Soft Future Singularities, Born-Infeld-like Fields and Particles

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   - Particles creation in curved spacetime
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   - The Tachyon Model
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   - Particles Behavior
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3. Phantom Divide Line Crossing

The Big Bang and the Big Crunch Singularities

- Until the end of 1990s almost all the discussions about classical and quantum cosmology of singularities were devoted to the big bang and big crunch singularities.
- Characterized by the vanishing value of the cosmological radius.
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The Big Rip Singularity

- This discovery was the starting point for the formulation of cosmological models containing dark energy, which, for its specific properties, was considered responsible for the accelerated expansion of the Universe.
- The Big Rip singularity arising in the models where the phantom energy is present.
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The Sudden (Soft) Singularities

- Another class of singularities includes the so-called soft or sudden singularities:

$$a \to a_S,$$
$$\dot{a} \to \dot{a}_S,$$
$$\ddot{a} \to -\infty.$$

- A particular case: the Big Brake singularity (Kamenshchik, 2004), characterized by $\dot{a} = 0$, $H = 0$ and $\rho = 0$.
- A very simple cosmological model, based on the anti-Chaplygin gas ($p = \frac{A}{\rho}$), leads unavoidably to the big brake singularity.

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Features of the Sudden Future Singularities I

- Softness: the Christoffel symbols depend only on the first derivative of the scale factor, they are regular at these singularities. Hence, the geodesics have a good behavior and they can cross the singularity.
- The particles crossing the singularity will generate the geometry of the spacetime, providing in such a way a soft rebirth of the universe after the singularity crossing.
- Note that the opportunity of crossing of some kind of cosmological singularities were noticed already in the early paper by Tipler (1977).

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Features of the Sudden Future Singularities II

- Another remarkable feature of the soft future singularities is their capacity to induce changes in the equations of state of the matter present in a universe under consideration.
- The form of the matter Lagrangian can also be changed.
- The effects of the matter transformation occur sometimes also without singularities, but only in the presence of some non-analyticities in the geometry of the spacetime.

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The General Procedure for the Definition of the Particles

- The Klein-Gordon equation for the minimally coupled scalar field ϕ in a flat Friedmann universe is

$$\Box\phi + V'(\phi) = 0. \quad (1)$$

- Consider a spatially homogeneous solution of this equation $\phi_0(t)$ as a classical background. A small deviation from this background solution can be represented as a sum of Fourier harmonics satisfying linearized equations

$$\ddot{\phi}\left(\vec{k},t\right) + 3\frac{\dot{a}}{a}\dot{\phi}\left(\vec{k},t\right) + \left(\frac{\vec{k}^2}{a^2} + V''(\phi_0(t))\right)\phi\left(\vec{k},t\right) = 0. \quad (2)$$

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$$\hat{\phi}(\vec{x},t)=\int d^3\vec{k}\left(\hat{a}\left(\vec{k}\right)u\left(\vec{k},t\right)e^{i\vec{k}\cdot\vec{x}}+a^{\dagger}\left(\vec{k}\right)u^{*}\left(\vec{k},t\right)e^{-i\vec{k}\cdot\vec{x}}\right), \tag{3}$$

where the creation and the annihilation operators satisfy the standard commutation relations

$$\left[\hat{a}\left(\vec{k}\right),a^{\dagger}\left(\vec{k}\right)\right]=0. \tag{4}$$

- The basis functions u satisfy the linearized equation (2).
- The Wronskian relation

$$W\left[u,u^{*}\right]=u(k,t)\,\dot{u}^{*}(k,t)-u^{*}(k,t)\,\dot{u}(k,t)=\frac{i}{(2\pi)^3 a^3(t)}. \tag{5}$$

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Vacuum States

- The linearized equation (2) has two independent solutions.
- Different linear combinations of u (chosen in such a manner that the Wronskian relation is satisfied) $\Rightarrow$ determine different choices of the operators $\hat{a}$ and $a^{\dagger}$ and different vacuum states on which the Fock spaces can be constructed.
- In the Minkowski spacetime: the plane waves.
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The Tachyon Model

- The Lagrangian of the tachyon field T is

$$L = -V(T)\sqrt{1 - g^{\mu\nu} T_{,\mu} T_{,\nu}}. \tag{6}$$

- What is called tachyon field is a modification of an old idea of Born and Infeld (1934), that the kinetic term of a field can have a non-polynomial form.
- The pressure is negative $p = -V(T)\sqrt{1 - \dot{T}^2}$.

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The Potential of the Tachyon Model

- The chosen potential

$$V(T) = \frac{\Lambda\sqrt{1-(1+w)\cos^2\left(\frac{3}{2}\sqrt{\Lambda(1+w)}T\right)}}{\sin^2\left(\frac{3}{2}\sqrt{\Lambda(1+w)}T\right)}. \qquad (7)$$

- The field equation

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V_{,T}}{V} = 0. \qquad (8)$$

- The case $w > 0$ is of a particular interest, because it reveals two unusual phenomena: a self-transformation of the tachyon into a pseudotachyon field and the appearance of the Big Brake cosmological singularity.

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Dynamical System

- Rewrite the Eq. (8) as a dynamical system of two first-order differential equations

$$\dot{T} = s, \tag{9}$$

$$\dot{s} = -3\sqrt{V}\left(1-s^2\right)^{\frac{3}{4}} s - \left(1-s^2\right)\frac{V_{,T}}{V}. \tag{10}$$

- One can show that the potential (7) is well defined inside the rectangle, where $-1 \leq s \leq 1$ and $T_3 \leq T \leq T_4$ with

$$T_3 = \frac{2}{3\sqrt{(1+w)\Lambda}}\arccos\frac{1}{\sqrt{1+w}}, \tag{11}$$

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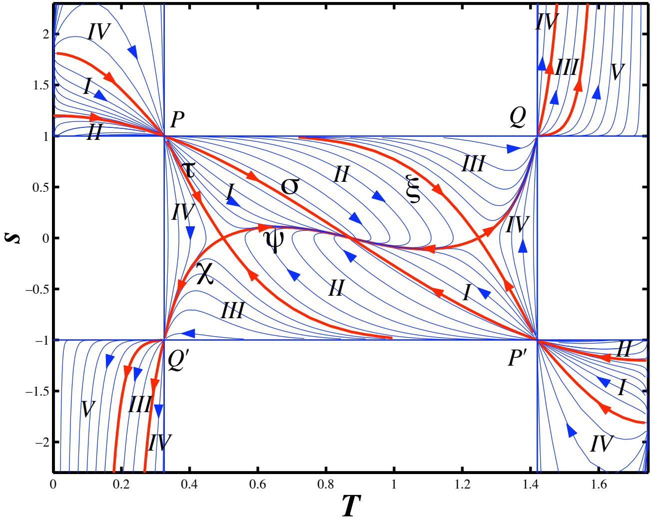
Phase Portrait of the Dynamical System



A Tachyon-Pseudotachyon Transformation

- The Lagrangian changes its form in such a way that the equations of motion conserve their form

$$L = W(T)\sqrt{g^{\mu\nu}T_{,\mu}T_{,\nu} - 1}. \tag{13}$$

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The Universe After the "Crossing the Corner"

- What happens with the universe after the "crossing the corner" and the transformation of the tachyon into the pseudotachyon?
- The universe in a finite moment of time $t = t_{BB}$ encounter the singularity, which is characterized by the following values of cosmological parameters

$$a(t_{BB}) = a_{BB} < \infty, \quad (15)$$

$$\dot{a}(t_{BB}) = 0, \quad (16)$$

$$\ddot{a}(t) \to -\infty, t \to t_{BB}. \quad (17)$$

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Particle Behavior Before Crossing

- What happens with particles during the transformation of the tachyon into the pseudotachyon?
- The field equation

$$\frac{\ddot{T}\left(1+\frac{T_{,\alpha}T_{,\alpha}}{a^2}\right)}{1-\dot{T}^2+\frac{T_{,\alpha}T_{,\alpha}}{a^2}}+3\frac{\dot{a}}{a}\dot{T}+\frac{V_{,T}}{V}.$$

$$+\frac{\dot{a}a\dot{T}T_{,\alpha}T_{,\alpha}-2a^2\dot{T}\dot{T}_{,\beta}T_{,\beta}+T_{,\alpha}T_{,\beta}T_{,\alpha\beta}}{a^4\left(1-\dot{T}^2+\frac{T_{,\alpha}T_{,\alpha}}{a^2}\right)}-\frac{\Delta T}{a^2}=0 \quad (18)$$

- From the Friedmann equation: $\frac{\dot{a}^2}{a^2}=\frac{V(T)}{\sqrt{1-\dot{T}^2}}$.

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The Equation for the Linear Perturbations

- Representing T as $T = T_0 + \tilde{T}$, where T_0 is the solution of the tachyon field equation for the spatially homogeneous background mode and $\tilde{T}$ is the linear perturbation, we obtain the following equation for the linear perturbations

$$\frac{\ddot{\tilde{T}}}{1-\dot{T}_0^2} + \left(\frac{2\ddot{T}_0\dot{T}_0}{\left(1-\dot{T}_0^2\right)^2} + \frac{3\sqrt{V}\left(2-\dot{T}_0^2\right)}{2\left(1-\dot{T}_0^2\right)^{5/4}}\right)\dot{\tilde{T}}$$

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- When the field crosses the corner, the spatially homogeneous part of the field T behaves as $T = T_3 + \bar{T}$, while $s = -1 + \bar{s}$.
- $\bar{T} = -t$, $\bar{s} = -Ct$.

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The Result

- The differential equation for the linear perturbations

$$\ddot{\tilde{T}} - \frac{1}{t}\dot{\tilde{T}} + \frac{C}{t}\tilde{T} = 0. \tag{20}$$

- The solution

$$\tilde{T} = c_1 t J_2\left(\sqrt{C}t\right) + c_2 t Y_2\left(\sqrt{C}t\right). \tag{21}$$

- Both solutions are regular at $t \to 0$ and the particles quietly pass through the corner.
- The same analysis can be carried out in the upper left corner, where the pseudotachyon field is transformed into the tachyon field while the universe is expanding.

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- The same analysis can be carried out in the upper left corner, where the pseudotachyon field is transformed into the tachyon field while the universe is expanding.

The Result

- The differential equation for the linear perturbations

$$\ddot{\tilde{T}} - \frac{1}{t}\dot{\tilde{T}} + \frac{C}{t}\tilde{T} = 0. \tag{20}$$

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Particle Behavior After Crossing

- The equation for the linear perturbation

$$\frac{\ddot{\tilde{T}}}{1-\dot{T}_0^2}+\left(\frac{2\ddot{T}_0\dot{T}_0}{\left(1-\dot{T}_0^2\right)^2}+\frac{3\sqrt{W}\left(2-\dot{T}_0^2\right)}{2\left(\dot{T}_0^2-1\right)^{5/4}}\right)\dot{\tilde{T}}$$
$$+\left(\frac{3W_{,T}\dot{T}_0}{2\sqrt{W}\left(\dot{T}_0^2-1\right)^{\frac{1}{4}}}+\frac{W_{,TT}}{W}-\frac{W_{,T}^2}{W^2}+\frac{k^2}{a_{BB}^2}\right)\tilde{T}=0. \quad (22)$$

- The homogeneous part of the field after the crossing the left lower corner is $T_0(t)=T_{BB}+\left(\frac{4}{3W(T_{BB})}\right)^{\frac{1}{3}}(-t)^{\frac{1}{3}}$, while the scale factor is $a(t)=a_{BB}-\frac{3}{4}a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{\frac{1}{3}}(-t)^{\frac{4}{3}}$.

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$$\ddot{\tilde{T}} + \frac{5}{3t}\dot{\tilde{T}} + \frac{B^2}{t^{\frac{5}{3}}}\tilde{T} = 0, \qquad (23)$$

where $B^2 = -\frac{W_{,T}(T_{BB})}{16}\left(\frac{4}{3W(T_{BB})}\right)^{\frac{4}{3}} > 0.$

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$$\tilde{T} = c_1 t^{-\frac{1}{3}} J_2\left(Bt^{\frac{1}{6}}\right) + c_2 t^{-\frac{1}{3}} Y_2\left(Bt^{\frac{1}{6}}\right). \qquad (24)$$

- The second term is singular at $t \to 0_-$ and we cannot use two independent solutions to construct the Fock space.
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The Paradox of Soft Singularity Crossing

- Suppose that our universe is filled not only with the tachyon field with the potential, described above, but also with some quantity of dust.
- In this case instead of the Big Brake singularity the universe will encounter a soft singularity of a more general kind.
- Due to the presence of dust, the energy density of the expanding universe cannot vanish and, hence, at the moment when the universe experiences an infinite deceleration its expansion should continue.
- This implies the appearance of some kind of contradictions, which can be resolved by transformation of the pseudotachyon field into another kind of Born-Infeld like field.

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The Quasitachyon Model

- The solution of this paradox was first found for the case of the anti-Chaplygin gas - perfect fluid with the equation of state

$$p = \frac{A}{\rho},\ A > 0, \tag{25}$$

which represents the simplest model, where the Big Brake singularity arises.

- The equation of state of this gas undergoes a transformation and it becomes the standard Chaplygin gas, but with a negative energy density.
- This solution was extended to the case of the pseudotachyon, which transforms itself into the *quasitachyon* with the Lagrangian

$$L = W(T)\sqrt{g^{\mu\nu}T_{,\mu}T_{,\nu}+1}. \tag{26}$$

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- When the universe is running towards the future soft singularity in the presence of dust, the pseudotachyon field behaves as $T(t) = T_S - \frac{2}{\sqrt{6H_S}}\sqrt{-t}$.
- The Friedmann equation in the presence of both the pseudotachyon field and dust is $\frac{\dot{a}^2}{a^2} = H_S^2 + \frac{W(T_0)}{\dot{T}^2 - 1}$.
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The Phenomenon

- The analysis of observation indicate the existence of the moment when the universe changes the value of the parameter w from the region $w > -1$ to $w < -1$.
- This transition is called "phantom divide line crossing".
- This phenomenon can be described by models, including two scalar fields - a standard one and a phantom.
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The Model

- The Lagrangian

$$L = -\frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi), \qquad (29)$$

where

$$V(\phi) = \frac{V_0}{\left(1 + V_1 \phi^{\frac{2}{3}}\right)^2}. \qquad (30)$$

- In this model we also find some transformation of matter properties induced by a change of geometry, even there is no cosmological singularity.
- In this aspect the phenomenon of the phantom divide line crossing in the model is analogous to the transformation between the tachyon and pseudotachyon field in the Born-Infeld model with the trigonometric potential considered earlier.

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The Cosmological Evolution

- The universe begins its evolution from the cosmological singularity of the "anti-Big Rip" type and its squeezing is driven by a scalar field with a negative kinetic term.
- Then at the moment t_1 the contraction of the universe is replaced by an expansion.
- At the moment $t = t_0 > t_1$ the kinetic term of the scalar field changes sign ("dephantomization").
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Perturbations

- The equation for linearized perturbations of the phantom field approaching the moment of the phantom divide line crossing is

$$\ddot{\tilde{\phi}} + \left(3\sqrt{-\frac{\dot{\phi}^2}{2} + \frac{V_0}{\left(1+V_1\phi^{\frac{2}{3}}\right)^2}} - \frac{3\dot{\phi}^2}{2\sqrt{-\frac{\dot{\phi}^2}{2} + \frac{V_0}{\left(1+V_1\phi^{\frac{2}{3}}\right)^2}}}\right)\dot{\tilde{\phi}}$$

$$+\left(\frac{4V_0V_1}{9\phi^{\frac{4}{3}}\left(1+V_1\phi^{\frac{2}{3}}\right)^3} + \frac{8V_0V_1^2}{3\phi^{\frac{2}{3}}\left(1+V_1\phi^{\frac{2}{3}}\right)^4}\right.$$

$$\left.-\frac{2V_0V_1\dot{\phi}}{\sqrt{-\frac{\dot{\phi}^2}{2} + \frac{V_0}{\left(1+V_1\phi^{\frac{2}{3}}\right)^2}}\,\phi^{\frac{1}{3}}\left(1+V_1\phi^{\frac{2}{3}}\right)^3} + \frac{k^2}{a^2}\right)\tilde{\phi} = 0.$$

The Result

- Using the background value for ϕ we reduce the equation to

$$\ddot{\tilde{\phi}} + 3\sqrt{V_0}\dot{\tilde{\phi}} + \frac{1}{4t^2}\tilde{\phi} = 0. \tag{31}$$

- The solution in the vicinity of $t = 0$ is

$$\tilde{\phi} = c_1\sqrt{-t} + c_2\sqrt{-t}\ln(-t). \tag{32}$$

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Summary

- We tried to answer a simple question: is it possible to conserve some kind of notion of particle corresponding to a chosen quantum field present in the universe when the latter is approaching the singularity?
- Tachyon field: both solutions of the corresponding differential equation are regular and the tachyons particles are transformed smoothly into the pseudotachyon particles.
- When the universe driven by the pseudotachyion field approaches the Big Brake singularity, one of the solutions is singular and the particles do not exist.
- The presence of dust works as a factor “normalizing” the passage through the singularity.
- Phantom divide line crossing: two regular solutions in the vicinity of the crossing point, but both of them tend to zero in the corresponding limit $\Longrightarrow$ some very special kind of particles.