Probing thermal fluctuations through scalar test particles

Alexsandre Leite Ferreira Junior

PPGCosmos - UFES

PPGF - UNIFEI

alexsandrej@gmail.com

March 19, 2021

G. Camargo, V. A. De Lorenci, A.L. Ferreira Junior, C. H. Ribeiro

arXiv:2010.07146

Stochastic motion on a charged particle due to the change in the background field

- Stochastic motion induced by a change in the background field, originally proposed by Ford & Yu (2004) [arXiv:0406122].
- Negative dispersion in the velocity $\rightarrow$ Subvacuum effects.
- General case of a massive scalar field in D spatial dimensions [Camargo et al. (2019) arXiv:1906.08322].
- In this paper we investigated temperature effects, and showed that increasing temperature facilitates the detection of subvacuum phenomena.

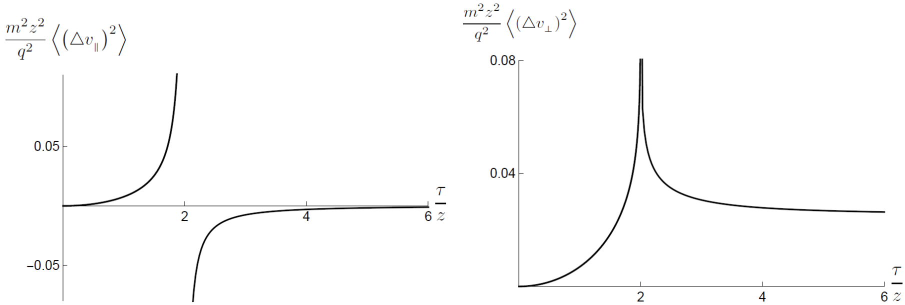


Figure: Figures taken from De Lorenci et al. (2014) arXiv:1404.3115

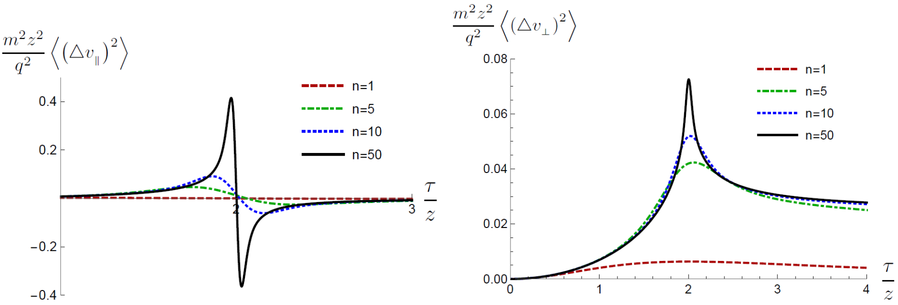


Figure: From De Lorenci et al. (2014) arXiv:1404.3115

Green Functions

Important features comes from the investigation of the field products expectation values in vacuum, which are related to its Green functions:

$$G^{+}(t,\vec{x};t',\vec{x}') = \langle 0|\,\phi(t,\vec{x})\phi(t',\vec{x}')\,|0\rangle = \int \frac{d^D\vec{k}}{2(2\pi)^D}\frac{\mathrm{e}^{i\vec{k}\cdot\Delta\vec{x}}\mathrm{e}^{-i\omega\Delta t}}{\omega}, \quad (1)$$

$$G^{-}(t,\vec{x};t',\vec{x}') = \langle 0|\,\phi(t',\vec{x}')\phi(t,\vec{x})\,|0\rangle = \int \frac{d^D\vec{k}}{2(2\pi)^D}\frac{\mathrm{e}^{i\vec{k}\cdot\Delta\vec{x}}\mathrm{e}^{i\omega\Delta t}}{\omega}, \quad (2)$$

denoted as the positive and negative frequency Wightman functions. Here $\omega = \sqrt{m^2 + k^2}$.

Green functions at finite temperature

To introduce temperature, the expectations values are taken over statistical ensembles. Then, following the procedure by Birrel & Davies (1984), the Wightman functions are:

$$G_{\beta}^{+}(t,\vec{x};t',\vec{x}') = \langle\phi(t,\vec{x})\phi(t',\vec{x}')\rangle_{\beta}, \tag{3}$$

$$G_{\beta}^{-}(t,\vec{x};t',\vec{x}') = \langle\phi(t',\vec{x}')\phi(t,\vec{x})\rangle_{\beta}. \tag{4}$$

Which obey the KMS condition

$$G_{\beta}^{+}(t,\vec{x};t',\vec{x}') = G_{\beta}^{-}(t+i\beta,\vec{x};t',\vec{x}'). \tag{5}$$

Some properties of the Green functions

- Vacuum divergence due to high energy modes.

- Non-Huygesian character of the massive fields, or when D is even: Signals propagate with an arbitrary low velocity.

$$v_g = k/\sqrt{k^2 + m^2}$$

- Infrared divergence for the massless field when $D = 1$, and, when temperature is present, also for $D = 2$.

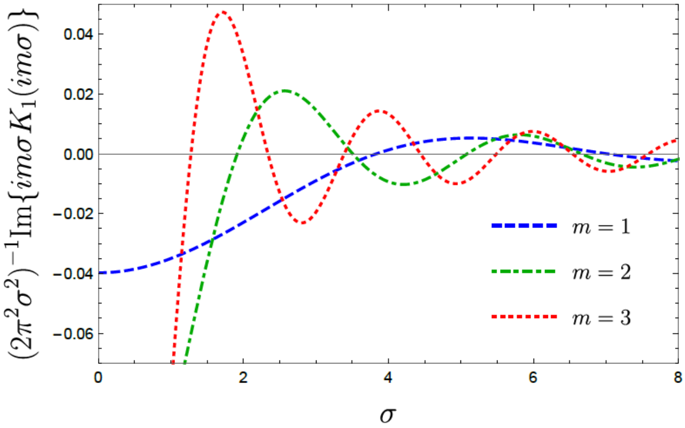


Figure: From Camargo et al. (2019) arXiv:1906.08322

Fluctuations in the presence of a perfectly reflective boundary

In the presence of an infinite Dirichlet's wall at $x_1 = 0$, where $\phi(t, x_1 = 0, x_2, ..., x_D) = 0$

$$G^{(1)}_{\beta,\mathrm{Ren}}(t, \vec{x}; t', \vec{x}') = -\frac{1}{\pi}\mathrm{Re}\left[\left(\frac{m}{2\pi i\sigma_0^+}\right)^{\frac{D-1}{2}} K_{\frac{D-1}{2}}(im\sigma_0^+)\right] + \frac{2}{\pi}\mathrm{Re}\sum_{l=1}^{\infty}\left[\left(\frac{m}{2\pi i\sigma_l}\right)^{\frac{D-1}{2}} K_{\frac{D-1}{2}}(im\sigma_l) - (\sigma_l \leftrightarrow \sigma_l^+)\right].$$

Here $\sigma_l^+ = [(\Delta t - i\beta l)^2 - (\hat{\Delta}\vec{x})^2]^{1/2}$, $\hat{\Delta}\vec{x}$ being $\Delta\vec{x}$ when $x_1' \to -x_1'$.

The interacting model

The interaction of a non-relativistic scalar charged test particle with the brackground field is given through

$$S[\phi, \partial_\mu \phi; \tau, \vec{x}] =\simeq S_F + \int d\tau \left[\frac{M}{2} v^2 - e\,\phi(\tau, \vec{x})\right] + \mathcal{O}(v^2/c^2). \tag{6}$$

Which gives

$$\frac{dv_i}{d\tau} = -g \frac{\partial \phi(\tau, \vec{x})}{\partial x_i}. \tag{7}$$

For the vacuum and thermal equilibrium $\langle v_i \rangle = 0$. So:

$$\langle (\Delta v_i)^2 \rangle_D = \frac{g^2}{2} \lim_{x \to x'} \left[\frac{\partial}{\partial x_i} \frac{\partial}{\partial x'_i} \int_0^\tau dt \int_0^\tau dt'\, G^{(1)}_{\beta, Ren}(t, \vec{x}; t', \vec{x}')\right]. \tag{8}$$

Henceforth, in order to describe a more realistic system, we introduce a switching function $F(t)$, such that

$$\int_{-\infty}^{\infty} dtF(t) = \tau. \tag{9}$$

The fluctuations are now given by

$$\langle(\Delta v_i)^2\rangle_D = \frac{g^2}{2} \lim_{x \to x'} \left[\frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i'} \int_{-\infty}^{\infty} dtF(t) \int_{\infty}^{\infty} dt' F(t') G^{(1)}_{\beta,\mathrm{Ren}}(t, \vec{x}; t', \vec{x}') \right]. \tag{10}$$

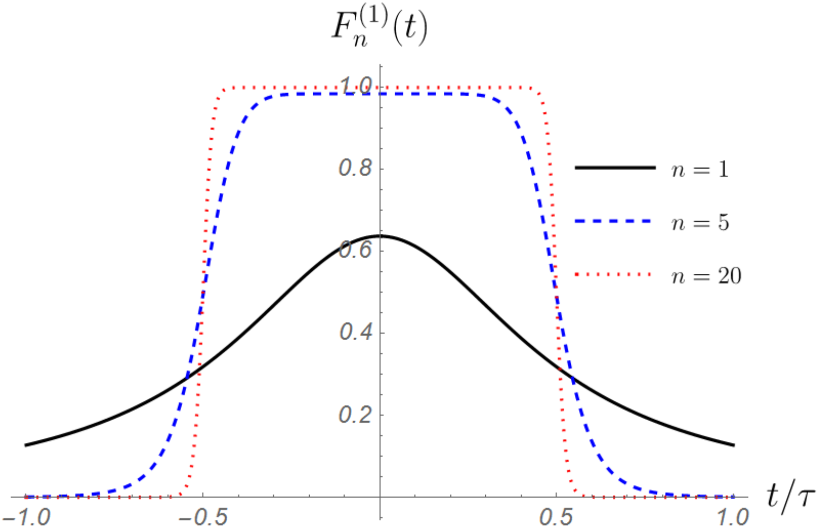
Switching Function

A suitable choice is the generalized Lorentzian

$$F_n^{(1)}(t) = \frac{c_n}{\left[1 + \left(\frac{2t}{\tau}\right)^{2n}\right]}, \tag{11}$$

with $c_n = (2n/\pi)\sin(\pi/2n)$. Which defines the transition time

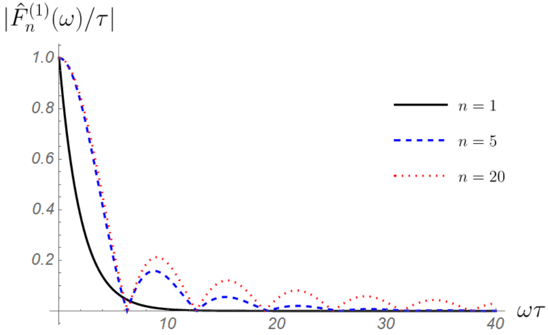
$$\tau_s = \frac{\tau}{2}\left(\frac{2n-1}{n+1}\right)^{\frac{1}{2n}} \left[\left(1 + \sqrt{1 - \frac{(n+1)(n-1)}{(2n+1)(2n-1)}}\right)^{\frac{1}{2n}} - \left(1 - \sqrt{1 - \frac{(n+1)(n-1)}{(2n+1)(2n-1)}}\right)^{\frac{1}{2n}}\right], \tag{12}$$


$F_n^{(1)}(t)$
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0
t/τ
$n = 1$
$n = 5$
$n = 20$

The Fourier transform of $F_n^{(1)}(t)$ is given by

$$\hat{F}_n^{(1)}(\omega) = \int_{-\infty}^{\infty} dt\, \mathrm{e}^{-i\omega t} F_n^{(1)}(t) = \frac{i\tau\pi c_n}{2n} \sum_{q=n}^{2n-1} \psi_{n,q} \mathrm{e}^{-i\omega\tau\psi_{n,q}/2}, \qquad (13)$$

with $\psi_{n,p} = \exp[i(\pi/2n)(1+2p)]$.



Another choice of switching function is

$$F^{(2)}_{\tau_s}(t) = \frac{1}{\pi}\left[\arctan\left(\frac{t}{\tau_s}\right) + \arctan\left(\frac{\tau - t}{\tau_s}\right)\right]. \qquad (14)$$

with fourier transform given

$$\hat{F}^{(2)}(\omega) = \frac{1}{i\omega}(1 - \mathrm{e}^{-i\omega\tau})\mathrm{e}^{-\tau_s|\omega|}. \qquad (15)$$

This switching function is useful because the transition time is a parameter and does not change with the interaction time.

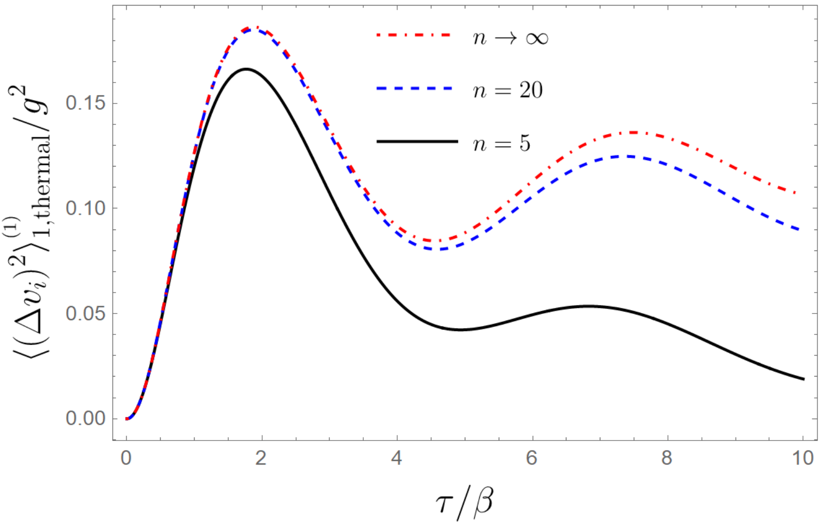
Dispersions due to a pure thermal bath

In order to obtain a closed expression we first choose the switching function $F_n^{(1)}(t)$. Which gives:

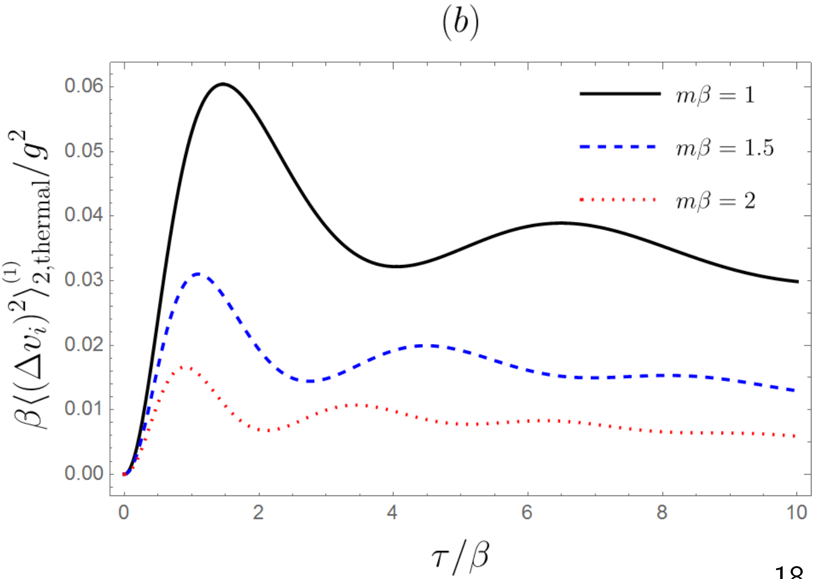
$$\langle(\Delta v_i)^2\rangle^{(1)}_{D,\text{thermal}} = \frac{2g^2}{\beta^{D-1}}\left[\frac{(\tau/\beta)\pi c_n}{2n}\right]^2 \sum_{p,q=n}^{2n-1}\sum_{l=1}^{\infty}\psi_{n,p}\psi^*_{n,q} \times \left(\frac{m\beta}{2\pi\sqrt{-a_l^2}}\right)^{\frac{D+1}{2}} K_{\frac{D+1}{2}}\left(m\beta\sqrt{-a_l^2}\right). \quad (16)$$

where $a_l = (\tau/2\beta)(\psi_{n,p} - \psi^*_{n,q}) - il$. Note that, when $\beta \to \infty$ or $m \to \infty$ the dispersions are exponentially suppressed.

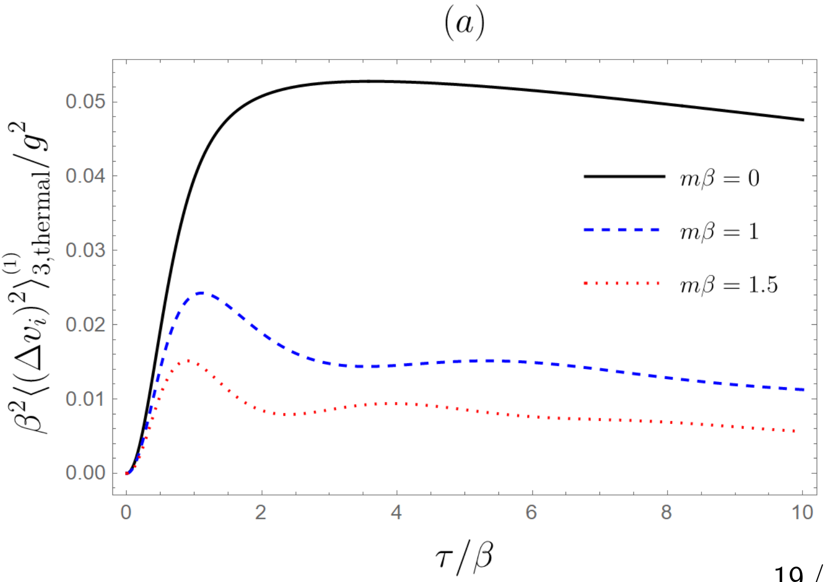
For $D = 1$, and $m\beta = 1$



Here $D = 2$, and $n = 20$.



Here $D = 3$, and $n = 20$.



Dispersions due to a boundary at finite temperature

In a physical setup, the thermal bath can be detached from the insertion of the boundary. Thence, we investigate the boundary contributions to the fluctuations, given by

$$\langle(\Delta v_i)^2\rangle_{D,\mathrm{boundary}} = \langle(\Delta v_i)^2\rangle_{D,\mathrm{vacuum}} + \langle(\Delta v_i)^2\rangle_{D,\mathrm{mixed}}. \tag{17}$$

The pure thermal contribution will be only the residual dispersions, which can be zero for τ_s long enough.

First, using the switching function $F_n^{(1)}(t)$, the mixed contribution to the fluctuations is

$$\langle(\Delta v_{\parallel})^2\rangle^{(1)}_{D,\mathrm{mixed}} = -\frac{2g^2}{x^{D-1}}\left(\frac{(\tau/x)\pi c_n}{2n}\right)^2 \times \sum_{p,q=n}^{2n-1}\sum_{l=1}^{\infty}\psi_{n,p}\psi^*_{n,q}\left(\frac{mx}{4\pi\sqrt{1-\gamma_l^2}}\right)^{\frac{D+1}{2}} K_{\frac{D+1}{2}}\left(2mx\sqrt{1-\gamma_l^2}\right), \quad (18)$$

$$\langle(\Delta v_{\perp})^2\rangle^{(1)}_{D,\mathrm{mixed}} = 8\pi x^2\langle(\Delta v_{\parallel})^2\rangle^{(1)}_{D+2,\mathrm{mixed}} - \langle(\Delta v_{\parallel})^2\rangle^{(1)}_{D,\mathrm{mixed}} \quad (19)$$

where we have defined $\gamma_l = (\tau/4x)(\psi_{n,p} - \psi^*_{n,q}) - il(\beta/2x)$.
This contributions vanishes as $\beta \to \infty$ or $m \to \infty$.

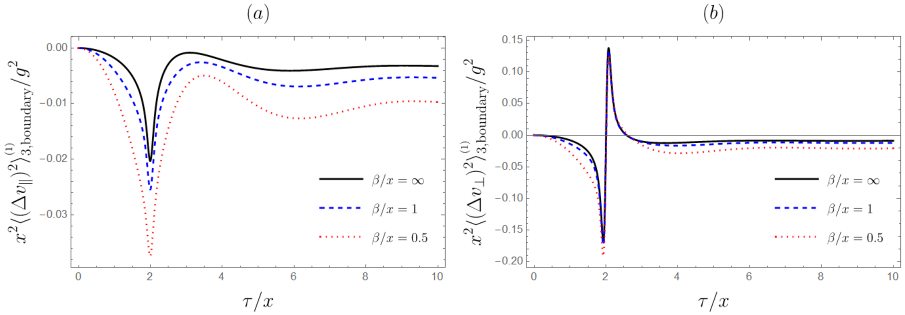
The vacuum contribution is just half the mixed term with $l = 0$. Thence:

$$\langle(\Delta v_{\|})^2\rangle^{(1)}_{D,\text{vacuum}} = -\frac{g^2}{x^{D-1}}\left(\frac{(\tau/x)\pi c_n}{2n}\right)^2 \times \sum_{p,q=n}^{2n-1}\psi_{n,p}\psi^*_{n,q}\left(\frac{mx}{4\pi\sqrt{1-\gamma_0^2}}\right)^{\frac{D+1}{2}} K_{\frac{D+1}{2}}\left(2mx\sqrt{1-\gamma_0^2}\right) \quad (20)$$

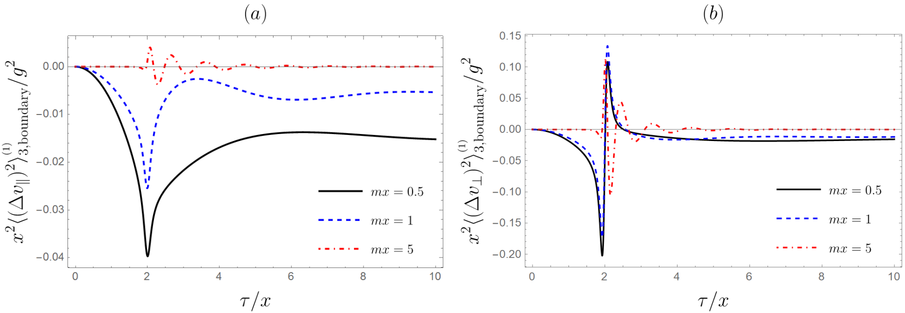
$$\langle(\Delta v_{\perp})^2\rangle^{(1)}_{D,\text{vacuum}} = 8\pi x^2\langle(\Delta v_{\|})^2\rangle^{(1)}_{D+2,\text{vacuum}} - \langle(\Delta v_{\|})^2\rangle^{(1)}_{D,\text{vacuum}}. \quad (21)$$

As $m \to \infty$ the vacuum term is also suppressed. However, as it is the zeroth order term of the summation, the temperature contributions disappear earlier.

Here $D = 3$, $m\beta = 1$, and $n = 20$



For $D = 3$, $\beta/x = 1$, and $n = 20$



Late-time dispersions due to a pure thermal bath

We find for the pure thermal bath at the late-time regime:

$$\lim_{\tau\to\infty}\langle(\Delta v_i)^2\rangle^{(2)}_{\rm thermal} = \frac{2g^2}{\pi\beta^{D-1}}\sum_{l=1}^{\infty}\left\{\left[\frac{m\beta}{2\pi(2\tau_s/\beta+l)}\right]^{\frac{D-1}{2}}\right.$$
$$\left.\times K_{\frac{D-1}{2}}\left(m\beta(2\tau_s/\beta+l)\right) - \frac{(2\tau_s/\beta+l)(m\beta)^D}{2^D\pi^{\frac{D}{2}-1}\Gamma(\frac{D}{2}+1)}I(D,m\beta(2\tau_s/\beta+l))\right\} \tag{22}$$

with

$$I(D,\alpha) = \int_1^\infty du\frac{(u^2-1)^{\frac{D}{2}}}{u}\mathrm{e}^{-\alpha u} = -\frac{\pi}{2}\mathrm{cosec}\left(\frac{\pi}{2}D\right)$$
$$-\frac{\alpha}{2\sqrt{\pi}}\Gamma\left(-\frac{D}{2}-1\right)\Gamma\left(\frac{D}{2}+1\right){}_1F_2\left[1/2;3/2,(D+3)/2;\alpha^2/4\right]$$
$$+\frac{1}{\alpha^D}\Gamma(D){}_1F_2\left[-D/2;(1-D)/2,1-D/2;\alpha^2/4\right]. \tag{23}$$

Late-time dispersions due to a boundary at finite temperature

For the mixed contribution we find:

$$\lim_{\tau\to\infty}\langle(\Delta v_{\parallel})^2\rangle^{(2)}_{D,\mathrm{mixed}} = -\frac{2g^2}{\pi x^{D-1}} \times\sum_{l=1}^{\infty}\left\{\left[\frac{mx}{4\pi\sqrt{1+\alpha_l^2}}\right]^{\frac{D-1}{2}} K_{\frac{D-1}{2}}\left(2mx\sqrt{1+\alpha_l^2}\right) - \frac{\alpha_l(mx)^{\frac{D}{2}}}{2^{D-1}\pi^{\frac{D}{2}-1}}\int_1^{\infty}du\frac{(\sqrt{u^2-1})^{\frac{D}{2}}}{u}\mathrm{e}^{-2mx\,\alpha_l}J_{\frac{D}{2}}(2mx\sqrt{u^2-1})\right\}, \quad (24)$$

$$\lim_{\tau\to\infty}\langle(\Delta v_{\perp})^2\rangle^{(2)}_{D,\mathrm{mixed}} = 8\pi x^2\lim_{\tau\to\infty}\langle(\Delta v_{\parallel})^2\rangle^{(2)}_{D+2,\mathrm{mixed}} - \lim_{\tau\to\infty}\langle(\Delta v_{\parallel})^2\rangle^{(2)}_{D,\mathrm{mixed}}, \quad (25)$$

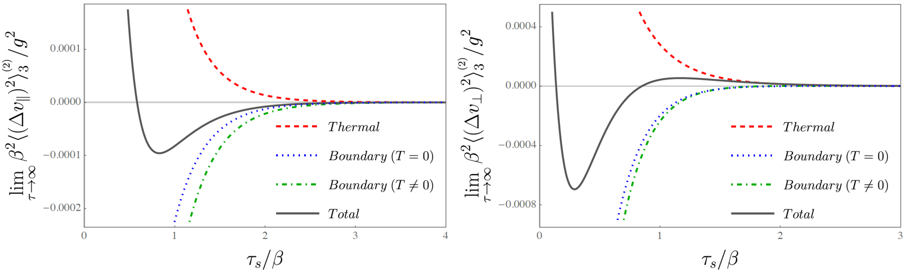
with $\alpha_l = \tau_s/x + l\beta/2x$.

The vacuum contribution is then:

$$\lim_{\tau\to\infty}\langle(\Delta v_{\parallel})^2\rangle^{(2)}_{D,\mathrm{vacuum}} = -\frac{g^2}{\pi x^{D-1}} \times\Bigg\{\left[\frac{mx}{4\pi\sqrt{1+\alpha_0^2}}\right]^{\frac{D-1}{2}} K_{\frac{D-1}{2}}\left(2mx\sqrt{1+\alpha_0^2}\right) - \frac{\alpha_0(mx)^{\frac{D}{2}}}{2^{D-1}\pi^{\frac{D}{2}-1}}\int_1^\infty du\frac{(\sqrt{u^2-1})^{\frac{D}{2}}}{u}\mathrm{e}^{-2mx\,\alpha_0}J_{\frac{D}{2}}(2mx\sqrt{u^2-1})\Bigg\}, \quad (26)$$

$$\lim_{\tau\to\infty}\langle(\Delta v_{\perp})^2\rangle^{(2)}_{D,\mathrm{vacuum}} = 8\pi x^2\lim_{\tau\to\infty}\langle(\Delta v_{\parallel})^2\rangle^{(2)}_{D+2,\mathrm{vacuum}} - \lim_{\tau\to\infty}\langle(\Delta v_{\parallel})^2\rangle^{(2)}_{D,\mathrm{vacuum}}, \quad (27)$$

Residual dispersions for $D = 3$, and $m\beta = 1$



Distance behavior of the dispersions

To investigate how the dispersions behaves with the distance to the plate, note that, when $x/\beta \ll 1$, we have

$$\sqrt{1-\gamma_l^2} = \frac{\beta}{2x}\sqrt{\frac{4x^2}{\beta^2} + \left(l + i\frac{\tau}{2\beta}(\psi_{n,p} - \psi^*_{n,q})\right)} \simeq \frac{\beta}{2x}\sqrt{l + i\frac{\tau}{2\beta}(\psi_{n,p} - \psi^*_{n,q})} = \sqrt{-\gamma_l^2}. \quad (28)$$

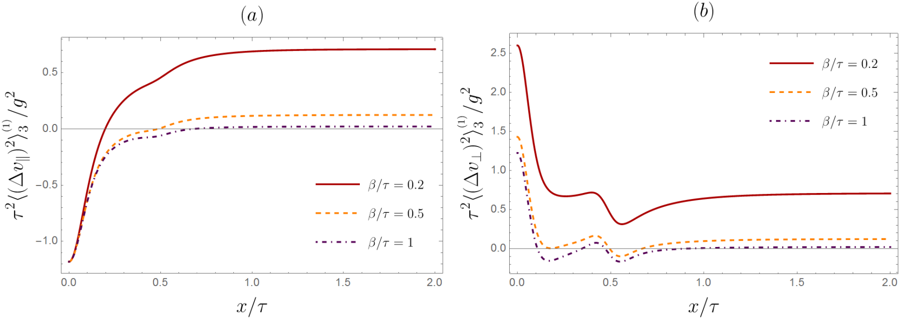
So that

$$\langle(\Delta v_{\|})^2\rangle^{(1)}_{D,\text{mixed}} \simeq -\langle(\Delta v_i)^2\rangle^{(1)}_{D,\text{thermal}},$$

and

$$\langle(\Delta v_{\perp})^2\rangle^{(1)}_{D,\text{mixed}} \simeq \langle(\Delta v_i)^2\rangle^{(1)}_{D,\text{thermal}} - 8\pi x^2\langle(\Delta v_i)^2\rangle^{(1)}_{D+2,\text{thermal}}.$$

For $D = 3$, $n = 5$ and $m\beta = 1$



Vacuum vs thermal dominance

To investigate the vacuum versus thermal dominance as the boundary is approached, we study the mean squared velocity:

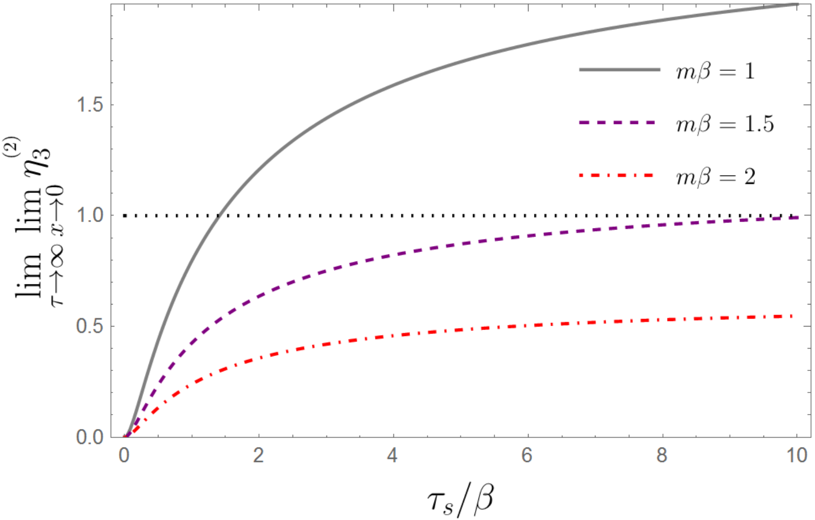
$$\langle v^2\rangle_{D,\beta} = \sum_i \langle v_i^2\rangle_{D,\beta} = (D-1)\langle(\Delta v)_{\parallel}^2\rangle_{D,\beta} + \langle(\Delta v)_{\perp}^2\rangle_{D,\beta}. \tag{29}$$

Then, defining the quantity

$$\eta_D = \left|\frac{\langle v^2\rangle_{D,\beta} - \langle v^2\rangle_D}{\langle v^2\rangle_D}\right|. \tag{30}$$

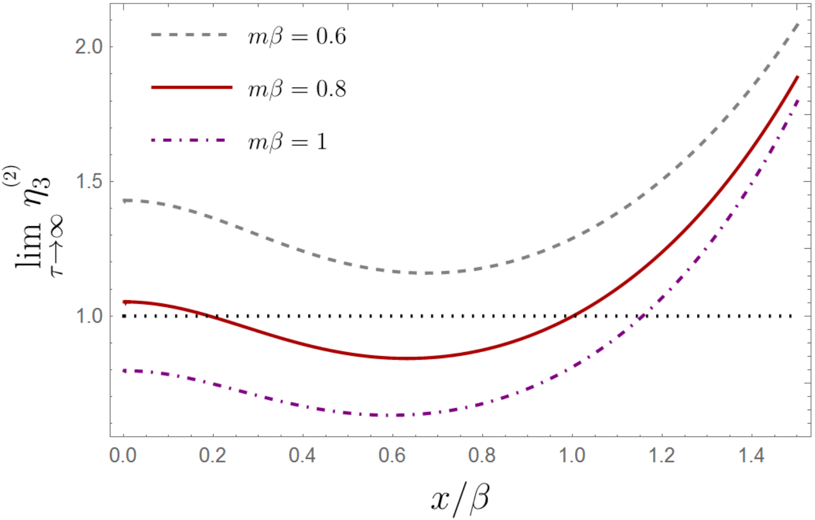
If $\eta_D > 1$ thermal effects dominate, if $\eta_D < 1$, the modified vacuum contribution is higher.

For $D = 3$



$\lim_{\tau\to\infty}\lim_{x\to 0}\eta_3^{(2)}$
τ_s/β
$m\beta = 1$
$m\beta = 1.5$
$m\beta = 2$
0.0
0.5
1.0
1.5
0
2
4
6
8
10

Here $D = 3$, and $\tau_s/\beta = 1$



Remarks on the assumptions of the model

Regarding the assumptions made in the model:

- Idealized boundary: Infinite potential barrier.
- Dirichlet's over Neumann's boundary conditions.
- $\langle(\Delta x_i)^2\rangle = \int_0^\tau d\,t \int_0^\tau d\,t' \langle v_i(t) v_i(t')\rangle \ll x_i^2$ [Yu & Ford (2004) [arXiv:0406122], De Lorenci et al. (2016) [arXiv:1606.09134]].
- Backreaction due to particle's emitted radiation [Yu & Ford (2004) [arXiv:0406122], De Lorenci et al. (2016) [arXiv:1606.09134]].

In Summary

- Subvacuum effects are present even at finite temperatures and at late times.
- In the absence of the thermal bath, temperature increases subvacuum effects, facilitating its detection.
- Thermal effects can dominated over the vacuum near the boundary, for switching times long enough.
- Mass creates an interesting pattern of domination as the wall is approached.

References

- H. Yu and L. H. Ford, Vacuum fluctuations and Brownian motion of a charged test particle near a reflecting boundary, Phys. Rev. **D 70**, 065009 (2004).
- M. Seriu and C.H. Wu, Switching effect on the quantum Brownian motion near a reflecting boundary, Phys. Rev. **A 77**, 022107 (2008).
- M. Seriu and C.H. Wu, Smearing effect due to the spread of a probe particle on the Brownian motion near a perfectly reflecting boundary, Phys. Rev. **A 80**, 052101 (2009).
- V.A. De Lorenci, E.S. Moreira Jr. and M.M. Silva, Quantum Brownian motion near a point-like reflecting boundary, Phys. Rev. **D 90**, 027702 (2014).
- V.A. De Lorenci, C.C.H. Ribeiro and M.M. Silva, Probing quantum vacuum fluctuations over a charged particle near a reflecting wall, Phys. Rev. **D 94**, 105017 (2016).
- C.H.G. Bessa, V.B. Bezerra and L.H. Ford, Brownian motion in Robertson-Walker spacetimes from electromagnetic vacuum fluctuations, J. Math. Phys. **50**, 062501 (2009).

References

- G.H.S. Camargo, V.A. De Lorenci, C.C.H. Ribeiro, F.F. Rodrigues and M.M. Silva, Vacuum fluctuations of a scalar field near a reflecting boundary and their effects on the motion of a test particle, JHEP **07**, 173 (2018).
- V.A. De Lorenci and L.H. Ford, Subvacuum effects on light propagation, Phys. Rev. **A 99**, 023852 (2019).
- L. H. Ford, Stochastic spacetime and Brownian motion of test particles, Int. J. Theor. Phys. **70**, 1753 (2005).
- H. Yu, J. Chen and P. Wu, Brownian motion of a charged test particle near a reflecting boundary at finite temperature, JHEP **02**, 058 (2006).
- V.A. De Lorenci and C.C.H. Ribeiro, Remarks on the influence of quantum vacuum fluctuations over a charged test particle near a conducting wall, JHEP **04**, 072 (2019).
- G.H.S. Camargo, V.A. De Lorenci, C.C.H. Ribeiro, and F.F. Rodrigues, Vacuum induced dispersions of the motion of test particles in $D+1$ dimensions, Phys. Rev. **D 100**, 065014 (2019).

Thank you!