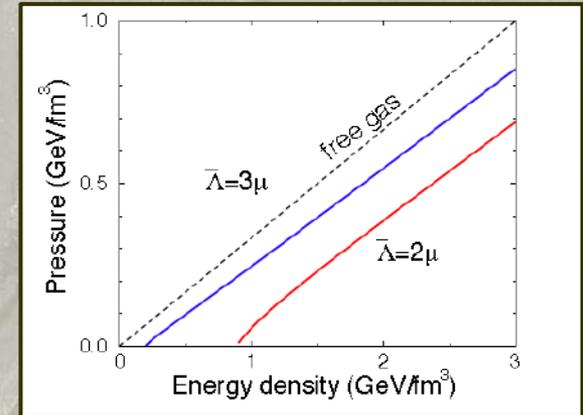


EoS and NS masses from cold ($T=0$) pQCD — 20 years ago:



[ESF, Pisarski & Schaffner-Bielich (2001)]

- Gas of massless u, d, s quarks: charge neutrality and β equilibrium achieved for $\mu_s = \mu_d = \mu_u$ (no need for electrons).
- Interaction taken into account perturbatively up to (RG running) α_s^2 , no bag constant.

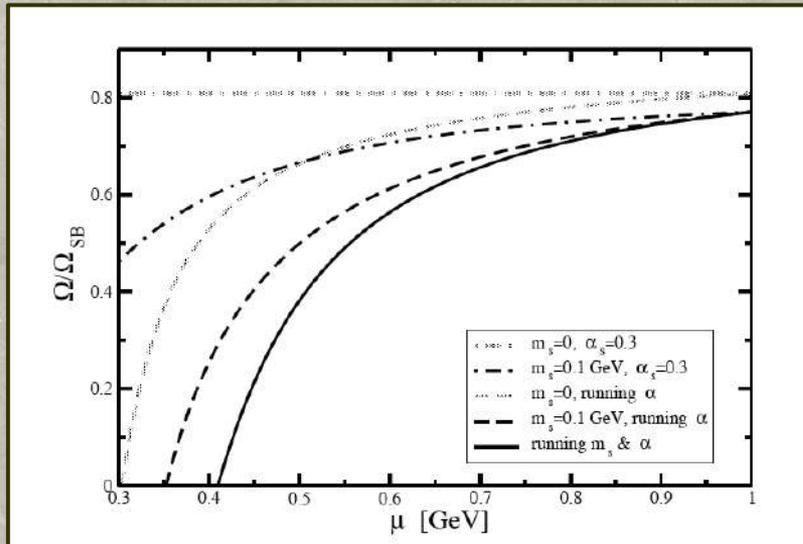


★ Large masses allowed for quark stars!



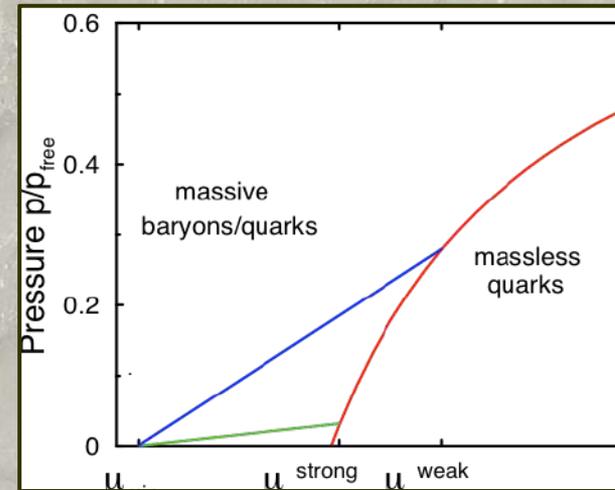
★ Strange quark mass correction:

[ESF & Romatschke (2005)]

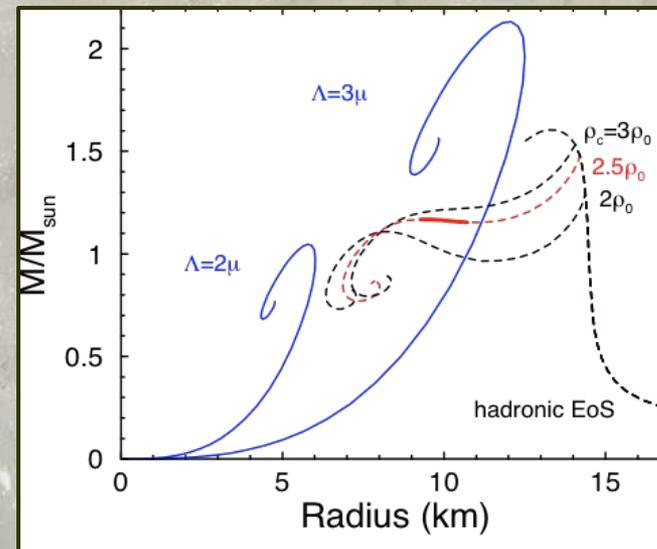


★ Matching and new class of stars (from pQCD + hadronic mantle):

[ESF, Pisarski & Schaffner-Bielich (2002)]



- Quark core with a hadronic mantle.
- Smaller, denser companion (twin) to a hybrid (more hadronic) star.
- Several results for twin stars in the literature.





Sometimes, after a long and subtle calculation, one realizes that **the complicated result can be very well adjusted** (EoS, 1st & 2nd derivatives) **by a simple and compact function:**

Previously: Fraga, Pisarski & Schaffner-Bielich (2001)
Alford et al. (2005)



effective bag model

Using the complete results from Kurkela, Romatschke & Vuorinen (2010):

[ESF, Kurkela & Vuorinen (2013)]

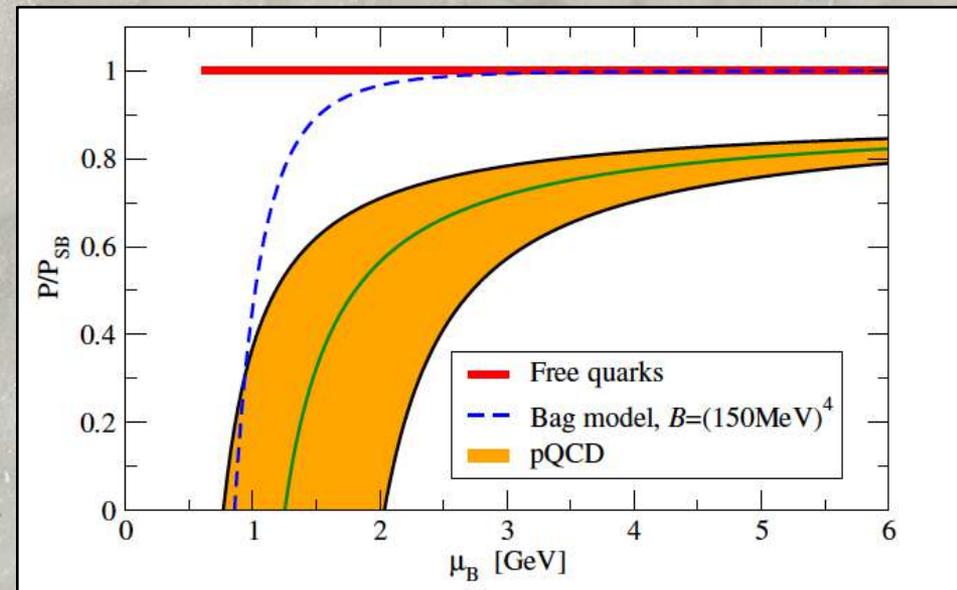
$$P_{\text{QCD}}(\mu_B) = P_{\text{SB}}(\mu_B) \left(c_1 - \frac{a(X)}{(\mu_B/\text{GeV}) - b(X)} \right)$$

$$a(X) = d_1 X^{-\nu_1}, \quad b(X) = d_2 X^{-\nu_2}$$

$$P_{\text{SB}} = \frac{3}{4\pi^2} (\mu_B/3)^4$$

$$X \equiv 3\bar{\Lambda}/\mu_B$$

Plug-and-play cold QCD EoS!





Caveats for the pQCD EoS

- ★ Validity is assured only for very high densities. Strictly speaking, way beyond the relevant region for NS.
- ★ However, this is the theory of strong interactions there.
- ★ There is always a dependence on the renormalization scale. This is unfortunate, of course, but it also provides a measure of our uncertainty (which is a good thing!).
- ★ Nevertheless, pQCD gives us something that no model can deliver: the correct behavior at large enough densities and robust constraints. And so we can ask different questions...



Asking questions we can answer using controlled approximations:

[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]

★ **A very important and challenging question:** is there deconfined quark matter in the core of compact stars?

Very hard to answer in a definite (model-independent) way.

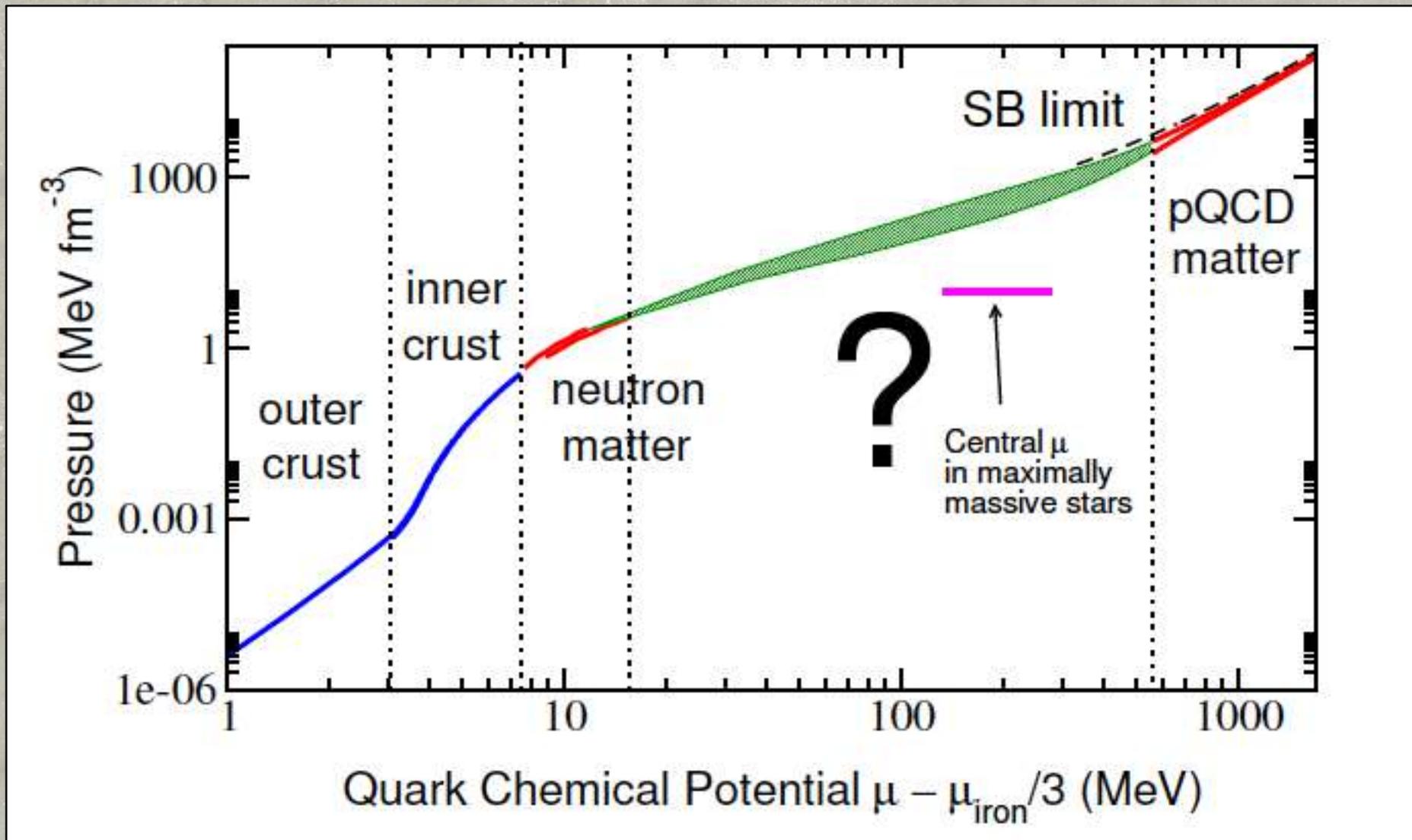
[However, see Annala et al (2019)]

★ **A related but totally different question:** does the fact that at asymptotically high densities one must have deconfined quark matter constrain the equation of state for compact stars?

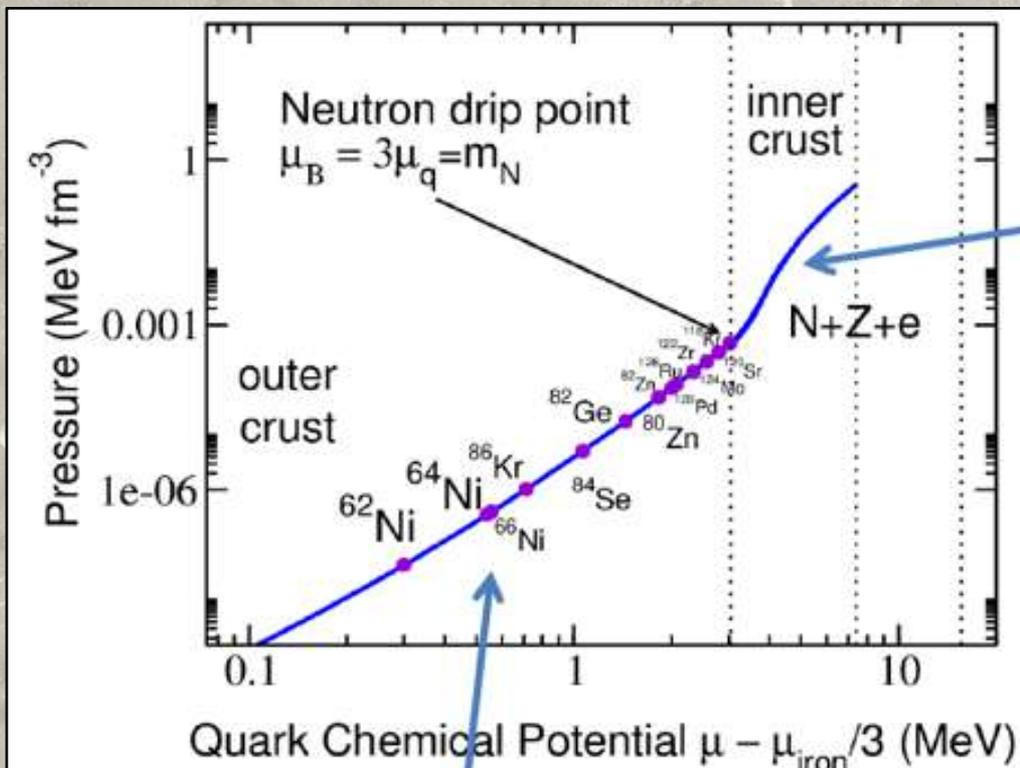
Yes!

★ **Main message:** even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities (which is known perturbatively) affects dramatically the EoS for compact stars!

In fact, now we know that the EoS is constrained to:



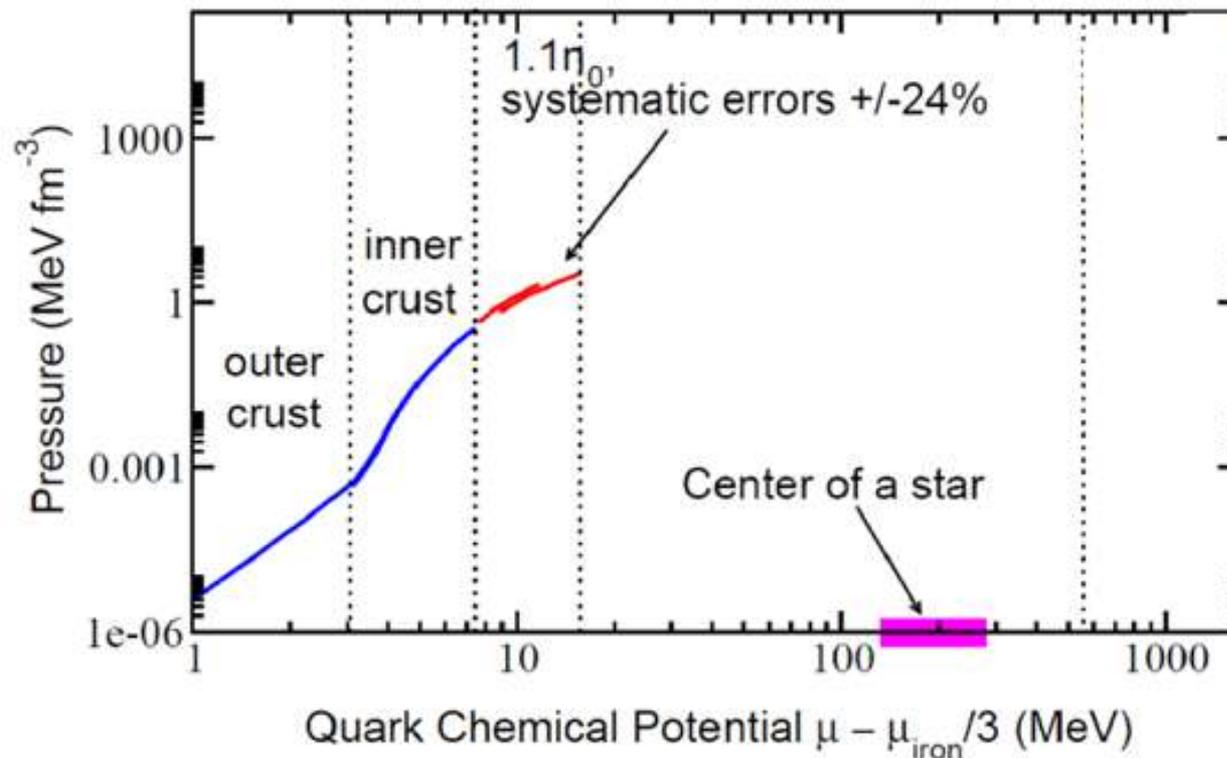
[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



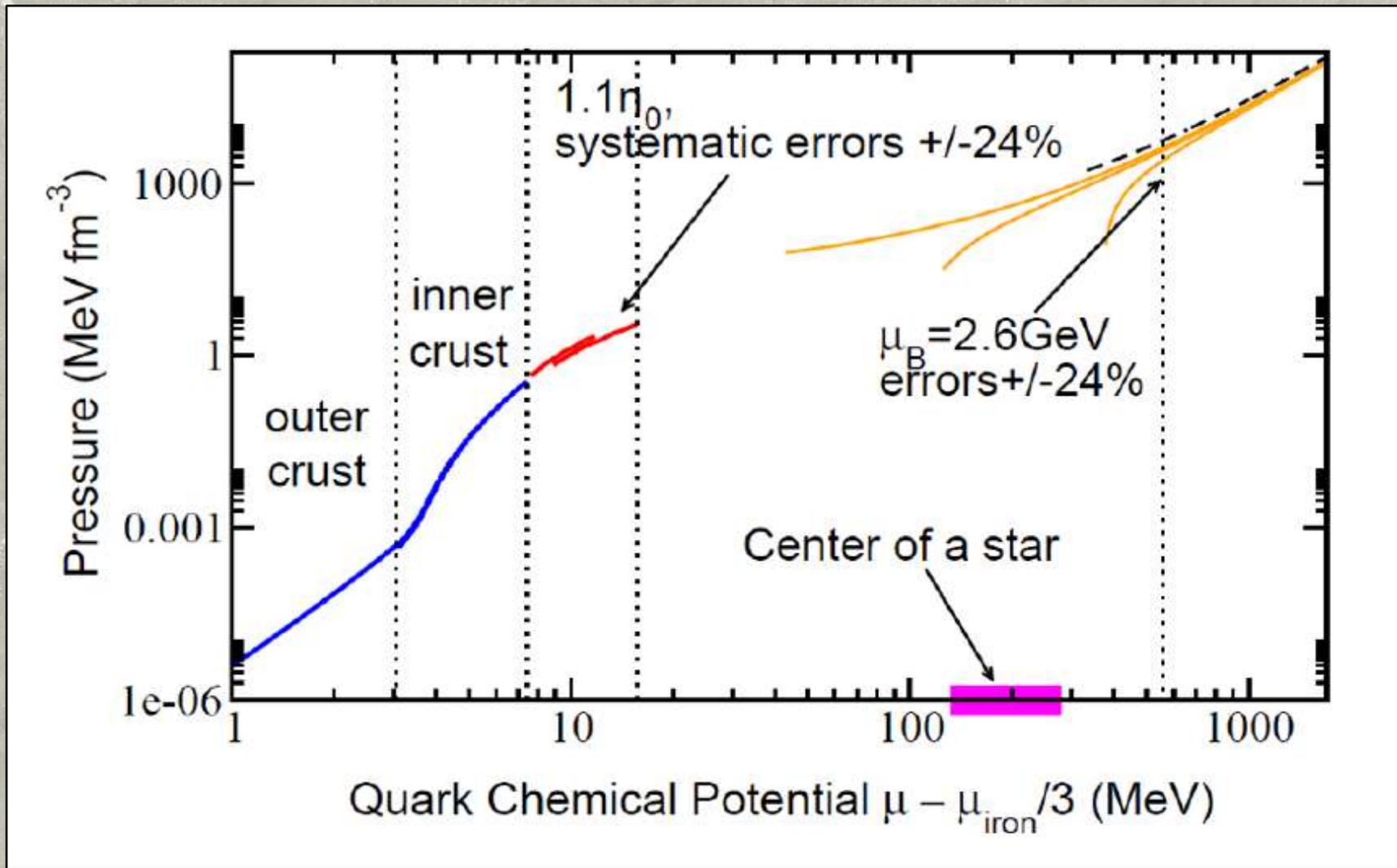
- Neutron gas with nuclei and electrons
- NN interactions important for collective properties; modeled via phenomen. potential models
- Eventually need 3N interactions, boost corrections,...

- Lattice of increasingly neutron rich nuclei in electron sea; pressure dominated by that of the electron gas
- At zero pressure nuclear ground state ^{56}Fe

Closer to saturation density:



- Closer to saturation density n_s , need many-body calculations within Chiral Effective Theory, including 3N and 4N interactions
- At $1.1n_s$, errors $\pm 24\%$ - mostly due to uncertainties in effective theory parameters
- State-of-the-art NNNLO Tews et al., PRL 110 (2013), Hebeler et al., APJ 772 (2013)



Uncertainties mostly from renormalization scale dependence, running of α_s & value of the strange quark mass.

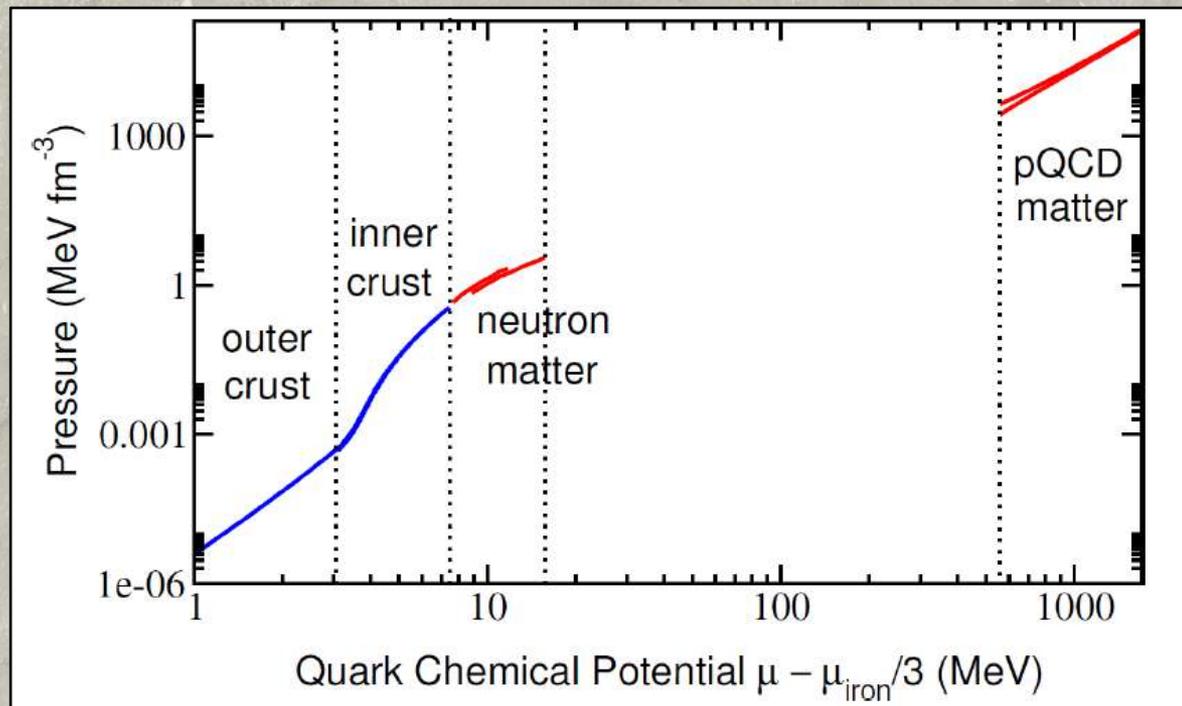
The two limits are given:



Any interpolation in between is bound to satisfying the extrema



strong constraints!



★ One can use different phenomenological models to study this region, satisfying the “boundary” constraints.

★ We chose, instead, to use a **multiple piecewise polytropic parametrization for the EoS**: $p_i(n) = \kappa_i n^{\gamma_i}$

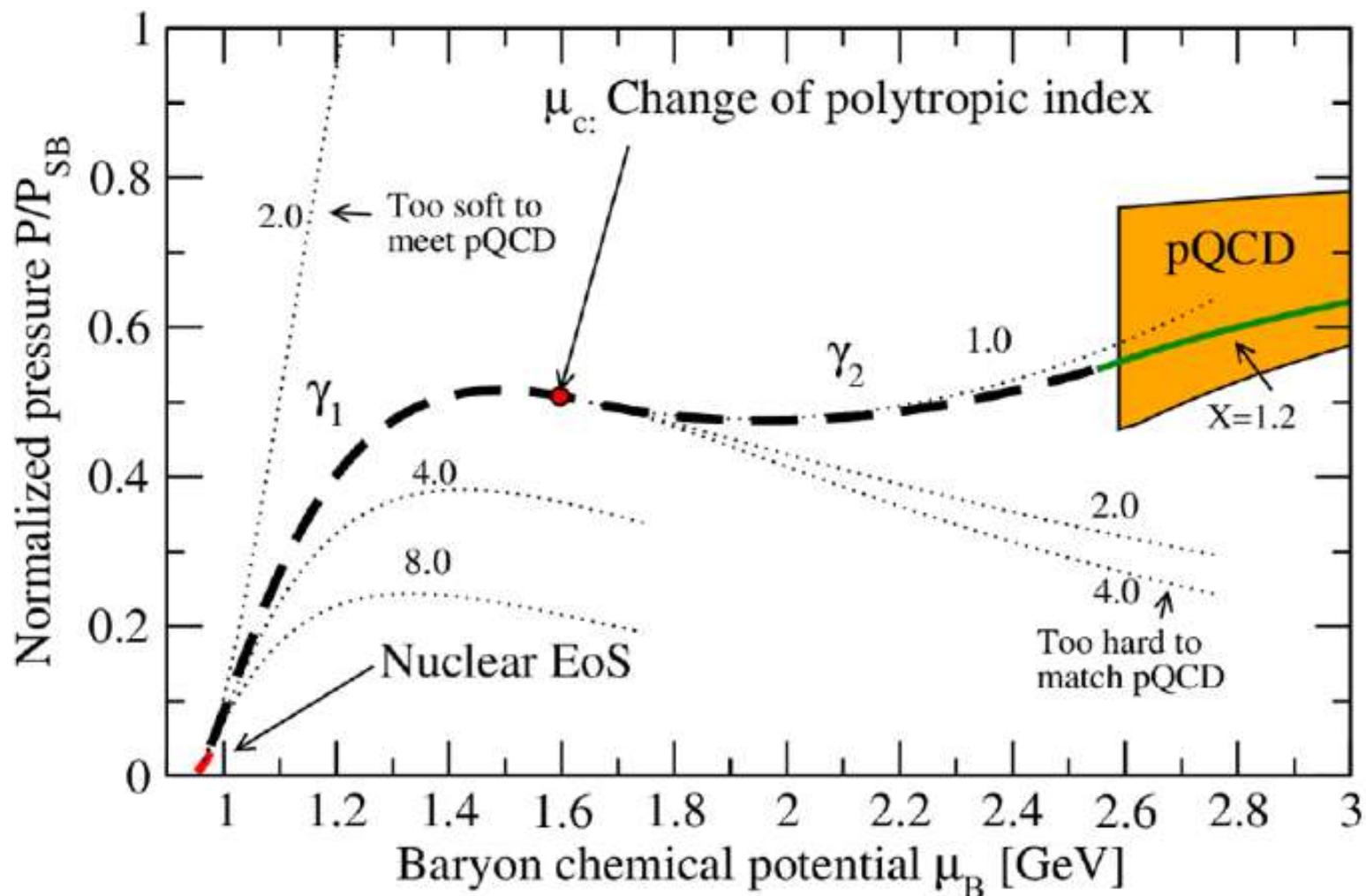
[Hebeler, Lattimer, Pethick & Schwenk (2013)]



So, by this parametrization, we quantify our ignorance by varying all parameters requiring the following:

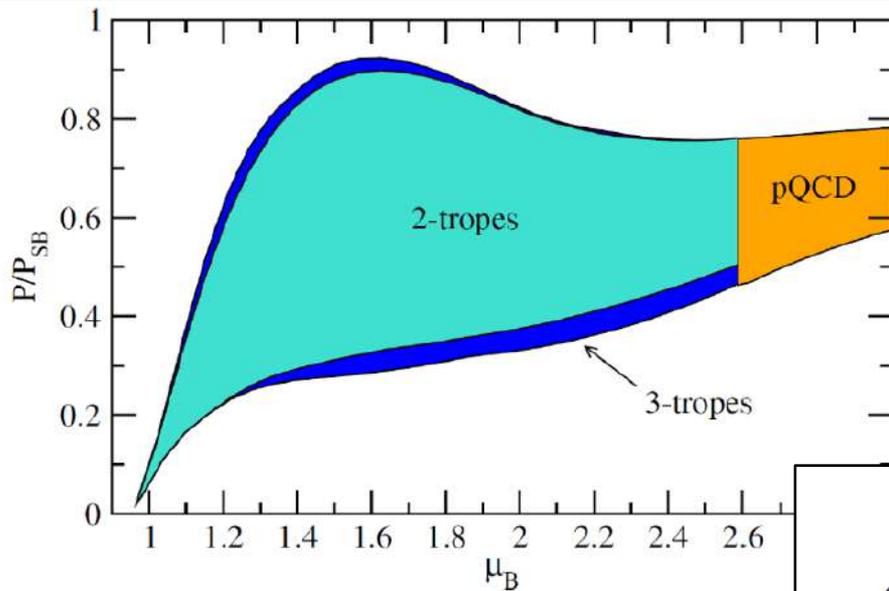
- ★ a smooth matching to nuclear and quark matter EoSs
- ★ **smoothness**: continuity of pressure & density when matching monotropes (can be relaxed)
- ★ **causality**: $c_s \leq 1$ (asymptotically equivalent to $\gamma \leq 2$)
- ★ possibility to support a **two solar mass star**

Illustration - 2 tropes:



Solutions exist for $\mu_c \in [1.08, 2.05]$ GeV, $\gamma_1 \in [2.23, 9.2]$, $\gamma_2 \in [1.0, 1.5]$

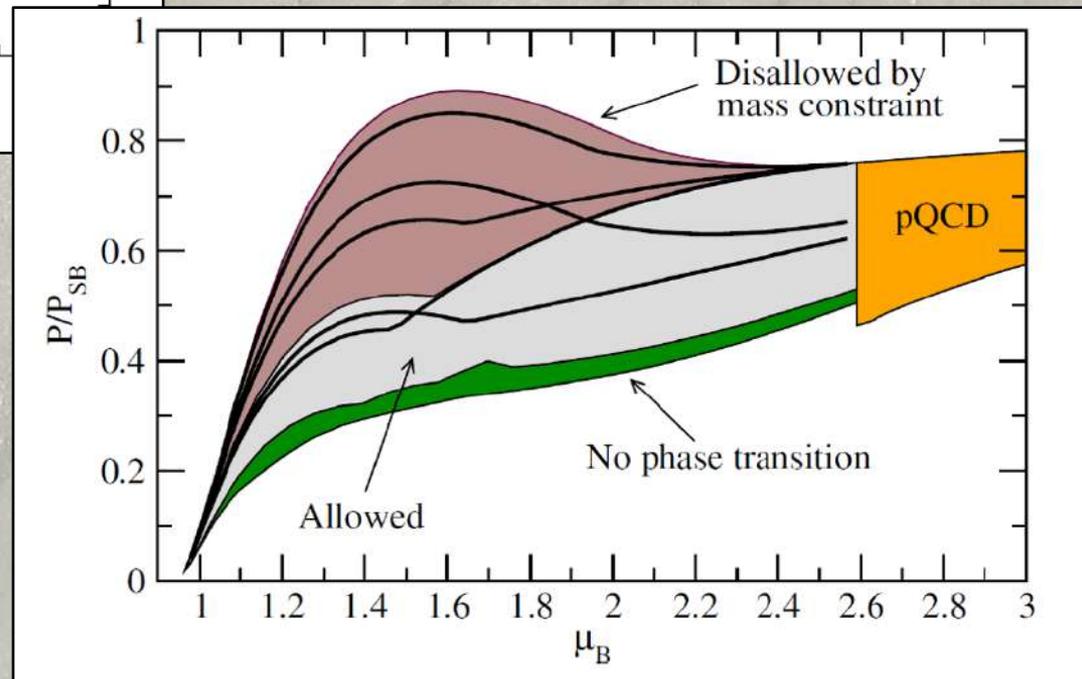
Constraining the EoS



★ the 2-tropes case is the minimum necessary and also seems to be enough to do the job.

★ Allowing for $2M_{\text{sun}}$ stars constrains dramatically the band!

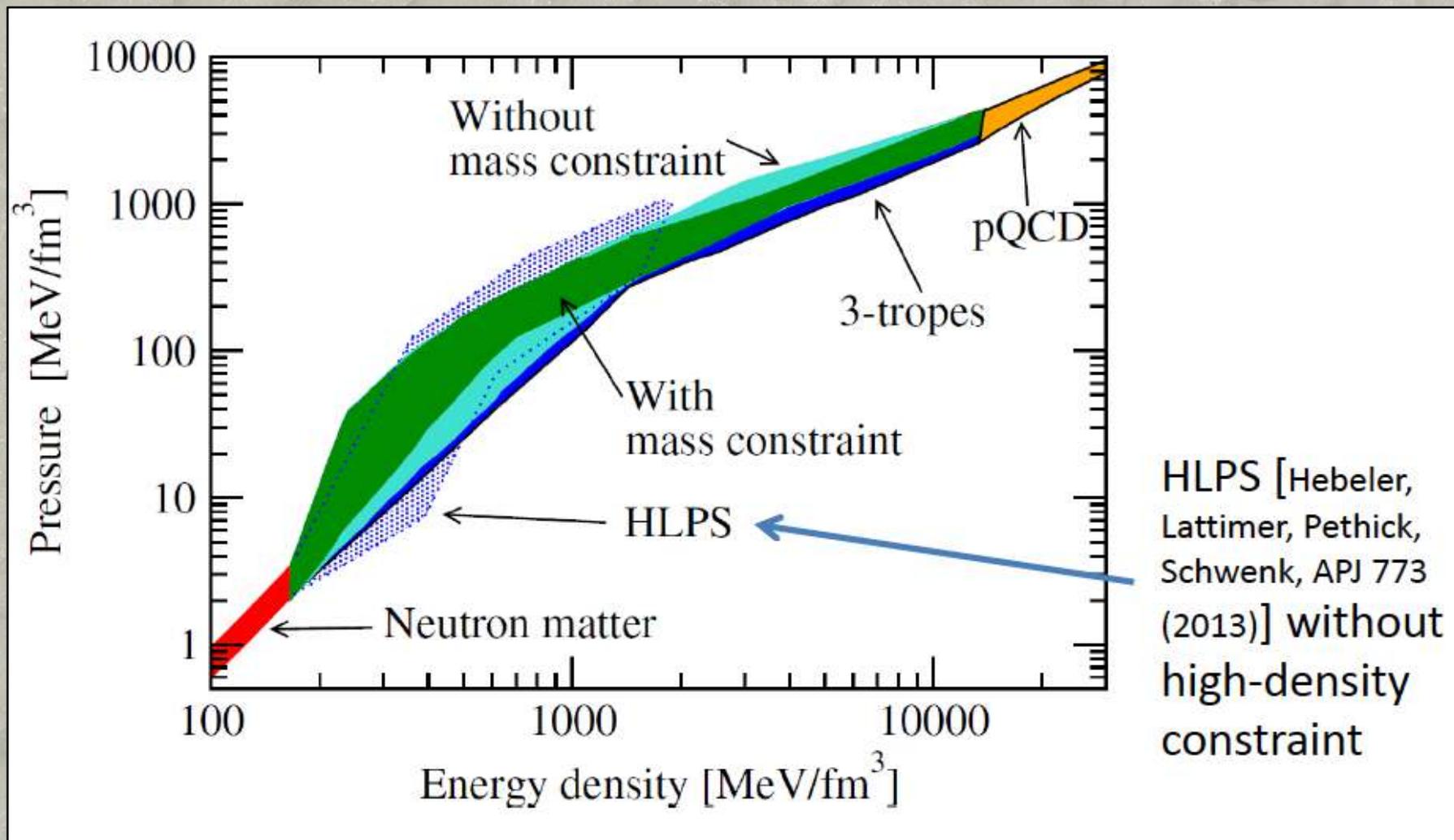
★ Implementing a 1st-order transition shrinks modestly the band.



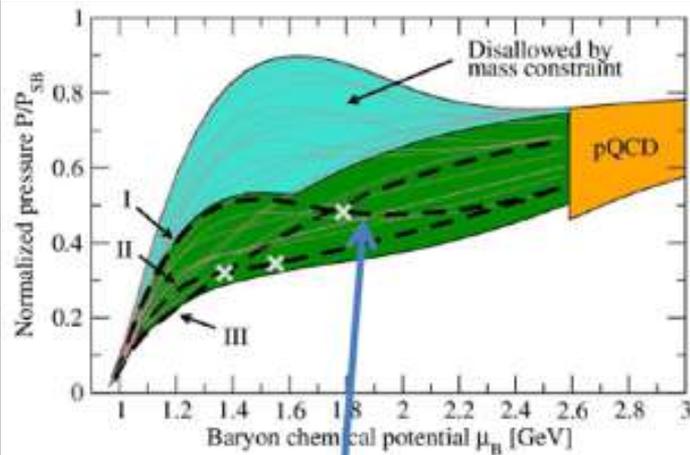
The band for pressure vs. energy density



[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



Implications for compact stars



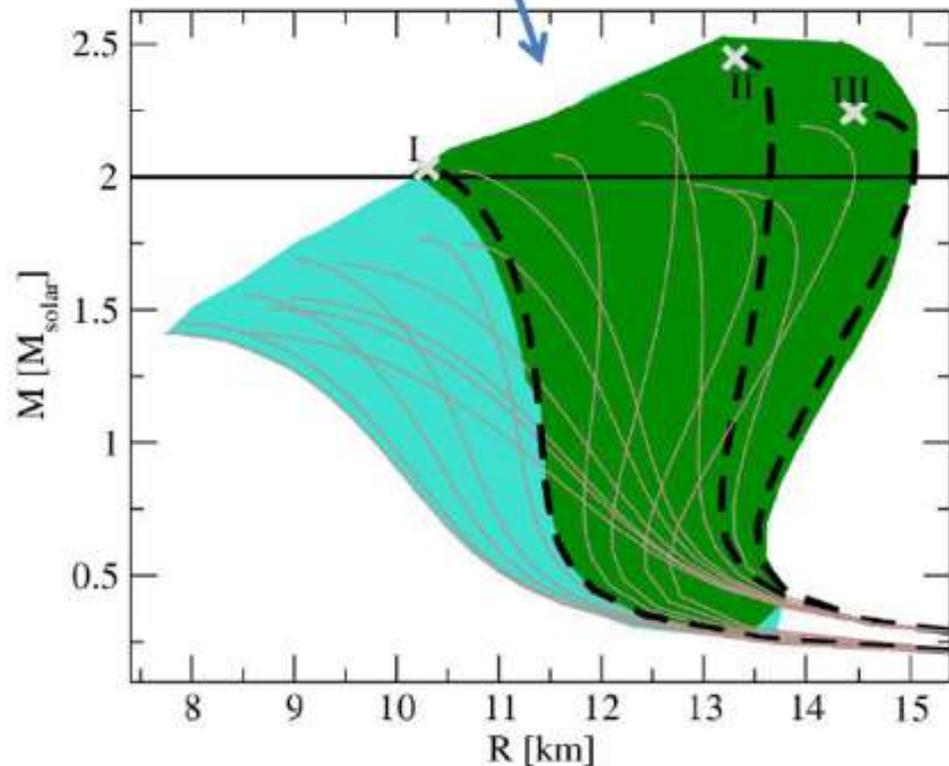
Maximal masses up to $2.5M_{\text{sun}}$!

Central densities

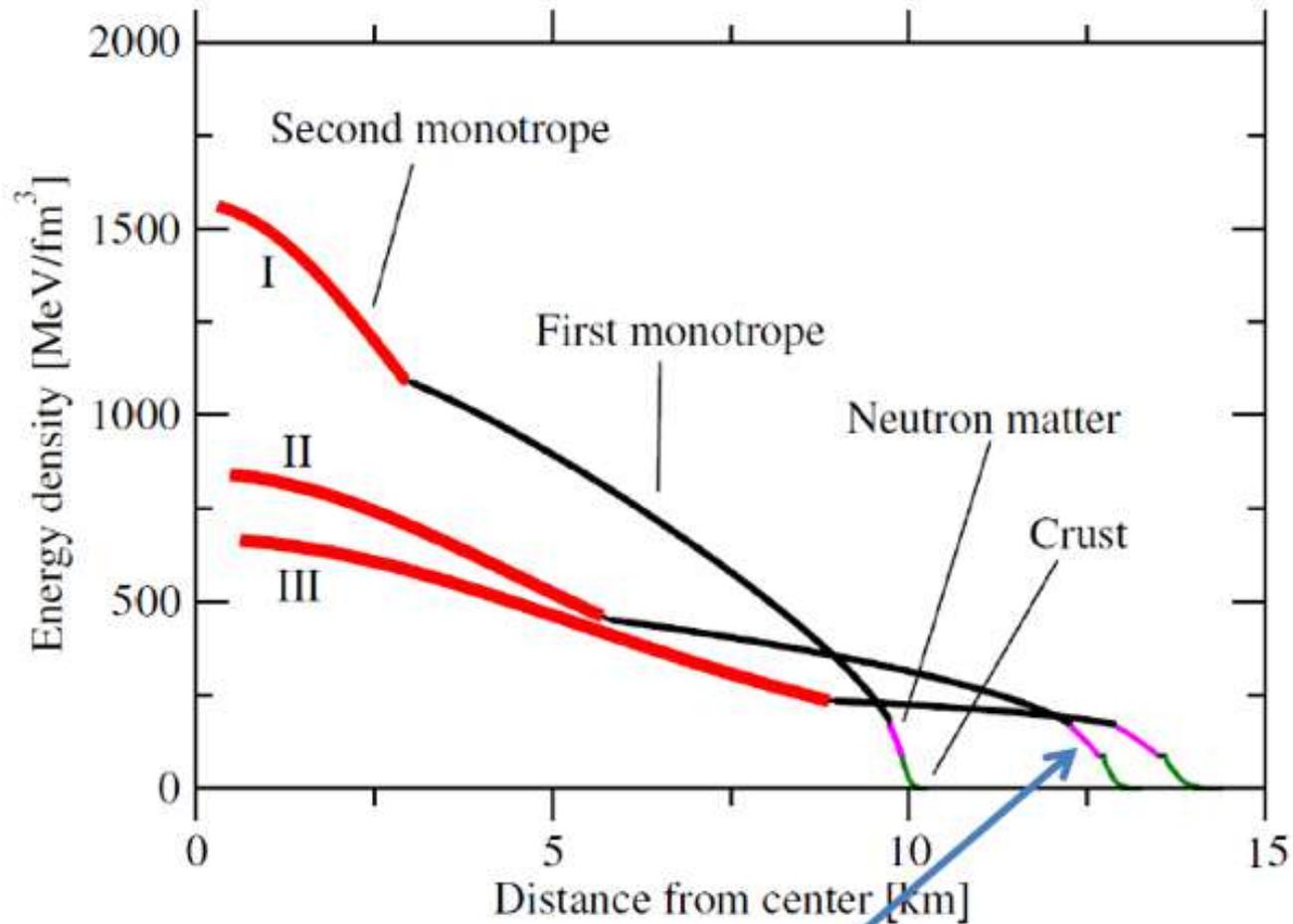
$$n_{\text{central}} \in [3.7, 14.3]n_s$$

Large reduction in EoS uncertainty due to tension from mass constraint: Large stellar masses require stiff EoS, matching to pQCD soft

\Rightarrow EoS uncertainty down to 30% at all densities



Energy density profile



Nuclear matter EoS only used very close to star surface – yet important effects from matching



Most recent results

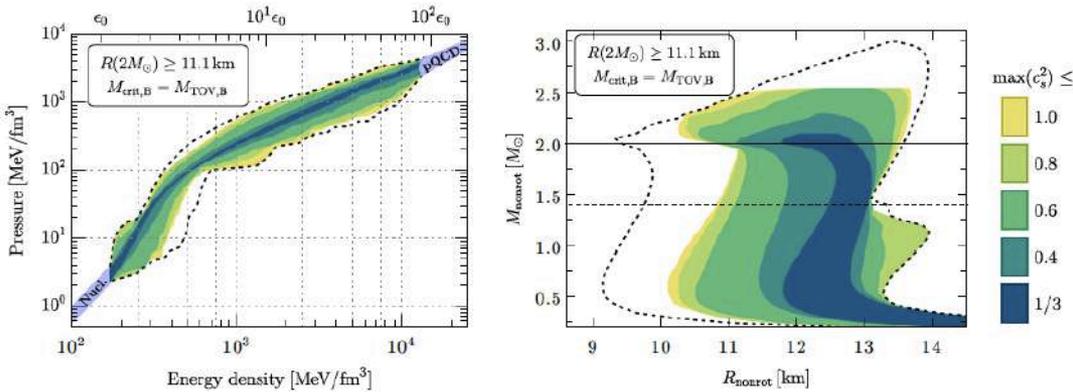


FIG. 1: The impact on the allowed EoS (left) and MR (right) regions arising from a radius measurement for PSR J0740+6620, implemented as $R_{\text{nonrot}}(2M_{\odot}) \geq 11.1$ km, and the GW170817 BH formation hypothesis with $M_{\text{crit,B}} = M_{\text{TOV,B}}$ (here, “nonrot” indicates that we ignore the rotation of the star). The resulting ensemble supports EoSs with moderate speeds of sound, which are known to be compatible with sizeable QM cores [46]. The color coding used here refers to the maximal value that the speed of sound squared c_s^2 reaches at any density, with the lower- c_s^2 regions drawn on top of the higher- c_s^2 regions. The EoS and MR bands obtained by only imposing $M_{\text{TOV}} \geq 2.0M_{\odot}$ and the GW170817 BH formation hypothesis ($\Lambda < 720$ (low-spin priors)). In the EoS figure, $\epsilon_0 \approx 150$ MeV/fm³ represents the nuclear saturation energy density. The blue regions illustrate the low- and high-density EoSs given by theoretical CET and pQCD calculations.

★ Effects from different allowed max speed of sound, different results from radius measurement, tidal deformability, etc

★ Randomly generated model-independent equations of state & corresponding rotating stellar structures without the use of quasi-universal relations.

★ Compatibility & impact of various hypotheses and measurements on the EoS (merger product in GW170817, binary merger components in GW190814, radius measurements of PSR J0740+6620).

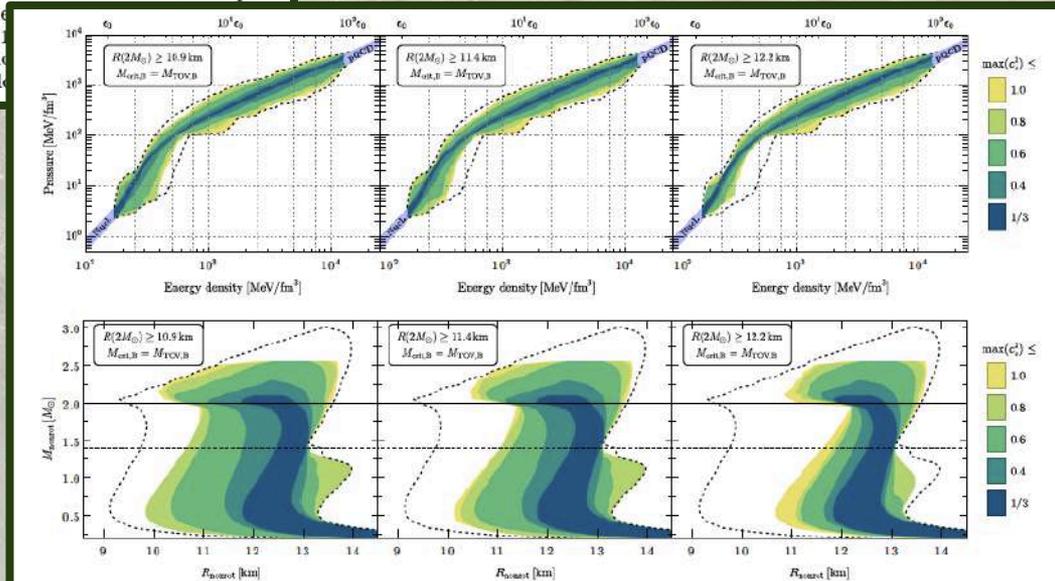


FIG. 7: The impact on the allowed EoS band and MR region from assuming radius measurements for PSR J0740+6620, implemented as $R_{\text{nonrot}}(2M_{\odot}) \geq 10.9$ km (left), $R_{\text{nonrot}}(2M_{\odot}) \geq 11.4$ km (middle), $R_{\text{nonrot}}(2M_{\odot}) \geq 12.2$ km (right). Here, we assume the GW170817 BH formation hypothesis with $M_{\text{crit,B}} = M_{\text{TOV,B}}$ and the constraints $\Lambda_{\text{GW170817}} < 720$ and $M_{\text{TOV}} \geq 2.0M_{\odot}$. Dotted lines correspond to the ensemble given in Fig. 2.



Final remarks

- ★ **Message 1:** Although strictly valid for very large densities, pQCD provides a controllable approximation for the EoS that can be systematically improved.
- ★ **Message 2:** even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities affects dramatically the EoS for compact stars! A robust constraint!
- ★ The existence of 2 solar mass stars strongly constrains the EoS. Having a 1st-order transition or a crossover not so much. Several new constraints not discussed here.
- ★ Fruitful interplay between particle/nuclear physics and neutron star physics!
- ★ Lots of new data coming from gravitational waves (LIGO-Virgo) & measurements of NS radii (NICER).
- ★ Very active field, in theory and observations. Exciting results appearing all the time now! **So, yes, neutron stars & particle physics! :-)**