Do Black Holes Fall in Love?

Marc Casals

Centro Brasileiro de Pesquisas Físicas (Brazil)
University College Dublin (Ireland)

In collaboration with Alexandre Le Tiec and Edgardo Franzin
Arxiv: 2007.00214 (accepted in PRL) and 2010.15795

UFES, 12/3/2021

Index

1. Love Numbers in *Newtonian* Gravity

2. Love Numbers in *General Relativity*

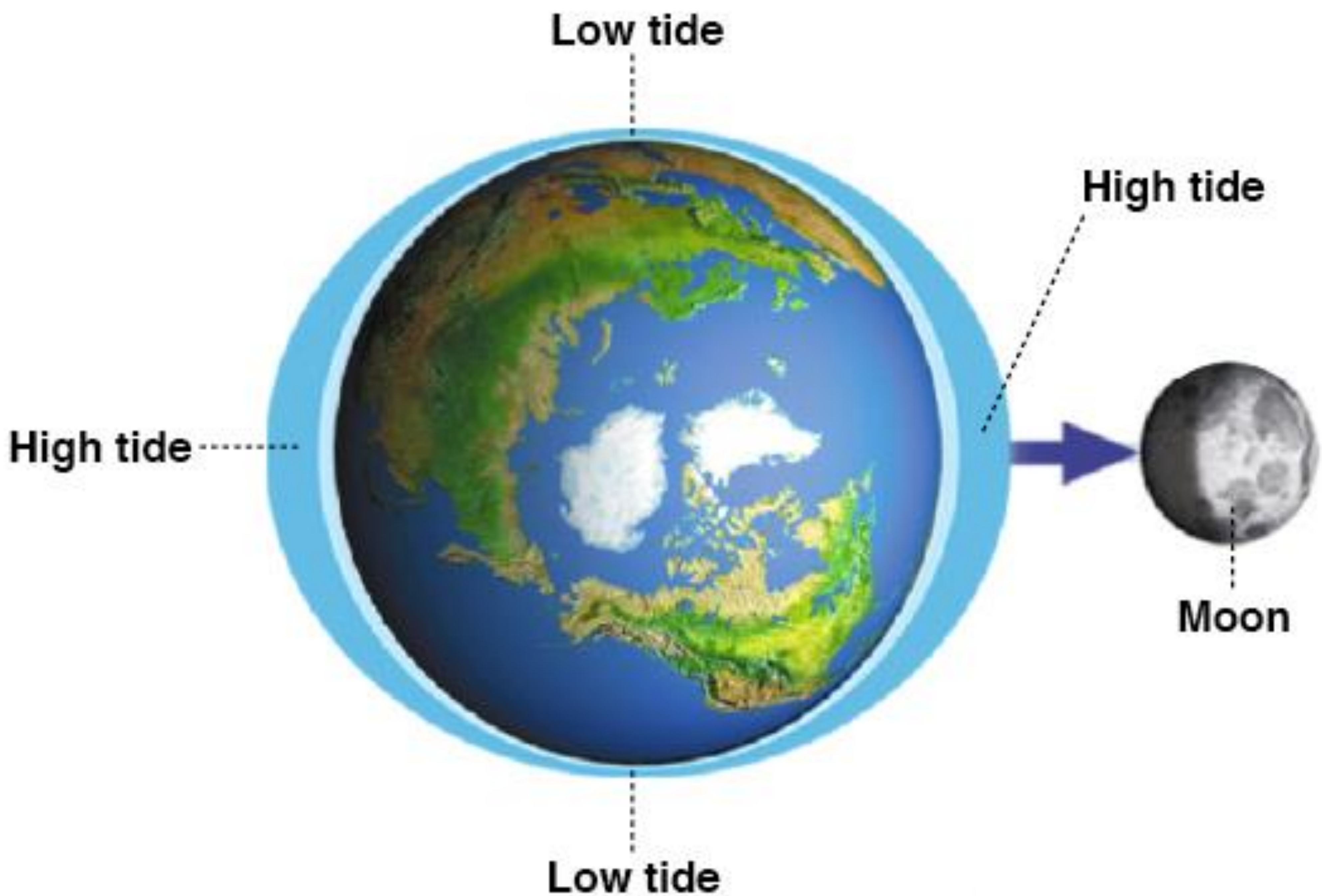
3. Love Numbers of *Rotating Black Holes*

4. Conclusions

1. Love Numbers in Newtonian Gravity

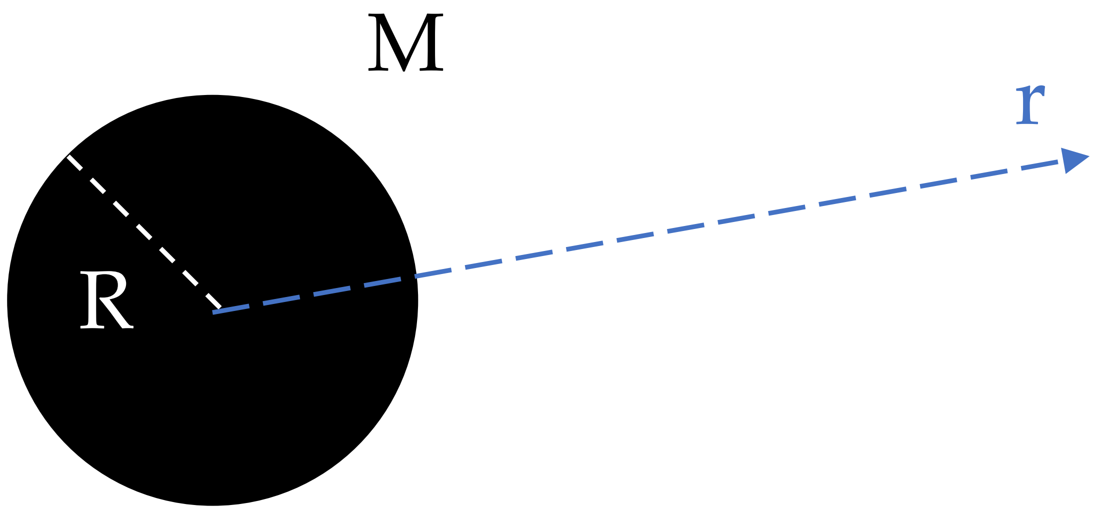
Earth & Moon – Love in Newtonian gravity

Augustus Love (1909) introduced numbers characterizing the Earth´s tides in its response to the Moon´s gravitational field



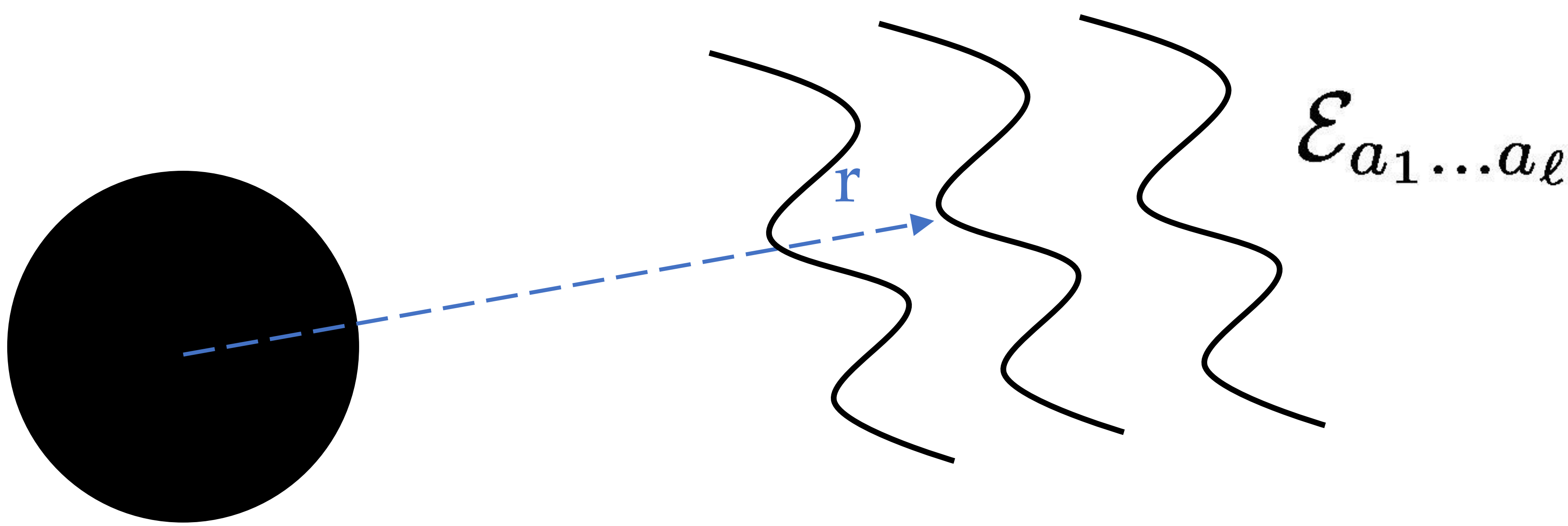
Tidal deformation in Newonian Gravity

Gravitational potential of a compact body in isolation



$$\mathring{U} = \frac{M}{r}$$

Gravitational potential of some external tidal field

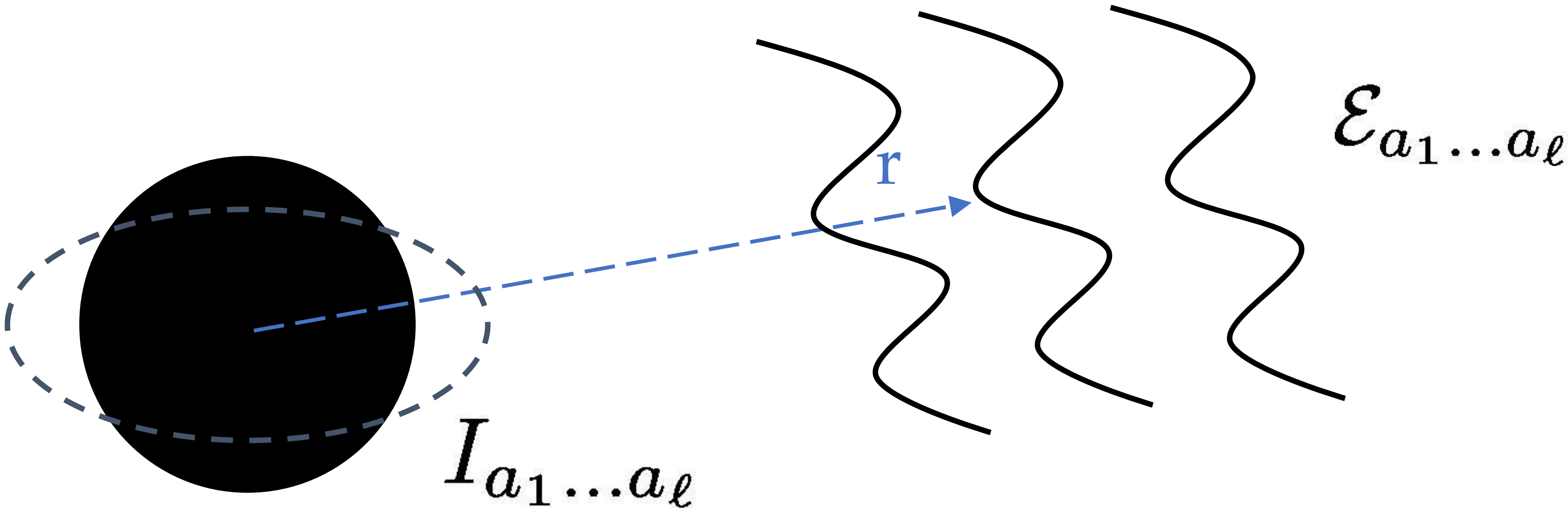


$$U^{\mathrm{tidal}} = -\sum_{\ell=2}^{\infty} \frac{(\ell-2)!}{\ell!} x_1^a \cdots x_\ell^a \, \mathcal{E}_{a_1 \ldots a_\ell}(t)$$

↑

tidal multipole moments

Deformation of non-rotating compact body in an external tidal field

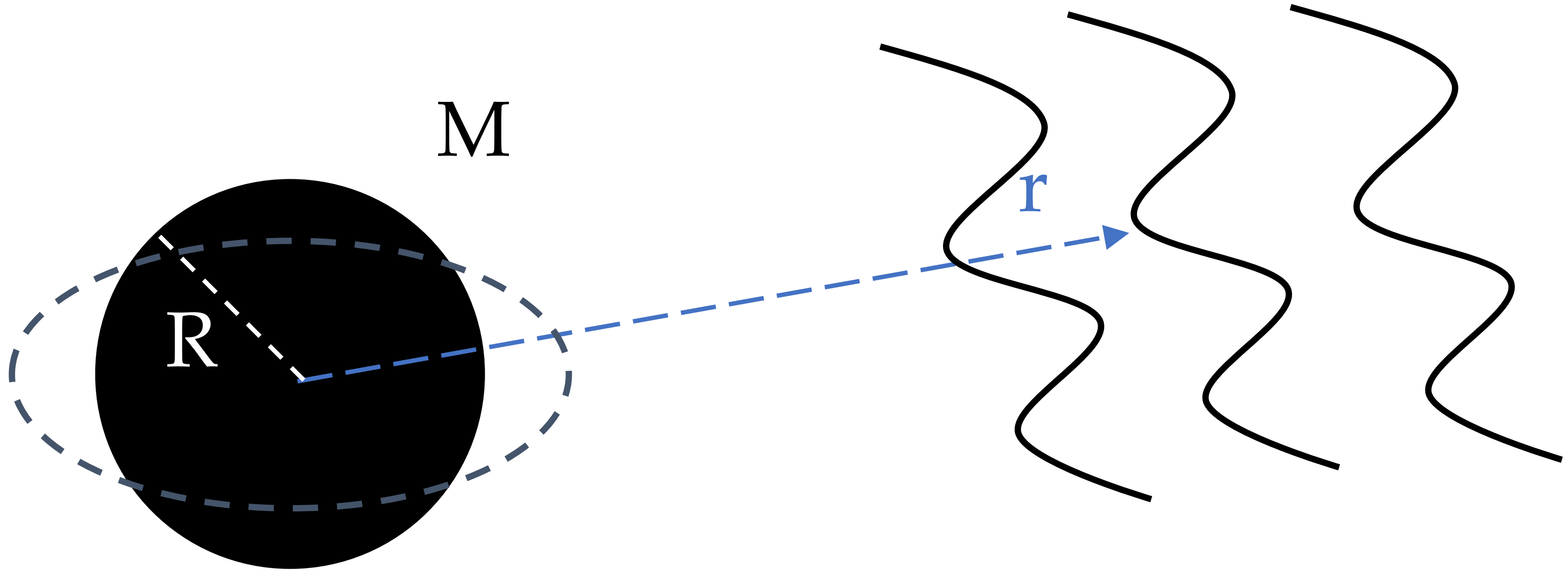


induced moments

$$U^{\mathrm{resp}} = \sum_{\ell=2}^{\infty} \frac{(2\ell-1)!!}{\ell!} \frac{x_1^a \cdots x_\ell^a I_{a_1\ldots a_\ell}(t)}{r^{2\ell+1}}$$

$$I_{a_1\ldots a_\ell} = \lambda_\ell \, \mathcal{E}_{a_1\ldots a_\ell}$$

tidal Love numbers (TLNs) of the compact body



Total gravitational potential by linearity (and decomposing into spherical harmonics $Y_{\ell m}(\theta, \varphi)$)

$$U = \mathring{U} + U^{\text{tidal}} + U^{\text{resp}} =$$

$$\frac{M}{r} - \sum_{\ell \geq 2} \sum_{m \leq |\ell|} \frac{(\ell-2)!}{\ell!} \mathcal{E}_{\ell m} \, r^{\ell} \left[1 + 2k_{\ell} \left(\frac{R}{r} \right)^{2\ell+1} \right] Y_{\ell m}$$

isolated

tidal

response

dimensionless TLNs: $k_{\ell} \equiv -\frac{(2\ell-1)!!}{2(\ell-2)!} \, \frac{\lambda_{\ell}}{R^{2\ell+1}}$

It´s convenient to use a curvature Weyl scalar

$$\psi_0 = C_{\alpha\beta\gamma\delta}\ell^\alpha m^\beta \ell^\gamma m^\delta = \sum_{\ell,m} \psi_0^{\ell m}$$

(projection of Weyl tensor $C_{\alpha\beta\gamma\delta}$ on some null vectors ℓ^α and m^β)

In the Newtonian limit,

$$\lim_{c\to\infty} c^2\psi_0 = \text{2nd order operator on } U$$

$$\lim_{c\to\infty} c^2\psi_0^{\ell m} \propto \mathcal{E}_{\ell m}\, r^{\ell-2}\left[1 + 2k_\ell \left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

tidal $\sim r^{\ell-2}$

response $\sim r^{-\ell-3}$

spin-weighted spherical harm.

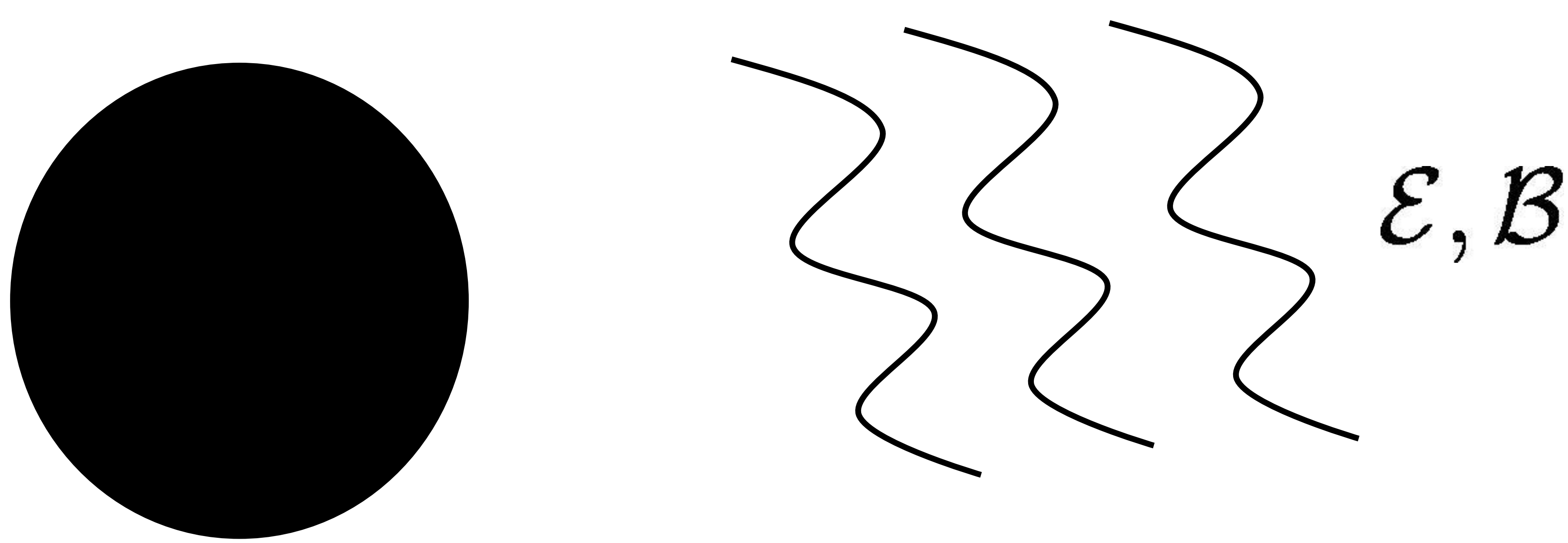
2. Love Numbers in General Relativity

Tidal moments

Consider a *slowly-varying* tidal environment. It can be described by two types of tidal moments constructed from the Weyl tensor:

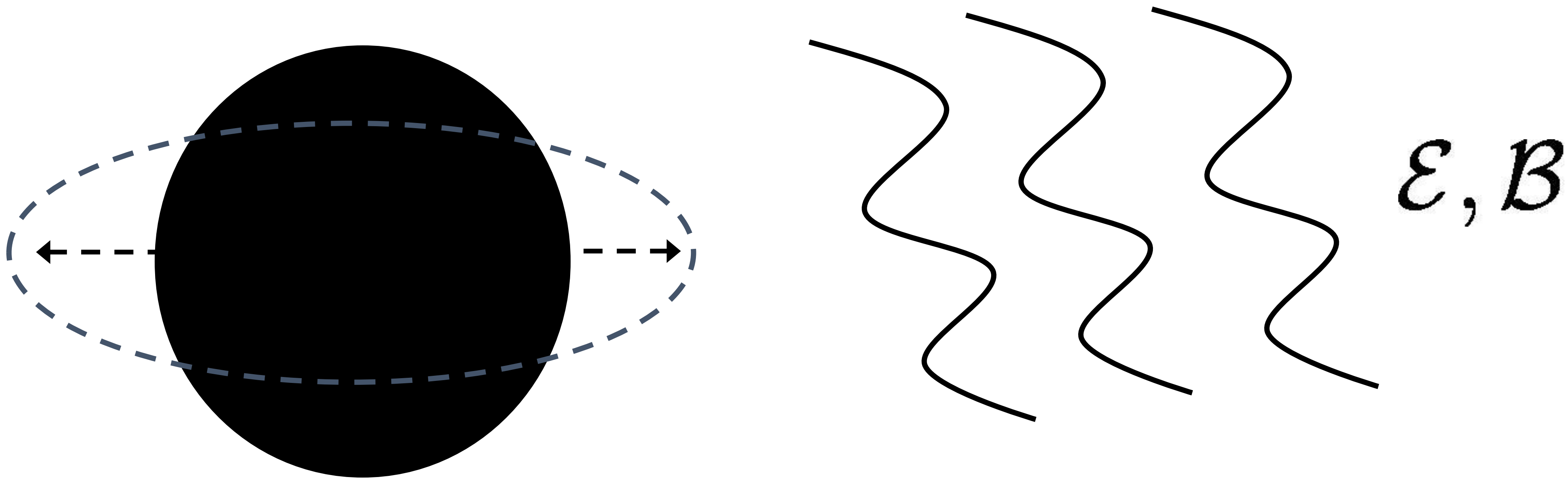
$$\mathcal{E}_{i_1 \ldots i_\ell} \propto C_{0\langle i_1 | 0 | i_2 ; i_3 \cdots i_\ell \rangle}$$ (electric-type)

$$\mathcal{B}_{i_1 \ldots i_\ell} \propto \varepsilon_{jk\langle i_1} C_{i_2 | 0jk | ; i_3 \cdots i_\ell \rangle}$$ (magnetic-type; absent in Newonian gravity)



Induced moments

Two types of moments for the compact body:



mass-type: $\mathring{M} + \delta M$

current-type: $\mathring{S} + \delta S$

$\mathring{S}$: background moments

δS: induced moments

Tidal moments: $\mathcal{E}, \mathcal{B}$

$$g_{\alpha\beta} \equiv \mathring{g}_{\alpha\beta} + h^{\text{resp}}_{\alpha\beta}$$

metric perturbation response

Geroch-Hansen moments (they´re *coordinate independent*):

$$M_{i_1 \ldots i_\ell} = \mathring{M}_{i_1 \ldots i_\ell} + \delta M_{i_1 \ldots i_\ell}$$

background moments

induced moments

$$S_{i_1 \ldots i_\ell} = \mathring{S}_{i_1 \ldots i_\ell} + \delta S_{i_1 \ldots i_\ell}$$

Tidal Love numbers

Expanding the moments $M_{i_1 \ldots i_\ell}$ and $S_{i_1 \ldots i_\ell}$ into modes:

$$M_{\ell m} = \mathring{M}_{\ell m} + \lambda^{M\mathcal{E}}_{\ell m}\,\mathcal{E}_{\ell m} + \lambda^{M\mathcal{B}}_{\ell m}\,\mathcal{B}_{\ell m}$$

$$S_{\ell m} = \mathring{S}_{\ell m} + \lambda^{S\mathcal{E}}_{\ell m}\,\mathcal{E}_{\ell m} + \lambda^{S\mathcal{B}}_{\ell m}\,\mathcal{B}_{\ell m}$$

$\lambda^{M\mathcal{E},M\mathcal{B},S\mathcal{E},S\mathcal{B}}_{\ell m}$: *four* types of TLNs, connecting electric/magnetic-type tidal moments with mass/current-type induced moments

TLNs of neutron stars

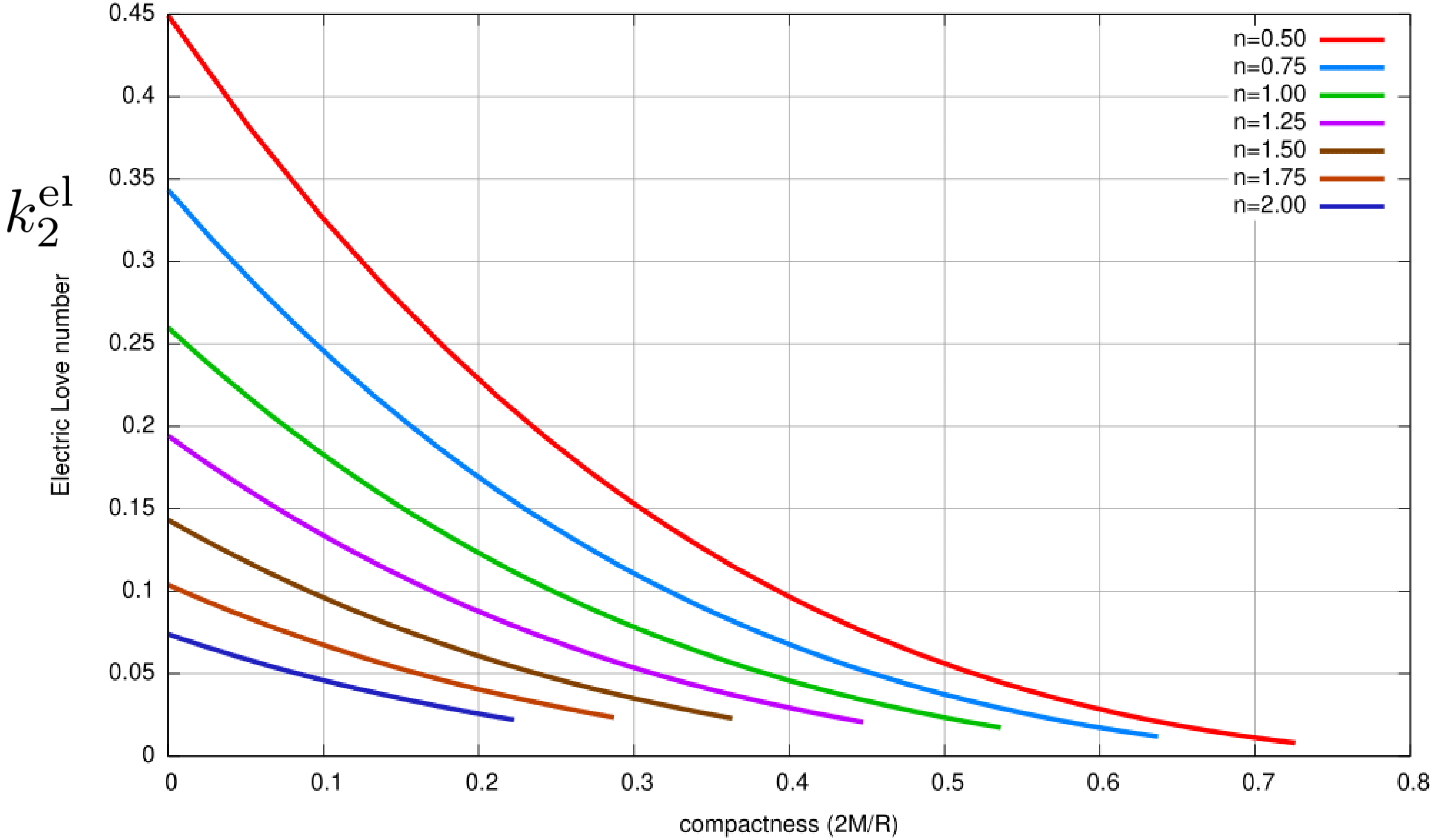
TLNs of neutron stars:

- are nonzero (even in the non-rotating case)

- depend on their equation of state

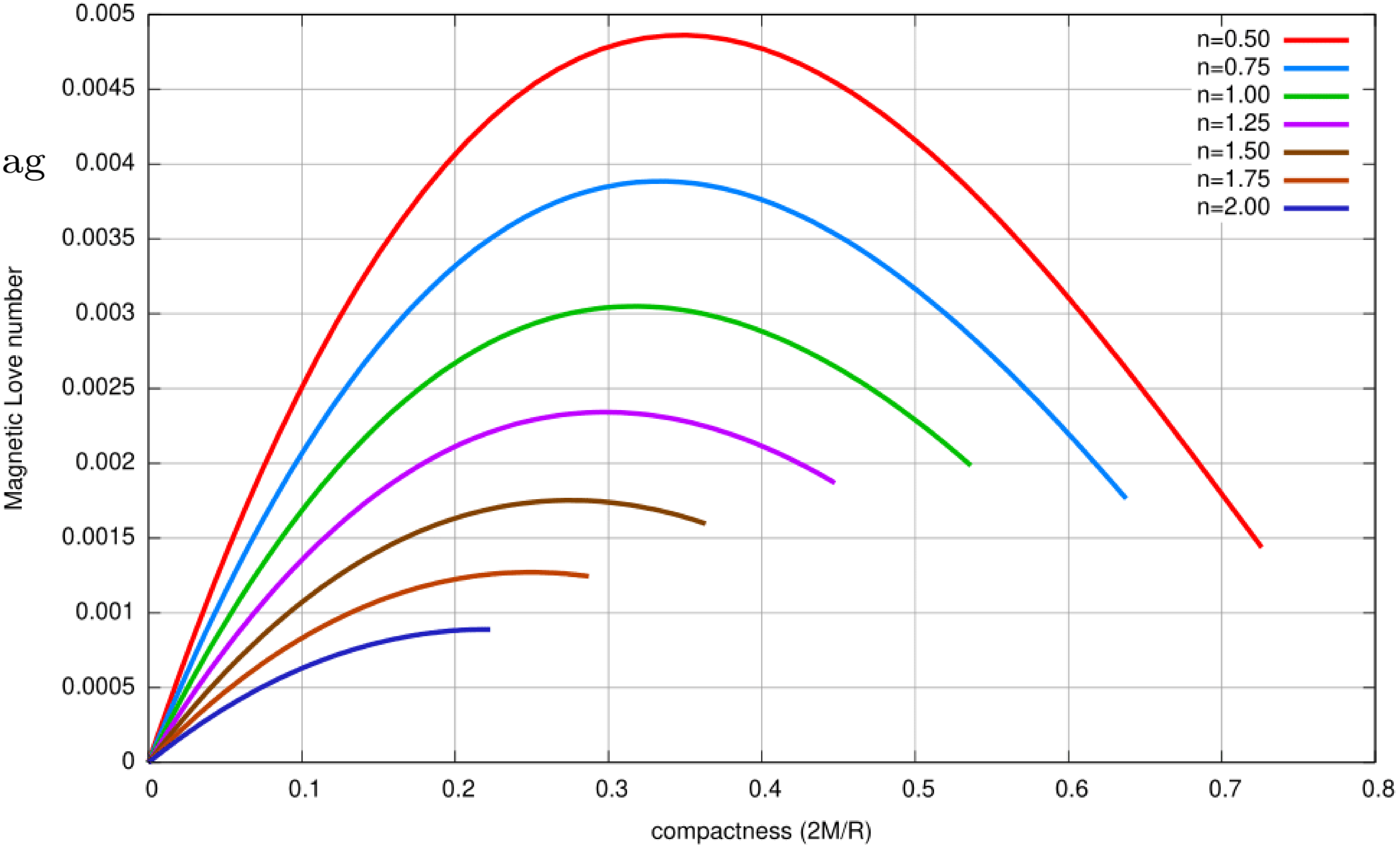
$$p = K\,\rho^{1+1/n}$$

pressure density



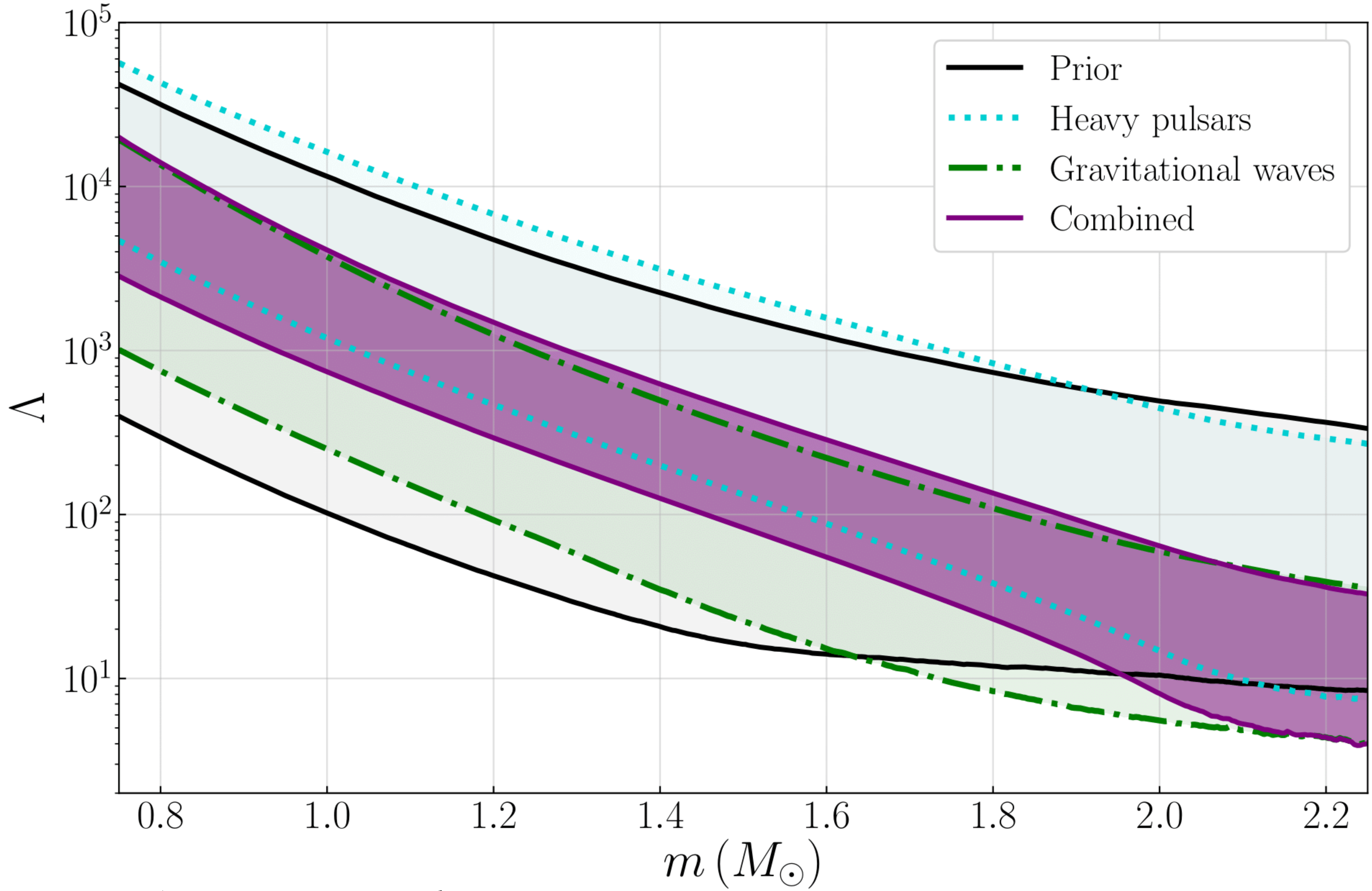
Binnington & Poisson´09

k_2^{mag}



Binnington & Poisson´09

TLNs of neutron stars have been constrained by gravitational wave observations by LIGO



Chatziioannou´20

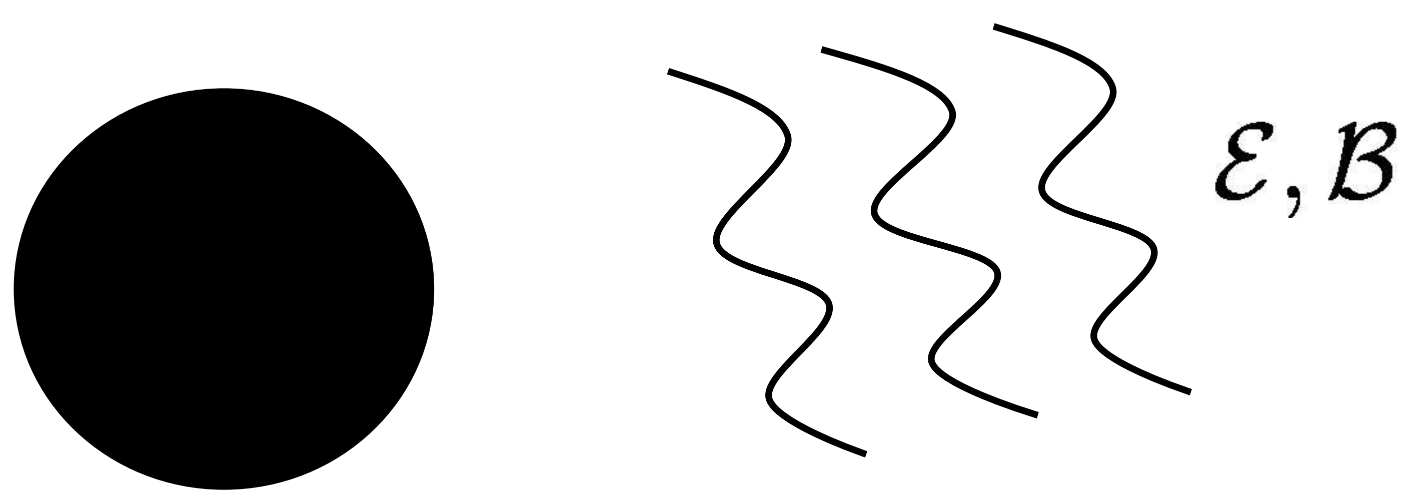
Love numbers of non-rotating black holes

Non-rotating BH are described by the Schwarzschild metric

It´s been shown that the (static) TLNs of Schwarzschild BHs are zero

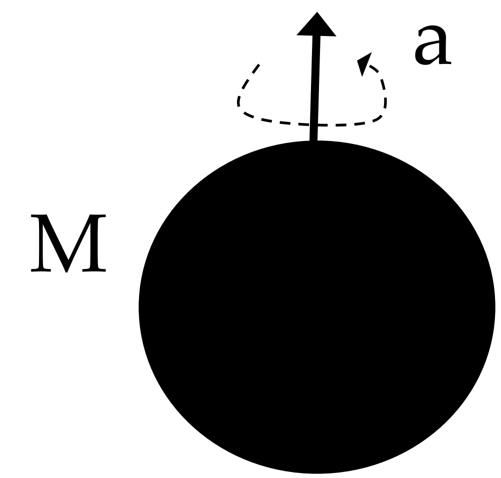
(Binnington & Poisson´09; Damour & Nagar´09)

So non-rotating BHs do *not* deform under an external (static) tidal field



3. Love Numbers of *Rotating* Black Holes

Kerr black hole



Kerr metric (in advanced coordinates)

$$\mathring{g}_{\alpha\beta}\, \mathrm{d}x^\alpha \mathrm{d}x^\beta = -\left(1 - \frac{2Mr}{\Sigma}\right) \mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r - \frac{4Mr}{\Sigma}\, a \sin^2\theta\, \mathrm{d}v\mathrm{d}\phi -$$

$$2a\sin^2\theta\, \mathrm{d}r\mathrm{d}\phi + \Sigma\, \mathrm{d}\theta^2 + \left(r^2 + a^2 + \frac{2Mr}{\Sigma}\, a^2 \sin^2\theta\right) \sin^2\theta\, \mathrm{d}\phi^2$$

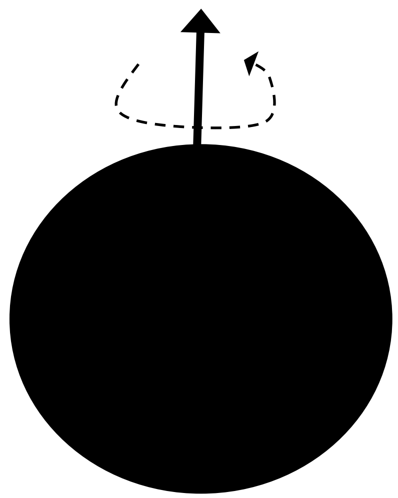
It describes the gravitational field of a *rotating* black hole with mass M and (intrinsic) ang. mom. a

It has an event horizon at $r = r_+$ and an inner horizon at $r = r_-$

All astrophysical BHs are expected to be rotating and so described by the Kerr metric

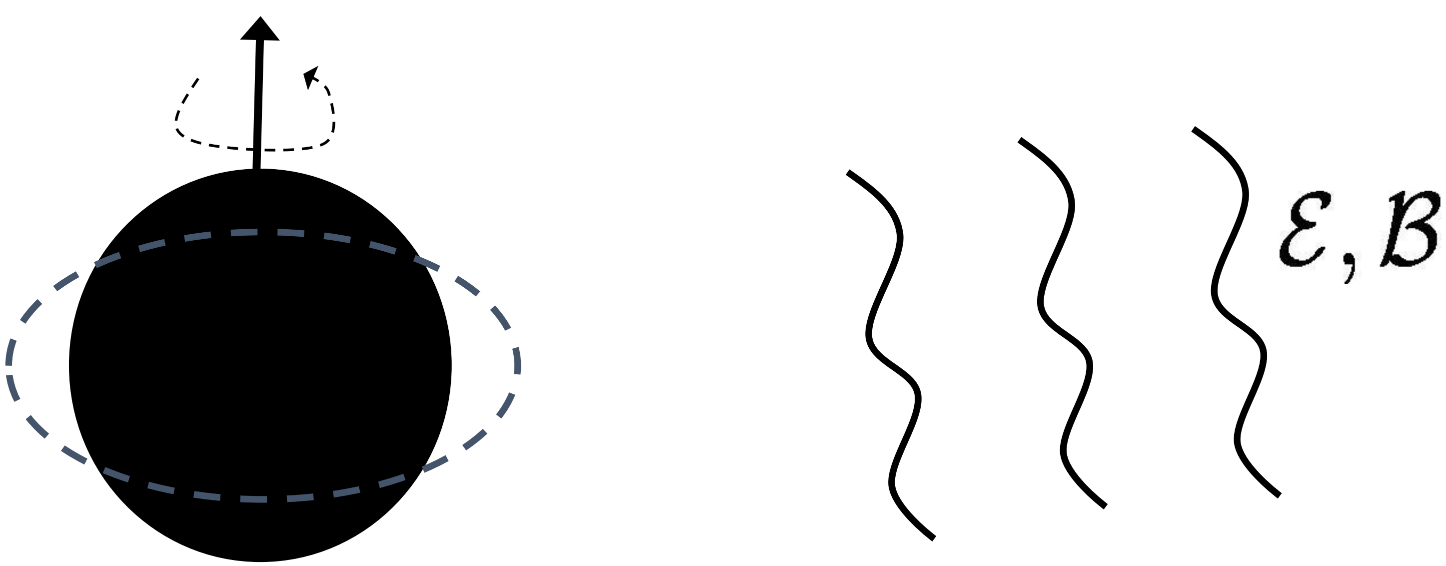
Kerr moments

Modes of the Geroch-Hansen moments of (*isolated*) Kerr black hole:



$$\mathring{M}_\ell + i\,\mathring{S}_\ell = M\,(i\,a)^\ell$$

What about the moments of Kerr in an external *tidal environment*?



Strategy for calculating the Kerr TLNs

Tidal moments: $\mathcal{E}, \mathcal{B}$

$\downarrow$

Induced response: $\psi_0^{\rm resp} \longrightarrow \Psi^{\rm resp} \longrightarrow h_{\alpha\beta}^{\rm resp}$

Weyl scalar — Hertz potential — Metric perturbation

Geroch-Hansen moments: TLNs

Perturbed Kerr metric

$$g_{\alpha\beta} \equiv \mathring{g}_{\alpha\beta} + h_{\alpha\beta}^{\rm resp} \longrightarrow \begin{cases} M_{i_1\ldots i_\ell} = \mathring{M}_{i_1\ldots i_\ell} + \delta M_{i_1\ldots i_\ell} \\ S_{i_1\ldots i_\ell} = \mathring{S}_{i_1\ldots i_\ell} + \delta S_{i_1\ldots i_\ell} \end{cases}$$

Modes of the Weyl tensor of perturbed Kerr background

$$\psi_0^{\ell m} \propto \left(\mathcal{E}_{\ell m}(v) + i\,\frac{\ell+1}{3}\,\mathcal{B}_{\ell m}(v) \right) R_{\ell m}(r)\,{}_2Y_{\ell m}(\theta,\phi)$$

after matching it at $r \to \infty$ with the tidal environment $\mathcal{E}, \mathcal{B}$

The radial factor satisfies the static ($\omega = 0$) Teukolsky eq.

$$x(x+1)R''_{\ell m} + (6x + 3 + 2im\gamma)R'_{\ell m} + \left[4im\gamma\,\frac{2x+1}{x(x+1)} - (\ell+3)(\ell-2)\right]R_{\ell m} = 0$$

$$x \equiv \frac{r - r_+}{r - r_-} \qquad \gamma \equiv \frac{a}{r_+ - r_-}$$

Sln. of Teukolsky eq.

The sln. can be obtained in terms of hypergeometric functions F:

$$R_{\ell m} = R_{\ell m}^{\text{tidal}} + 2k_{\ell m}\ R_{\ell m}^{\text{resp}}$$

$$R_{\ell m}^{\text{tidal}} \propto \frac{x^{\ell}}{(1+x)^2} F\left(-\ell-2, -\ell-2im\gamma, -2\ell; -\frac{1}{x}\right) \underset{r\to\infty}{\sim} r^{\ell-2}$$

$$R_{\ell m}^{\text{resp}} \propto \frac{x^{-(\ell+1)}}{(1+x)^2} F\left(\ell-1, \ell+1-2im\gamma, 2\ell+2; -\frac{1}{x}\right) \sim r^{-(\ell+3)}$$

$$k_{\ell m} \equiv -i\, m\, \gamma\ \left(1-(a/M)^2\right)^{\ell+1/2} \frac{(\ell-2)!(\ell+2)!}{2(2\ell)!(2\ell+1)!} \prod_{n=1}^{\ell} (n^2+4m^2\gamma^2)$$

The large-r behaviour of Weyl tensor modes of perturbed Kerr is thus

$$\psi_0^{\ell m} \underset{r\to\infty}{\sim} \left[\mathcal{E}_{\ell m} + i\,\frac{\ell+1}{3}\,\mathcal{B}_{\ell m}\right] r^{\ell-2} \left[1 + 2k_{\ell m}\left(\frac{2M}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

tidal

response

"Newtonian" TLNs

Cf. the modes in the Newtonian theory:

$$\lim_{c\to\infty} c^2 \psi_0^{\ell m} \propto \mathcal{E}_{\ell m}\, r^{\ell-2} \left[1 + 2k_{\ell}\left(\frac{R}{r}\right)^{2\ell+1}\right] {}_2Y_{\ell m}$$

Kerr TLNs

We calculated the *quadrupole* ($\ell = 2$) modes of the induced moments and the TLNs to *linear order in ang. mom. a*

$$M_{2m} + i\,S_{2m} \doteq \frac{8}{45}\, i\, m\, a\, M^4 (\mathcal{E}_{2m} + i\,\mathcal{B}_{2m})$$

$$\parallel$$

$$\lambda_{2m}^{M\mathcal{E}} = \lambda_{2m}^{S\mathcal{B}}$$

No parity mixing: $\lambda_{2m}^{M\mathcal{B}} = \lambda_{2m}^{S\mathcal{E}} = 0$

quadrupole TLNs

The corresponding dimensionless TLNs are

$$k_{2m}^{M\mathcal{E}} = k_{2m}^{S\mathcal{B}} = -\frac{i\ m}{120}\frac{a}{M} \qquad k_{2m}^{M\mathcal{B}} = k_{2m}^{S\mathcal{E}} = 0$$

(1) They are zero for:

(i) rotating BH in axisymmetric tidal field ($m = 0$)

(ii) non-rotating BH ($a = 0$)

(2) For, e.g., $a = 0.1M$ it´s

$|k_{2,\pm 2}| \approx 2 \times 10^{-3}$ ⟶ Kerr BHs are "rigid"

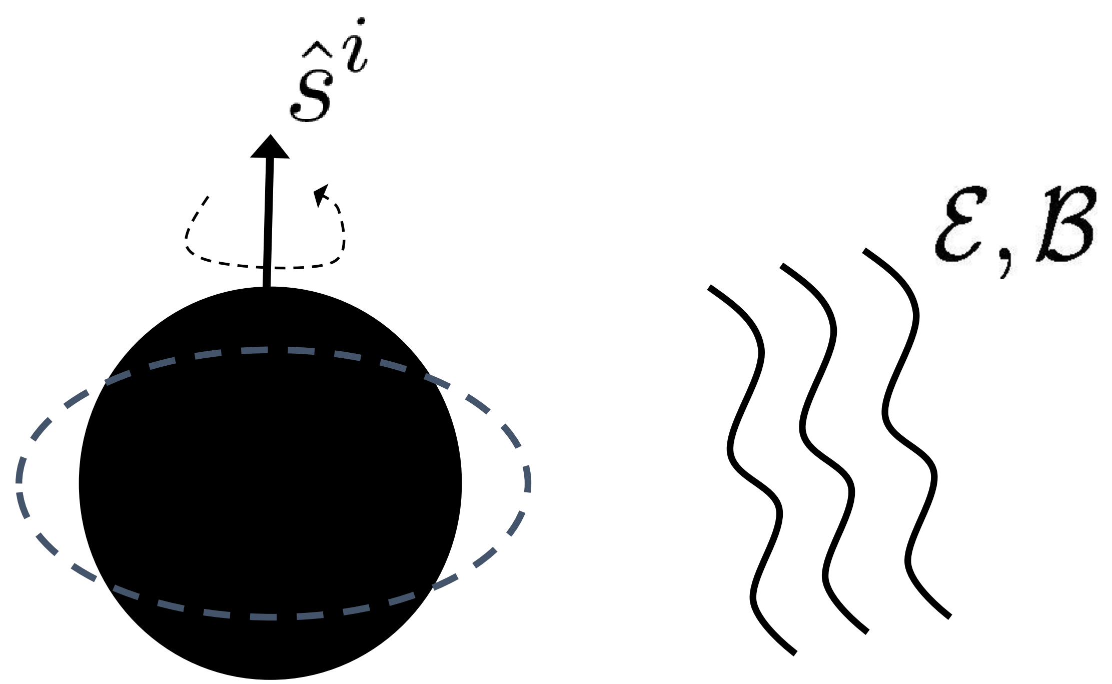
(3) TLNs purely imaginary ⟶ the BH tidal bulge is rotated by 45° in relation to the tidal perturbation (tidal lag)

Induced moments

We calculated the tidally-induced quadrupole moments

$$\delta M_{ij} \doteq \lambda_{ijkl}\,\mathcal{E}^{kl} \doteq \frac{16}{45}\,a\,M^4\,\mathcal{E}^k{}_{(i}\,\varepsilon_{j)kl}\,\hat{s}^l$$

$$\delta S_{ij} \doteq \lambda_{ijkl}\,\mathcal{B}^{kl} \doteq \frac{16}{45}\,a\,M^4\,\mathcal{B}^k{}_{(i}\,\varepsilon_{j)kl}\,\hat{s}^l$$

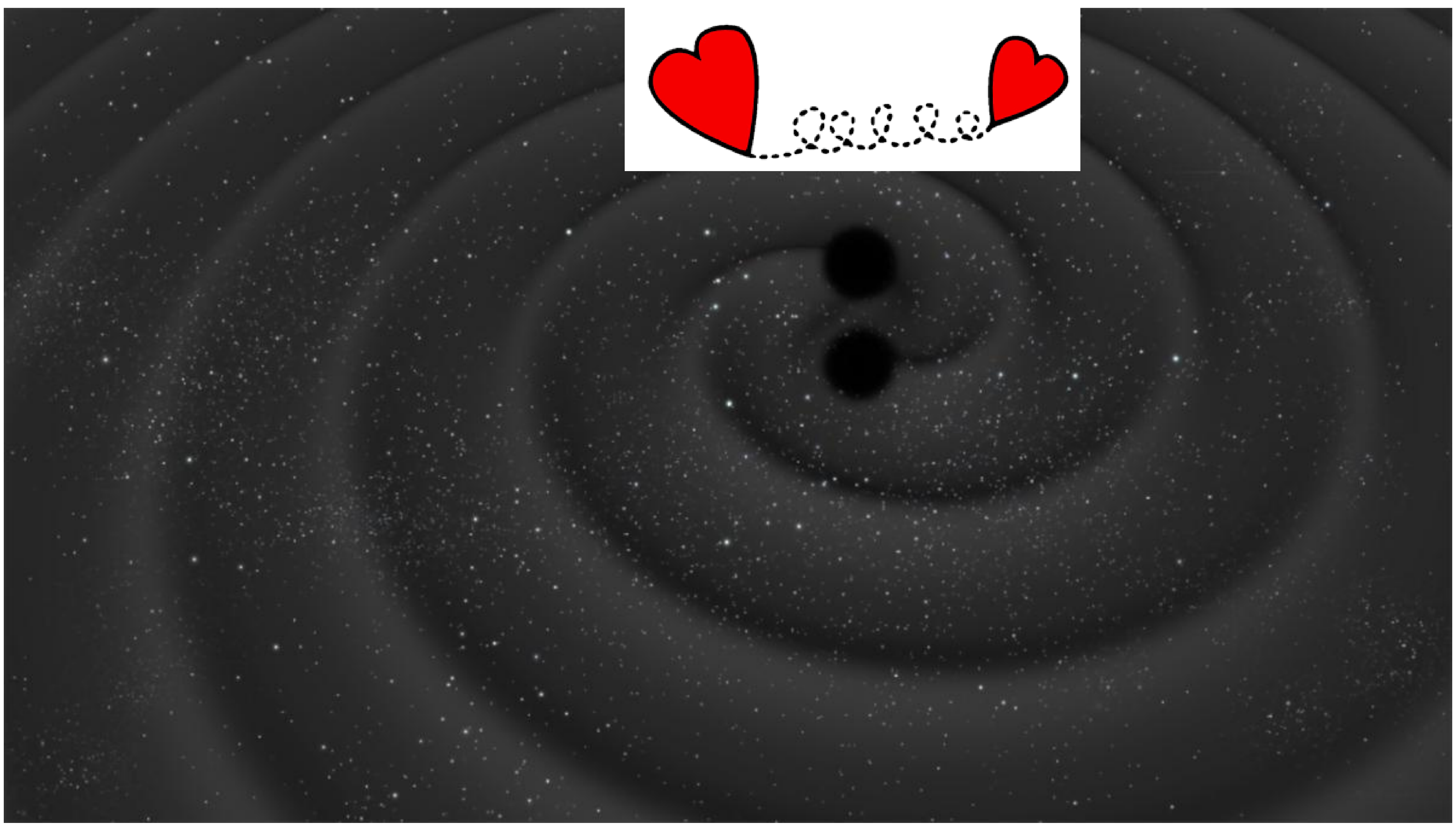


where the tidal Love *tensor* is

$$\lambda_{ij\langle kl\rangle} \doteq -\frac{16}{45}\,a\,M^4\,\delta_{(i|\langle k}\,\varepsilon_{l\rangle|j)q}\,\hat{s}^q$$

So rotating BHs *do* deform under an external (static) tidal environment (as opposed to non-rotating BHs)

In particular, during the inspiral of two rotating BHs, each one acts as a tidal environment for the other one and so each one "*falls in Love*" with its companion



Credit:
ESA-C.Carreau

Tidal torquing

Consider an *arbitrary* spinning body in a tidal environment $\mathcal{E}, \mathcal{B}$

The average rate of change of its angular momentum (tidal torquing) is (Thorne&Hartle´80):

$$M\langle \dot{a} \rangle = -\varepsilon^{ijk}\hat{s}_i \langle M_{jl}\mathcal{E}^l{}_k + S_{jl}\mathcal{B}^l{}_k \rangle$$

Introducing $M_{jl},\ S_{jl}$ in it by our values for the induced Kerr moments

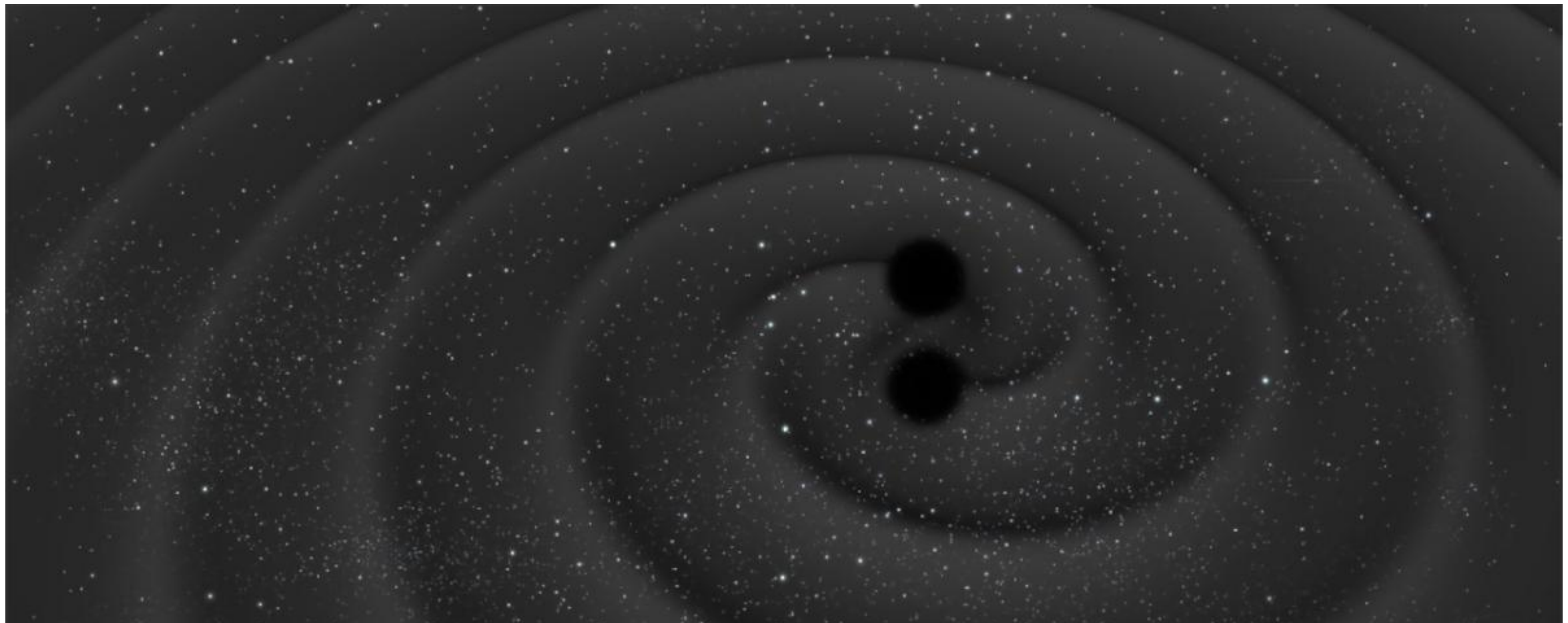
$$M\langle \dot{a} \rangle \doteq -\frac{8}{45}M^4\,a\left[2\langle \mathcal{E}_{ij}\mathcal{E}^{ij} + \mathcal{B}_{ij}\mathcal{B}^{ij}\rangle - 3\langle \mathcal{E}_{ij}\hat{s}^j\mathcal{E}^{ik}\hat{s}_k + \mathcal{B}_{ij}\hat{s}^j\mathcal{B}^{ik}\hat{s}_k\rangle\right]$$

Purely dissipative

So the induced Kerr moments that we found lead to a dissipative tidal torquing effect. In principle, it´s possible that they also contain conservative effects

However, since our results, it´s been shown within Effective Field Theory that our effect is purely dissipative (eg, Chia´21; Goldberger, Li & Rothstein´21)

This means that this Kerr tidal deformation is probably too small to be observed by LIGO or LISA during a black hole binary inspiral



Credit:
ESA-C.Carreau

Conclusions

TLNs tell us how much a compact object deforms under a tidal field

TLNs of neutron stars have been constrained by LIGO, thus providing information about their equation of state

Non-rotating BHs do *not* tidally deform (their static TLNs are zero)

Rotating BHs *do* tidally deform (their static TLNs are nonzero)

This tidal deformation induces torquing and is a purely *dissipative* effect

Obrigado!