Post-Newtonian γ-like parameters and the gravitational slip in scalar-tensor and $f(R)$ theories

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MOTIVATION

- Parameterized Post-Newtonian (PPN) formalism provide practical and rigorous procedure to infer observational constraints for alternative theories of gravity.
- However, there are limitations since several of the alternative theories cannot be parametrized according to the original formalism.
- In those cases, one can work with an extended PPN formalism, but care must be taken when using the PPN bounds.
- Scalar-tensor and f(R) theories are cases where is not difficult to find in the literature different definitions of an effective gamma, which can lead to distinct constraints for the model.

GOAL

- We start by reviewing the fundamentals of the standard PPN formalism, with emphasis to its γ parameter, which is directly associated with tests concerning the propagation of electromagnetic waves.
- Based on that, we will explore an extended PPN, suitable for scalar-tensor theories, and discuss two others γ-like parameters that emerge.
- We also address the similarities and differences between the gammas and the gravitational slip parameter (η).
- Finally, we apply these discussions to massive Brans-Dicke, *f(R)* and Horndeski formulations.

The PPN formalism and the physical meaning of γ

PPN FORMALISM HYPOTHESIS

- The matter of the system can be described as a perfect fluid.
- The relevant spacetime for the system is asymptotically flat.
- A well defined Newtonian limit must exist.
- To obtain PN corrections to propagation of light we only need first order metric perturbations. The PPN metric, in this case, reads

$$g_{00}^{PPN} = -1 + 2U + O(v^4/c^4)$$

$$g_{0i}^{PPN} = 0 + O(v^3/c^3) \longrightarrow$$

$$g_{ij}^{PPN} = (1 + 2\gamma U)\delta_{ij} + O(v^4/c^4)$$

$$U(x, t) \equiv \int \frac{\rho(x', t)}{|\mathbf{x} - \mathbf{x}''|} d^3x'$$

and we use units such that

$$G = c = 1$$

PHYSICAL MEANING OF γ

- Considering the PPN framework into the geodesic equation for photons, one can obtain the relations,

$$\mathrm{v}^i = \left[1 - (1+\gamma)U\right] n^i$$

$$\frac{dn^i}{dt} = (1+\gamma)(\delta^{ij} - n^i n^j)\partial_j U$$

v^i is the photon four-

n^i is an unitary

- The relevant equations for the deflection of light by a static body, as well as the Shapiro time-delay effect, are directly obtained from above. Both phenomena can be used to put bounds on γ

- But the PPN hypothesis must hold. Among them, we recall that only the Newtonian potential is present at the O(2) and that γ is a constant. Scalar-tensor theories need not to satisfy these conditions.

EXTENDED PPN METRIC

- We want to consider PN equations of motion for photons in an extended PPN metric

$$g_{00} = -1 + 2\alpha_e U + O(v^4/c^4)$$
$$g_{0i} = 0 + O(v^3/c^3)$$
$$g_{ij} = (1 + 2\gamma_e U)\delta_{ij} + O(v^4/c^4)$$

α_e and γ_e are functions of coordinates

α_e can at most be eliminated locally

- From geodesic equation for photons, one finds

$$\mathrm{v}^i = \left[1 - \left(\alpha_e + \gamma_e\right) U\right] n^i,$$
$$\frac{dn^i}{dt} = (\delta^{ij} - n^i n^j)\partial_j[(\alpha_e + \gamma_e)U]$$

…these equations with $\alpha_e + \gamma_e = 2$ are the same of GR for light. This holds even if $\gamma_e \not\approx 1$, or even if α_e and γ_e are spacetime functions that change considerably locally.

THE γ_Σ PARAMETER

- A direct comparison between the PN equations of motion of the PPN and EPPN allows one to identify the effective gamma parameter

$$\gamma_\Sigma = \alpha_e + \gamma_e - 1$$

$$\mathrm{v}^i = \left[1 - (1 + \gamma_\Sigma)U\right] n^i$$

$$\frac{dn^i}{dt} = (\delta^{ij} - n^i n^j)\partial_j[(1 + \gamma_\Sigma)U]$$

- Indeed, this observation was done by Berry and Gair, in the specific context of *f(R)* theories (arXiv:1104.0819).

- Note that, whenever γ_Σ is a constant, it has exactly the same role that γ has for light propagation

WHAT ABOUT THE OBSERVATIONAL BOUNDS ?

- The constraints on γ cannot be applied to γ_e.

- The γ bounds cannot be immediately applied to γ_Σ, since it is required $\gamma_\Sigma = cte$ and a valid Newtonian limit (which is important for the gravitational mass definition).

- On the other hand, even without the knowledge of the mass of the Sun, one could test how the deflection angle changes for different impact parameter values.

$$\delta\theta = \left(\frac{1+\gamma_\Sigma}{2}\right)\frac{4M}{d}\left(\frac{1+\cos\theta_0}{2}\right) .$$

THE GRAVITATIONAL SLIP

- Within the cosmological context,

$$ds^2 = a^2(\tau)\left[-(1-2\psi)d\tau^2 + (1+2\phi)d\mathbf{x}^2\right]$$

- a comparison between lensing effects generated by a given system with the non-relativistic internal motion of the same system could be used to test the difference between the scalar potentials above.

$$\eta \equiv \frac{\phi}{\psi} \xrightarrow{\text{Solar system context}} \eta_{EPPN} = \frac{\gamma_e}{\alpha_e}$$

- For the standard PPN metric, one has $\eta_{PPN} = \gamma$.

- The point to be stressed is that the equality between slip and gamma is wrong in general, but it holds for the PPN.

THE GAMMA IN MASSIVE BRANS-DICKE THEORIES

- We start by considering the following action,

$$S = \int \frac{\sqrt{-g}}{2\kappa} \left[\Phi R + 2\frac{\omega(\Phi)}{\Phi} X - V(\Phi) \right] d^4x + S_m,$$

- And the PN approximation scheme,

$$\Phi = \varphi_0 + \varphi, \text{ with } \varphi_0 > 0 \text{ and } \varphi \sim O(2)$$

$$V(\Phi) \approx V_0 + V_1\varphi + V_2\varphi^2$$

$$\omega(\Phi) \approx \omega_0 + \omega_1\varphi + \omega_2\varphi^2$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$T^{00} \approx \rho$$

THE GAMMA IN MASSIVE BRANS-DICKE THEORIES

- Solving the equation for the scalar field, one finds

$$\varphi = \frac{\kappa}{4\pi(3+2\omega_0)} \int \frac{\rho(x',t)}{|\mathbf{x}-\mathbf{x}'|} e^{-m_\varphi|\mathbf{x}-\mathbf{x}'|} d^3x' \quad \text{with} \quad m_\varphi^2 = \frac{2\left(V_2\varphi_0 - V_1\right)}{3+2\omega_0}$$

- The metric equations yields

$$\alpha_e = \frac{\kappa}{16\pi\varphi_0}\left(a_1 + a_2\frac{\varphi}{U}\right) \quad \text{with} \quad a_1 \equiv 2 - \frac{2}{3+2\omega_0}\frac{V_1}{m_\varphi^2} \quad \text{and} \quad a_2 \equiv \frac{8\pi}{\kappa}\left(1 + \frac{V_1}{m_\varphi^2}\right)$$

$$\gamma_e = \frac{\kappa}{16\pi\varphi_0}\left(a_3 - a_2\frac{\varphi}{U}\right), \quad \text{with} \quad a_3 \equiv 2 + \frac{2}{3+2\omega_0}\frac{V_1}{m_\varphi^2} = 4 - a_1\,.$$

THE GAMMA IN MASSIVE BRANS-DICKE THEORIES

- One then obtains

$$\gamma_\Sigma = \alpha_e + \gamma_e - 1 = \frac{\kappa}{4\pi\varphi_0} - 1 \quad \text{and} \quad \eta = \frac{\gamma_e}{\alpha_e} = -1 + \frac{4}{a_1 + a_2\varphi/U}$$

- The γ_Σ parameter in generalized Brans-Dicke theories is always a <u>constant</u> and its value depends on the theory's coupling constant κ.

- This means that its numerical value is influenced by Newtonian limit, or, in the absence of a Newtonian limit, by the definition of the mass (gravitational constant).

- The gravitational slip η on other hand, is a spacetime function whose value is independent from κ.

ANOTHER PARAMETRIZATION COMMONLY USED

- There are references working with

$$g_{00} = -1 + 2G_{eff}U + O(v^4/c^4)$$

$$g_{0i} = 0 + O(v^3/c^3)$$

$$g_{ij} = (1 + 2G_{eff}\,\gamma_{eff}U)\delta_{ij} + O(v^4/c^4)$$

$$\Rightarrow \quad \alpha_e = G_{eff} \qquad \gamma_e = G_{eff}\gamma_{eff} \quad \Rightarrow \quad \gamma_{eff} = \frac{\gamma_e}{\alpha_e} = \eta$$

Considerations on the Newtonian limit

NEGLIGIBLE MASS SCALAR FIELD

- For a given mass m_φ and inside the system of length scale ℓ, with $m_\varphi^2 \ell^2 \sim O(v/c)$, one has $\varphi \propto U$ and

$$\alpha_e \approx \frac{\kappa}{4\pi\varphi_0}\left(\frac{2+\omega_0}{3+2\omega_0}\right)$$

- Therefore, to satisfy the Newtonian limit the right hand side above must be equal to 1, which implies

$$\frac{\kappa}{4\pi\varphi_0} = \frac{3+2\omega_0}{2+\omega_0} \quad \Rightarrow \quad \gamma_\Sigma = \frac{1+\omega_0}{2+\omega_0}.$$

- It is worth to reinforce that the γ_Σ expression above is valid even where the Newtonian limit does not hold.

LARGE MASS SCALAR FIELD

- For a given mass m_φ and inside the system of length scale ℓ, with $e^{-m_\varphi \ell} \sim O(1)$, one has $\varphi \ll U$ and

$$\alpha_e \approx \frac{\kappa a_1}{16\pi\varphi_0}$$

- Therefore, to satisfy the Newtonian limit the right hand side above must be equal to 1, which implies

$$\frac{\kappa}{4\pi\varphi_0} = \frac{4}{a_1} \quad \Rightarrow \quad \gamma_\Sigma = \frac{(3+2\omega_0)m_\varphi^2 + V_1}{(3+2\omega_0)m_\varphi^2 - V_1}$$

- The GR result $\gamma_\Sigma = 1$ is found if either $m_\varphi \to \infty$, $\omega_0 \to \infty$ or $V_1 \to 0$.

INTERMEDIATE MASS SCALAR FIELD

- In the two cases before, the bounds on γ can be applied to γ_Σ
- If there is no Newtonian correspondence within the considered system, γ_Σ still parametrizes light trajectories, but how the mass of Sun is determined? This question was considered by Alsing et al. (arXiv: 1112.4903)
- To exemplify, consider $V_1 = 0, \quad \rho = M_\odot \delta^3(\mathbf{x})$ and $\mathbf{x} = r$. One finds

$$\alpha_e = \frac{\kappa}{8\pi\varphi_0}\frac{M_\odot}{r}\left(1 + \frac{1}{3+2\omega_0}e^{-m_\varphi r}\right) \xrightarrow{\text{Newtonian limit}} \alpha_e(r_\oplus) = 1$$

And this imply $$\gamma_\Sigma = \frac{1 - \frac{1}{3+2\omega_0}e^{-m_\varphi r_\oplus}}{1 + \frac{1}{3+2\omega_0}e^{-m_\varphi r_\oplus}}$$ $\Rightarrow$ This is similar to the slip $[\eta(r_\oplus) = \gamma_\Sigma]$ but still a true constant

$f(R)$ theories

METRIC *f(R)*

- The interpretation of a metric *f(R)* model as an effective scalar-tensor theory is done through

$$\Phi \equiv \frac{df(R)}{dR}, \quad V(\Phi) \equiv R\Phi - f(R) \quad \text{and} \quad \omega(\Phi) = 0.$$

- Which gives $\varphi_0 = f_1, \;\; V_1 = 0, \;\; V_2 = \dfrac{1}{4f_2}$ and $m_\varphi^2 = \dfrac{1}{6}\dfrac{f_1}{f_2}$.

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi f_1} - 1\,, \text{ in general}\,; \\ \frac{1}{2}\,, \quad \text{if } \frac{f_1 \ell^2}{6 f_2} \sim O(1)\,; \\ 1\,, \quad \text{if } e^{-\sqrt{f_1 \ell^2/(6 f_2)}} \sim O(1)\,. \end{cases}$$

PALATINI *f(R)*

- Palatini formulations are a special case where $\omega = -3/2$, implying that the scalar field is not dynamical,

$$\varphi = \frac{2\kappa f_2}{f_1}\rho \quad \text{Which gives} \quad \alpha_e = \frac{\kappa}{8\pi f_1}\left(1 + 16\pi f_2 \frac{\rho}{U}\right)$$

- The last term indicates that Newtonian gravity is violated inside matter. However, this violation is relevant only if the pressure and the internal energy are known from first principles.

- Thus, the Newtonian limit of Palatini *f(R)* theories is well posed if one sets $\kappa/4\pi f_1 = 2$, and consequently $\gamma_\Sigma = 1$.

- For details on Palatini gravity please see Toniato, Rodrigues and Wojnar, arXiv:1912.12234

Extension to Horndeski theories

HORNDESKI THEORIES

Action:

$$S = \sum_{i=2}^{5} \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, \mathscr{L}_i + S_m$$

$$\mathcal{L}_2 = K(\Phi, X), \quad \mathcal{L}_3 = -G_3(\Phi, X)\Box\Phi,$$

$$\mathcal{L}_4 = G_4(\Phi, X)R + G_{4X}(\Phi, X)\left[(\Box\Phi)^2 - (\nabla_\mu\nabla_\nu\Phi)^2\right],$$

$$\mathcal{L}_5 = G_5(\Phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\Phi - \frac{1}{6}G_{5X}(\Phi, X)\big[(\Box\Phi)^3 - 3\Box\Phi(\nabla_\mu\nabla_\nu\Phi)^2 + 2(\nabla_\mu\nabla_\nu\Phi)^3\big].$$

Expansion:

$$\xi(\varphi, X) \approx \xi_{(0,0)} + \xi_{(1,0)}\varphi + \xi_{(0,1)}X + \ldots,$$

$$\xi = (K, G_i)$$

The gamma:

$$\gamma_\Sigma = \begin{cases} \frac{\kappa}{4\pi\, G_{4(0,0)}} - 1\,, & \text{in general,} \\ \dfrac{W - G_{4(1,0)}^2}{W + G_{4(1,0)}^2}, & \text{if } m_\varphi^2\ell^2 \sim O(1), \\ \dfrac{W m_\varphi^2 - K_{(1,0)}G_{4(1,0)}}{W m_\varphi^2 + K_{(1,0)}G_{4(1,0)}}, & \text{if } e^{-m_\varphi \ell} \sim O(1). \end{cases}$$

$$W = G_{4(0,0)}\left(K_{(0,1)} - 2G_{3(1,0)}\right) + 3G_{4(1,0)}^2,$$

CONCLUSIONS

- The differences between the gamma from PPN, its possible extensions and the gravitational slip are subtle but with important consequences to the physical bounds, as here discussed.

- The importance of precise statements about them becomes higher with the crescent use of PPN and related formalisms in the context of extragalactic astronomy.

- Essentially: the bounds on γ cannot be applied to η, in general. They can be applied to γ_Σ when i) it is a constant and ii) the Newtonian limit is valid (or if the Keplerian mass can be corrected).

- The distinction between γ_Σ and η is clear in scalar-tensor and *f(R)* theories.

- Care must be taken in applying PPN bounds to theories which does not have a well defined Newtonian limit.