The Geometric Scalar Gravity Theory

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Outline

Nordström theory

In 1912 Nordström achieved the first well acceptable generalization of Newton's gravitation.

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- Mass was no longer a constant $\rightarrow \quad m = m_0\, \Phi$

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Although this is essentially the same theory that of Nordström, it represents the first time that gravitational interactions has a purely geometric description.

Basic problems in scalar gravity

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- The source of the gravitational field is the trace of the energy-momentum tensor
- Conformally flat geometry
- Violation of the relativity of inertia

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$\Longrightarrow$ Geometric Scalar Gravity (GSG) does not take in to account these assumptions, overcoming the attendant problems.

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Example: Electromagnetic field

The electromagnetic part of the Lagrangian is given by $F = F_{\alpha\mu}F_{\beta\nu}q^{\alpha\beta}q^{\mu\nu}$. The corresponding field equation obtained by the variational principle

$$\delta\,\frac{1}{4}\int\sqrt{-q}\,F\,d^4x = 0\,, \tag{6}$$

is given by

$$F^{\mu\nu}{}_{;\nu} = 0\,, \tag{7}$$

where the semicolon represents the covariant derivative w.r.t. the gravitational metric.

Hidden geometries in non linear scalar field theory

- Our main inspiration comes from the following paper,

IOP PUBLISHING CLASSICAL AND QUANTUM GRAVITY

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Main statement

The dynamics of a scalar field endowed with a Lagrangian $L(w, \varphi)$ in a given background can be mapped in an different spacetime in which its metric is constructed by the field φ itself (and it derivatives).

An specific geometrization of a scalar field

- We consider the following Lagrangian in flat Minkowski spacetime,

$$L = V(\Phi)\, w\,, \tag{8}$$

where $w = \eta^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi$.

- The field equations is

$$\frac{1}{\sqrt{-\eta}}\partial_\mu\left(\sqrt{-\eta}\,\eta^{\mu\nu}\,\partial_\nu\Phi\right) + \frac{1}{2}\,\frac{V'}{V}\,w = 0\,. \tag{9}$$

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- But,

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where the $\Box$ represents the d'Alambertian operator calculated in terms of the metric,

$$q^{\mu\nu} = \alpha(\Phi)\,\eta^{\mu\nu} + \frac{\beta(\Phi)}{w}\,\partial^\mu\Phi\partial^\nu\Phi\,, \tag{10}$$

with

$$\alpha + \beta = \alpha^3 V\,. \tag{11}$$

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- Thus, the dynamic of Φ in the curved spacetime, given by $q_{\mu\nu}$, is

$$\Box\Phi = 0\,. \tag{12}$$

Specifying the coefficients α and the potential V

- First, we consider the weak field regime in the case of a test particle, in order to have Newtonian limit for the theory, we have that

$$\frac{d^2x^i}{dt^2} = -\,\Gamma^i_{00} = -\,\partial^i\,\Phi_N\,. \quad (13)$$

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- Now, it only remains to determine the potential.

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- So, assuming $\Phi = \Phi(r)$, where r is the radial coordinate, the gravitational metric takes the form

$$ds^2 = \frac{1}{\alpha} dt^2 - \frac{1}{\alpha^2 V}\left(1 - r\frac{d\Phi}{dr}\right) dr^2 - r^2\, d\Omega^2\,. \qquad (16)$$

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- In order to have a Schwarzschild-like geometry, we impose $q_{11} = -1/q_{00}$. Applying in the dynamical equation $\Box \Phi = 0$, we get

$$\Phi(r) = \frac{1}{2} \ln \left(1 - \frac{r_H}{r} \right) , \quad V = \frac{(3-\alpha)^2}{4\alpha^3} \, . \tag{17}$$

with $r_H = 2MG/c^2$, and the final metric will be

$$ds^2 = \left(1 - \frac{r_H}{r} \right) dt^2 - \left(1 - \frac{r_H}{r} \right)^{-1} dr^2 - r^2 d\Omega^2 \, . \tag{18}$$

Final theory

Now the theory is complete.

The dynamic of the gravitational field is given by the Lagrangian

$$L = V(\Phi)\, w \,, \tag{19}$$

where the potential is

$$V = \frac{(3-\alpha)^2}{4\alpha^3}, \tag{20}$$

with

$$\alpha = e^{-2\Phi} \,. \tag{21}$$

The gravitational metric generated by Φ is

$$q^{\mu\nu} = \alpha\, \eta^{\mu\nu} + \beta\, \frac{\partial^\mu \Phi\, \partial^\nu \Phi}{w} \,, \tag{22}$$

where the following expression determines the value of β,

$$\alpha + \beta = \alpha^3\, V \,. \tag{23}$$

Action principle

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- The action of the scalar field is easily described in the curved spacetime,

$$S_\Phi = \int \sqrt{-\eta}\, V w \, d^4x = \int \sqrt{-q}\, \Omega \sqrt{V} d^4x \,, \quad \text{with} \quad \Omega \equiv q^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi \,. \tag{24}$$

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- From the variational principle, we get

$$\delta\, S_\Phi = -\, 2 \int \sqrt{-q}\, \sqrt{V}\, \Box\Phi\, \delta\Phi\, d^4x\,. \tag{25}$$

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- Action corresponding to the matter,

$$S_m = \int \sqrt{-q}\,L_m d^4x\,, \tag{24}$$

which returns that,

$$\delta S_m = -\frac{1}{2}\int \sqrt{-q}\,T^{\mu\nu}\delta q_{\mu\nu}\,d^4x\,, \quad \text{where} \quad T_{\mu\nu} \equiv \frac{2}{\sqrt{-q}}\frac{\delta(\sqrt{-q}\,L_m)}{\delta q^{\mu\nu}}\,. \tag{25}$$

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- But the metric $q_{\mu\nu}$ is not the fundamental quantity. We have to vary it as a function of Φ.

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- Matter variational principle:

$$\delta S_m = \int \sqrt{-q}\left[\left(\frac{3+\alpha}{3-\alpha}\right)E - T - C^\lambda{}_{;\lambda}\right]\delta\Phi\,d^4x\,, \tag{24}$$

$$T \equiv T^{\mu\nu}\,q_{\mu\nu}\,, \quad E \equiv \frac{T^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi}{\Omega}\,, \tag{25}$$

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- The quantity χ involves a non-trivial coupling between the gradient of the scalar field and the energy-momentum tensor of the matter that allows the electromagnetic field to interact with the gravitational field.
- The Newtonian limit gives the identification

$$\kappa \equiv \frac{8\pi G}{c^4}\,. \tag{25}$$

Final comments

- By assuming Einstein's idea, that gravity is a metrical phenomenon, and using the geometrization technique for a scalar field, we showed that is possible to construct a gravitational theory that overcomes the traditional drawbacks existing for the old scalar theories of gravity.
- In a first moment, we choose to determine all theory's parameters using the Newtonian limit and the solar system as a reference.
- Our results was published in JCAP 06(2013) 014.
- Some important issues are already being investigated by our co-workers, such as
  - Cosmology (Toniato and Novello)
  - Gravitational waves (Moschella and Bittencourt)
  - Interior solutions (Rua and Novello)

THANKS!