

# Inverno Astrofísico



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## Lecture 1

Indications of the existence of dark energy

SNIa, CMB, BAO

## Lecture 2

The cosmological constant problem

Effects of the cosmological constant

## Lecture 3

Models of dark energy...?

# Luminosity distance

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Photons move on null geodesics

$$ds^2 = -c^2 dt^2 + a(t)^2 dr^2 = 0$$

we are looking to a radial geodesic

# Luminosity distance

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$$\frac{\delta t_{rec}}{a(t_{rec})} = \frac{\delta t_{em}}{a(t_{em})}$$

$$\delta t = \lambda / c$$


$$\frac{\lambda_{rec}}{a(t_{rec})} = \frac{\lambda_{em}}{a(t_{em})}$$

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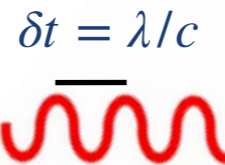
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$$\frac{\delta t_{rec}}{a(t_{rec})} = \frac{\delta t_{em}}{a(t_{em})}$$



$$\frac{\lambda_{rec}}{a(t_{rec})} = \frac{\lambda_{em}}{a(t_{em})}$$

We define the redshift as

$$z = \frac{\lambda_{rec} - \lambda_{em}}{\lambda_{em}} = \frac{a(t_{rec})}{a(t_{em})} - 1$$

$$1 + z = \frac{a(t_{rec})}{a(t_{em})} \equiv \frac{1}{a(t)}$$

$$a(t) = \frac{1}{1 + z}$$

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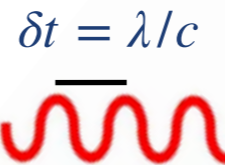
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$$E = \frac{hc}{\lambda}$$

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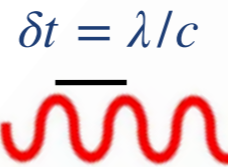
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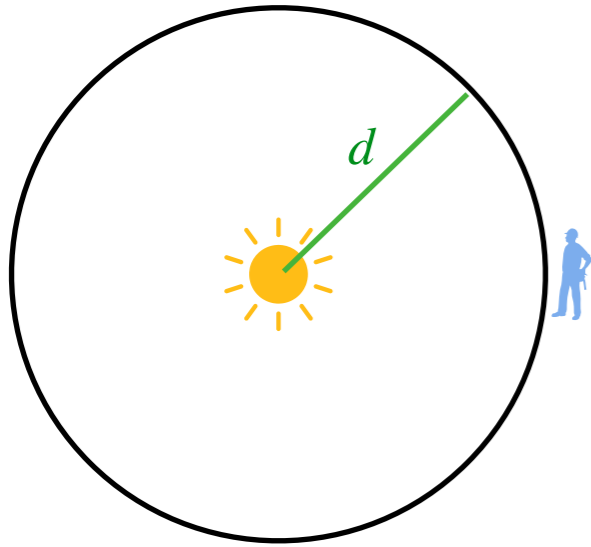
$$a(t) = \frac{1}{1 + z}$$

$$1 + z = \frac{\lambda_{rec}}{\lambda_{em}} = \frac{\delta t_{rec}}{\delta t_{em}} = \frac{E_{em}}{E_{rec}}$$

$$\begin{aligned} E_{em} &= (1 + z)E_{rec} \\ \delta t_{rec} &= (1 + z)\delta t_{em} \end{aligned}$$

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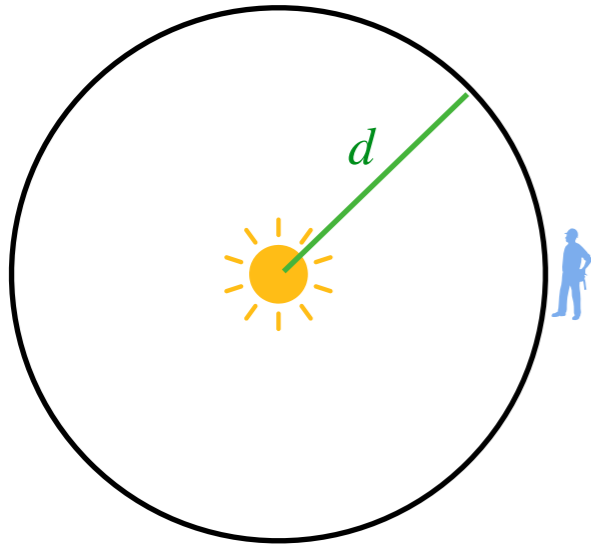


Absolute luminosity ( $L$ ): radiated power

Apparent luminosity ( $\ell$ ): power per unit area (flux density)

$$\ell = \frac{L}{4\pi d^2}$$

# Luminosity distance



**Absolute luminosity ( $L$ ): radiated power**

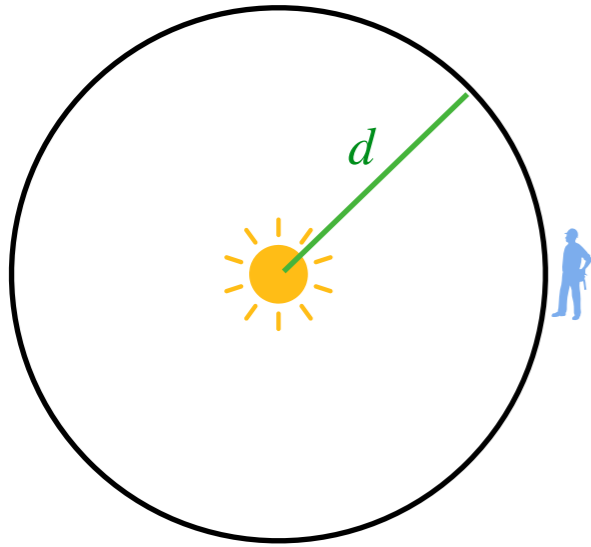
**Apparent luminosity ( $\ell$ ): power per unit area (flux density)**

$$\ell = \frac{L}{4\pi d^2}$$

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Distance modulus

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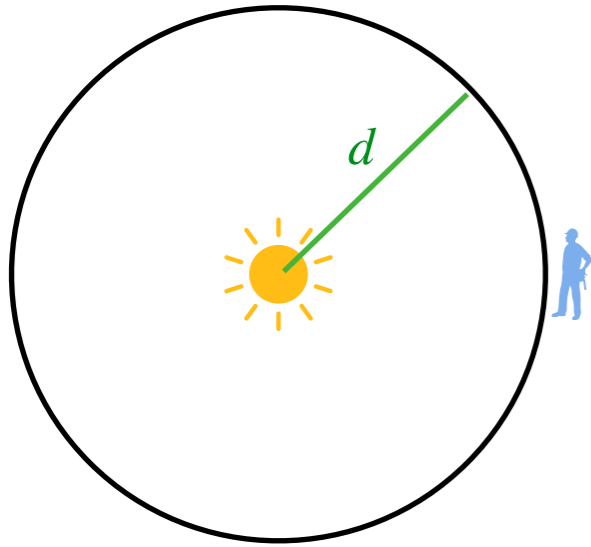
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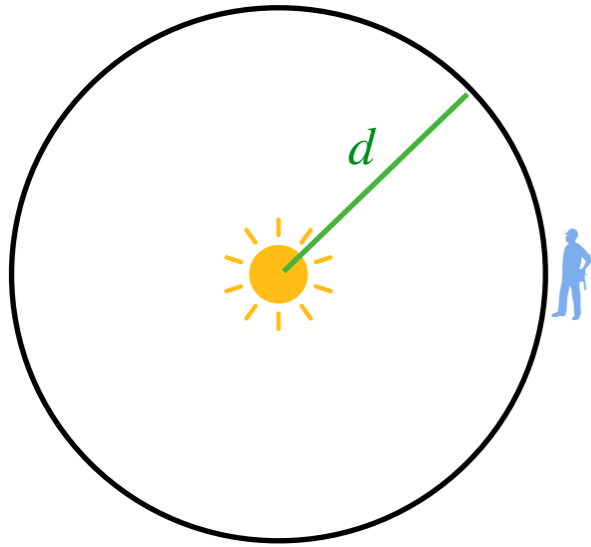
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Is chosen such that  $\mu = 0$  for  $d = 10\text{pc}$

$$\mu = 5 \log_{10} d(\text{pc}) - 5$$



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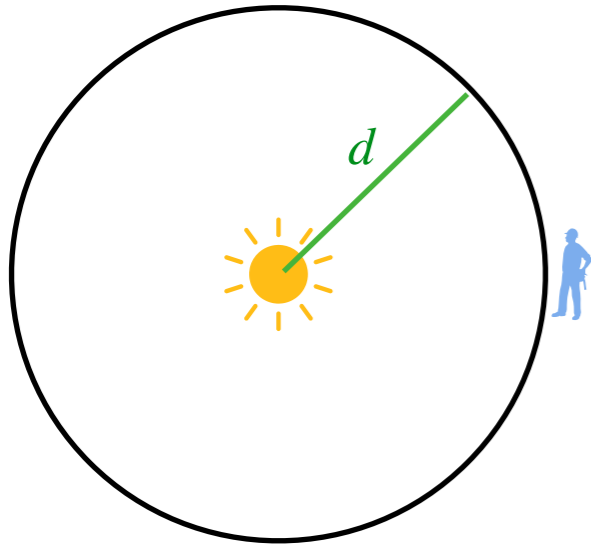
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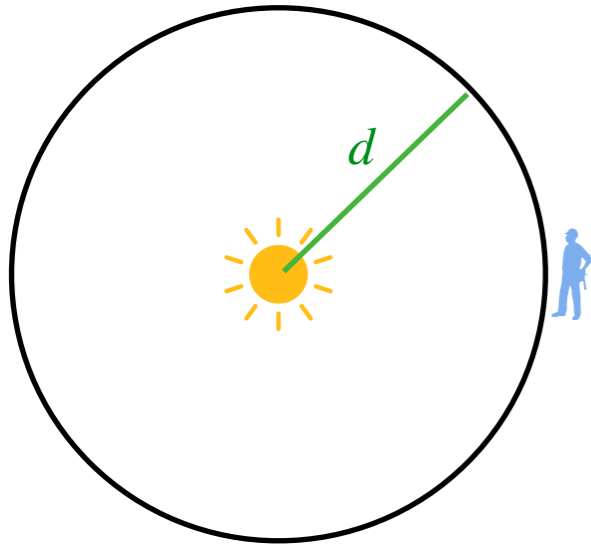
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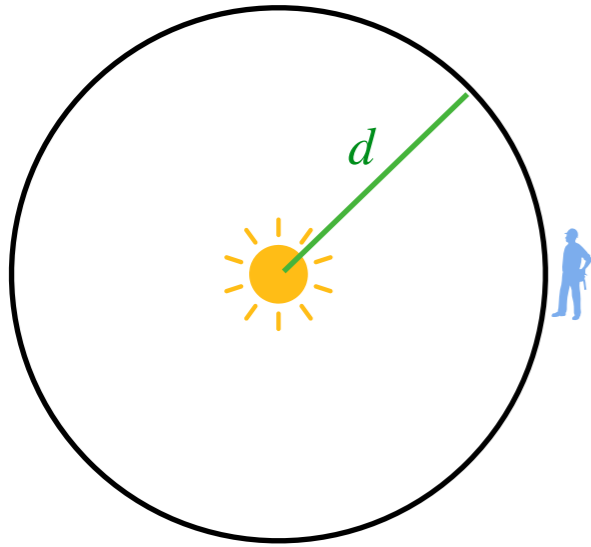
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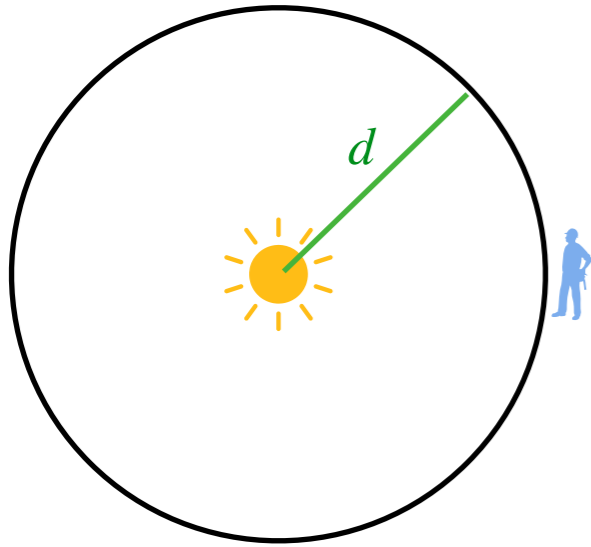
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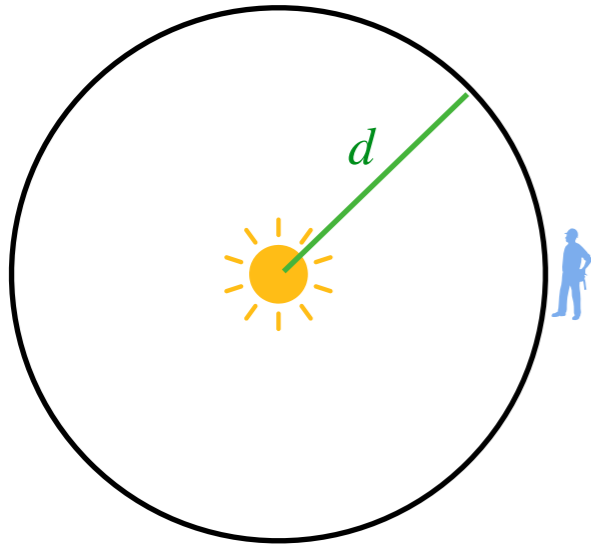
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$$d_L = (1+z) \sqrt{\frac{A}{4\pi}}$$

## Luminosity distance

---

How to calculate the superficie  $A$  ... from the metric

$$ds^2 = -c^2 dt^2 + a(t)^2 dr^2 + a(t)^2 r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

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$\rightarrow A = 4\pi r^2 a(t) = 4\pi r^2$

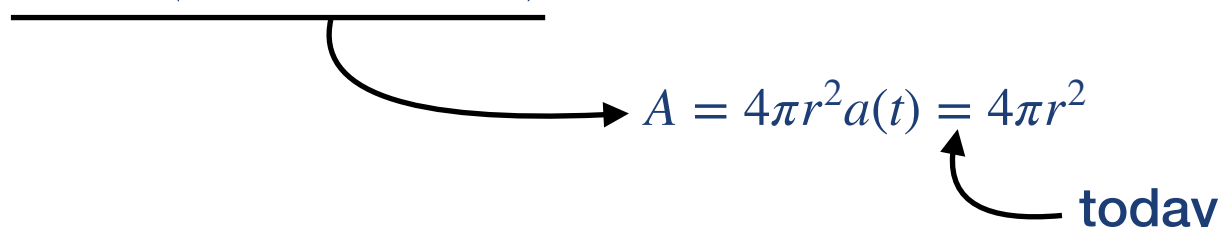
$\uparrow$  today



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But we know that

$$r = c \int_t^{t_0} \frac{dt}{a}$$

today

and

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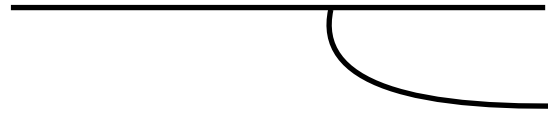
$$\frac{dt}{a} = -\frac{dz}{H(z)}$$

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$$d_L = (1+z)r$$

But we know that

$$r = c \int_t^{t_0} \frac{dt}{a}$$

today

and

$$a(t) = \frac{1}{1+z}$$

$$\dot{a} = -\frac{1}{(1+z)^2} \frac{dz}{dt}$$

$$r = -c \int_z^0 \frac{dz}{H(z)}$$

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$$H \equiv \frac{\dot{a}}{a} = -\frac{1}{1+z} \frac{dz}{dt}$$

$$r = c \int_0^z \frac{dz}{H(z)}$$

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$$\mu = 5 \log_{10} \left[ c(1+z) \int_0^z \frac{dz}{H(z)} \right] - 5$$



# Luminosity distance

How to calculate the superficie  $A$  ... from the metric

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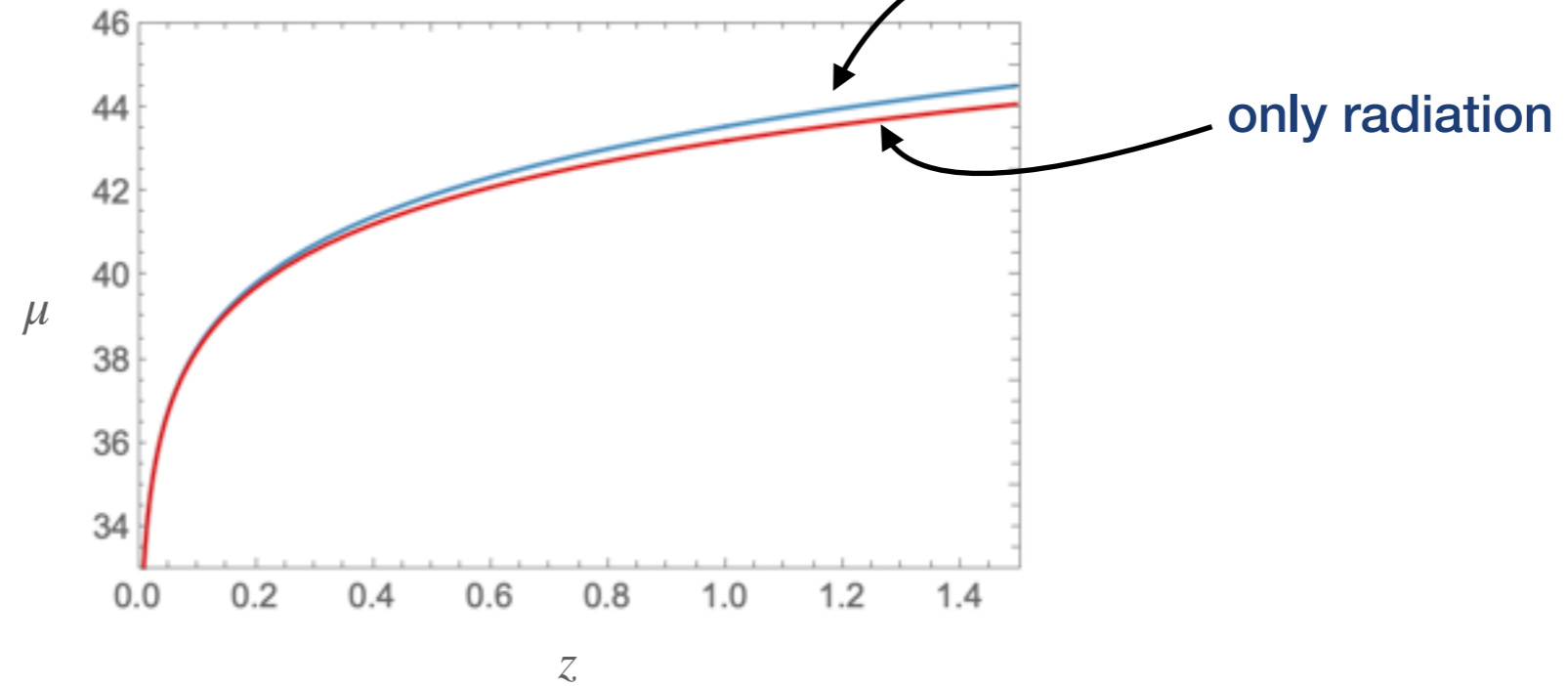
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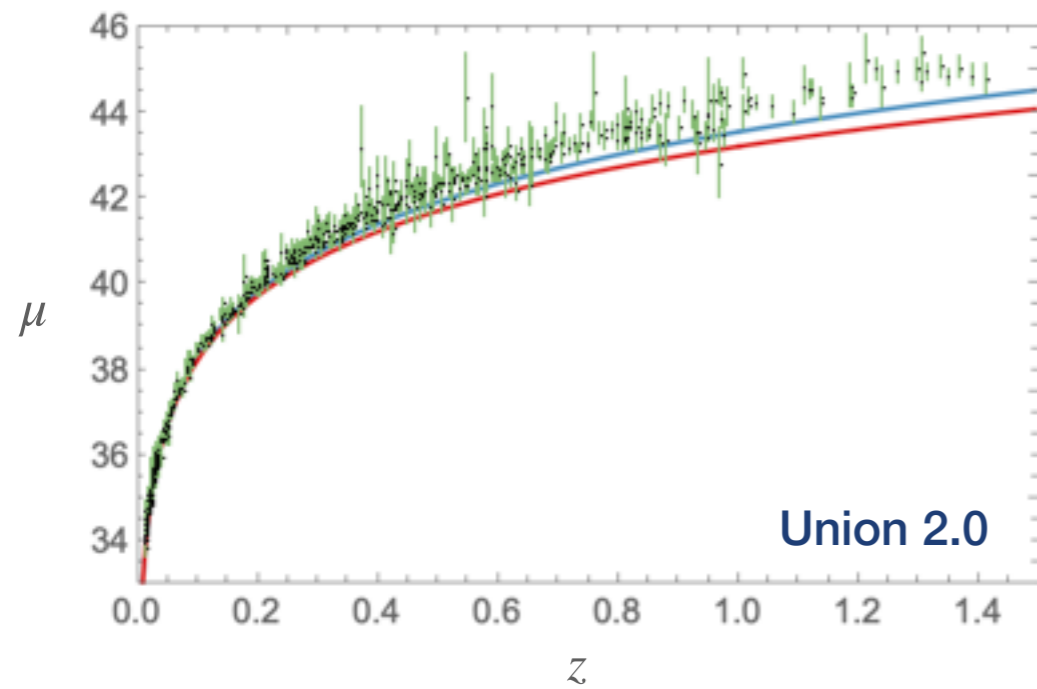
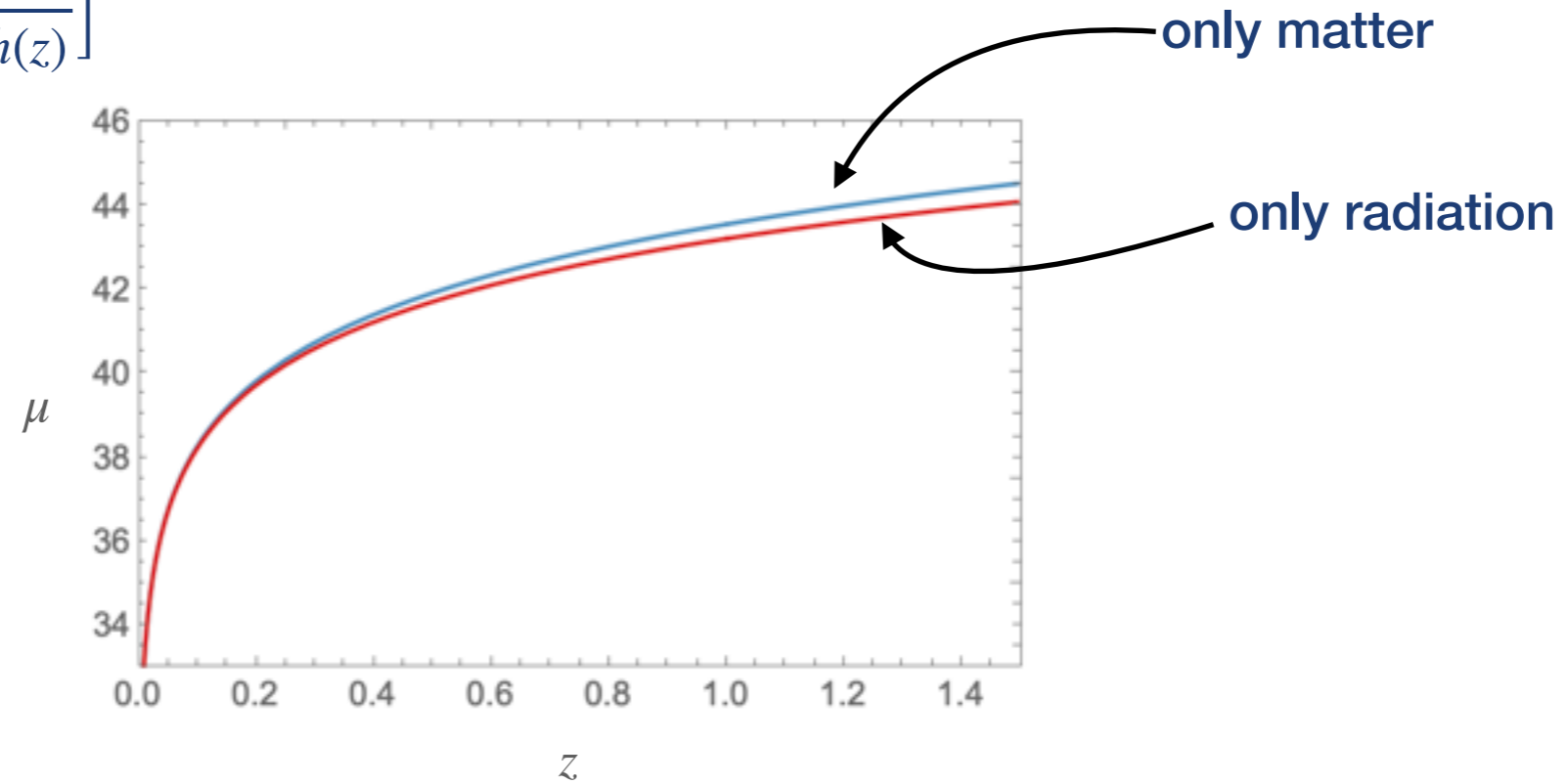
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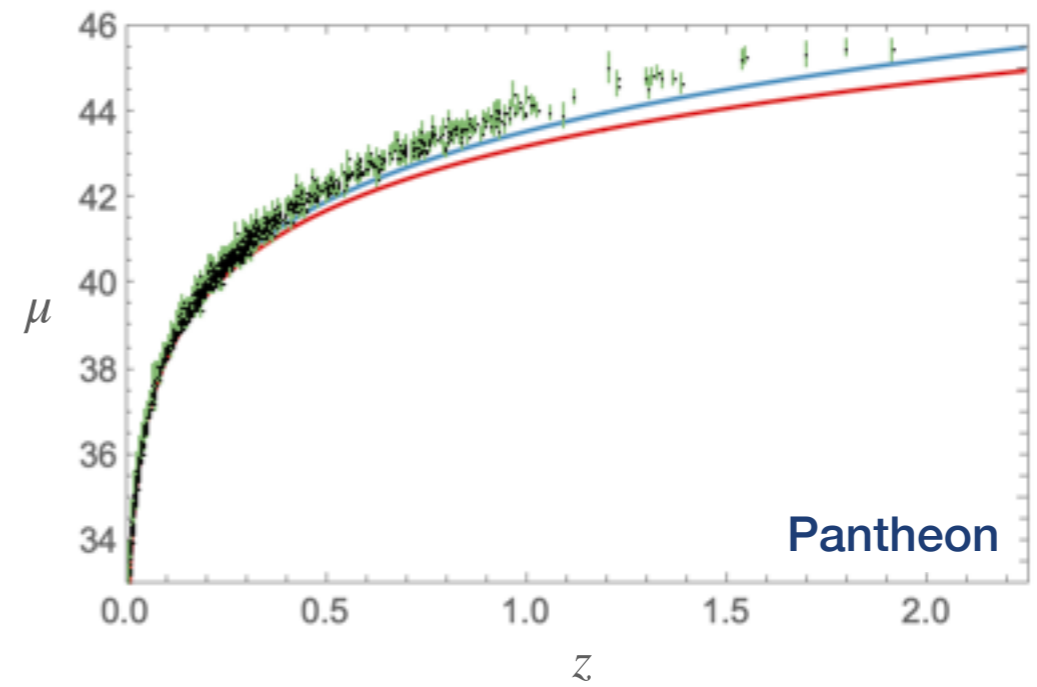
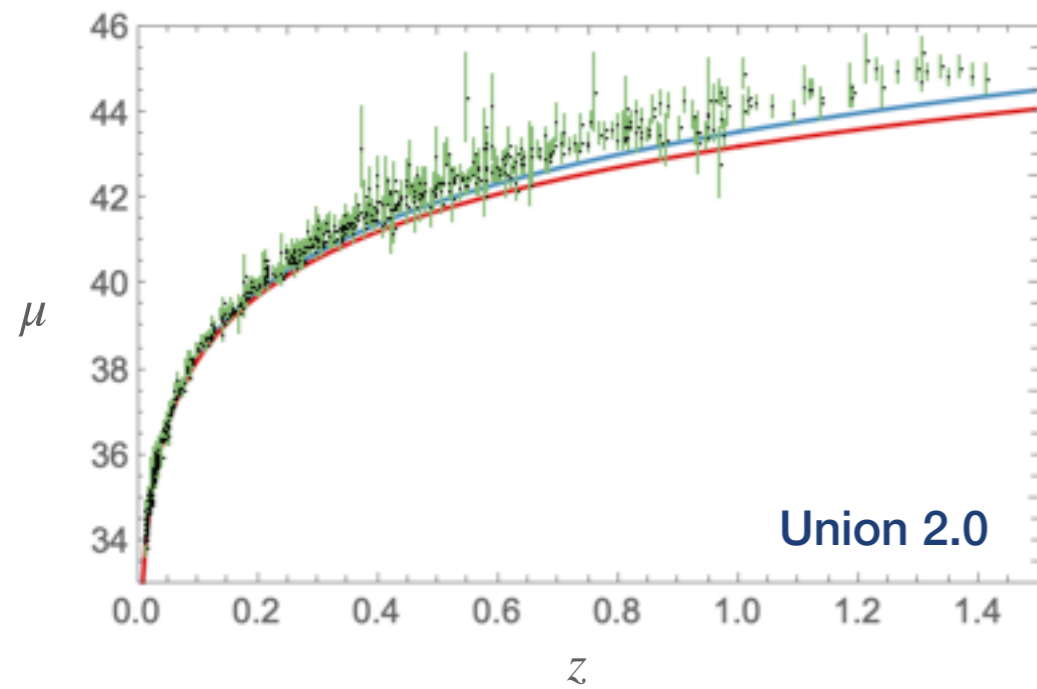
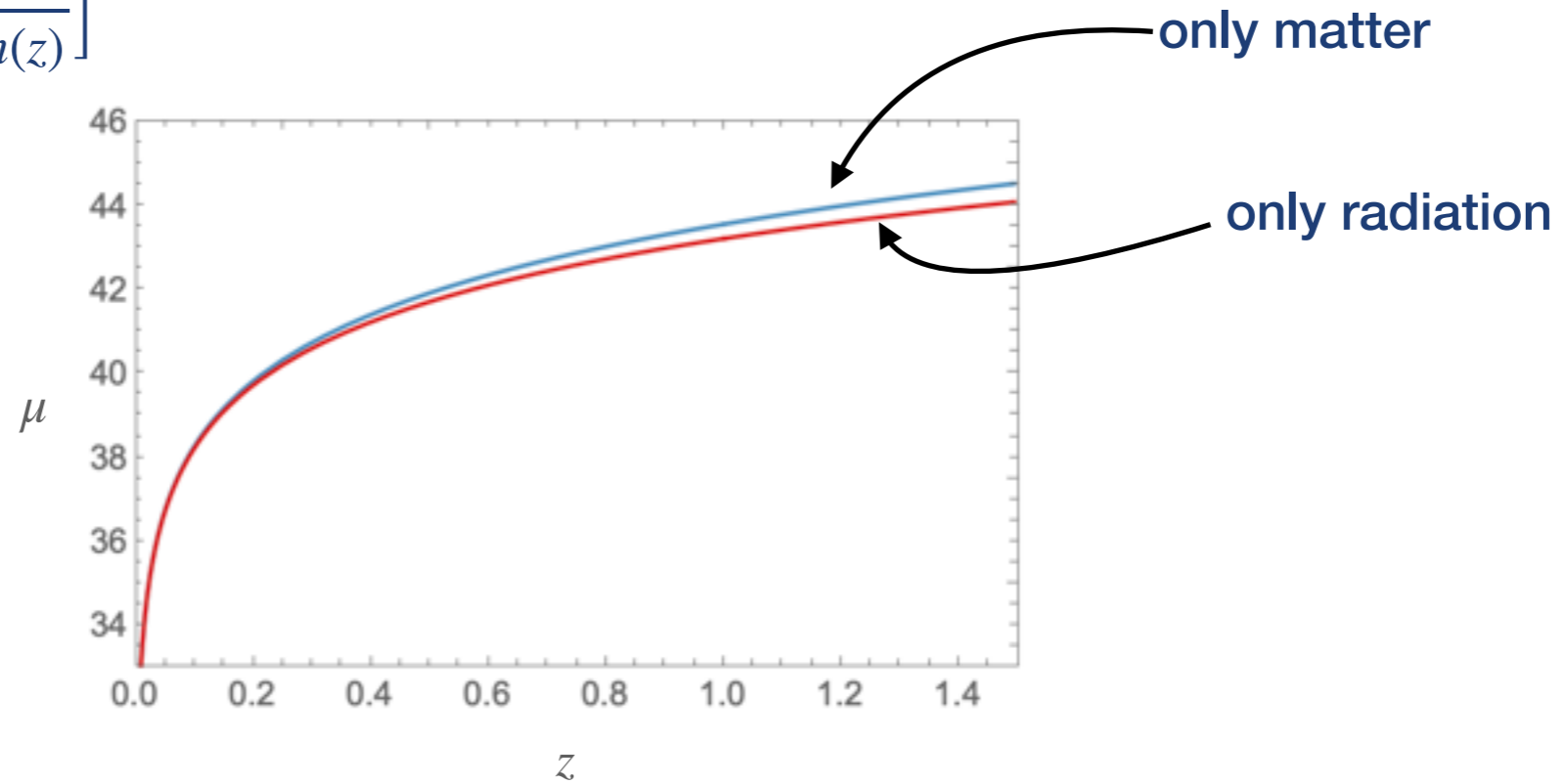
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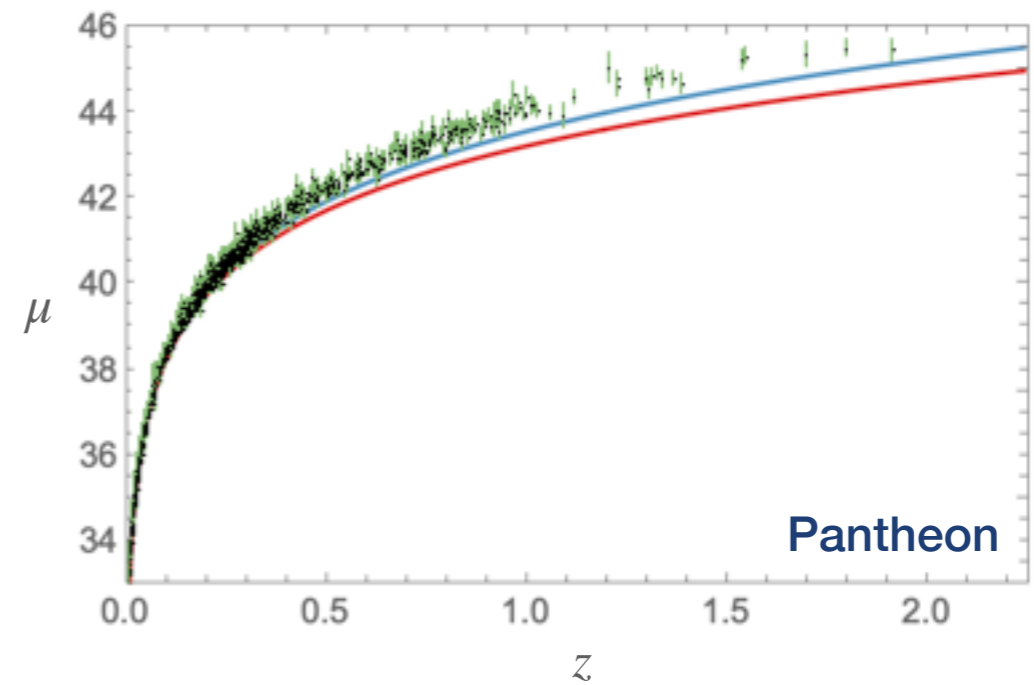
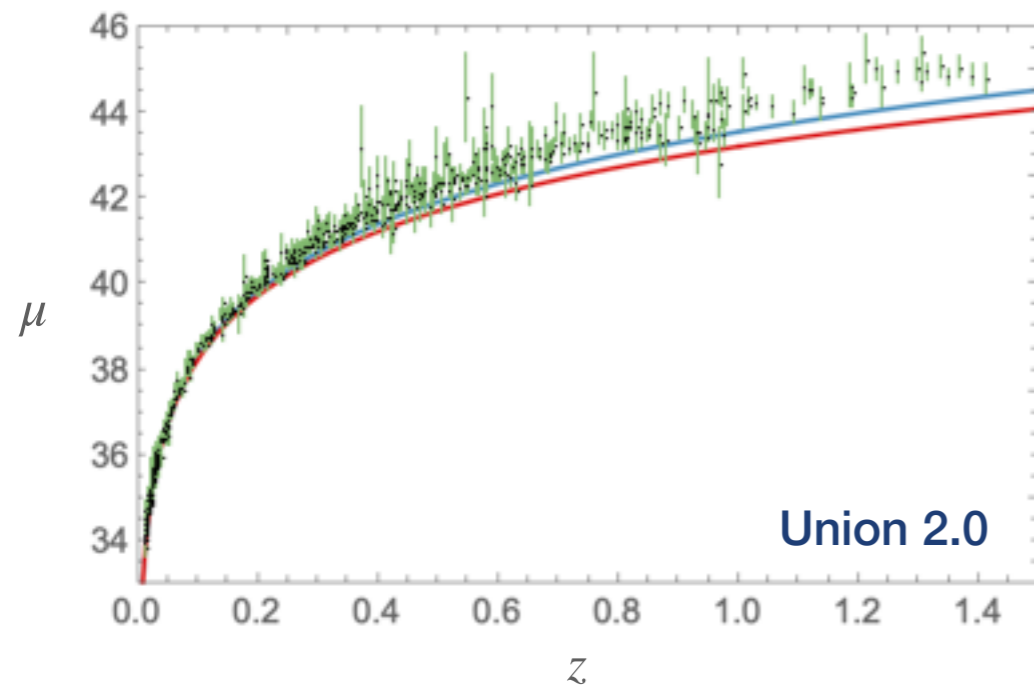
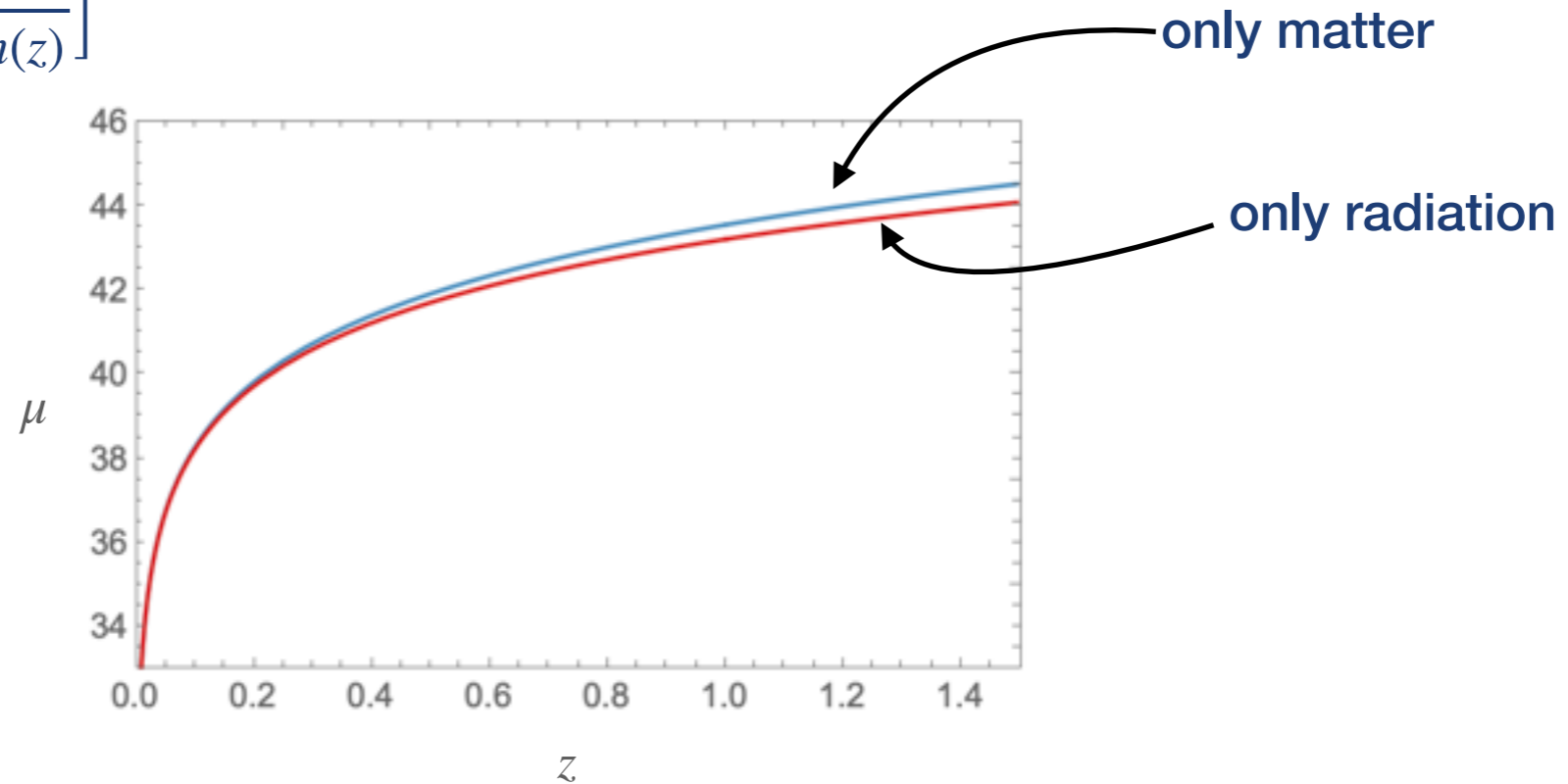
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What are these data?

How to fit them with a better model?

What are these data?

They come from supernovae of type Ia



There are two very different mechanisms leading to SN explosions:

The gravitational collapse of the core of a star, once the nuclear fuel that feeds the thermonuclear reactions inside the core is exhausted. Depending on the properties of the progenitor, this leads to events classified as type Ib, Ic or type II SNe, and leaves behind a compact remnant, usually a neutron star or possibly a black hole

The thermonuclear explosion of a white dwarf that accretes mass from a companion, going beyond its Chandrasekhar limit (in reality it never reaches it, but we have increase of temperature in the core, which leads to carbon fusion leading to an explosion). This gives rise to type Ia SNe. In this case the star that explodes is dispersed in space and its remnant is not a compact object.

Release an energy  $\sim 10^{56}$  GeV

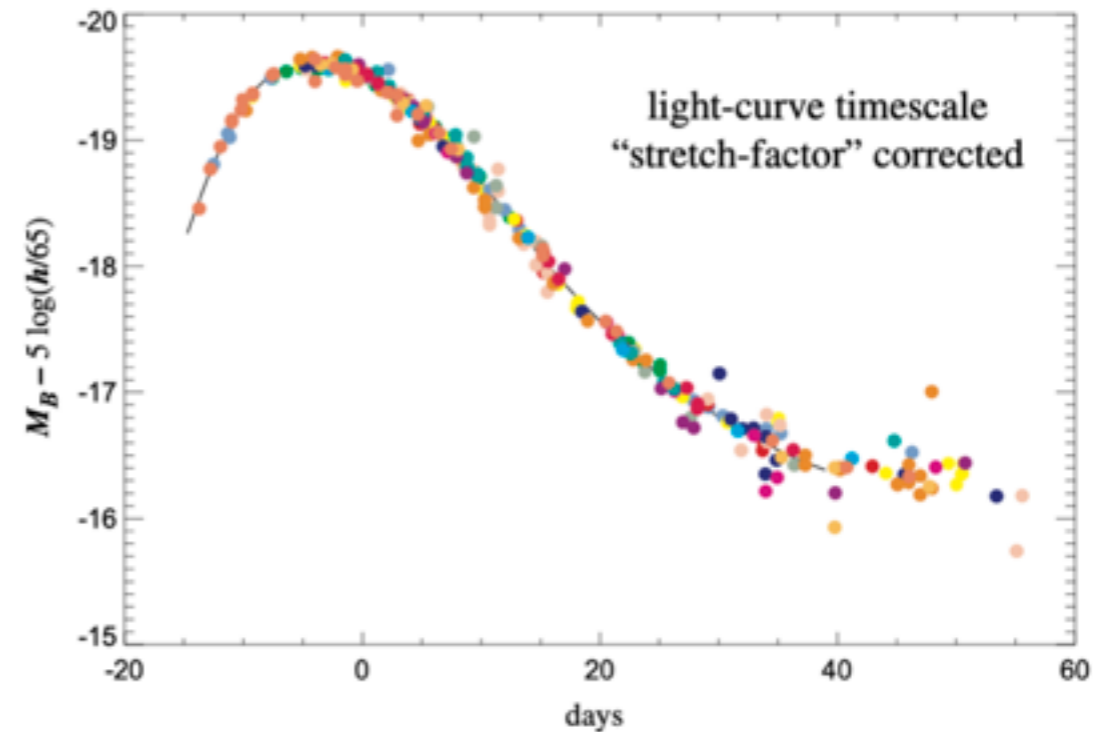
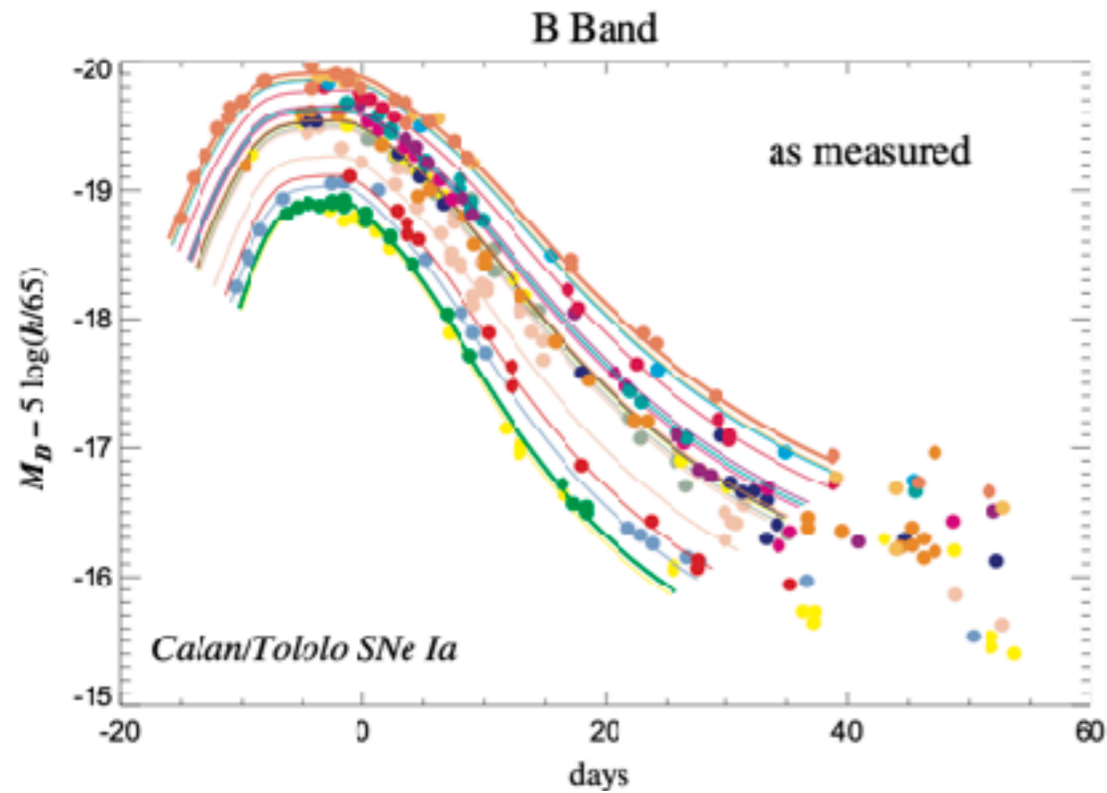
99% are neutrinos

1% goes into kinetic energy of the ejected material

less than 0.01%, i.e. about  $10^{52}$  GeV, is released in photons

The corresponding peak luminosity in photons can be of order a few times  $10^9 L_{\odot}$  or higher. Thus, a typical core-collapse SN at its peak has an optical luminosity that rivals the cumulative light emitted by all the stars in its host galaxy





SN Ia at small distance, so their relative distance can be found from redshift, so the relative brightness

They don't really have the same luminosity

Because of different composition

Because of intergalactic medium

But we see that they look the same, some stretch factor should correct it

The Pskovskii–Phillips relation

$$(M_B)_{peak} = -21.727 + 2.698\Delta m_{15}(B) \quad \text{with similar relations in V- and I- bands}$$

because of composition: more  $^{56}\text{Ni}$ , implies higher peak, so higher temperature and opacity, therefore a slower decline of the light curve

+ some other corrections, we can make them standard candles

Therefore after corrections, we can know their absolute magnitude  $M$ , we measure  $m$  and redshift ( $z$ ) so we obtain  $\mu(z)$

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Very luminous, they reach an absolute magnitude of  $M \simeq -19$  corresponding to  $10^{10} M_{\odot}$

Relative small dispersion of the peak absolute magnitude

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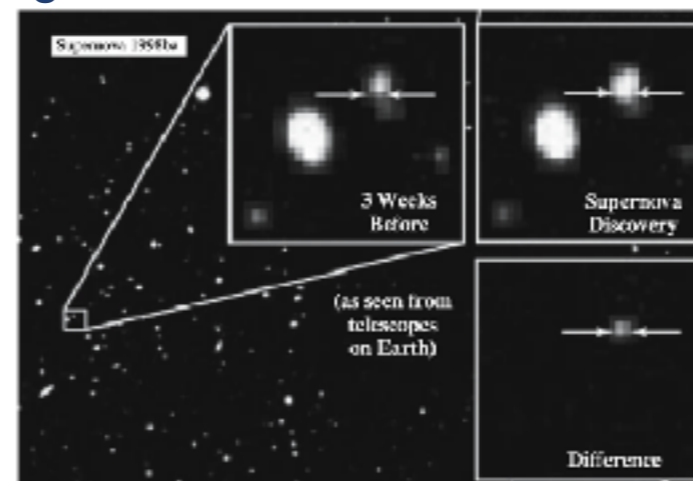
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So we need a better strategy than luck to find them

Observe a large part of the sky

Observe it again  $\simeq 3$  weeks after

Do the difference between images and follow the differences which are supernovae



Perlmutter et al. 1997

They found 7 SN Ia for  $0.35 < z < 0.65$

They concluded that it is consistent with a Universe with matter and radiation

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A Universe with only matter and radiation was ruled out at 99% confidence level

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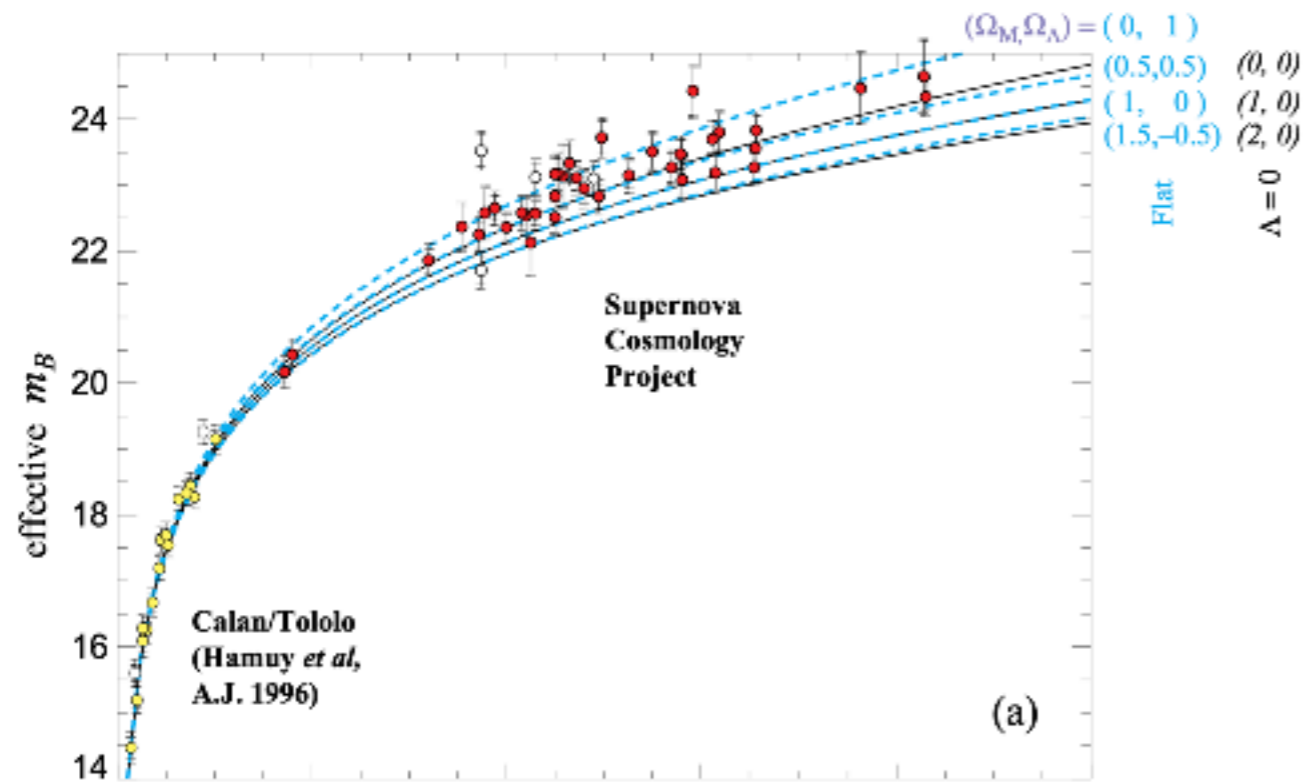
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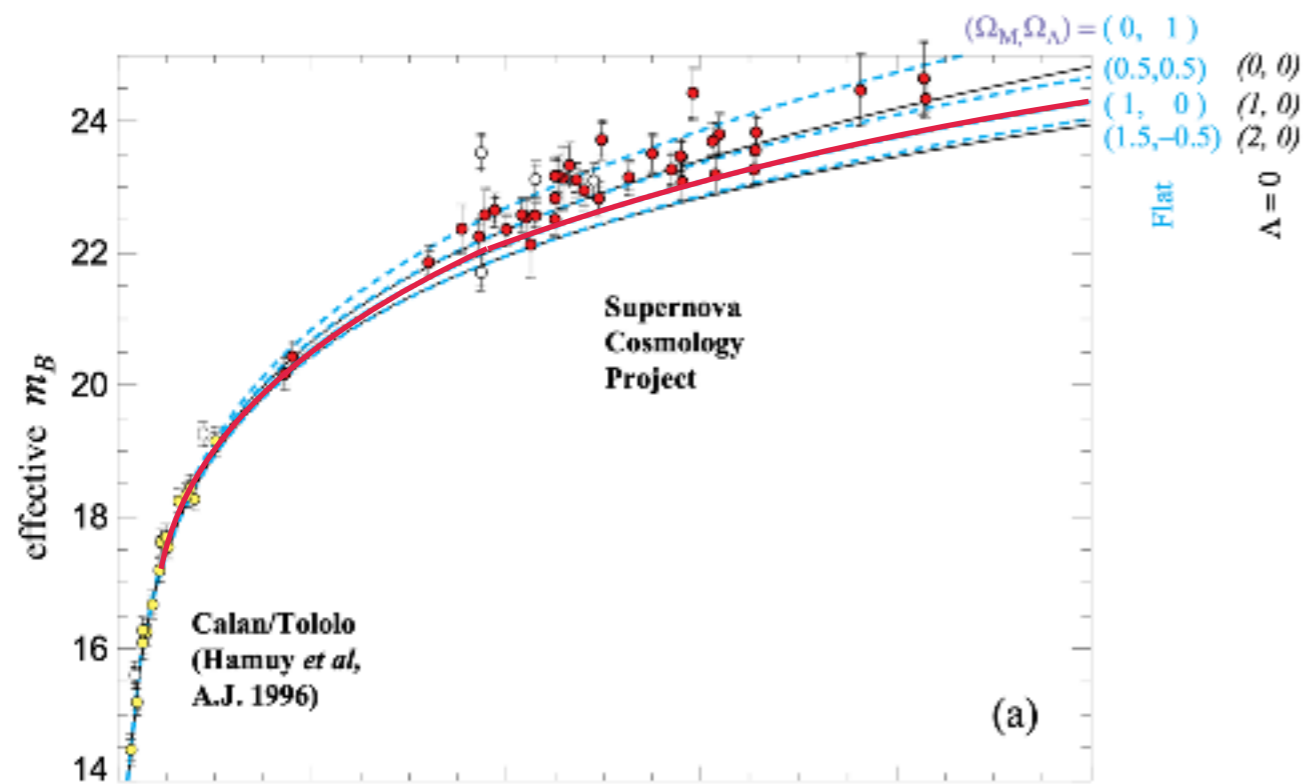
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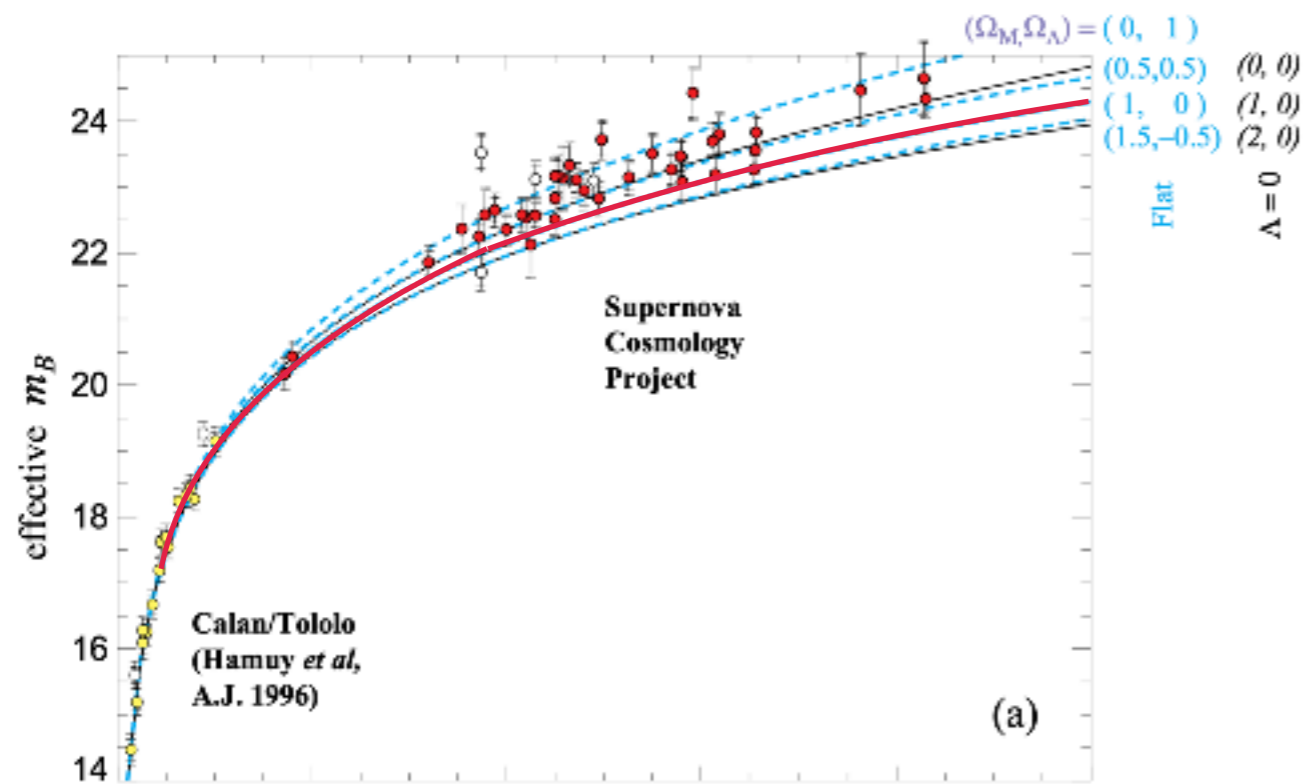
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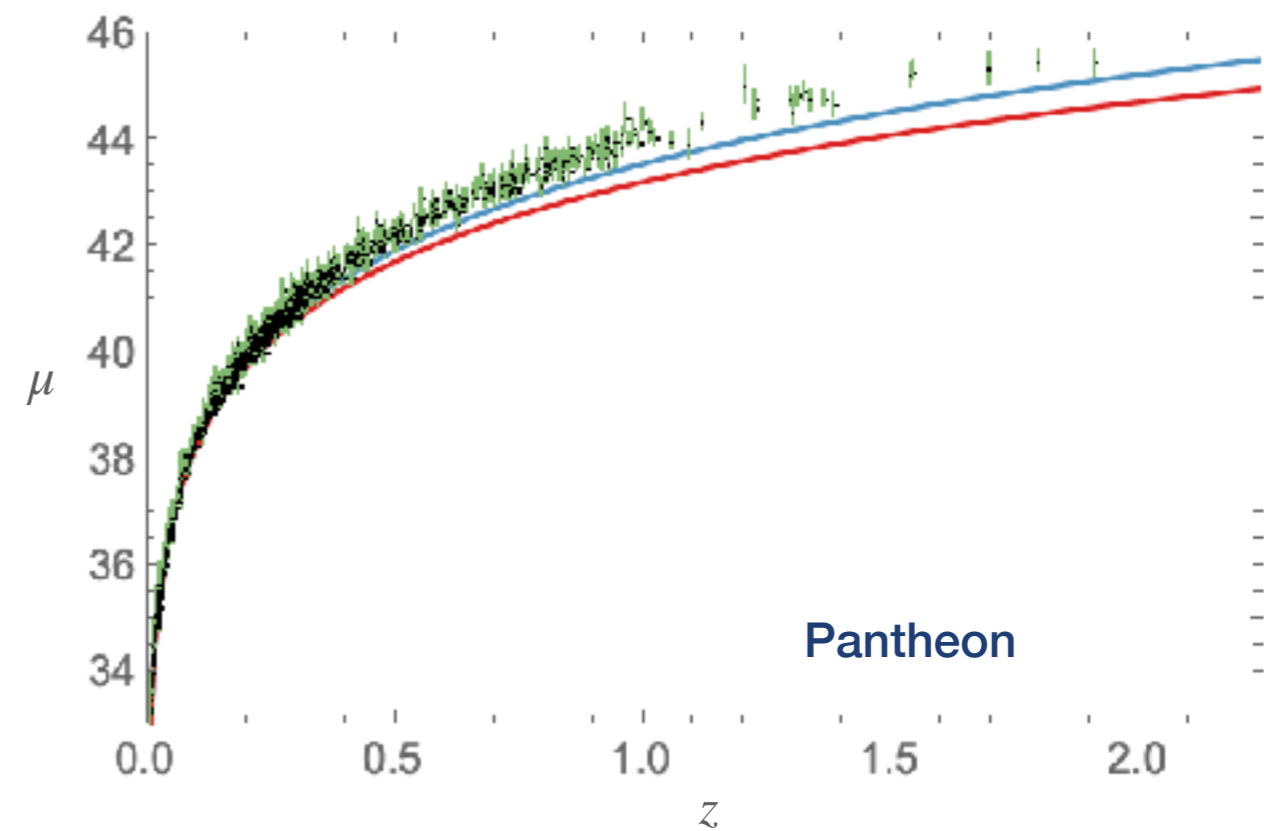
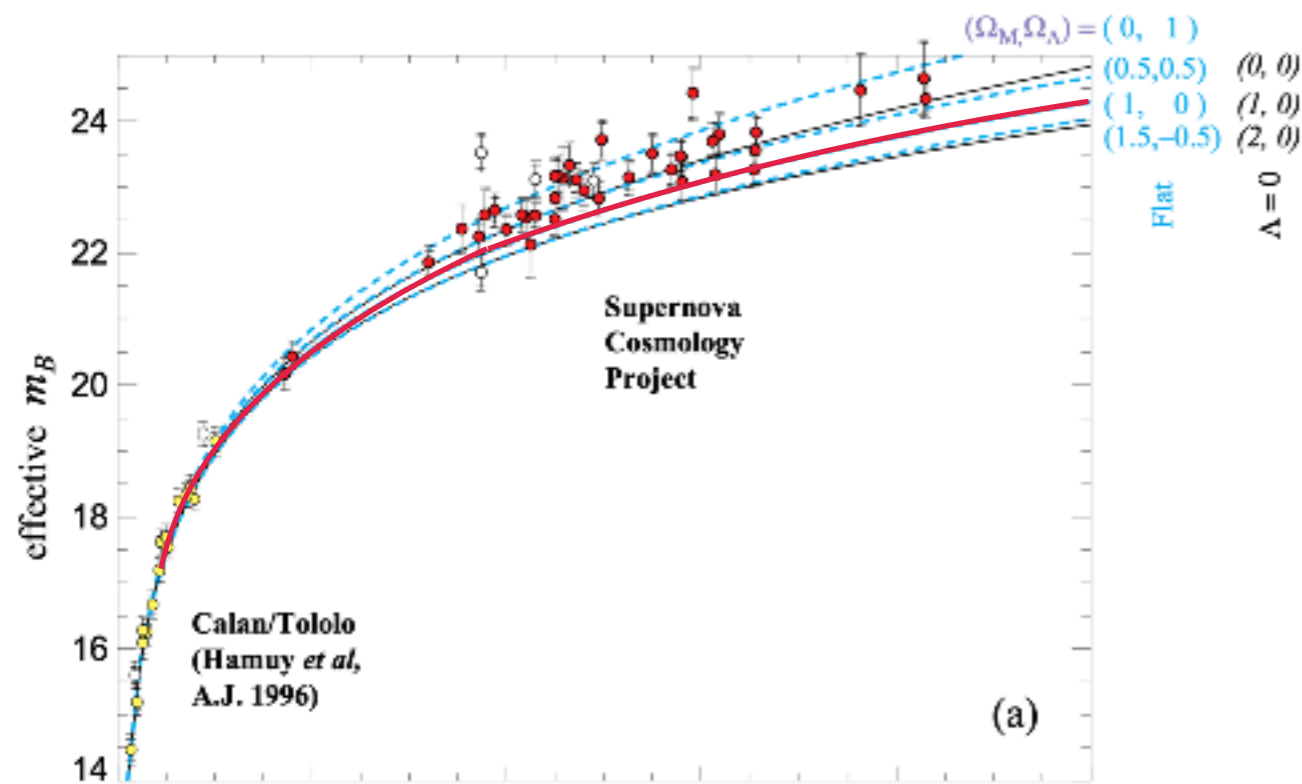
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Let's go back to the modulus distance

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We obtain the Friedmann equations

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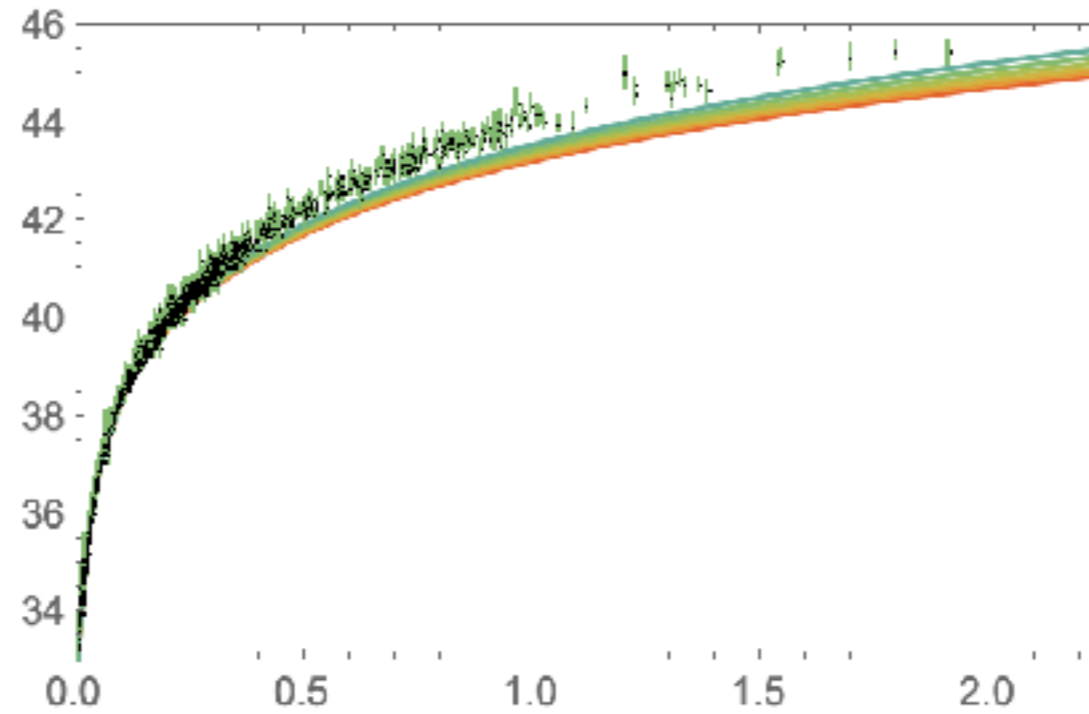
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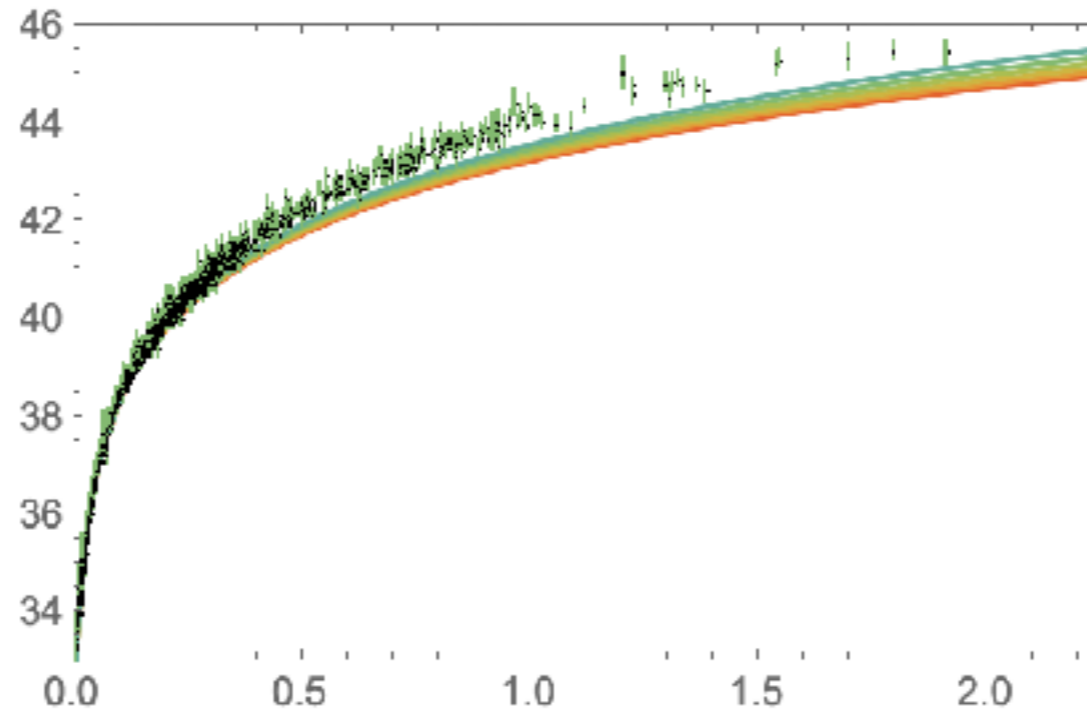


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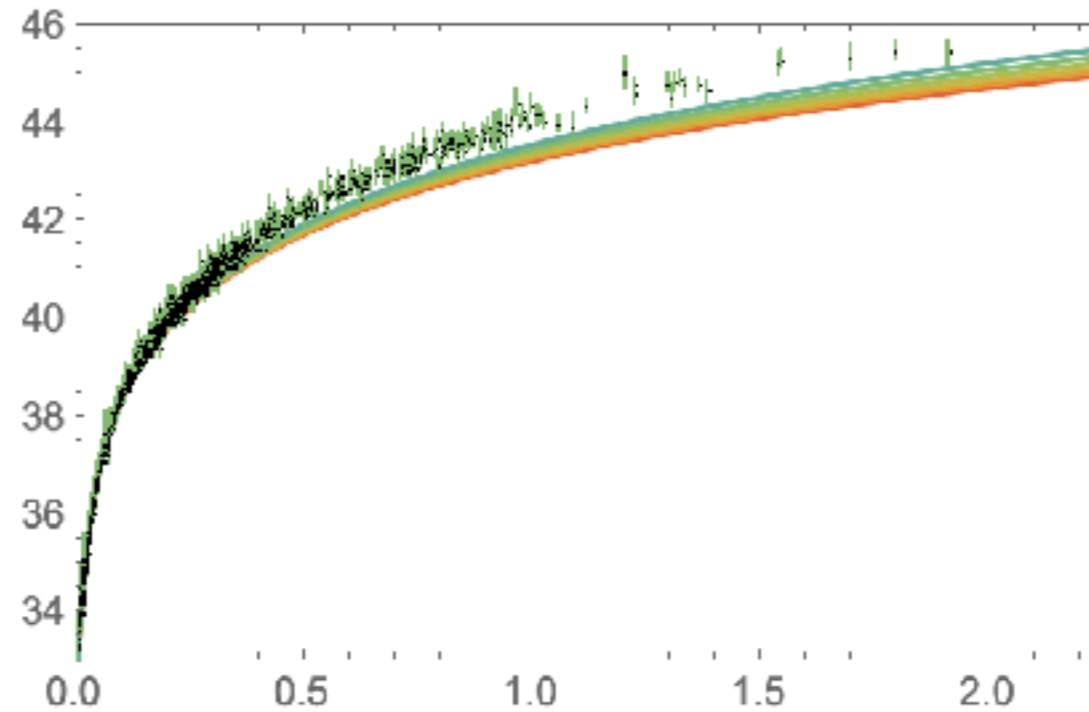
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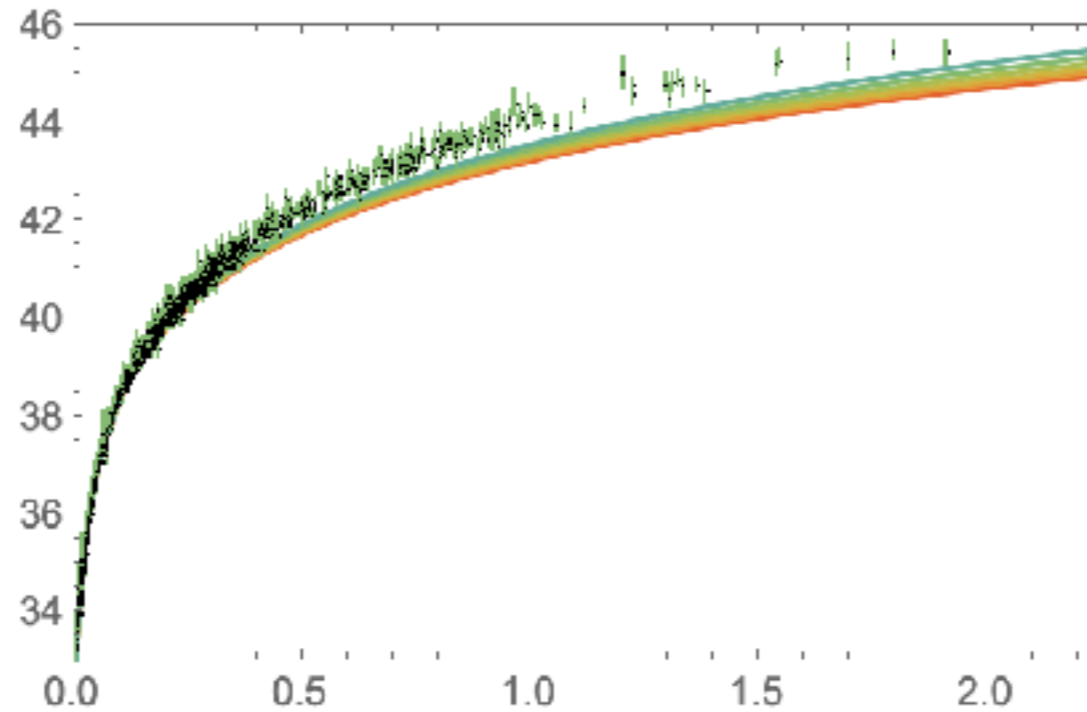
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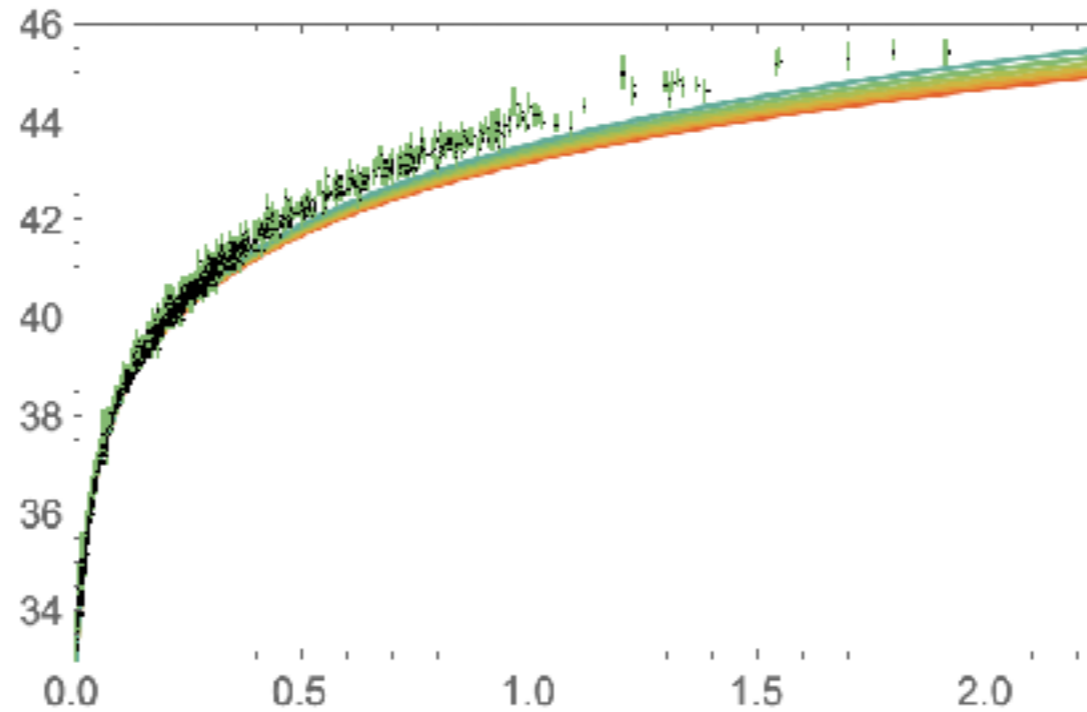
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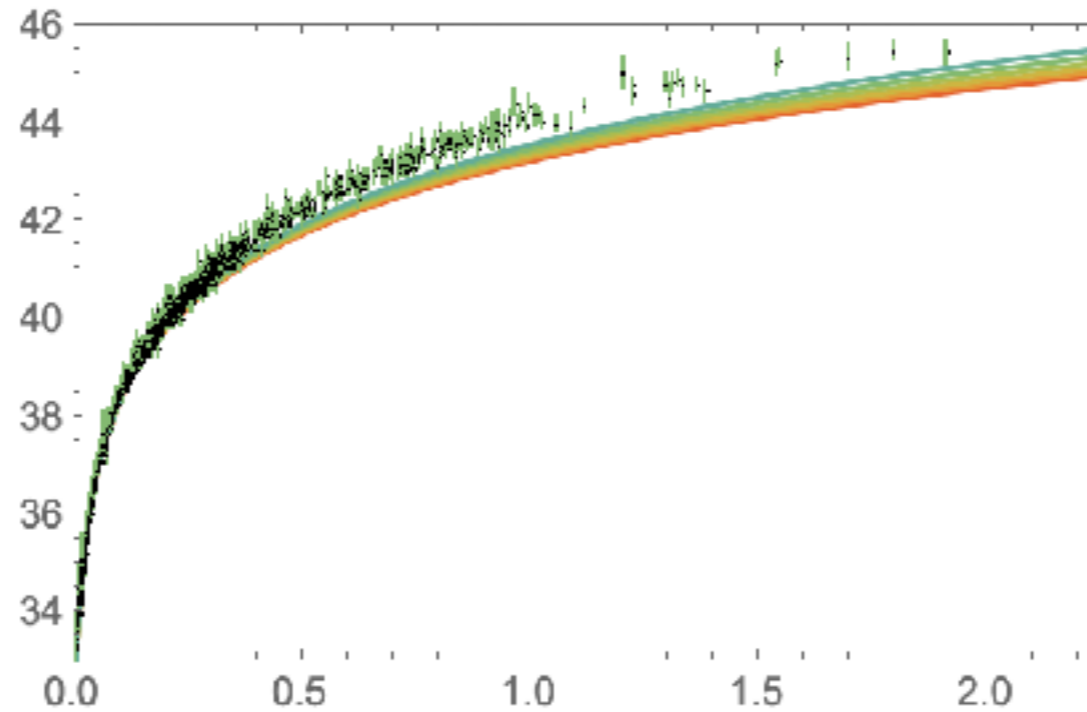
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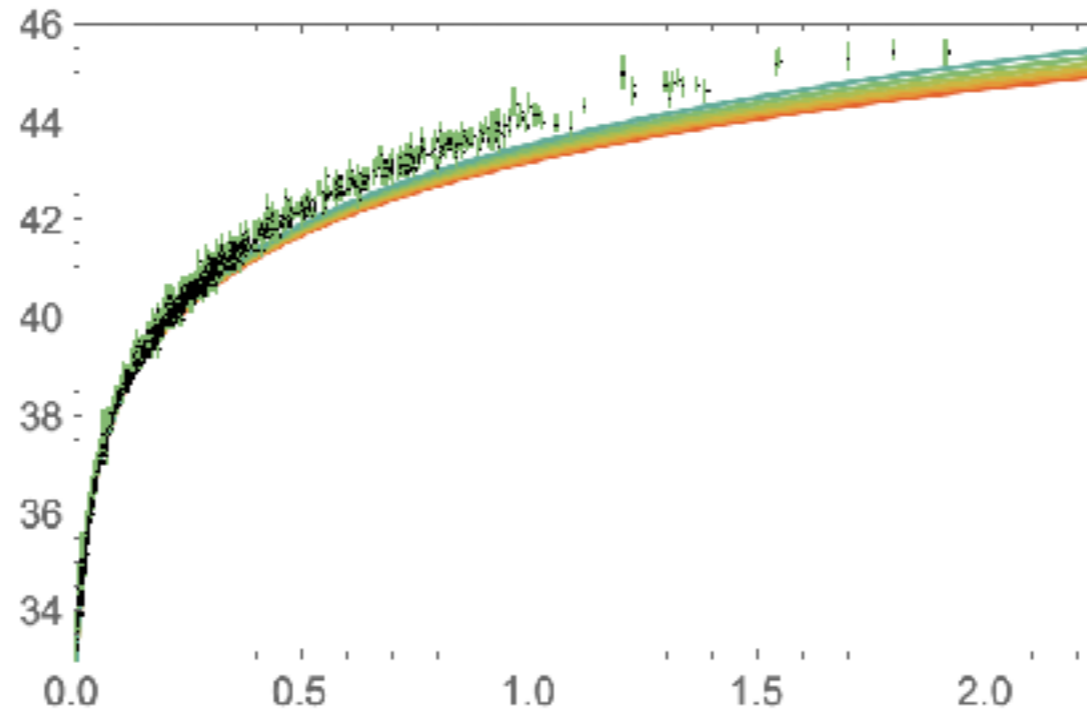
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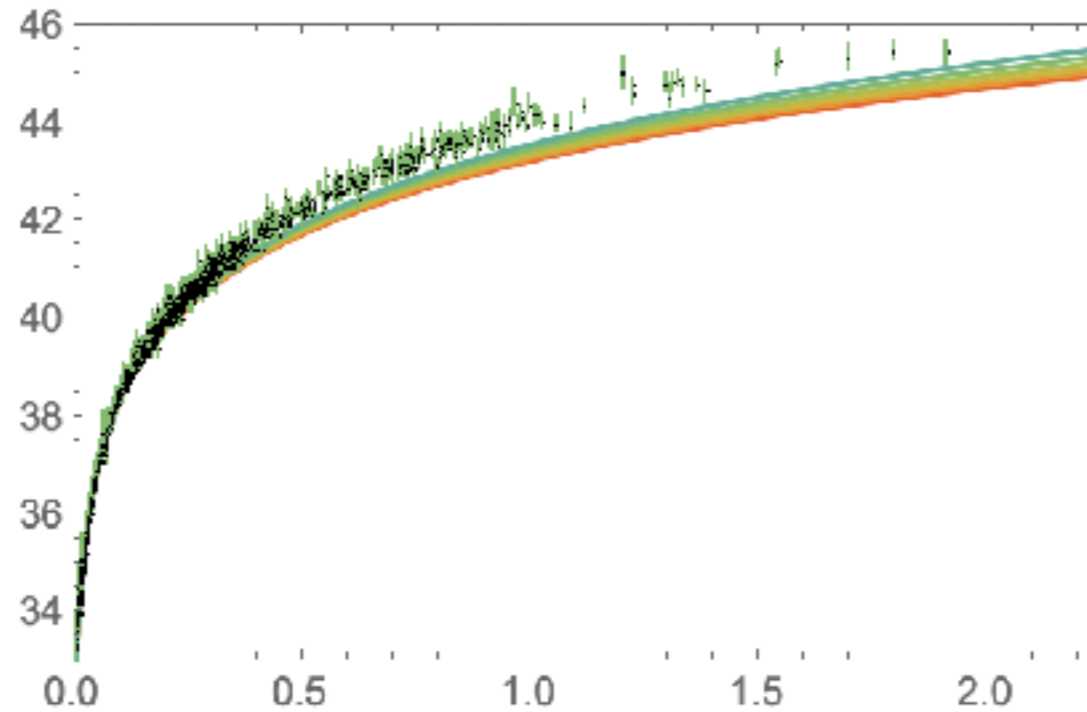
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Let's neglect radiation for simplicity

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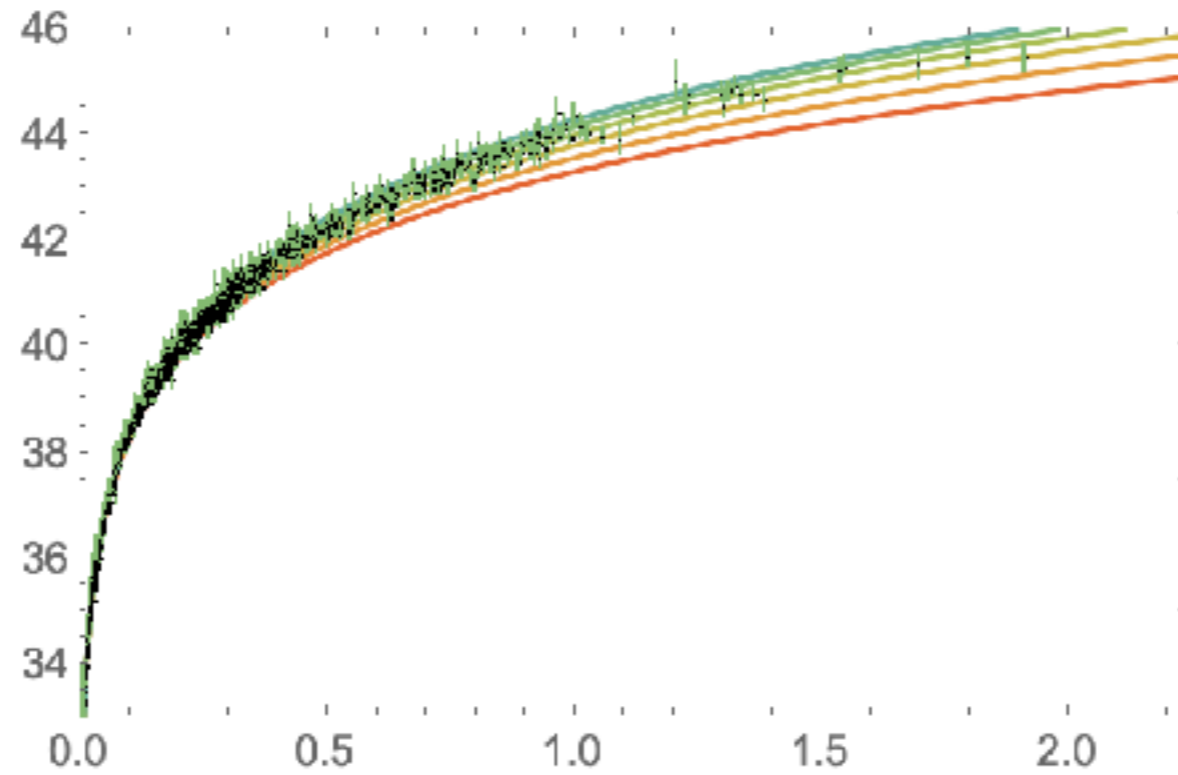
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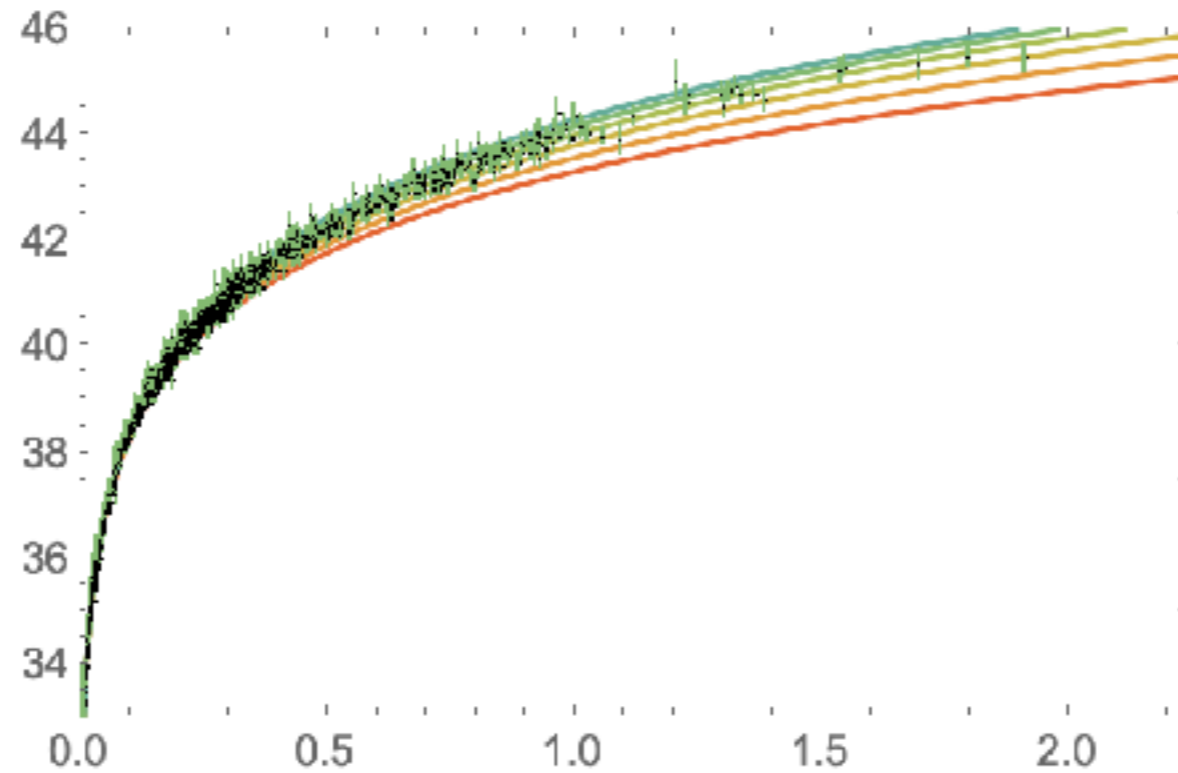
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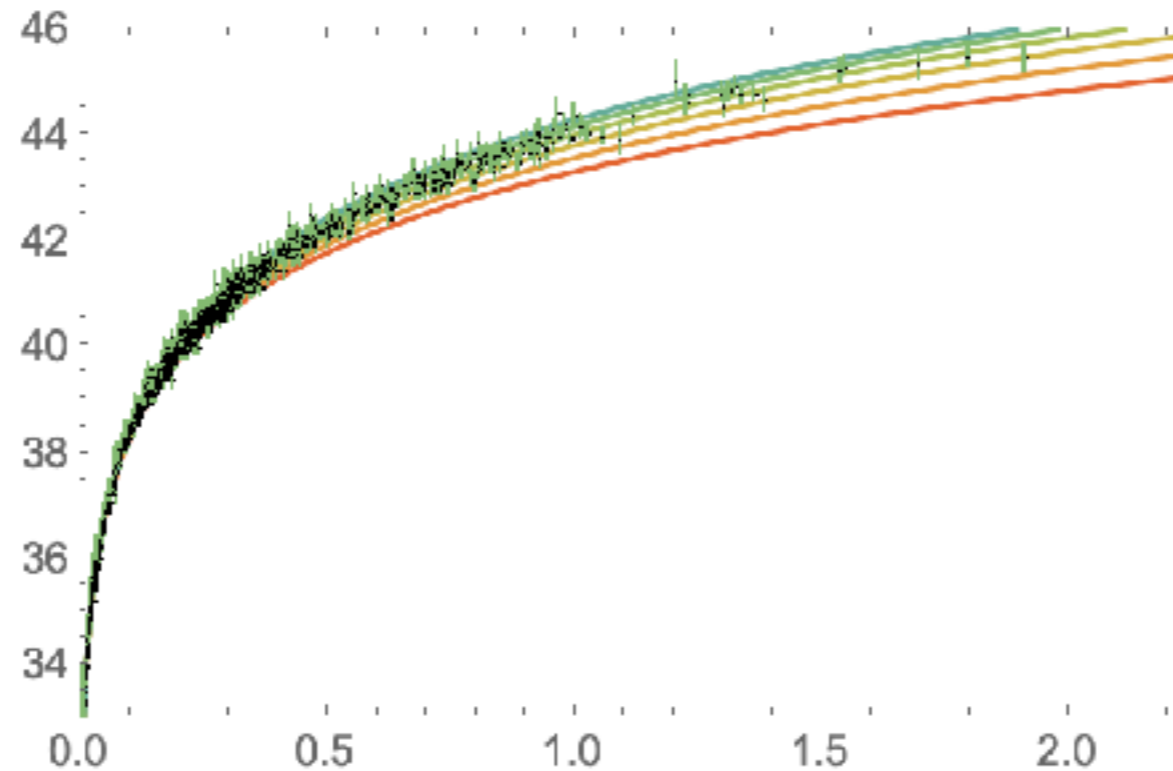
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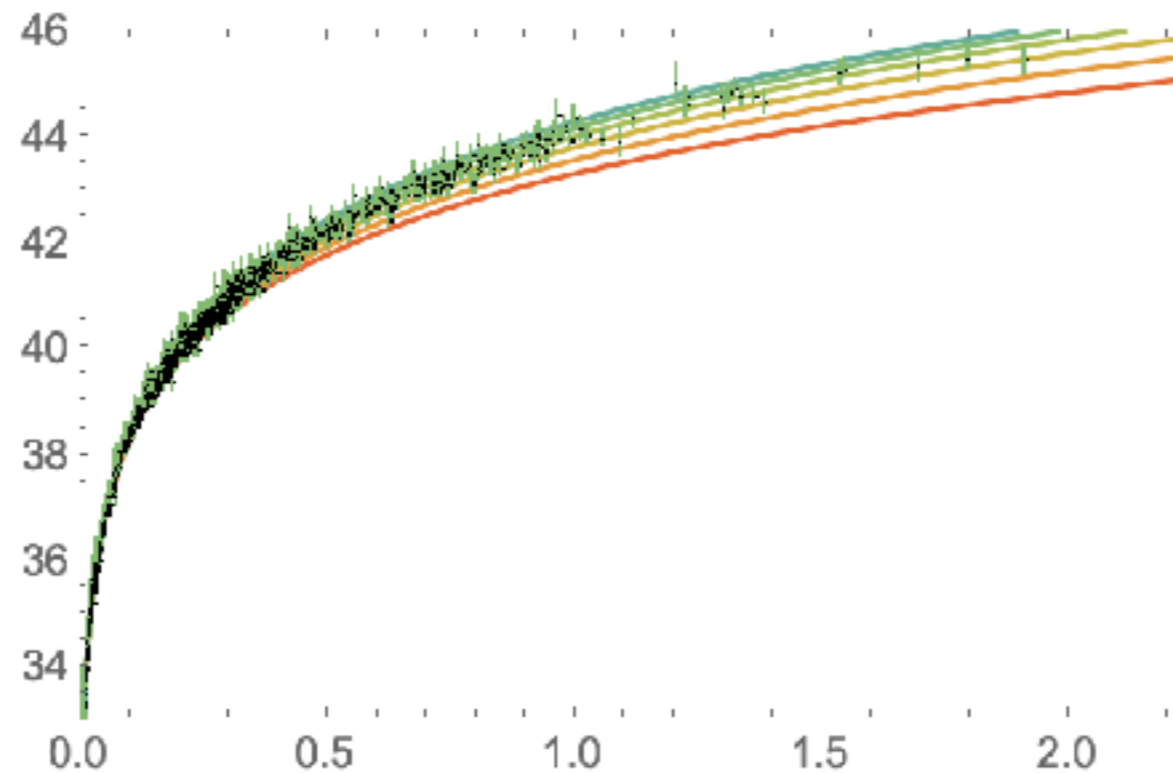
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$X$  is known as the cosmological constant, and we write it  $\Lambda$  (instead of  $X$ )

## A brief history of the Universe

	Time	Energy	
Planck Epoch?	$< 10^{-43}$ s	$10^{18}$ GeV	
String Scale?	$\gtrsim 10^{-43}$ s	$\gtrsim 10^{18}$ GeV	
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Due to the phase transition, a huge amount of energy was released in the Universe

The release of the energy of the vacuum transformed the virtual particles, into real particles

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In all cases, we end this period with a hot Universe filled with interacting particles

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The Universe contained plasma, an incandescent and opaque soup of photons, protons and electrons

Because of these continuous reactions, light underwent continuous deviations and reflections and was therefore trapped in the plasma

The Universe was opaque and dark but its temperature continues to decrease

At sufficient low temperature, electrons were captured by helium and hydrogen: the so called Recombination

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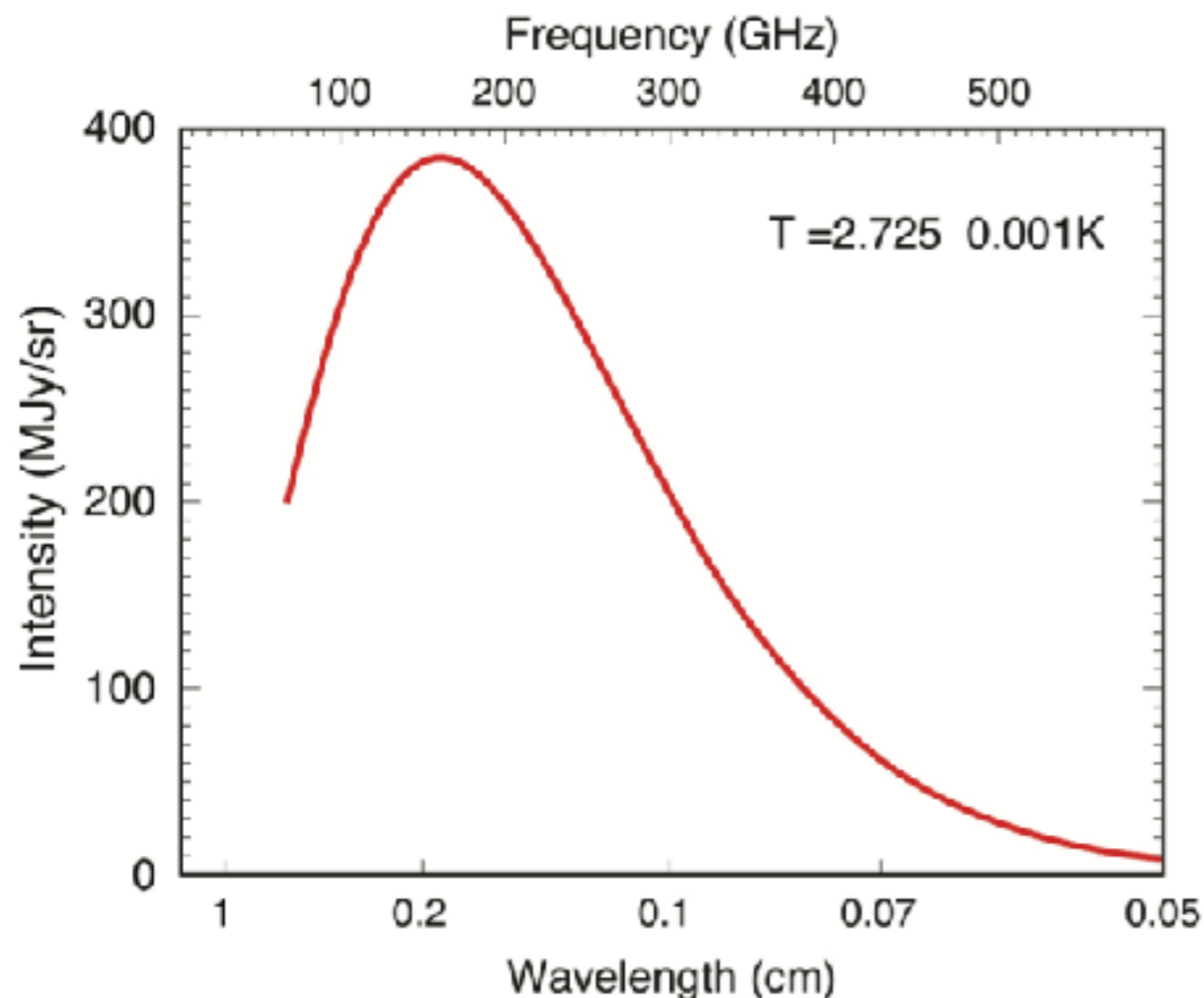
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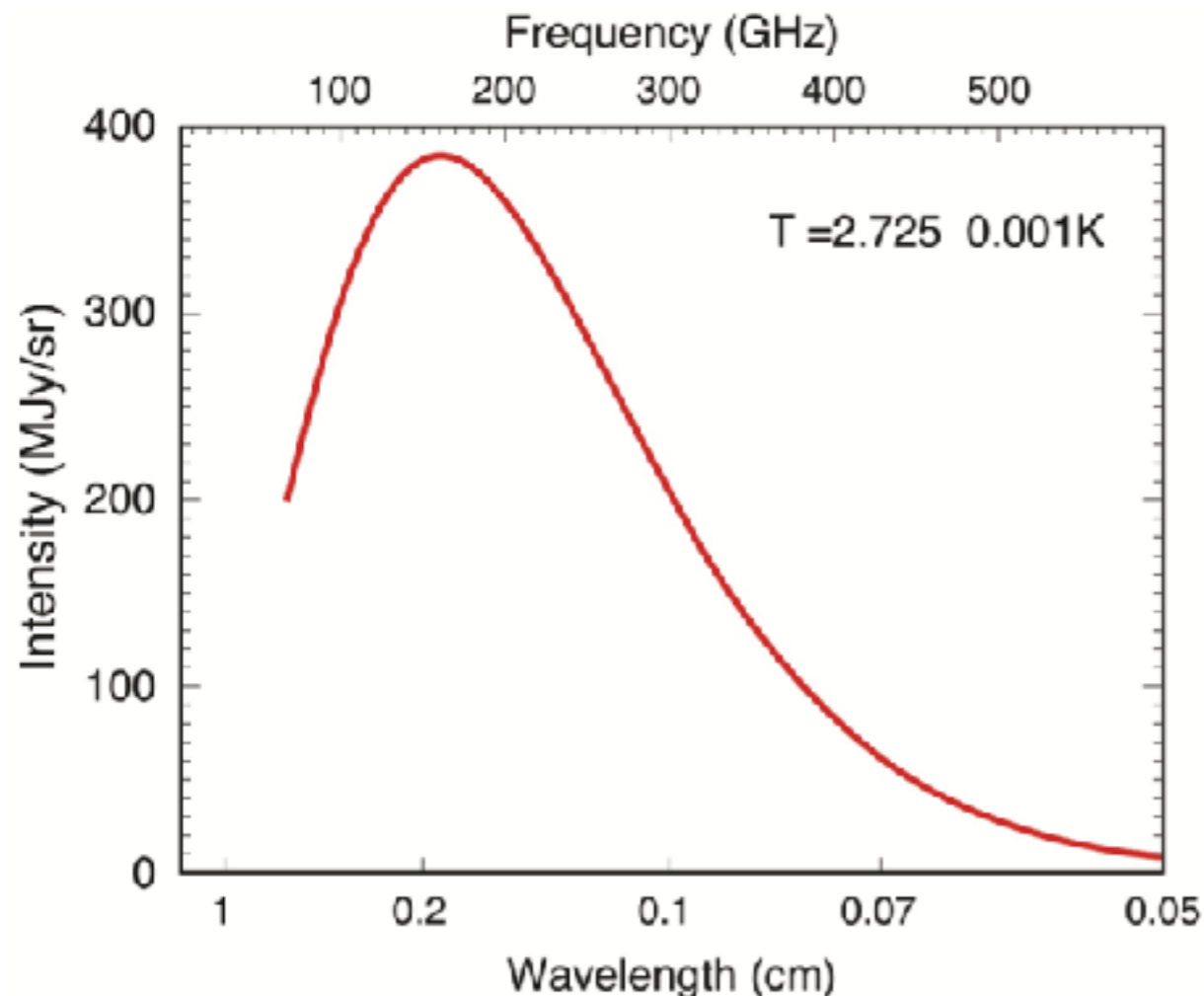
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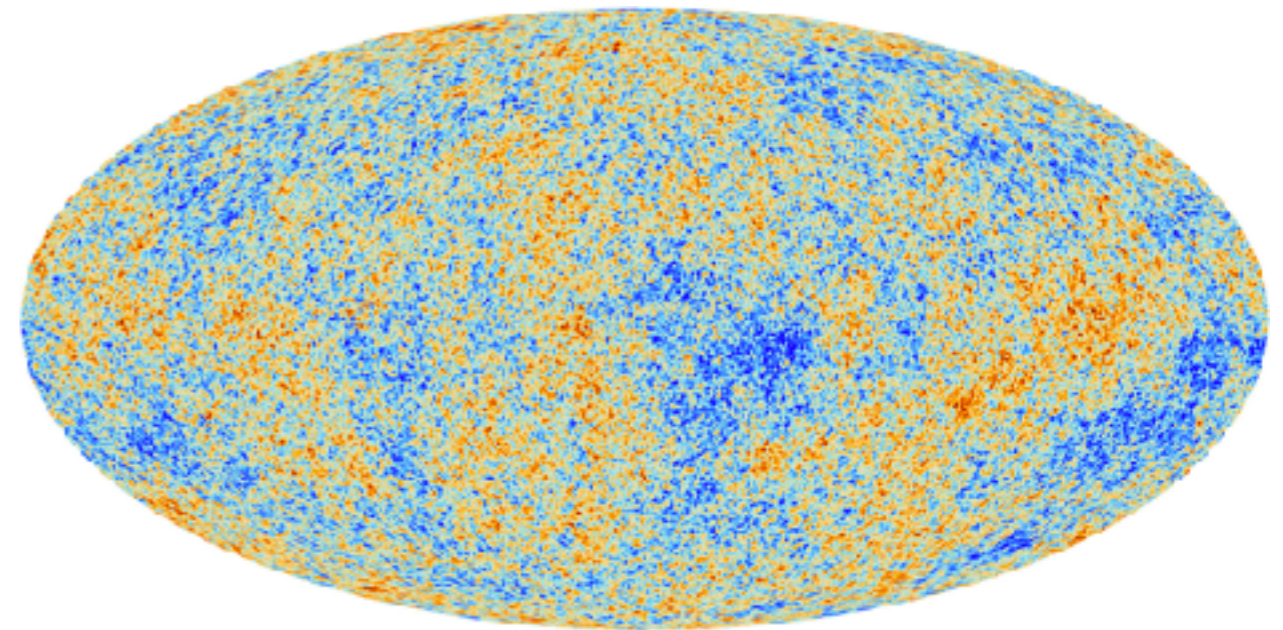
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Where do the fluctuations come from?

They come from inflation, during which quantum fluctuations are stretched which caused the variations in density

One important ingredient is dark matter which interacts only gravitationally, so they tend to accumulate in potential wells

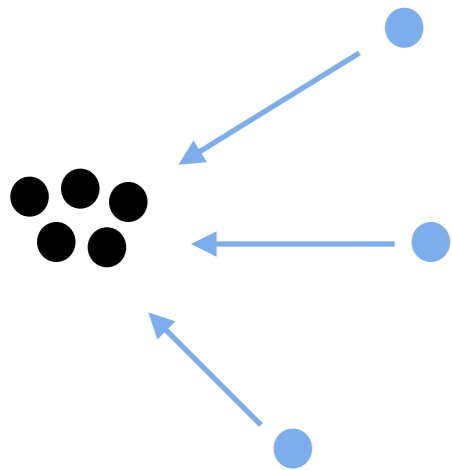
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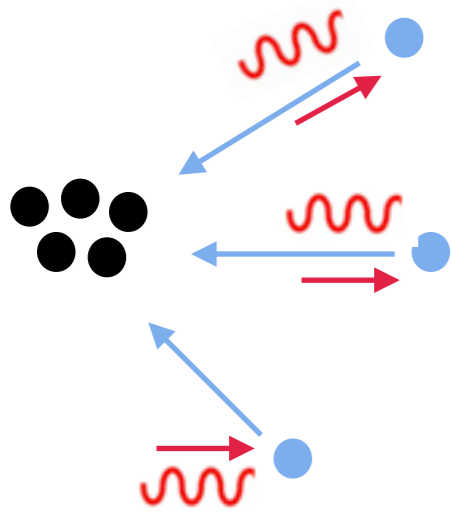
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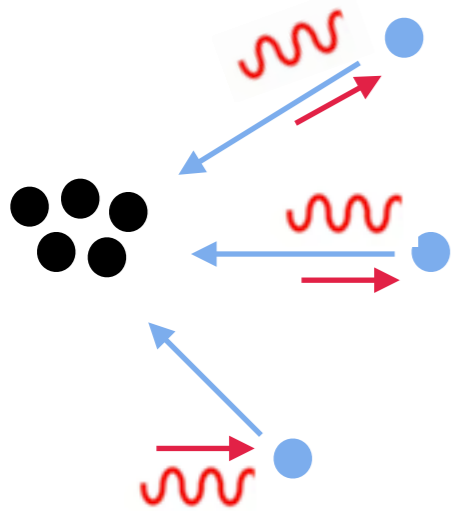
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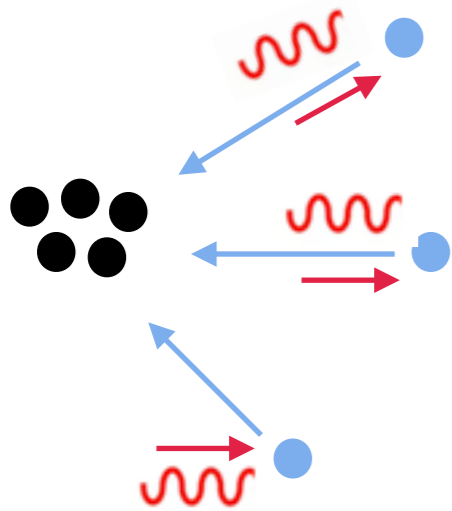
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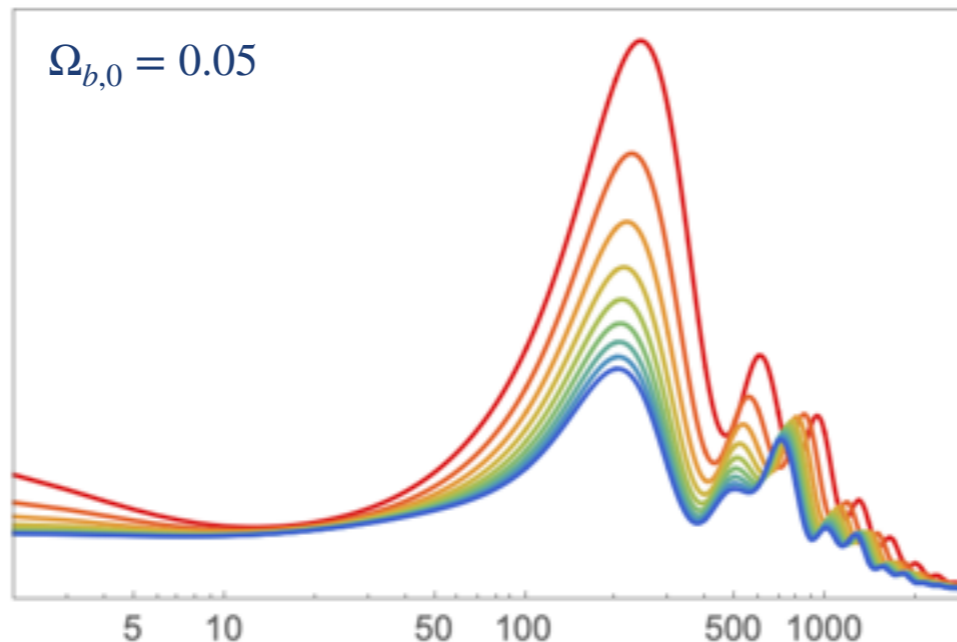
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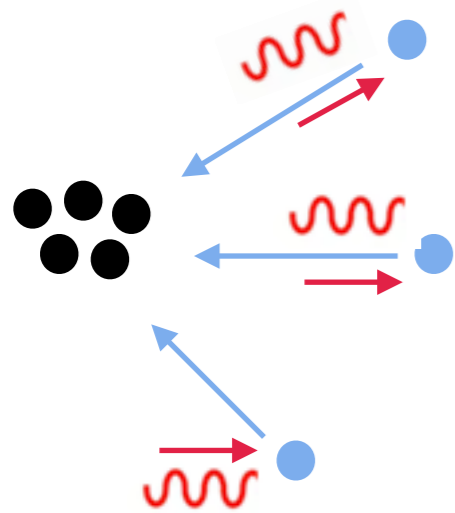


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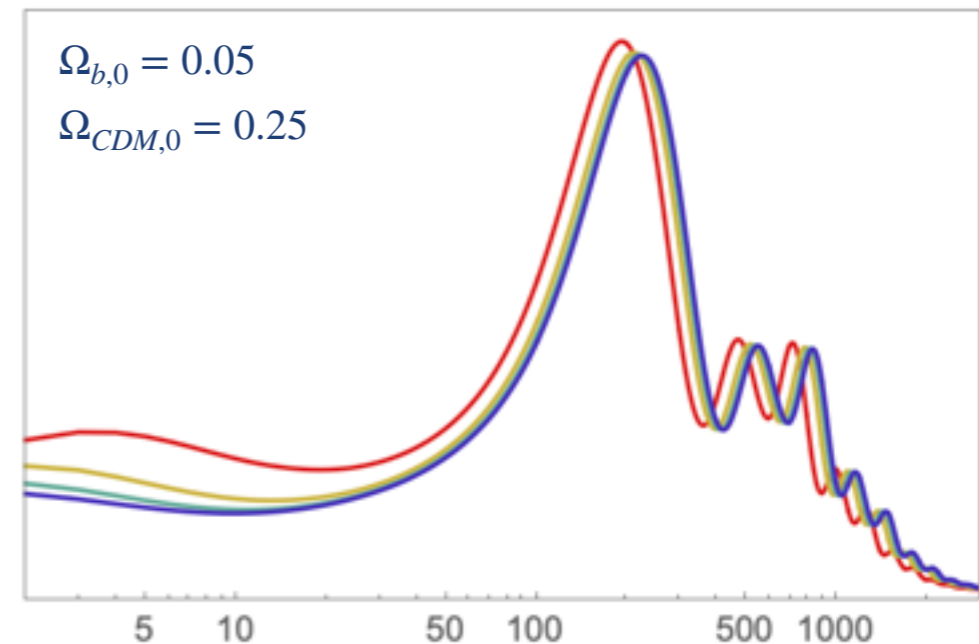
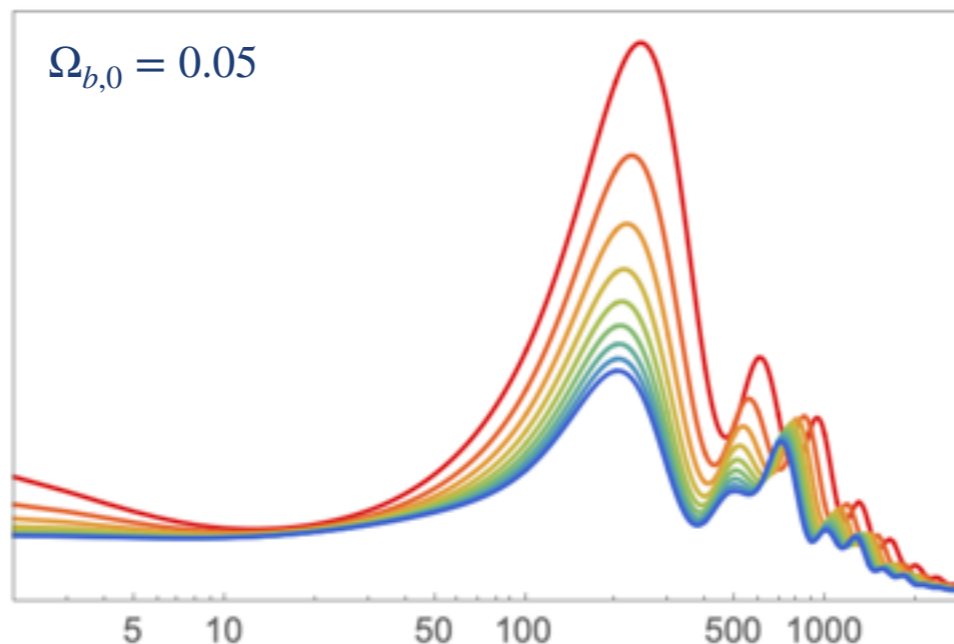
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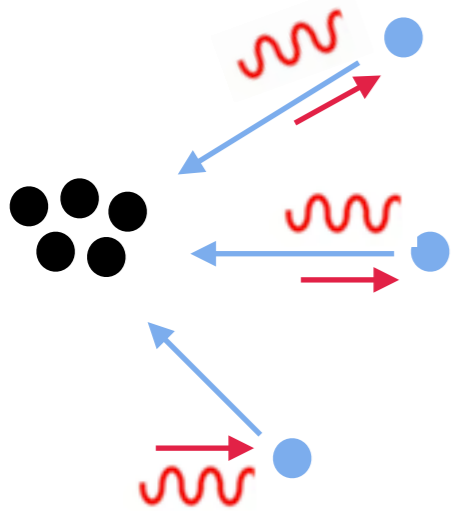
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$\Omega_{CDM,0} = 0.7$	$\Omega_{\Lambda,0} = 0.25$
$\Omega_{CDM,0} = 0.8$	$\Omega_{\Lambda,0} = 0.15$
$\Omega_{CDM,0} = 0.9$	$\Omega_{\Lambda,0} = 0.05$



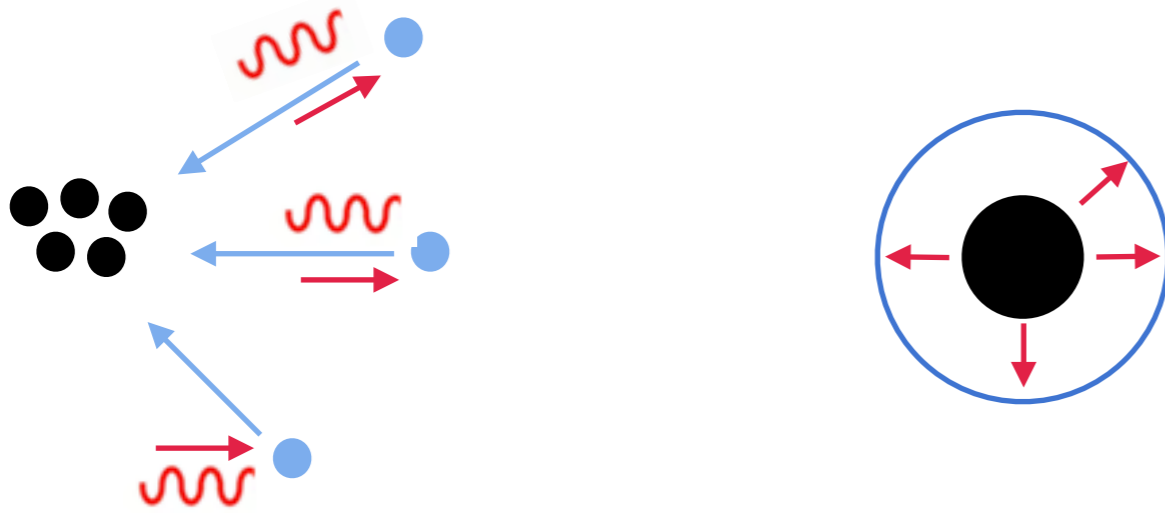
$w = -1/3$   
 $w = -2/3$   
 $w = -1$   
 $w = -4/3$



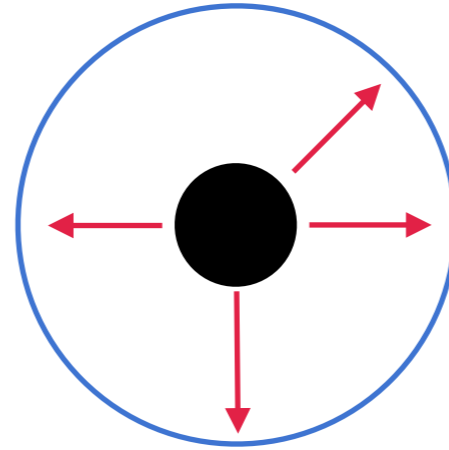
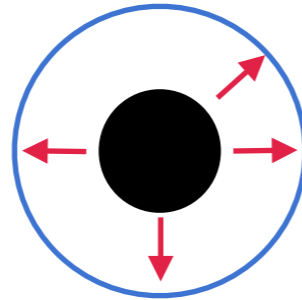
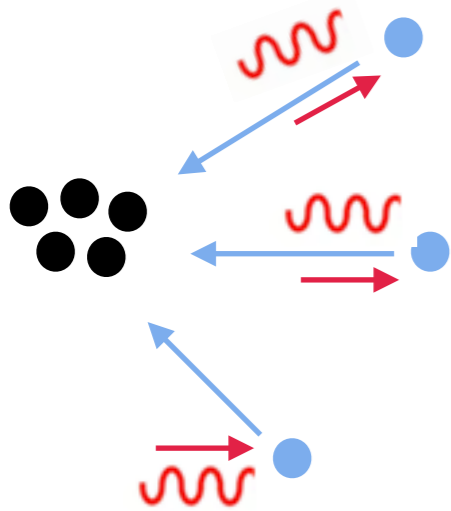
What is the **Baryon Acoustic Oscillation**?



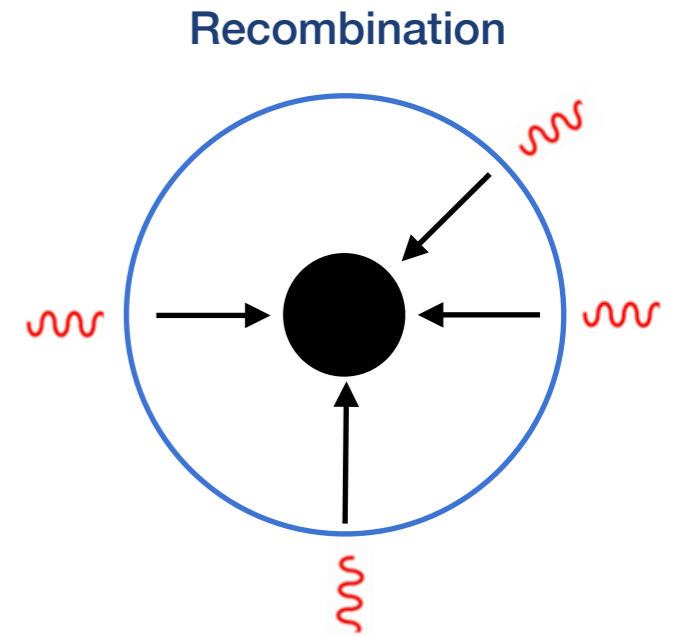
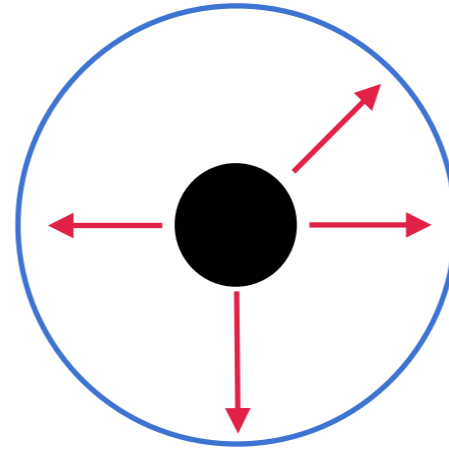
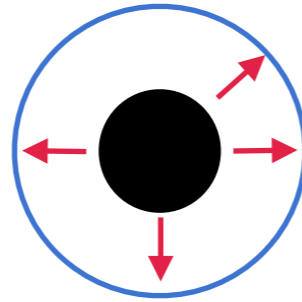
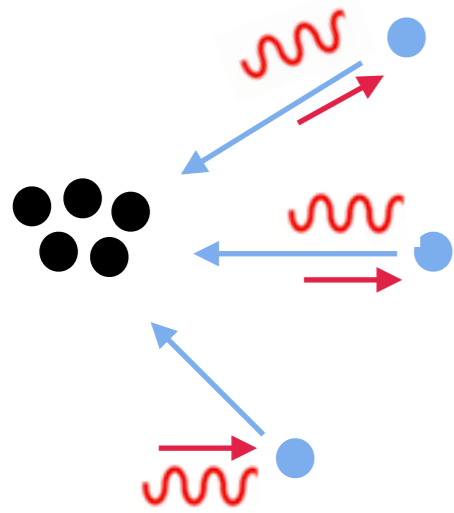
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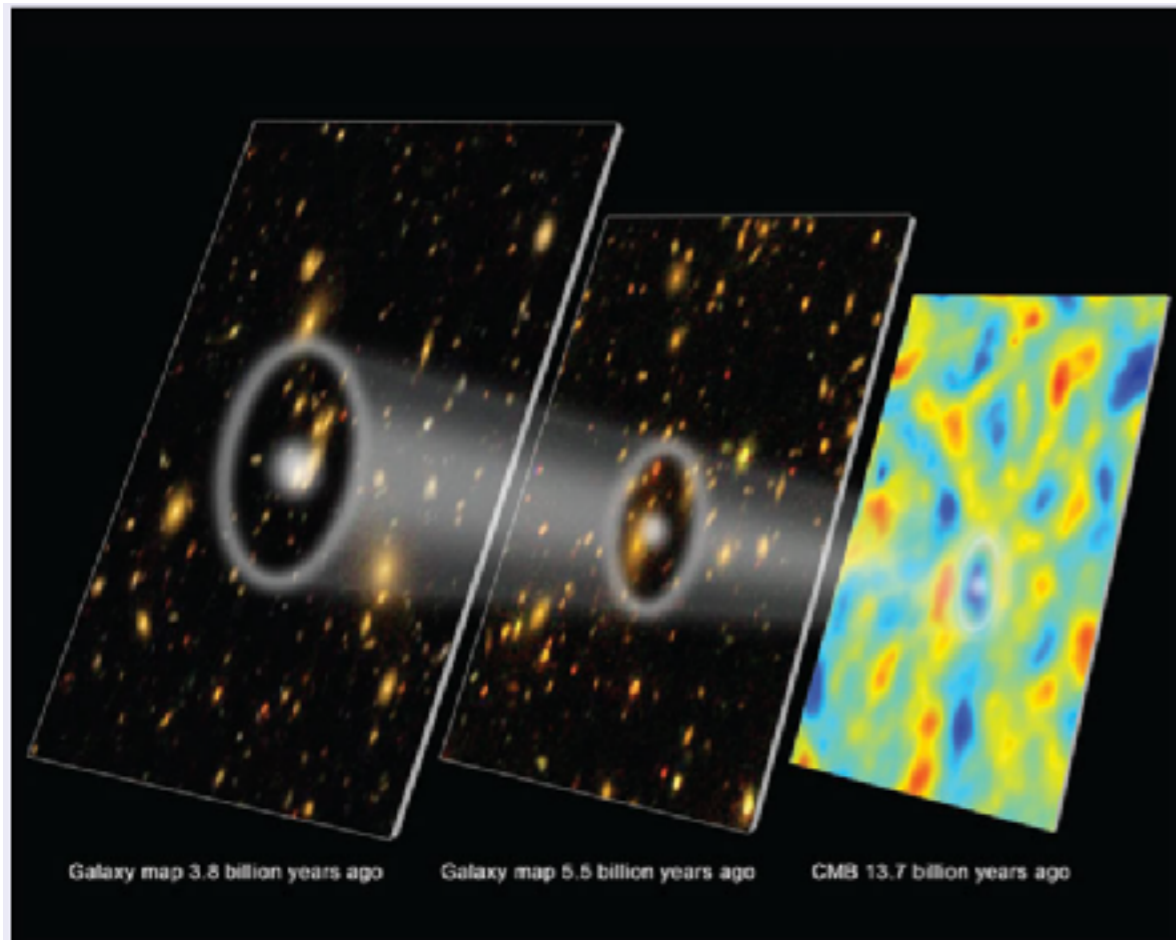
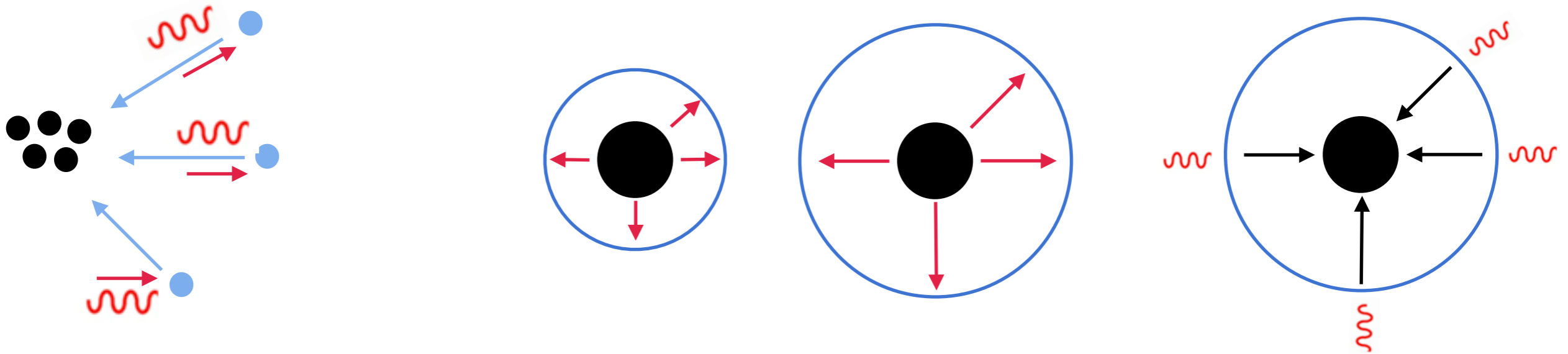
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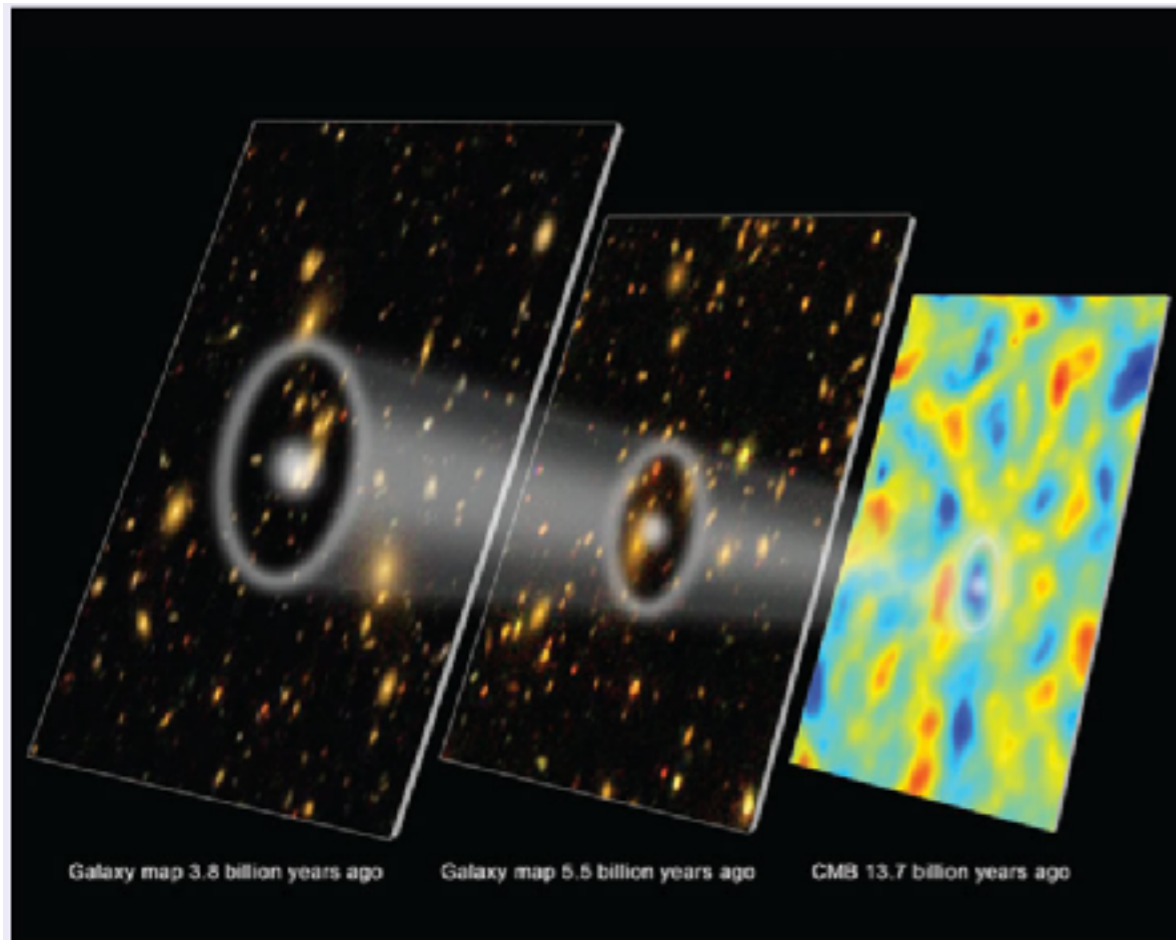
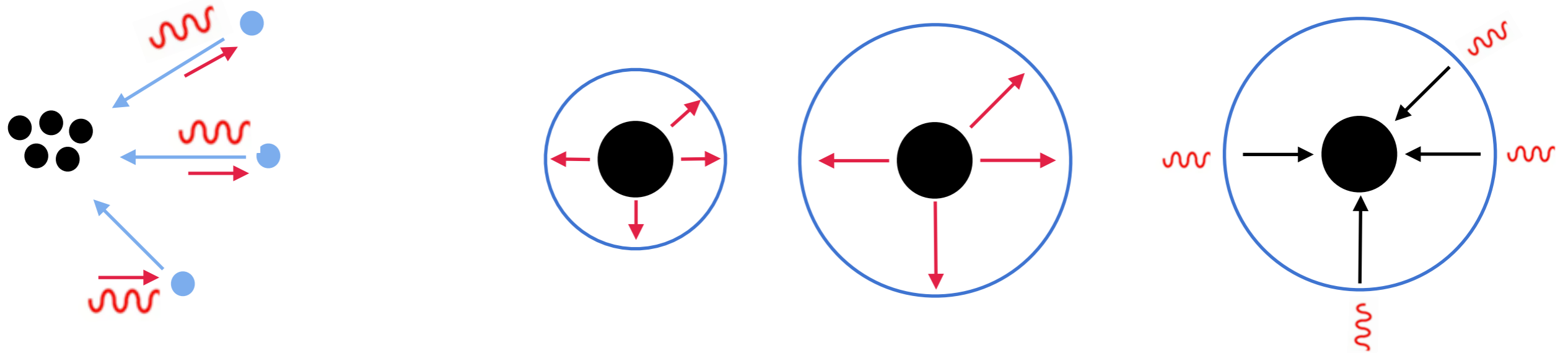


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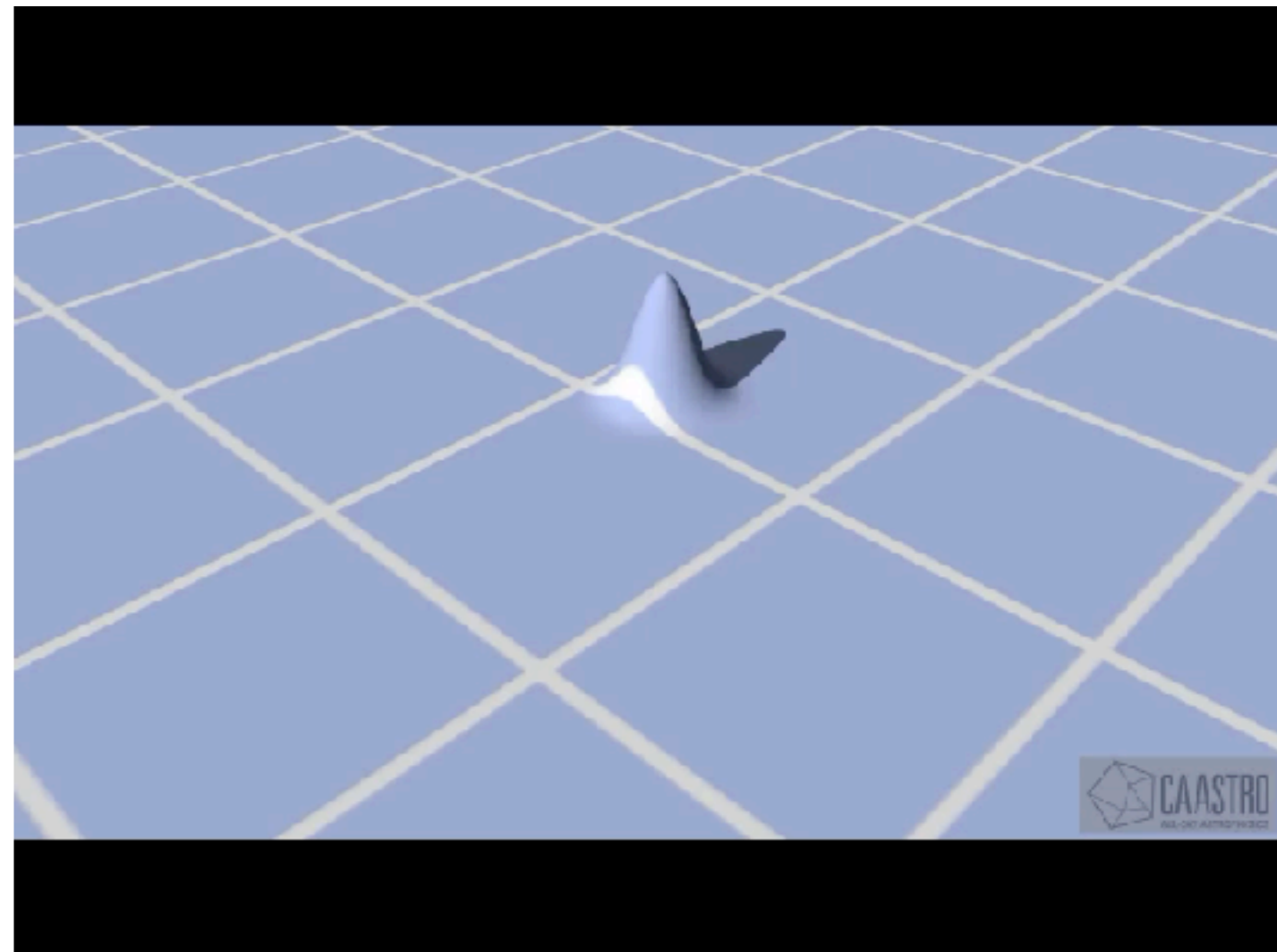


Credit: SDSS-III, South Pole Telescope

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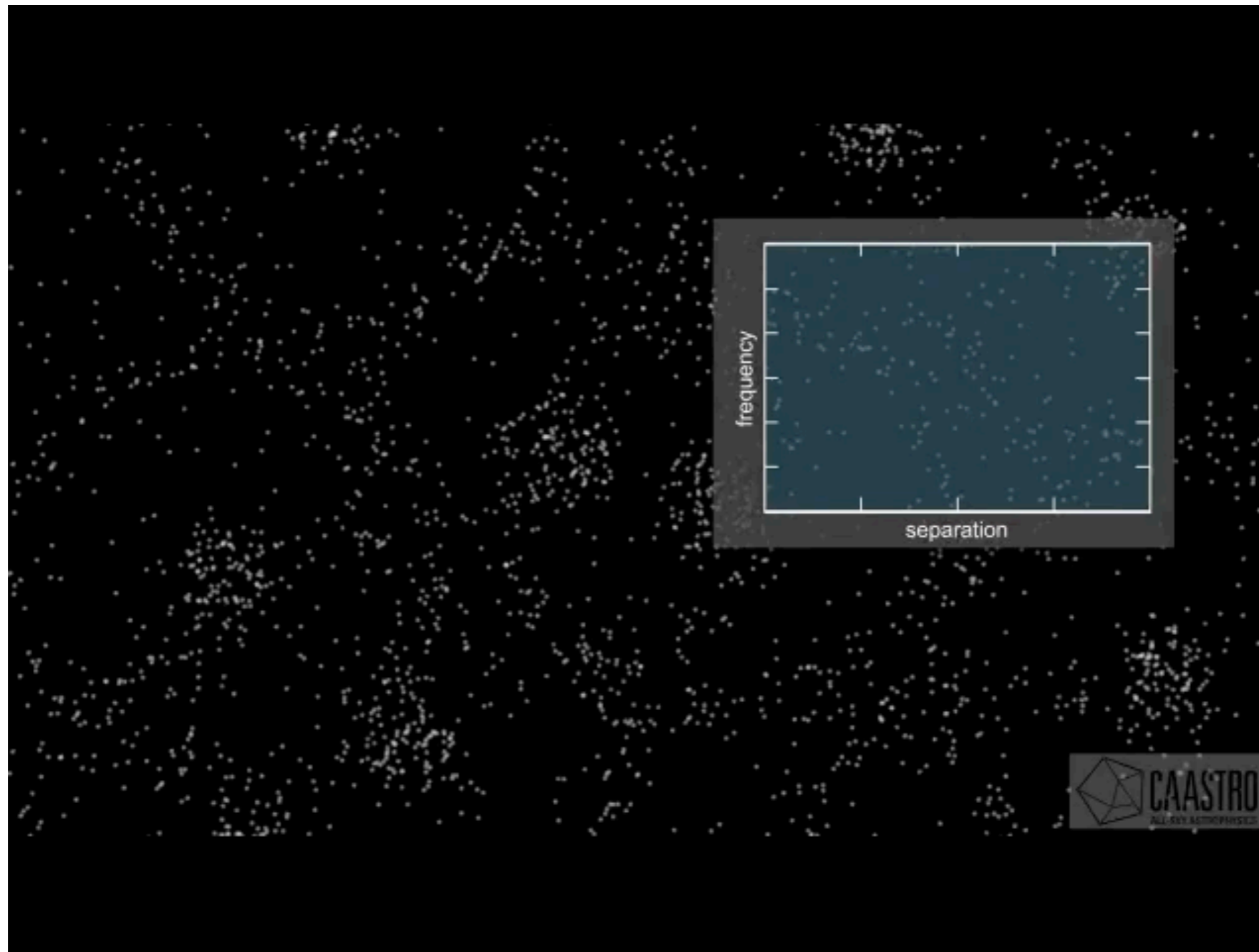


Credit: SDSS-III, South Pole Telescope



Credit: <http://caastro.org/>

How do we measure BAO?



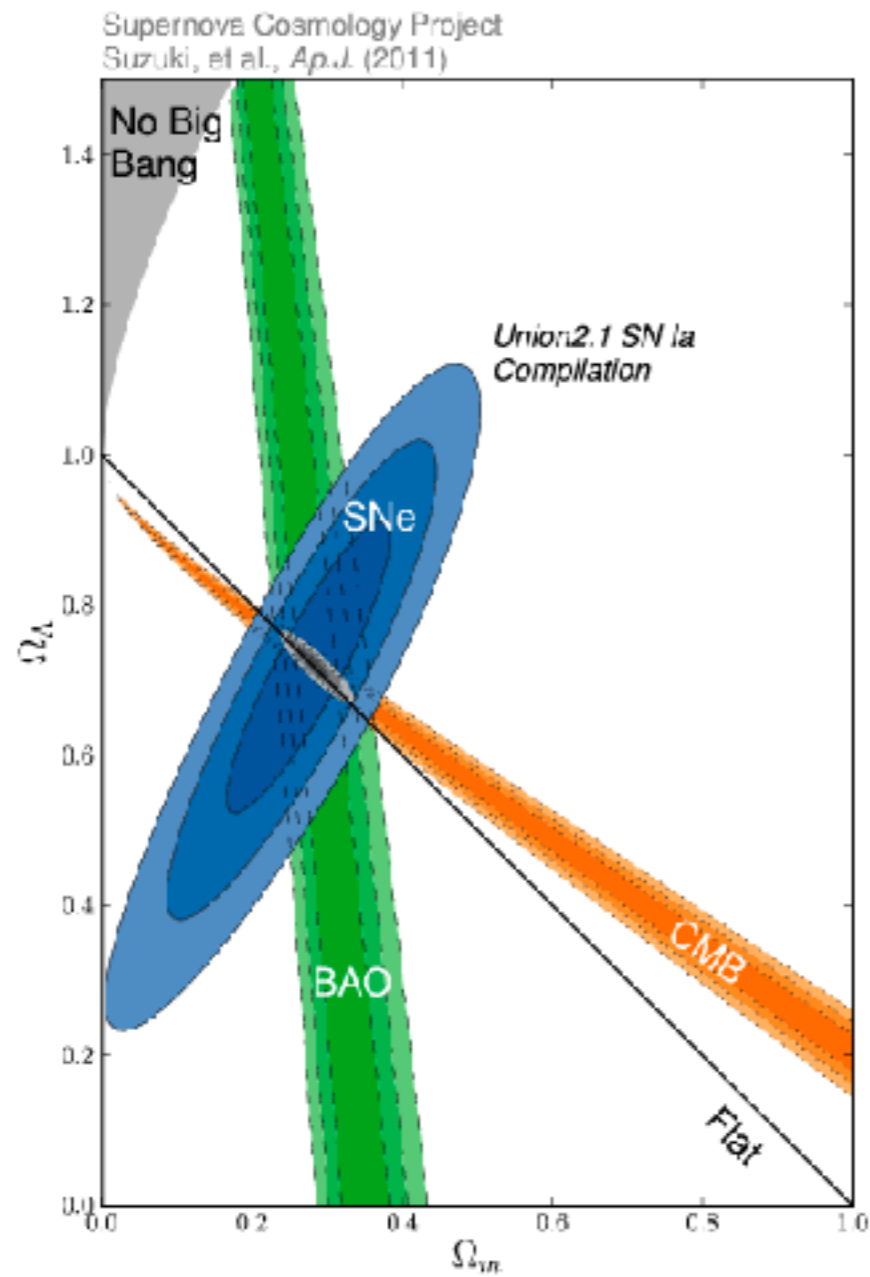
Credit: <http://caastro.org/>

The BAO can be measured at a given redshift and it will depend on the cosmological model

# All observations

Considering all observations, and if we assume  $w = -1$ , we find

$$\Omega_{m,0} \simeq 0.31 \quad \Omega_{\Lambda,0} \simeq 0.69$$

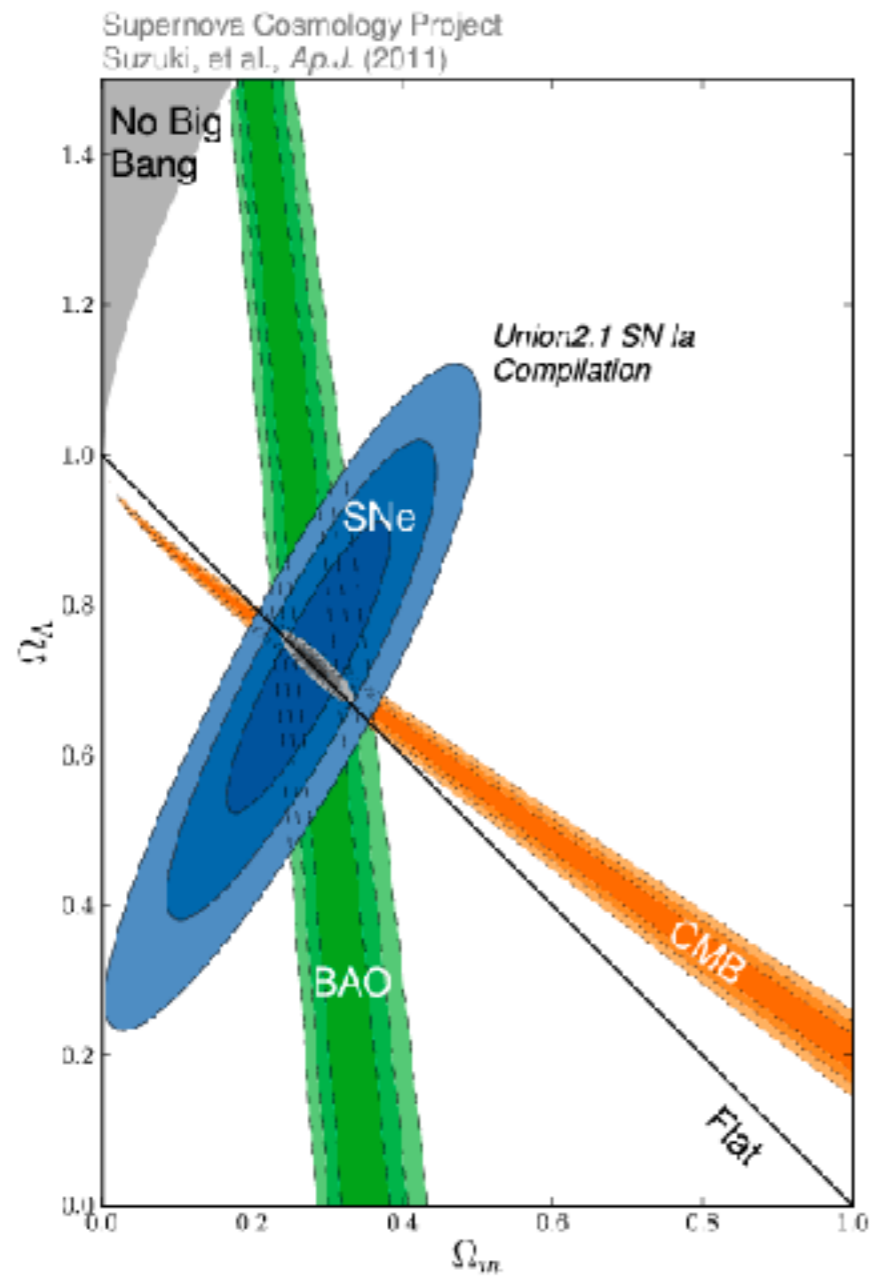




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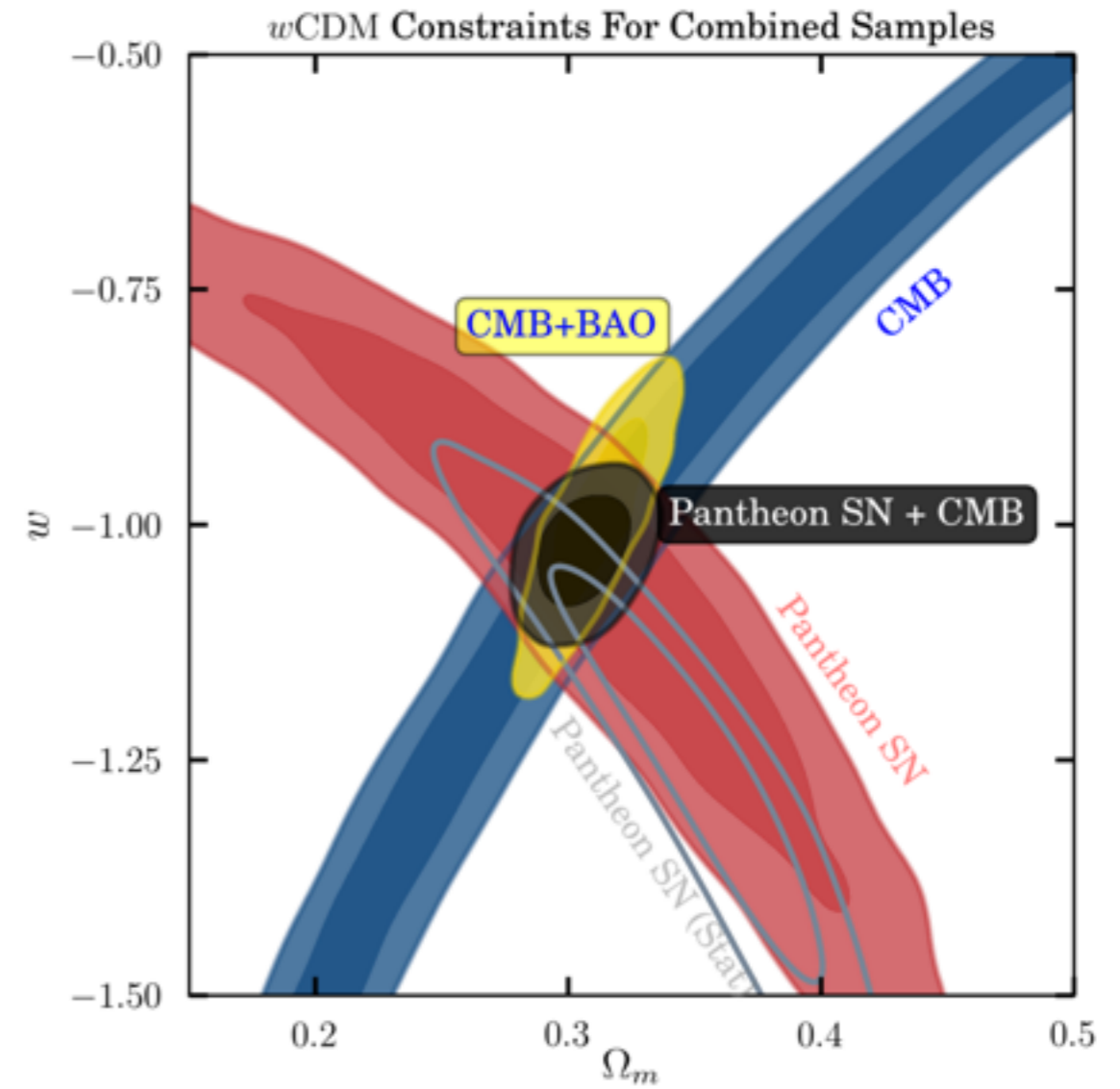
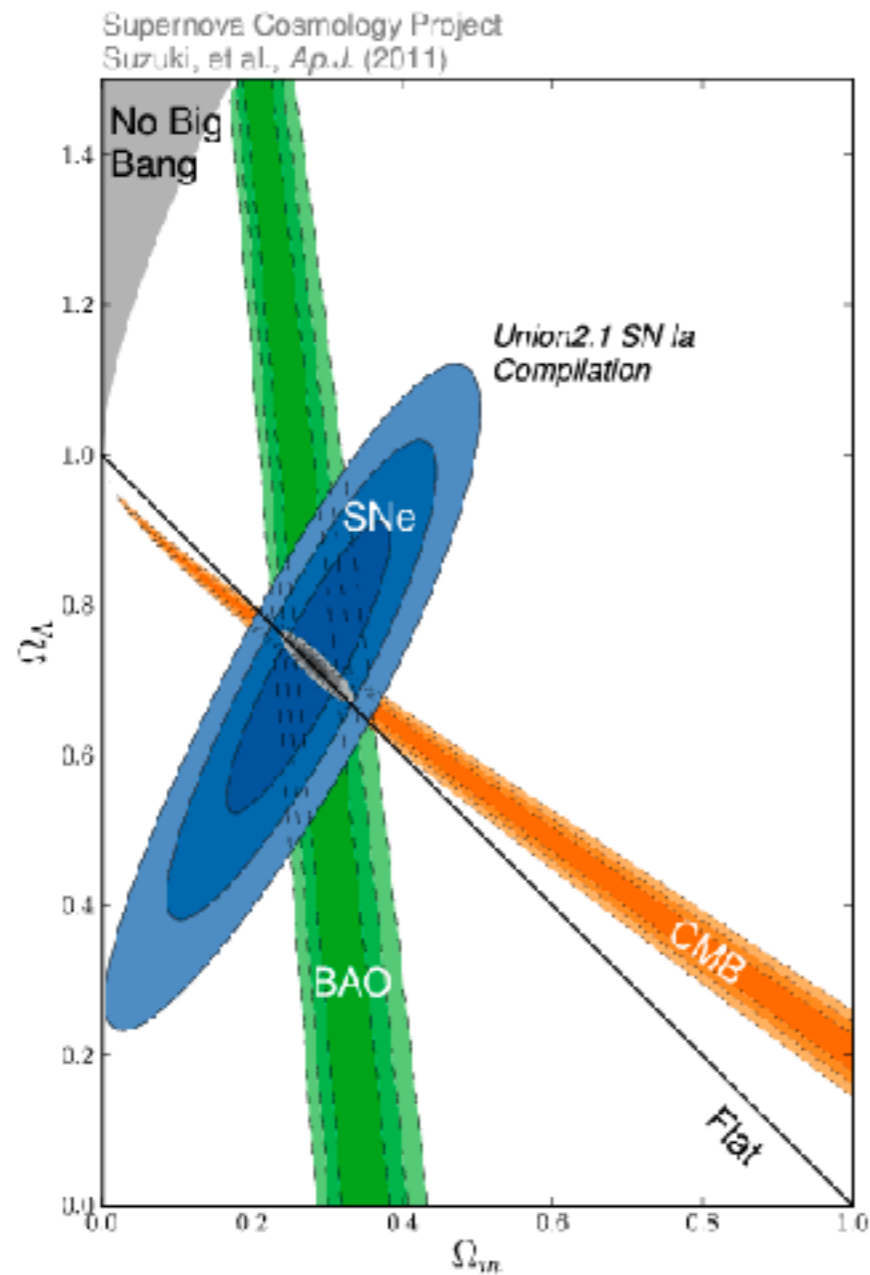


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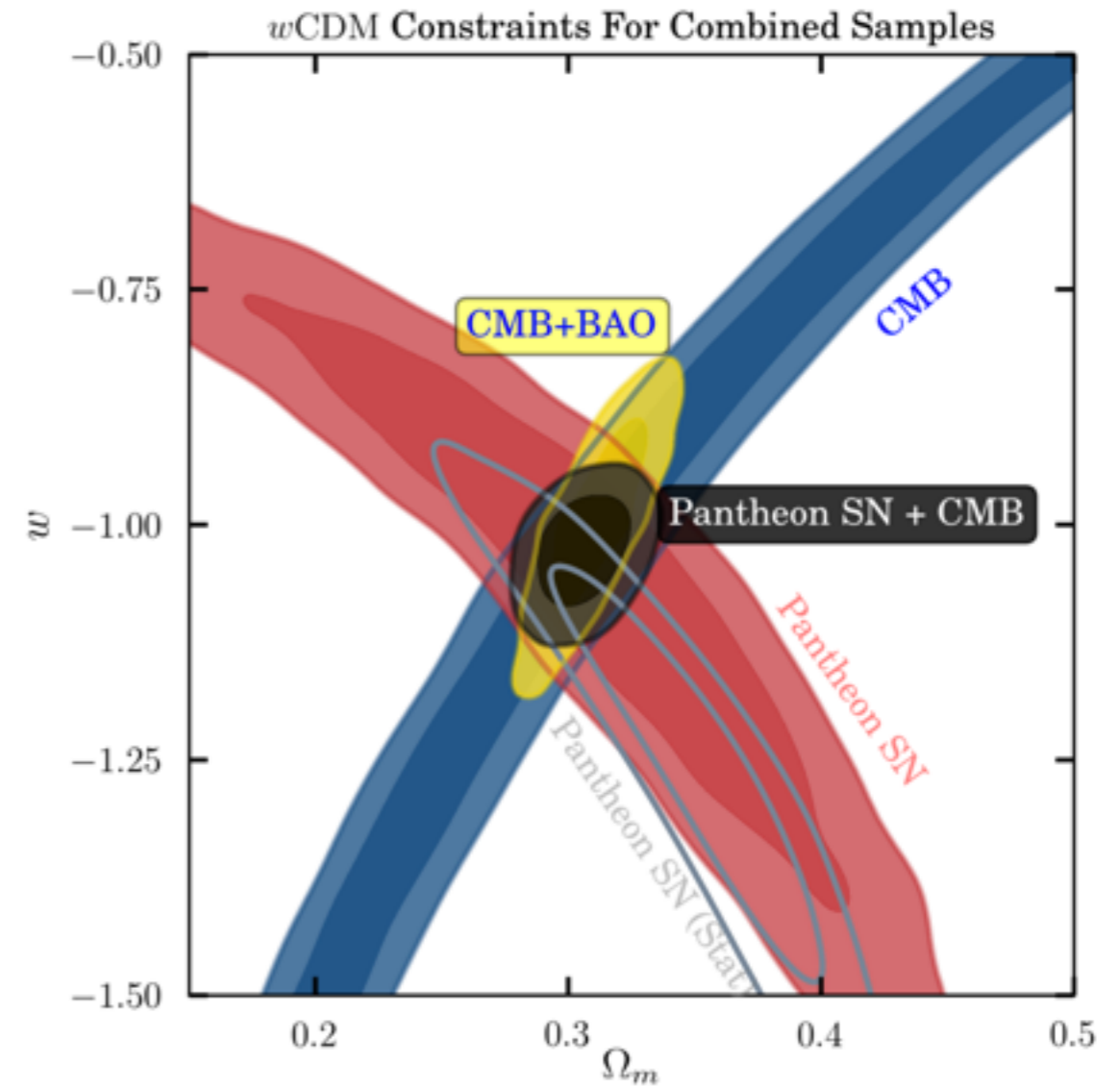
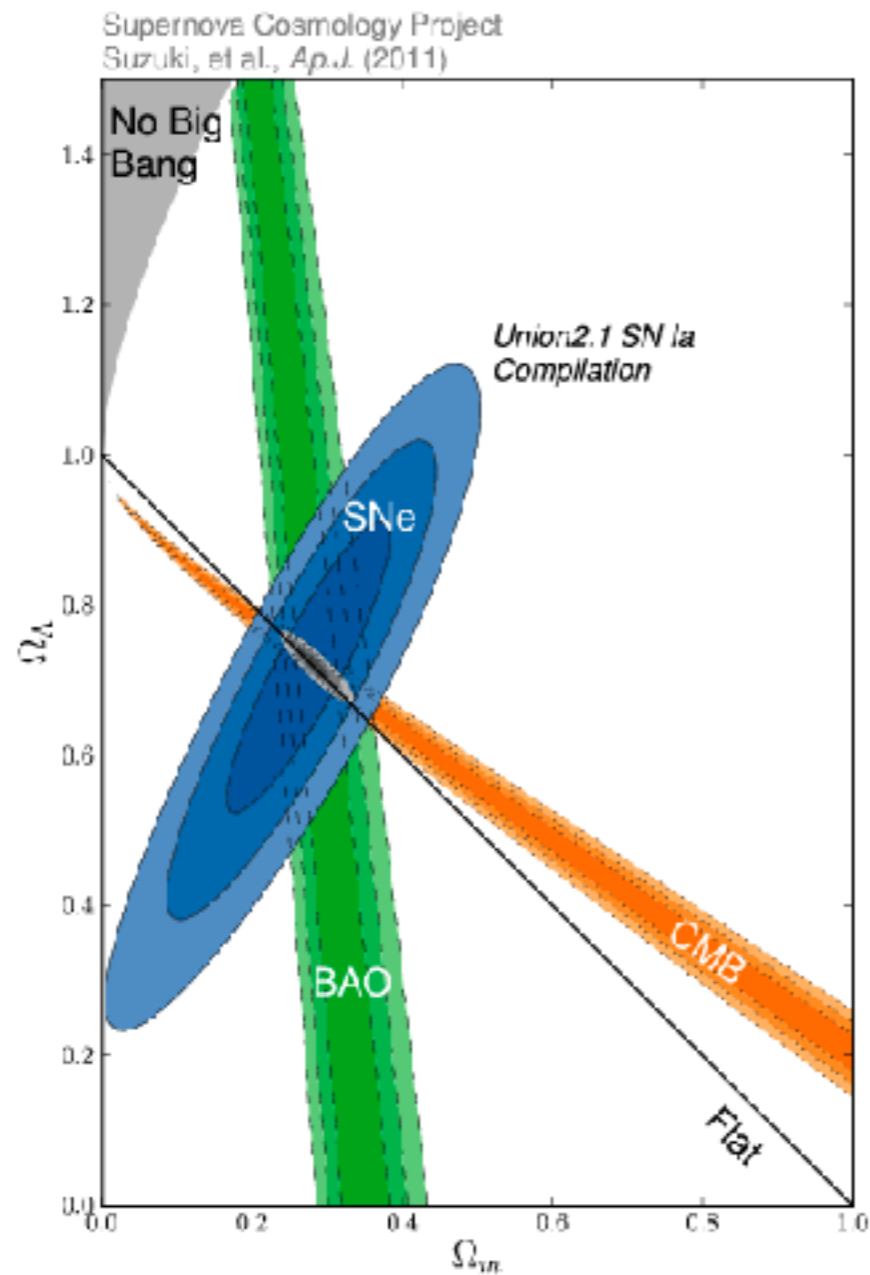
If we assume  $w$  constant

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If we assume  $w$  constant

It is possible that the “X” known as dark energy is the cosmological constant but not sure

# Effects of the cosmological constant

---

## Cosmological effects

### Accelerate the Universe


$$H \equiv \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}}$$

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The Universe started accelerating 6,1  $10^9$  years ago

(depends a lot on  $H_0$ )

# Effects of the cosmological constant

---

## Cosmological effects

Modification of the age of the Universe

$$dt = - \frac{dz}{(1+z)H(z)}$$

$$\text{Age of the Universe} = \int_0^{t_0} dt = - \int_{\infty}^0 \frac{dz}{(1+z)H(z)} = \int_0^{\infty} \frac{dz}{(1+z)H(z)}$$

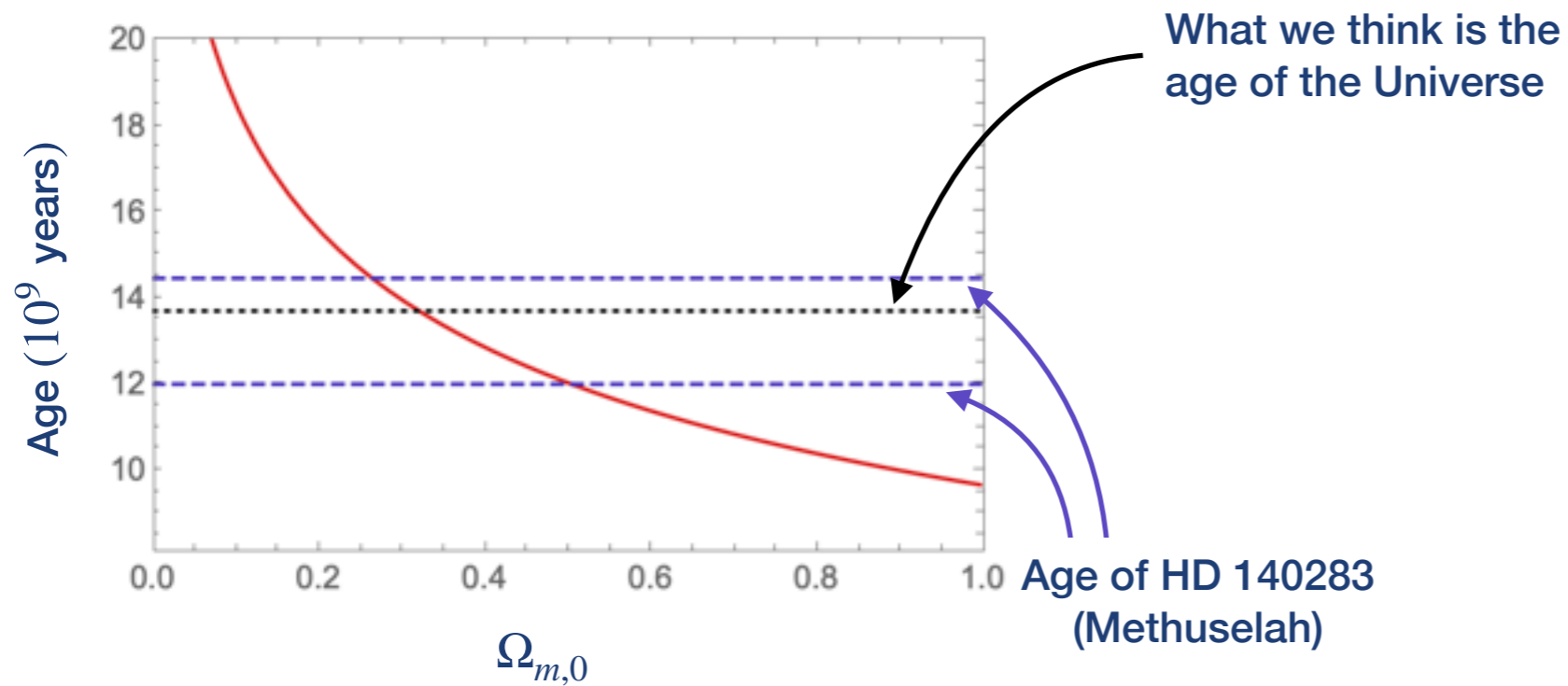
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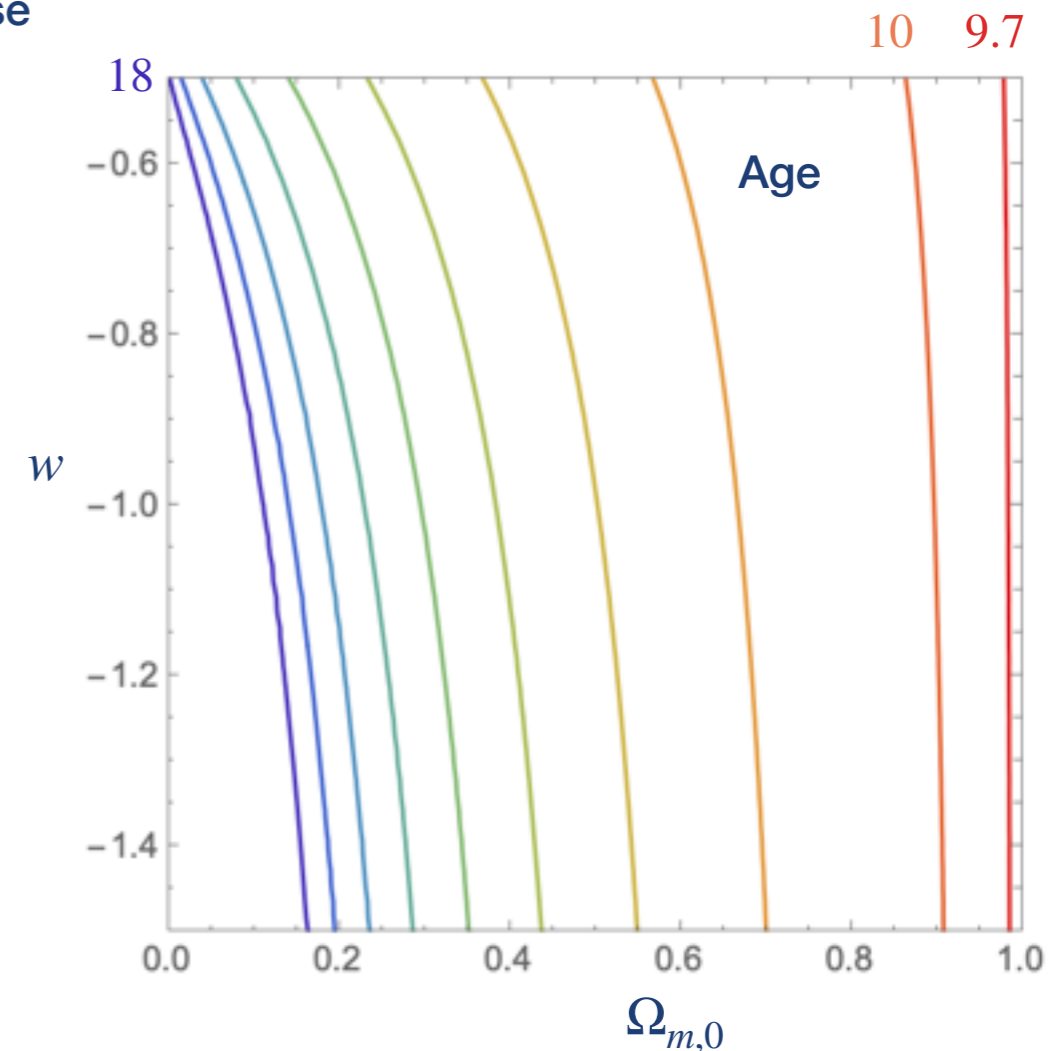
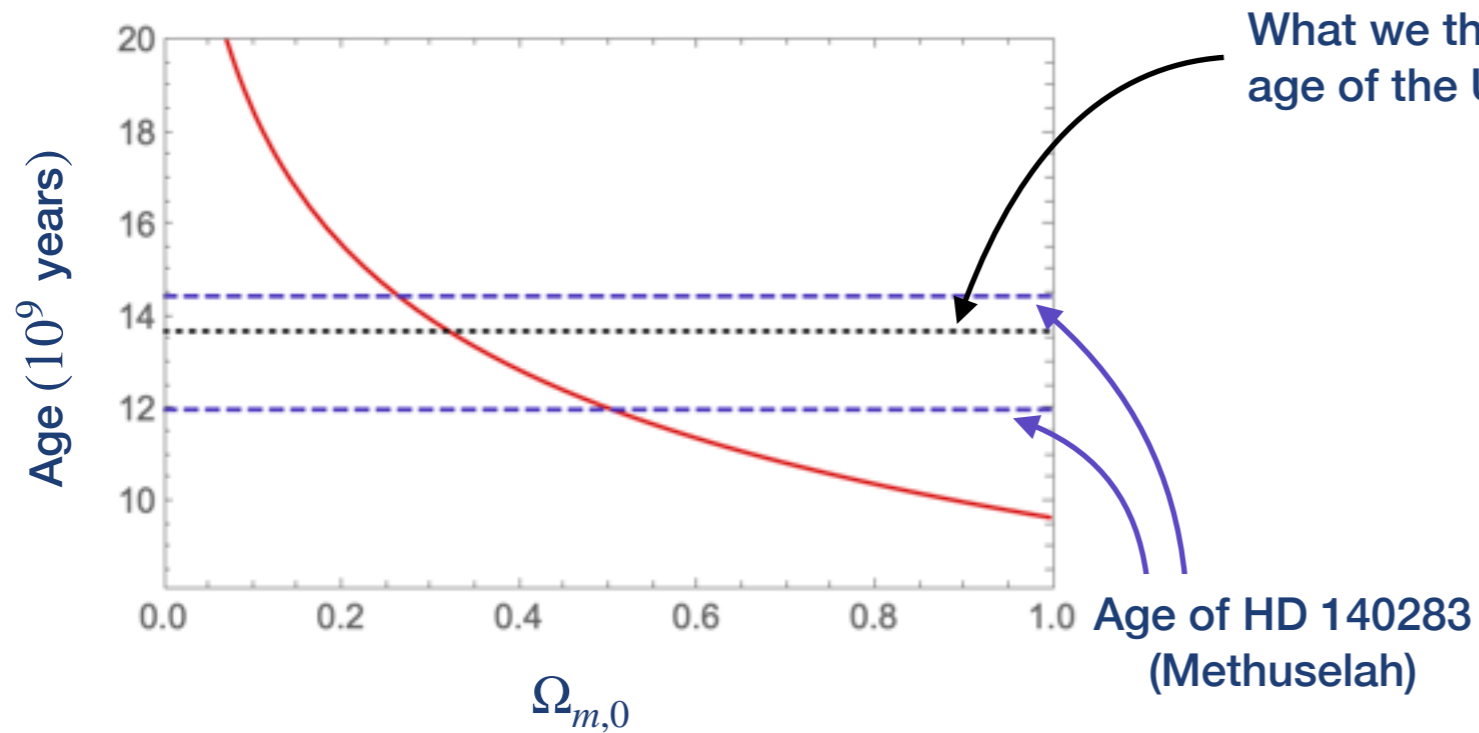
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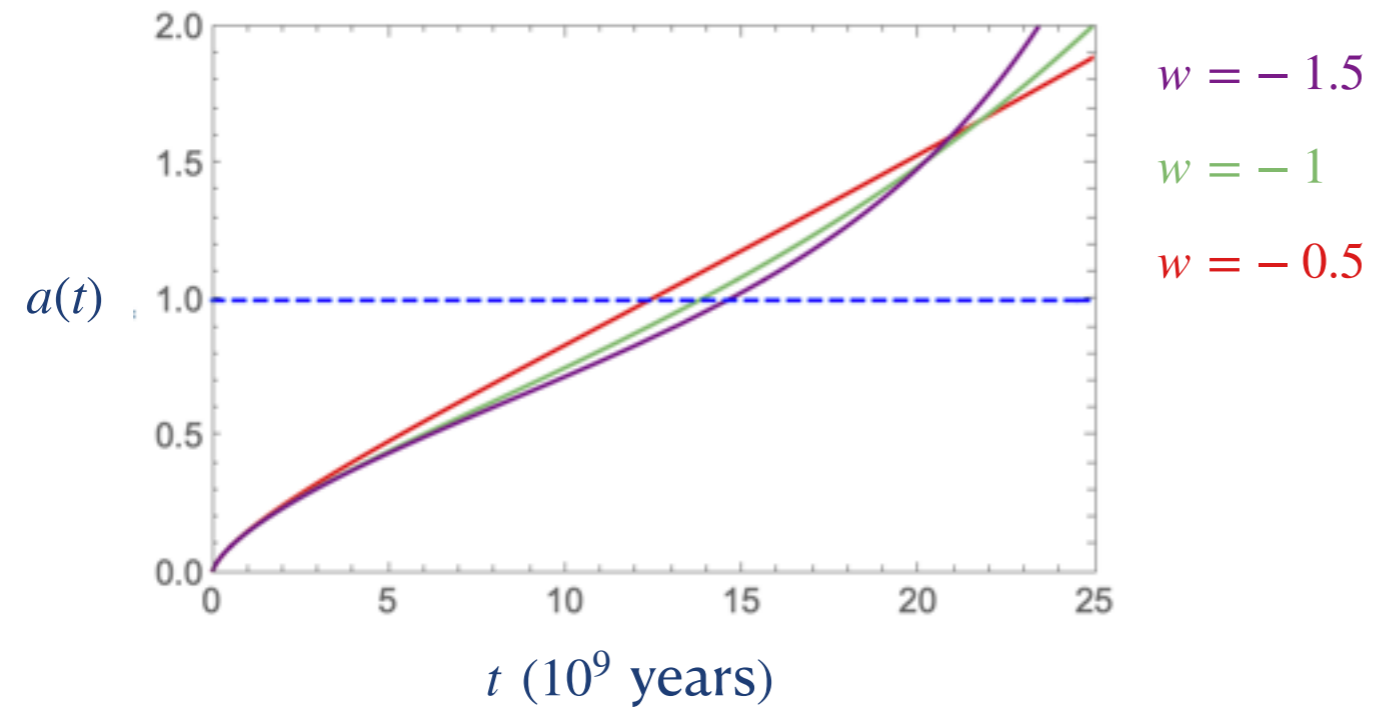
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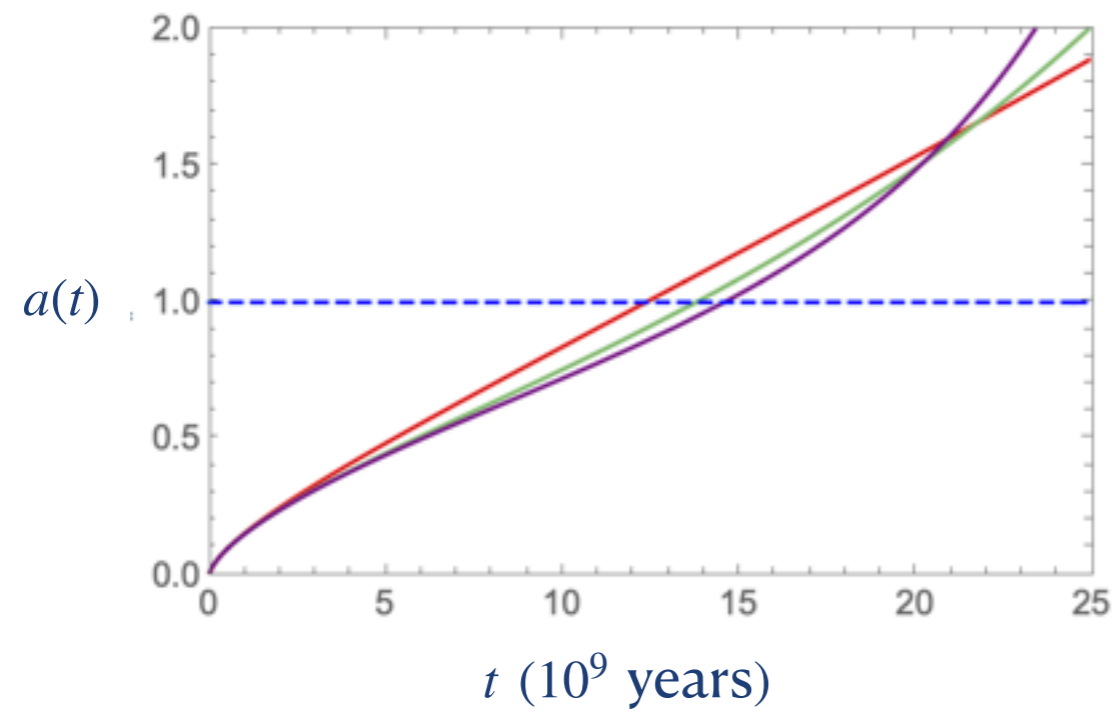
Evolution of the Universe



# Effects of the cosmological constant

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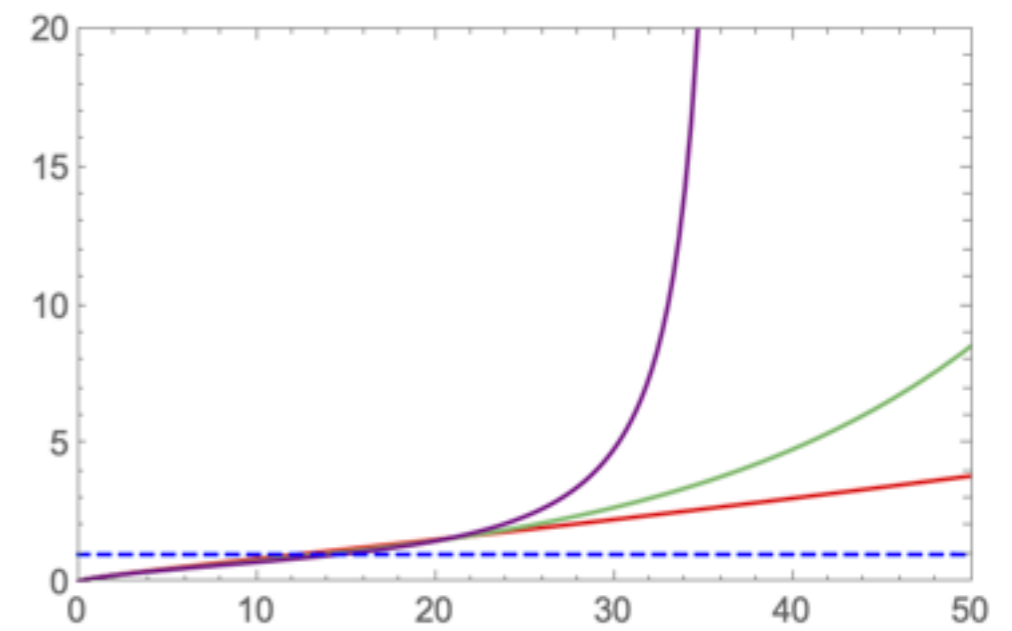
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$w = -1.5$

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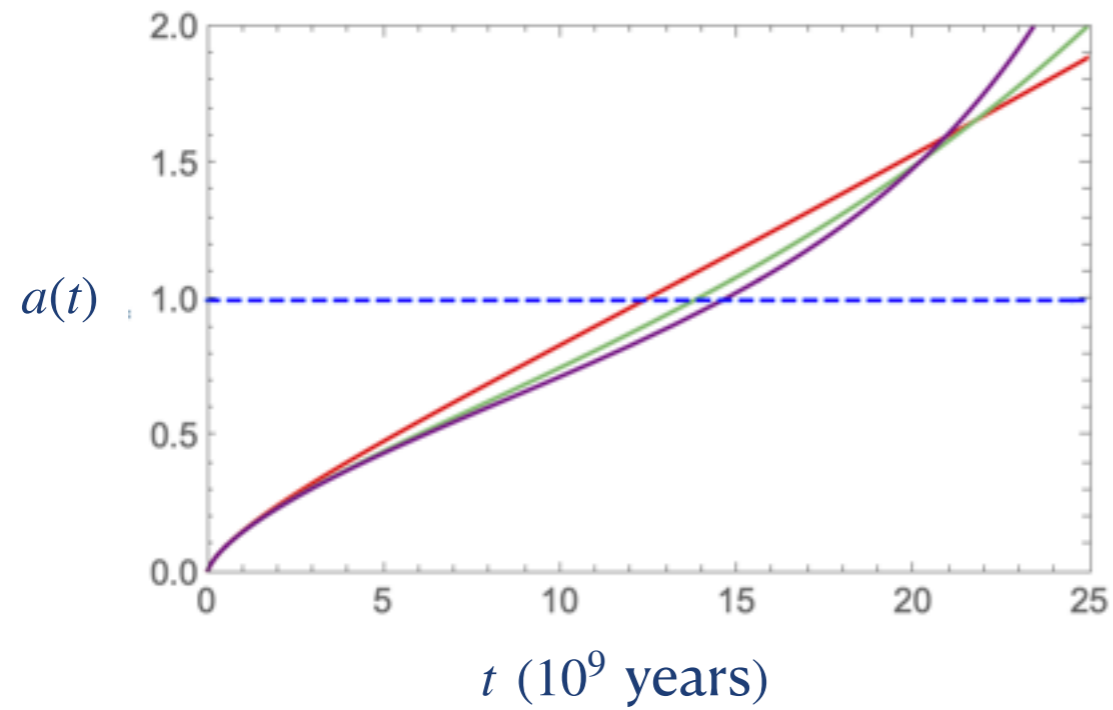




# Effects of the cosmological constant

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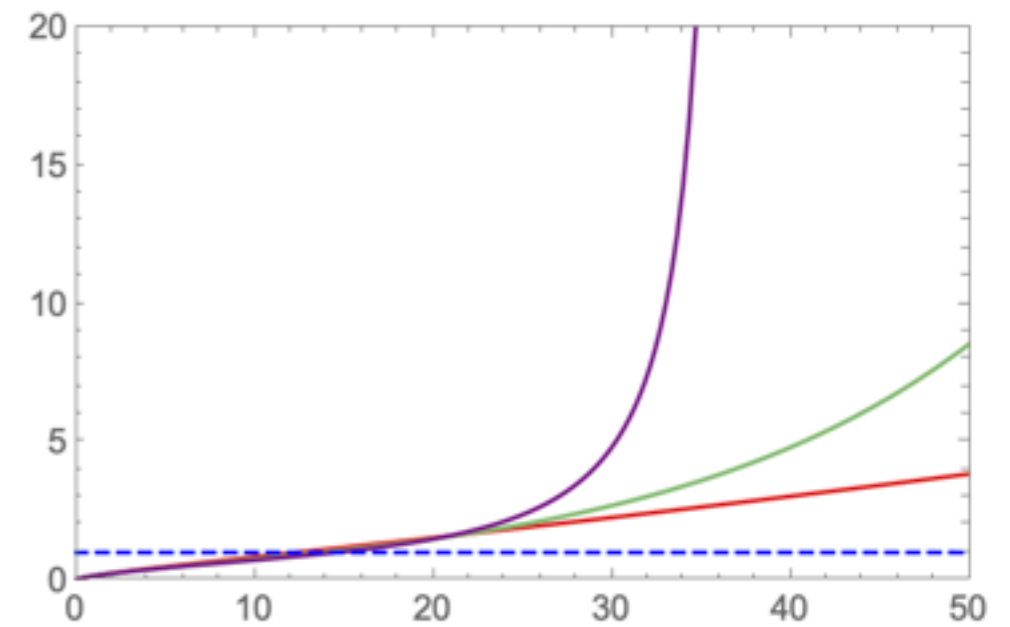
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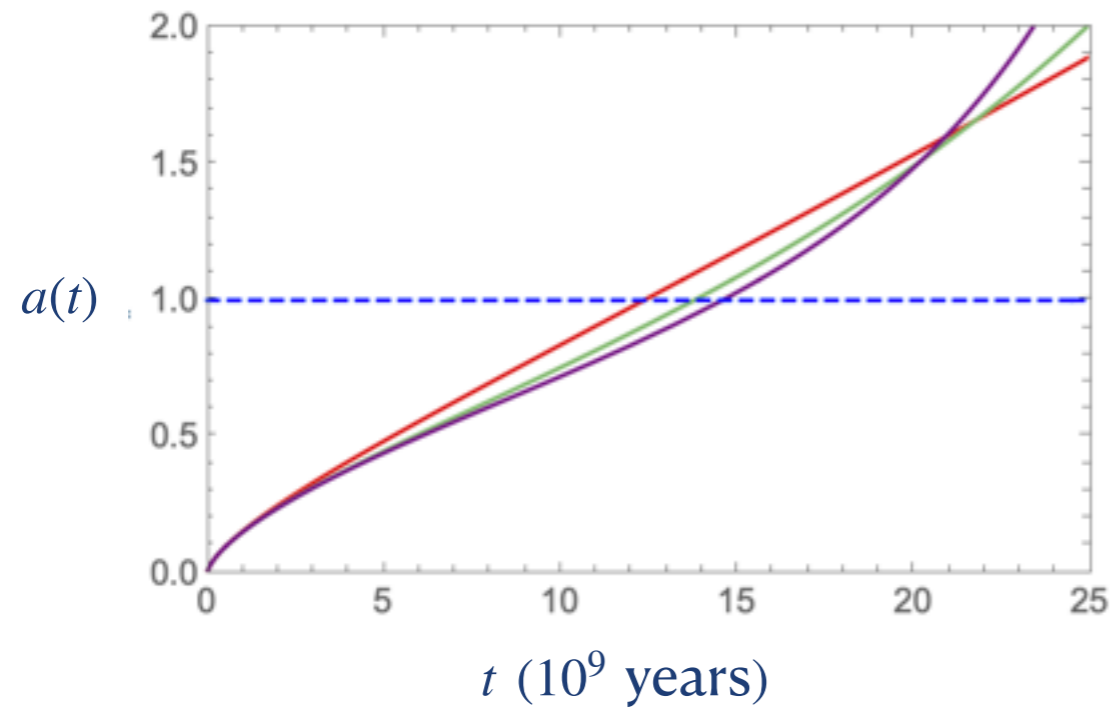


For a cosmological constant, the Universe expands exponentially in the future

# Effects of the cosmological constant

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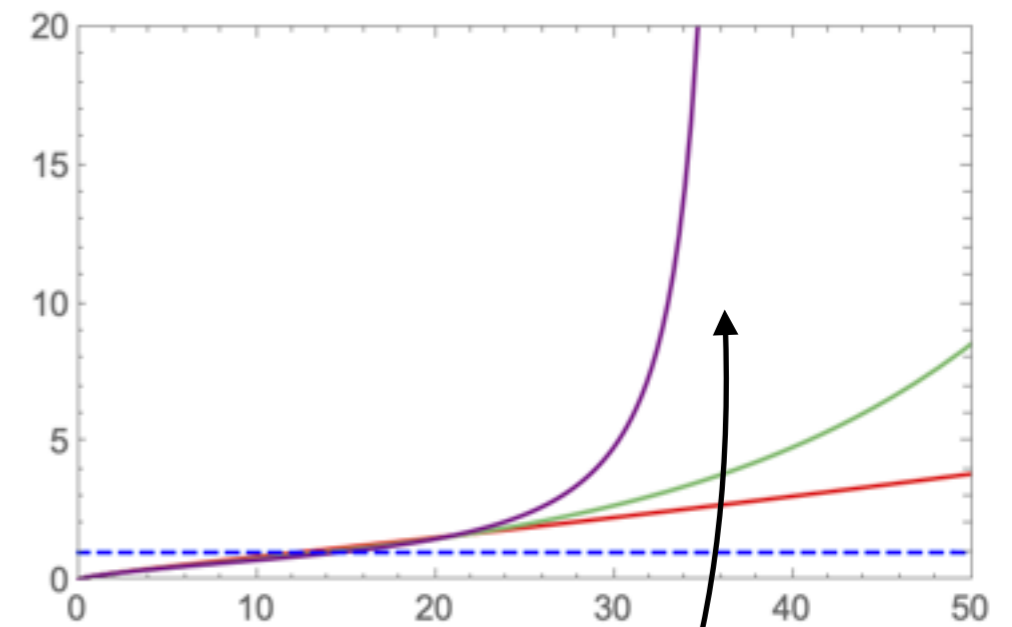
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For a cosmological constant, the Universe expands exponentially in the future

For  $w < -1$  (phantom energy), the scale factor diverges after a finite time (Big Rip)

## Black hole effects

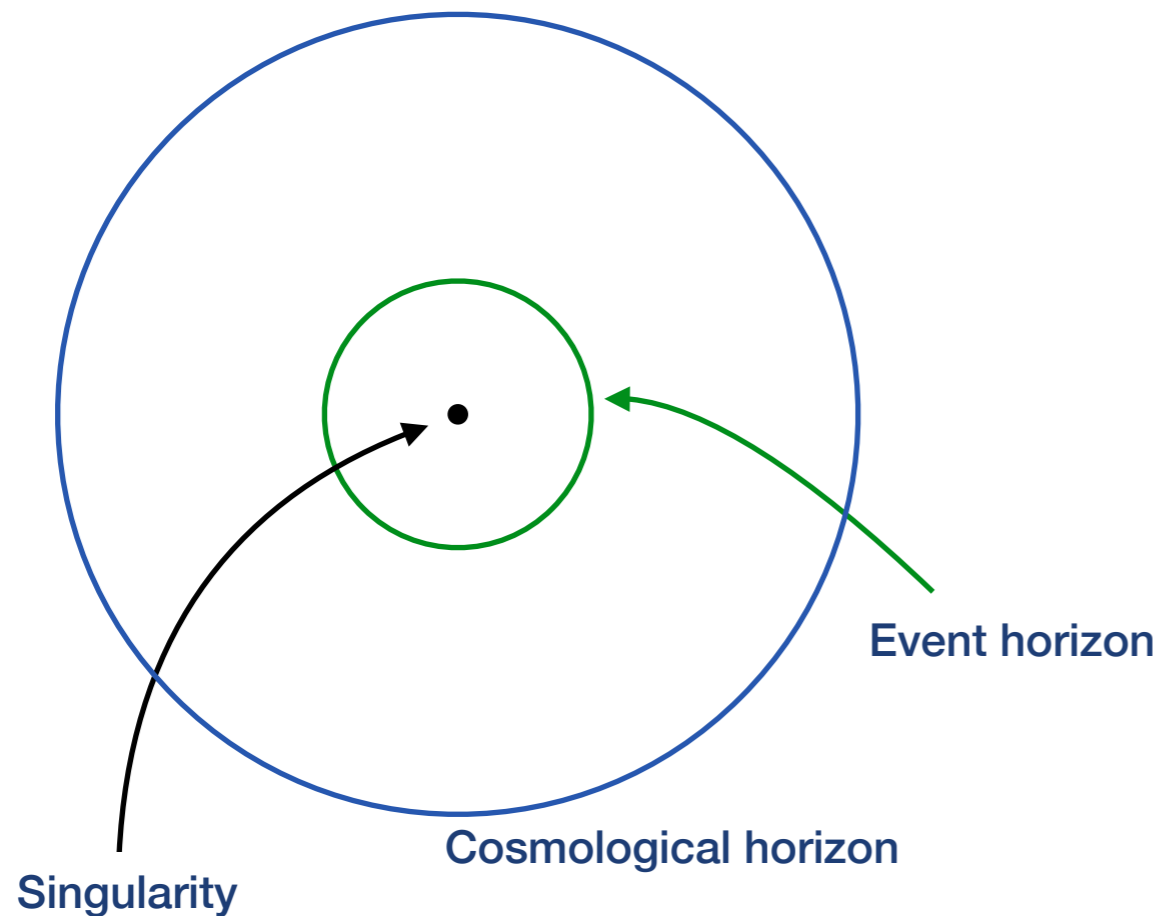
$$ds^2 = - \left( 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2 \right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

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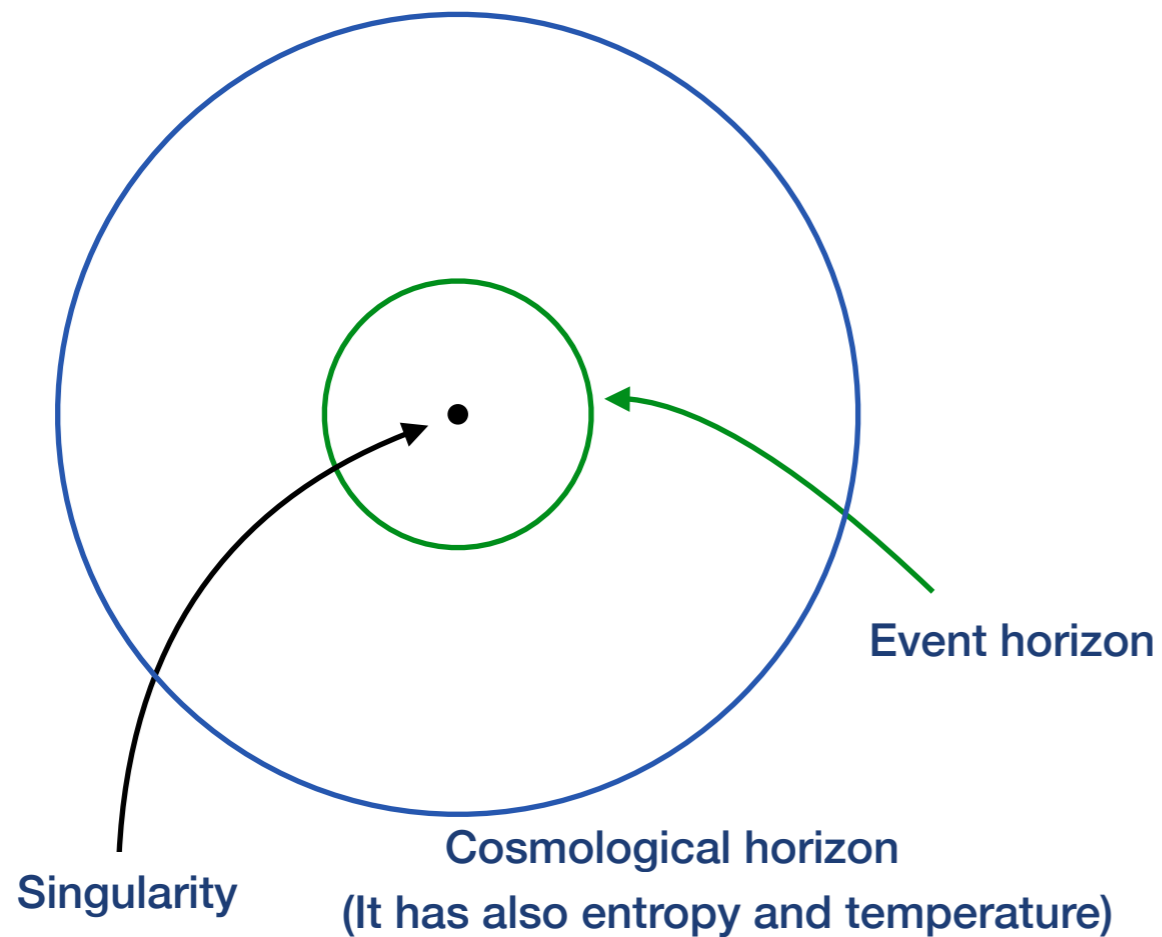


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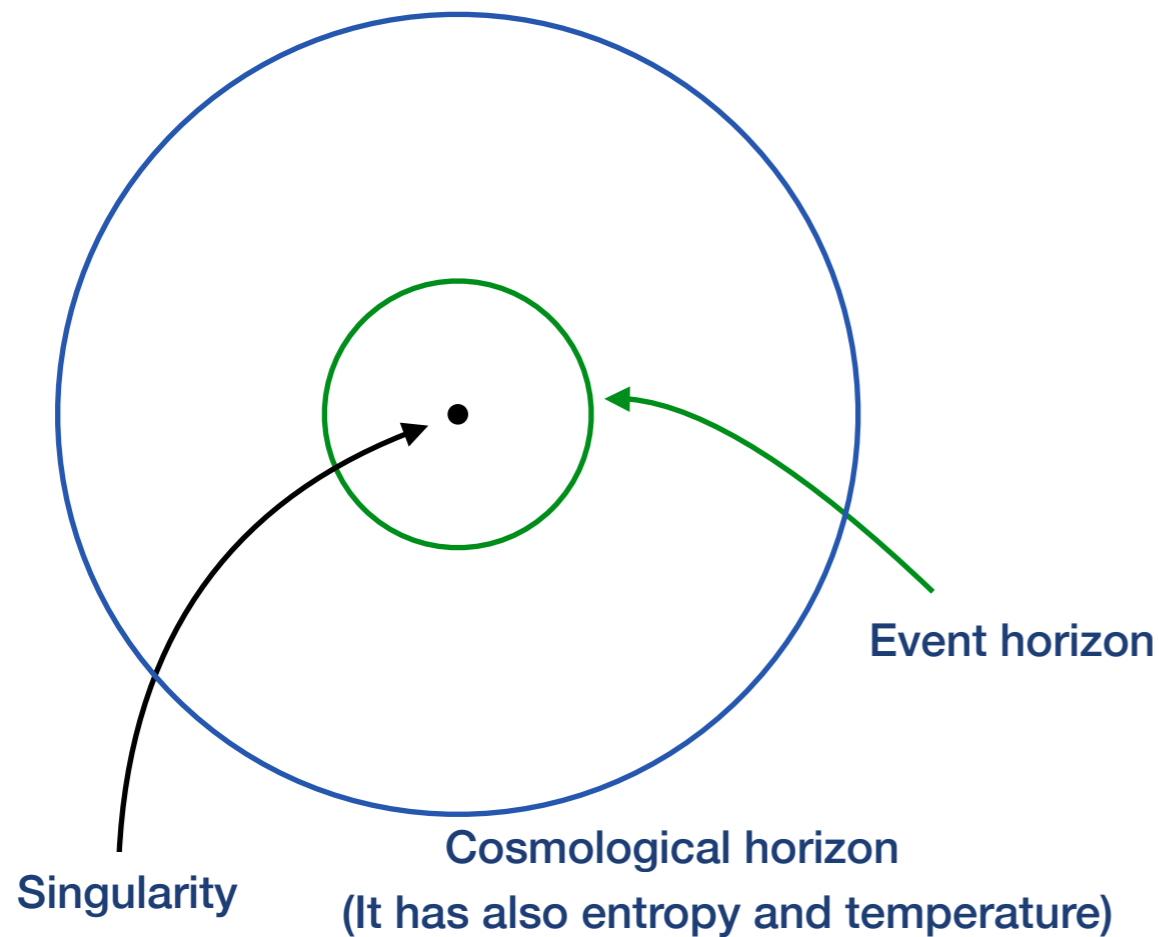
Even without mass we have a horizon, we can't see points at infinity

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Newtonian limit

$$m \vec{a} = \left( \frac{m\Lambda}{3} r - \frac{GMm}{r^2} \right) \vec{e}_r$$

Repulsive force

Dominant at large distances

Even without mass we have a horizon, we can't see points at infinity

# Cosmological Constant Problem

---

## Quantum mechanics

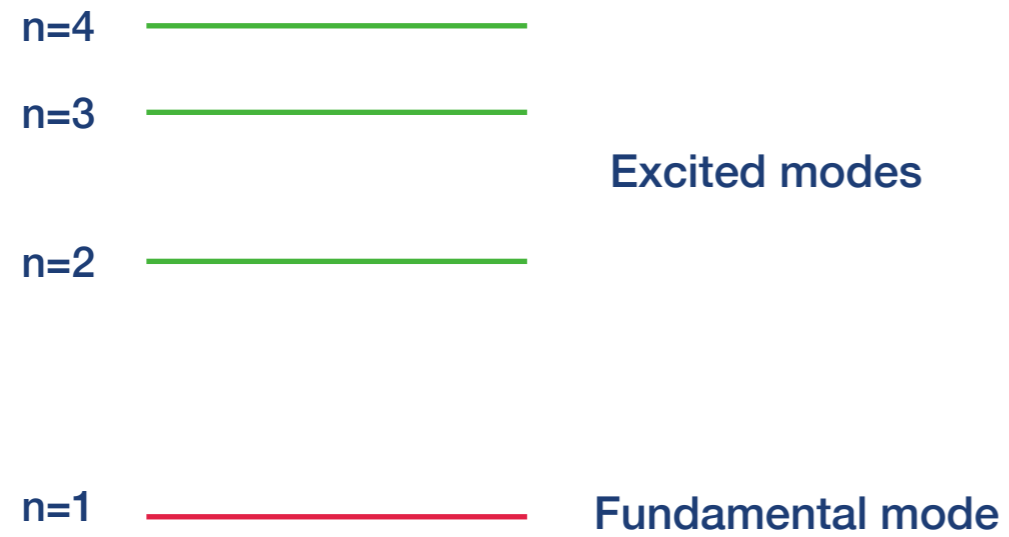
### Electron of the hydrogen atom



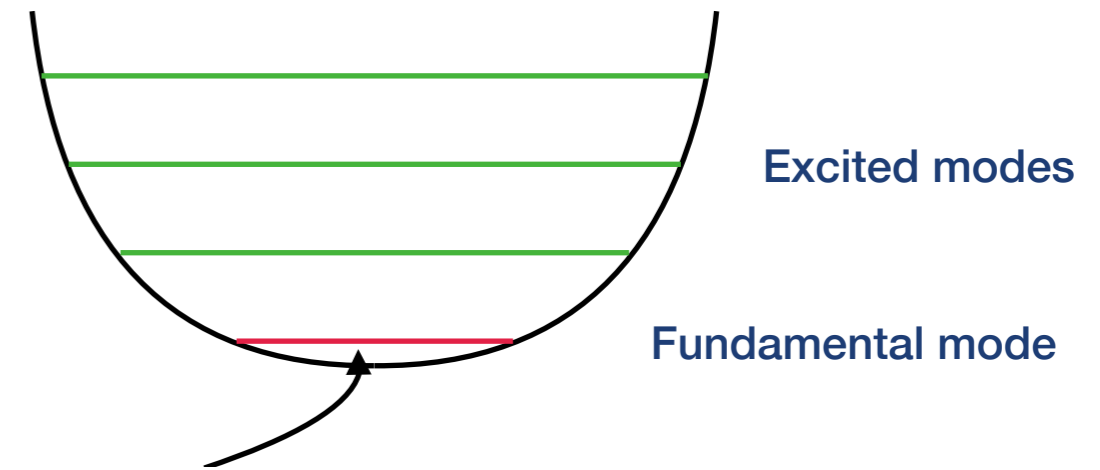
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## Quantum mechanics

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### Harmonic oscillator



The energy is not zero because of the Heisenberg Principle



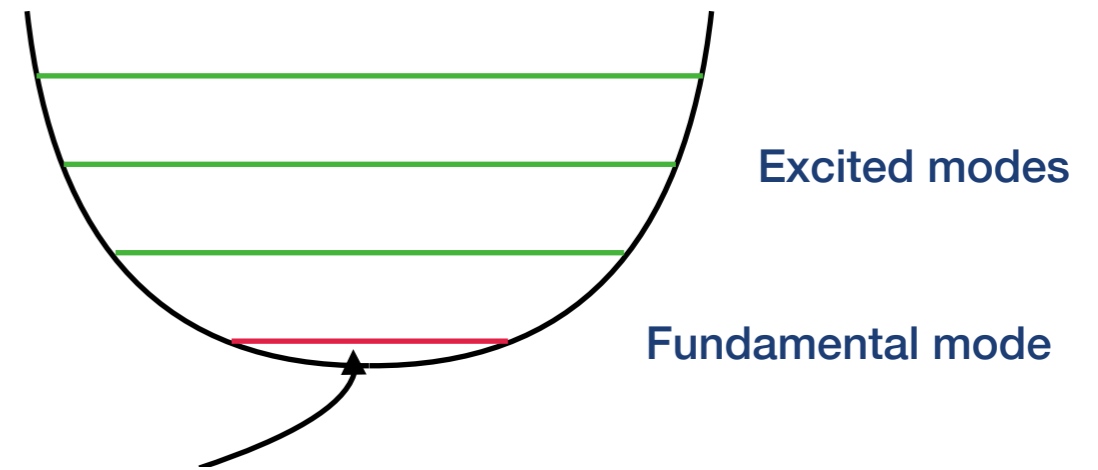
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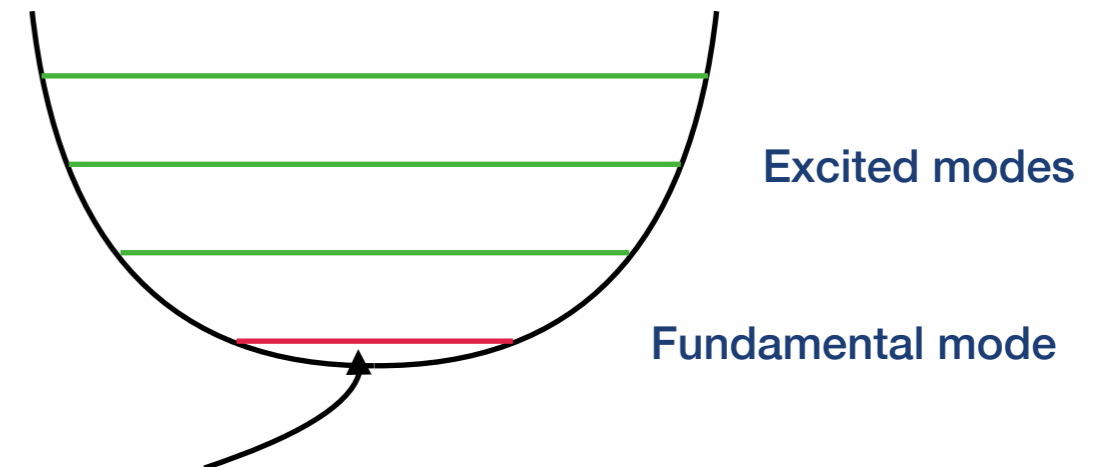
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## Quantum mechanics

### Electron of the hydrogen atom



### Harmonic oscillator



The energy is not zero because of the Heisenberg Principle

## Quantum Field Theory

The number of particles is not fixed

We have a field which has a fundamental mode and excited states

Excited states represent the creation of particles

The fundamental mode is the absence of particles, known as vacuum and as quantum mechanics it has energy

So vacuum of each field has energy... and it has pressure such that  $P = -\rho$

# Cosmological Constant Problem

---

They are various fields in Nature and each one has some vacuum energy

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Lamb Shift

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electron



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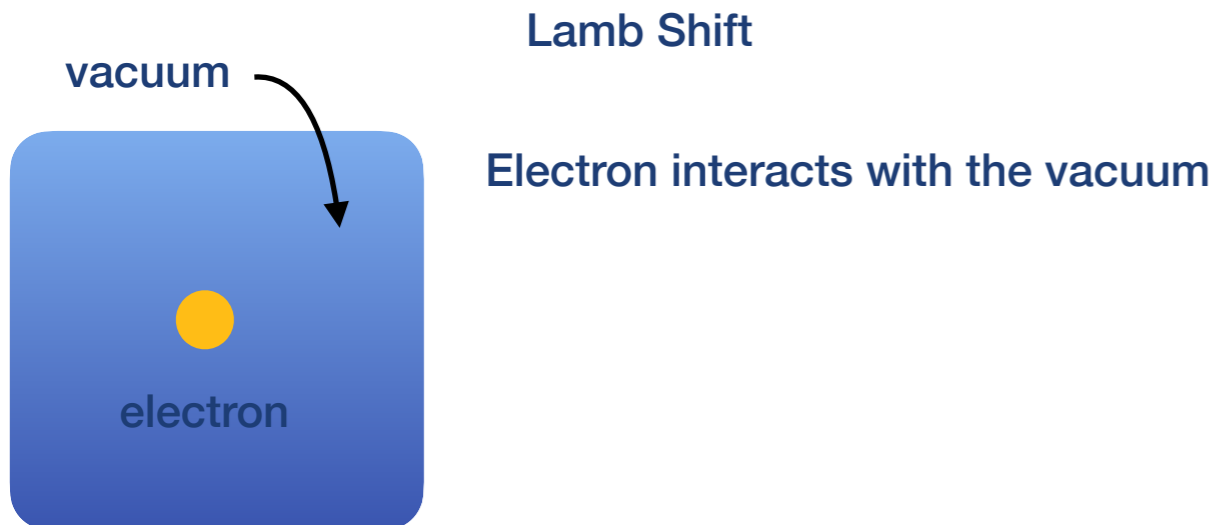
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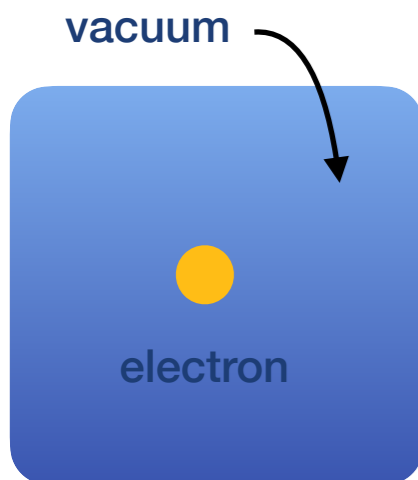
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Lamb Shift

Electron interacts with the vacuum

Which modifies the “position” of the electron

And therefore its energy

# Cosmological Constant Problem

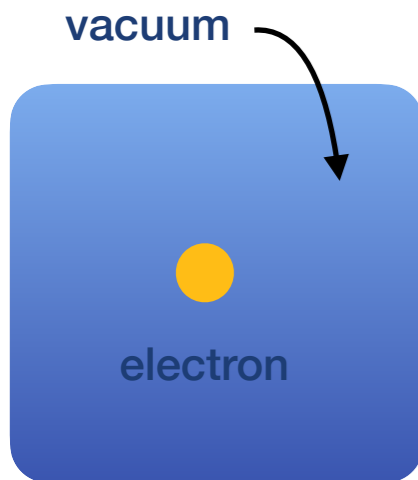
They are various fields in Nature and each one has some vacuum energy

Fermion fields have a negative vacuum energy

Boson fields have a positive vacuum energy

We do not have in Nature, 1 boson for 1 fermion, so they do not cancel each other and we need to calculate it

Do vacuum fluctuations really exist? **Yes**



## Lamb Shift

Electron interacts with the vacuum

Which modifies the “position” of the electron

And therefore its energy

For the state  $n = 2$  and  $\ell = 0$   
we have  $\Delta E \simeq 2.8 \mu eV$

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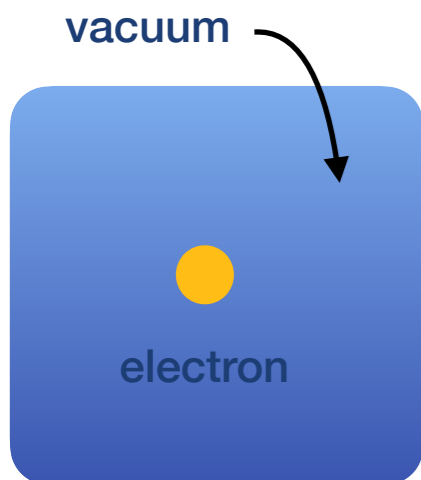
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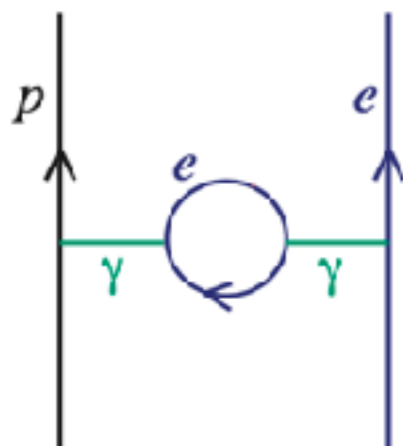
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one-loop effect of  
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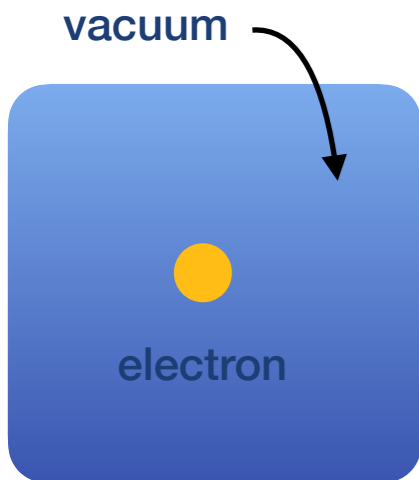
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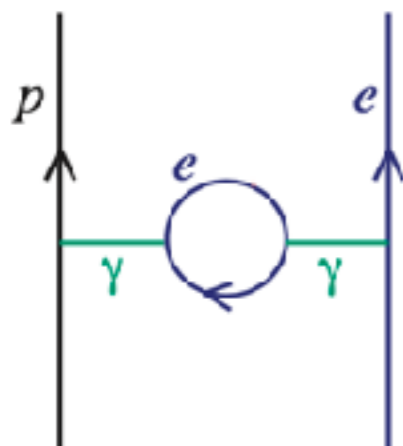
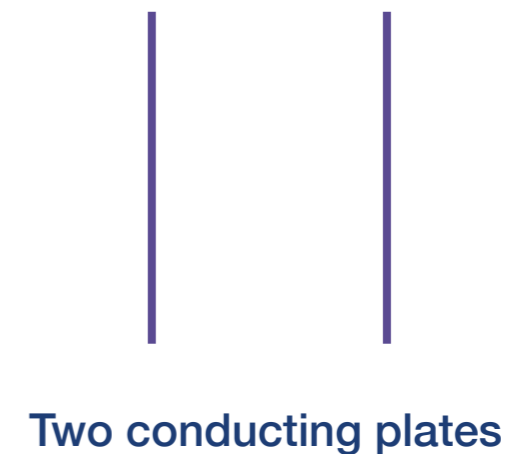
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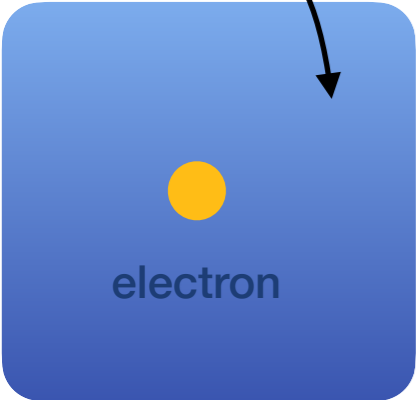
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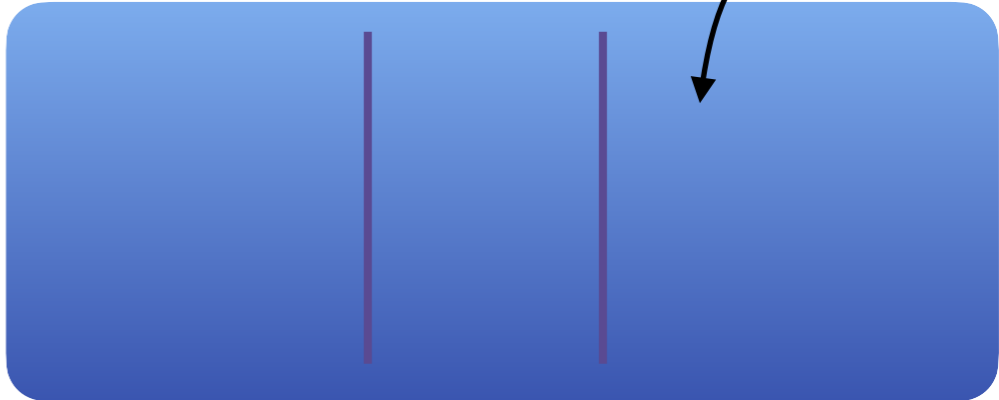
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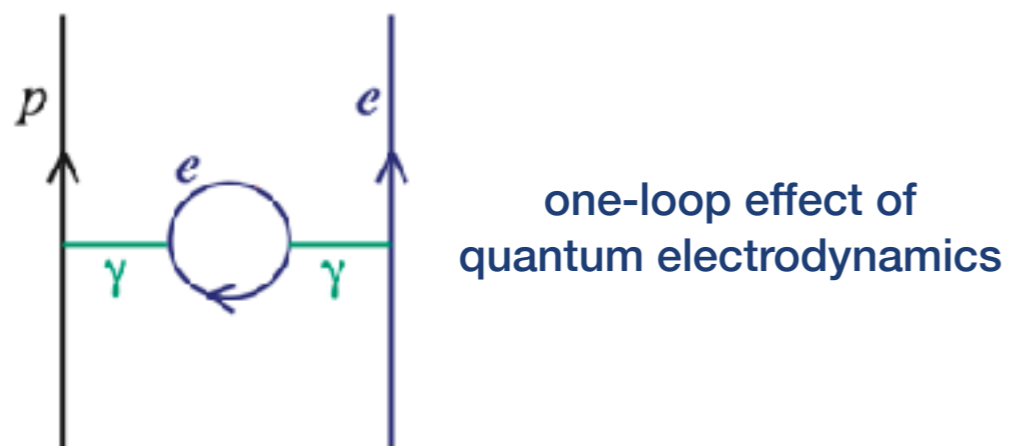
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Two conducting plates

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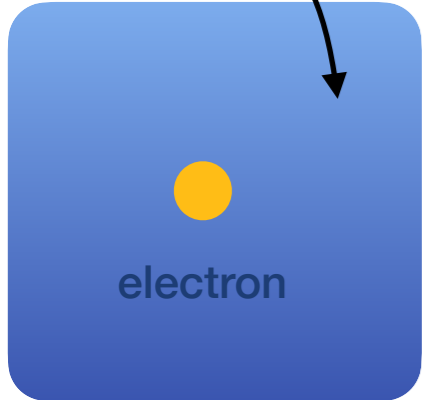
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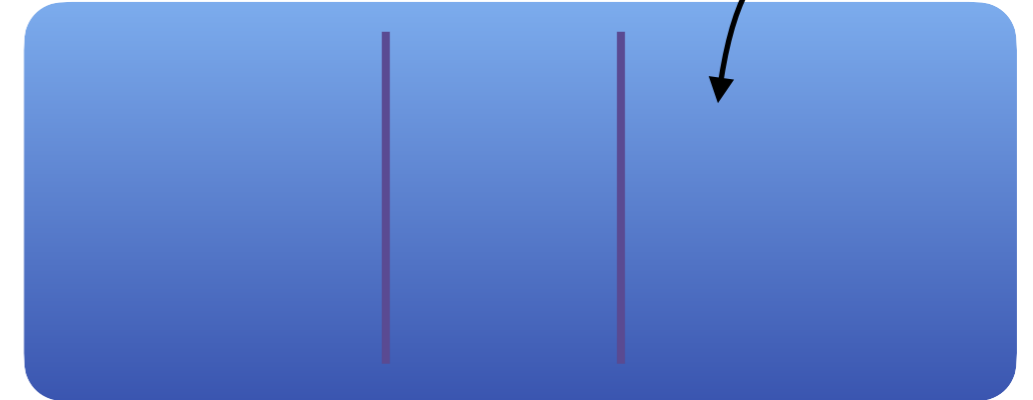
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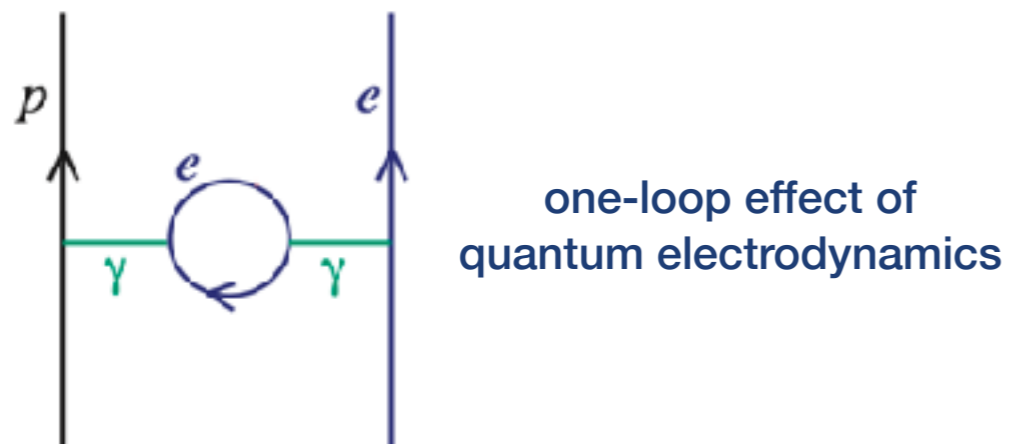
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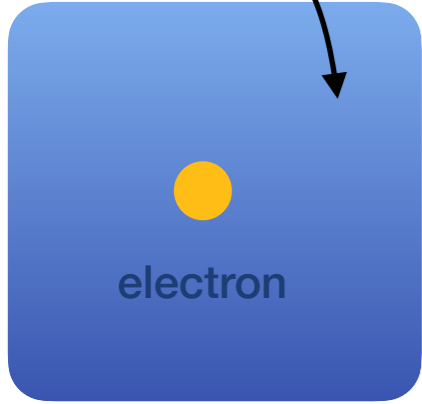
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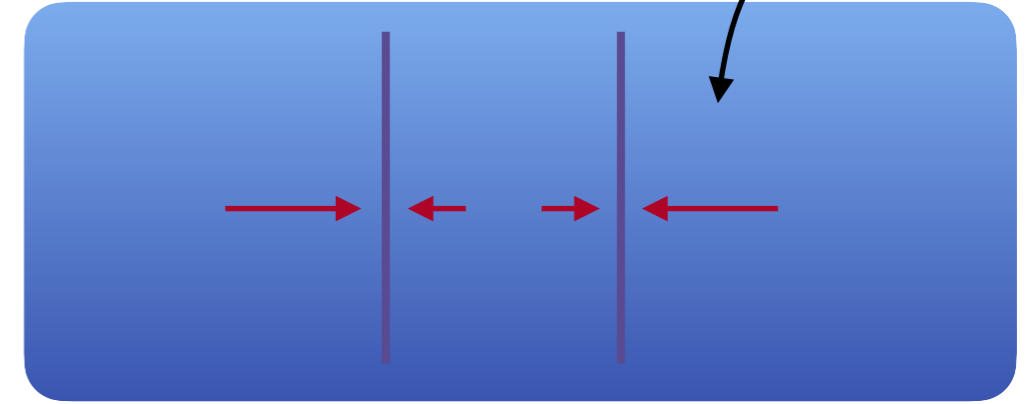
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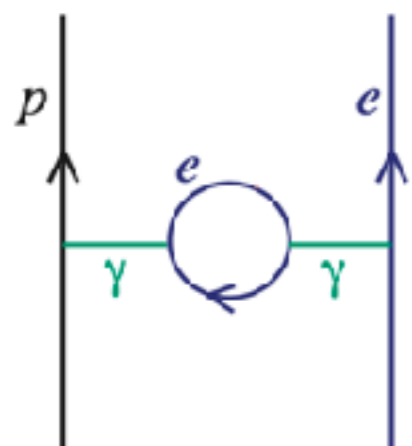


vacuum

Two conducting plates

The diagram shows a blue rounded rectangle representing a vacuum. Inside, two vertical purple lines represent 'Two conducting plates'. Red arrows point towards each other between the plates, indicating an attractive force. An arrow points from the word 'vacuum' to the rectangle.

They move under some pressure due to vacuum energy



one-loop effect of quantum electrodynamics

The diagram is a Feynman diagram showing two vertical lines representing fermions. The left line is labeled 'p' and the right line is labeled 'e'. Two horizontal lines labeled 'γ' connect the two vertical lines, forming a loop. A circular arrow inside the loop indicates a fermion loop.



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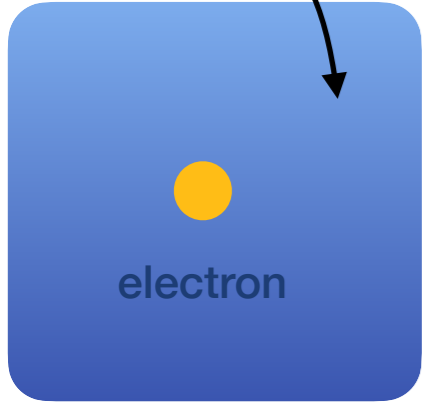
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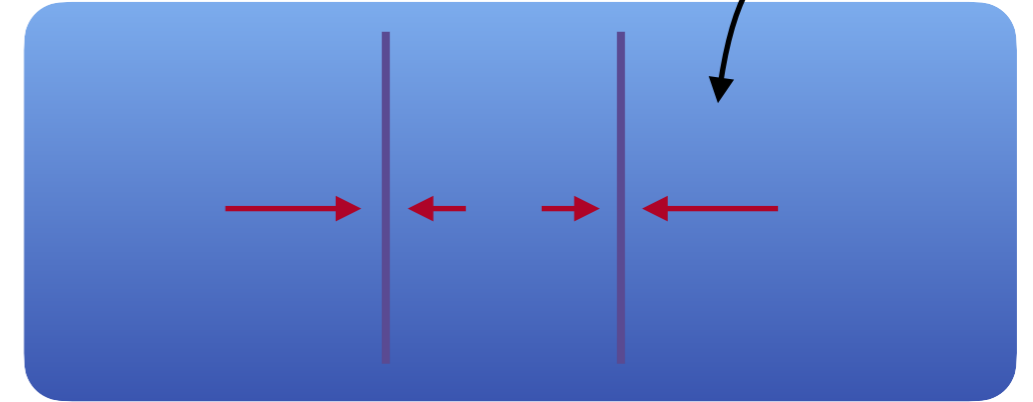
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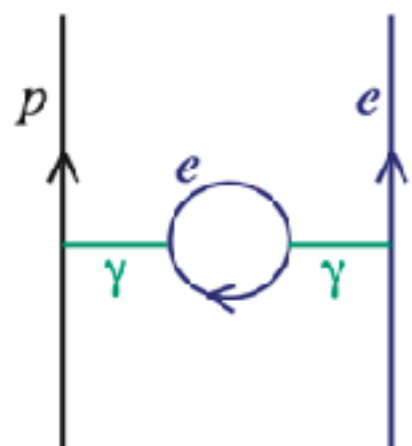
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**Vacuum energy exists**

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de Sitter Universe

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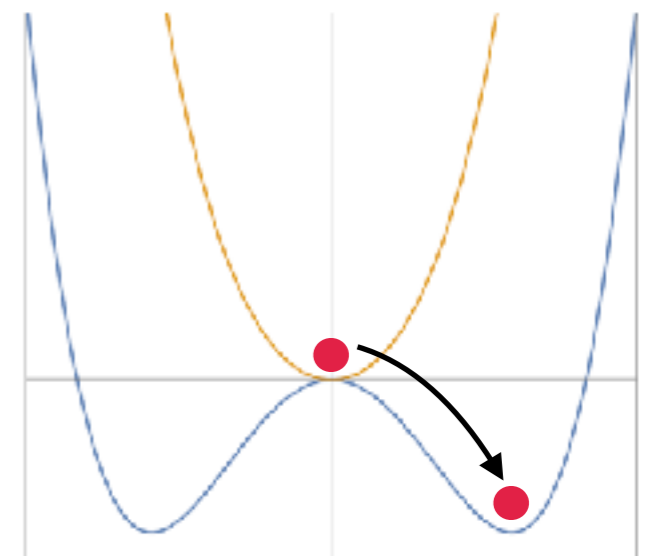
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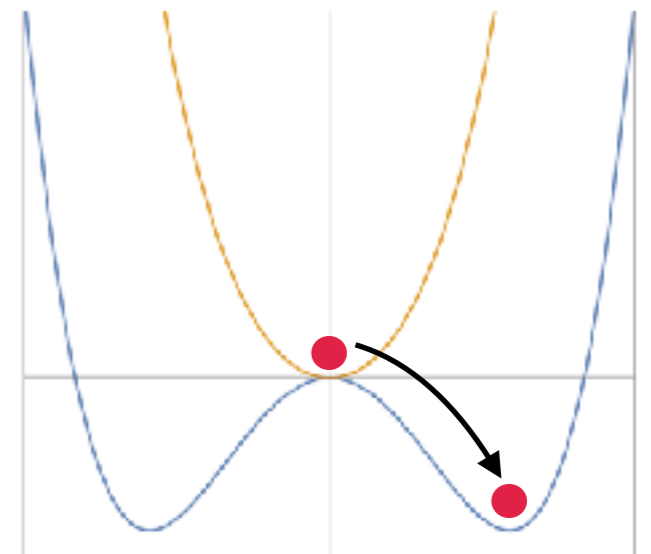
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The cosmological constant problem is a problem of fine-tuning





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There is no positive cosmological constant in String Theory

String Theory is a theory which is the best candidate for a theory describing all forces

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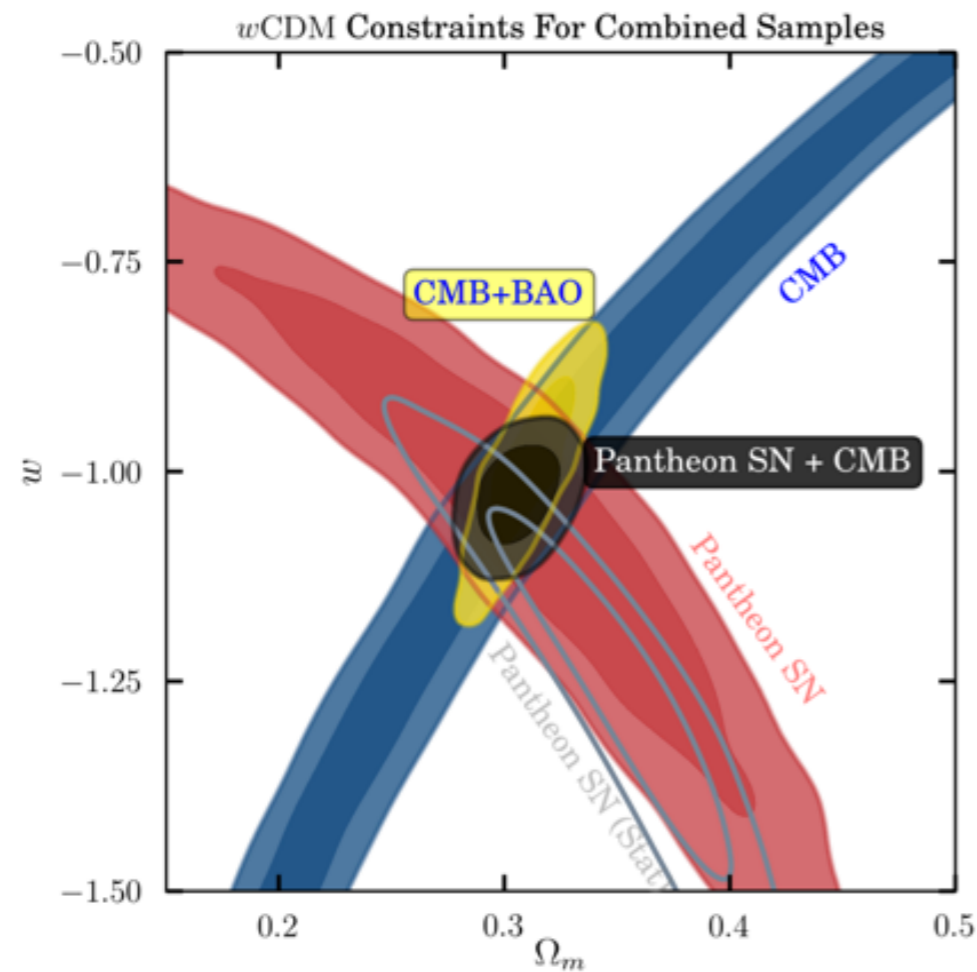
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w constant

# Cosmological Constant Problem

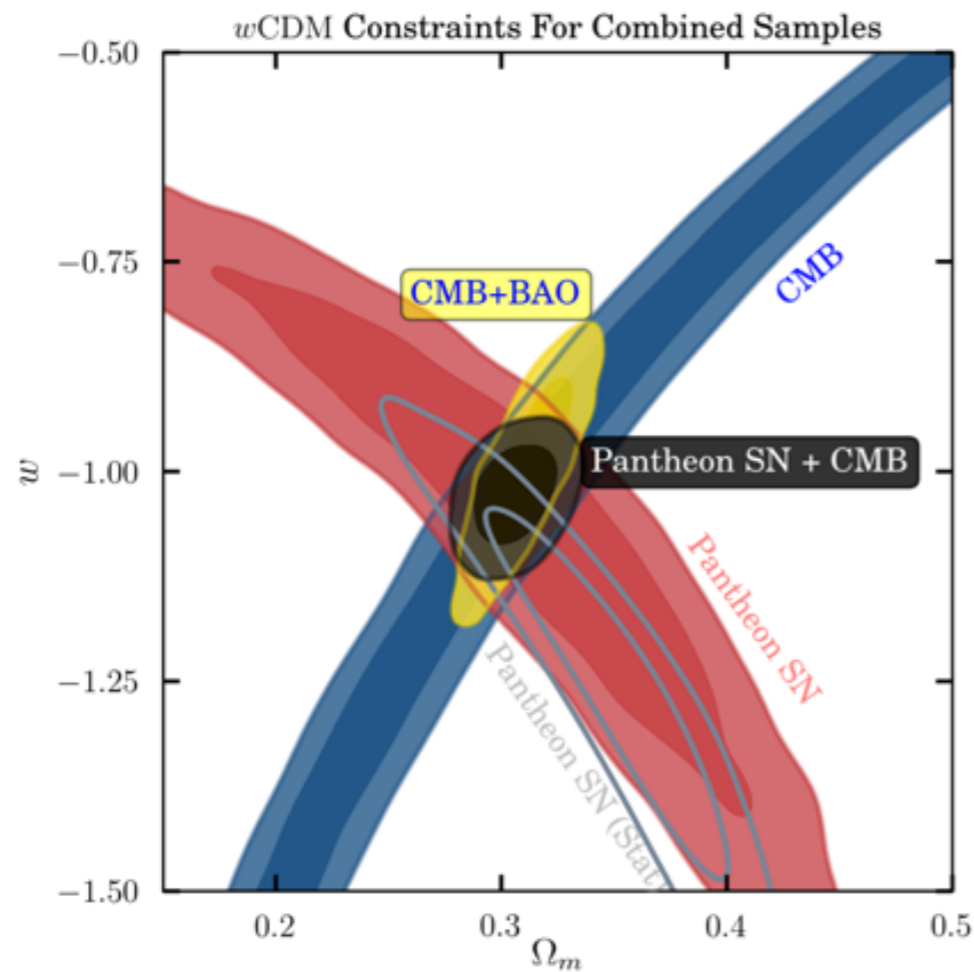
There is no positive cosmological constant in String Theory

String Theory is a theory which is the best candidate for a theory describing all forces

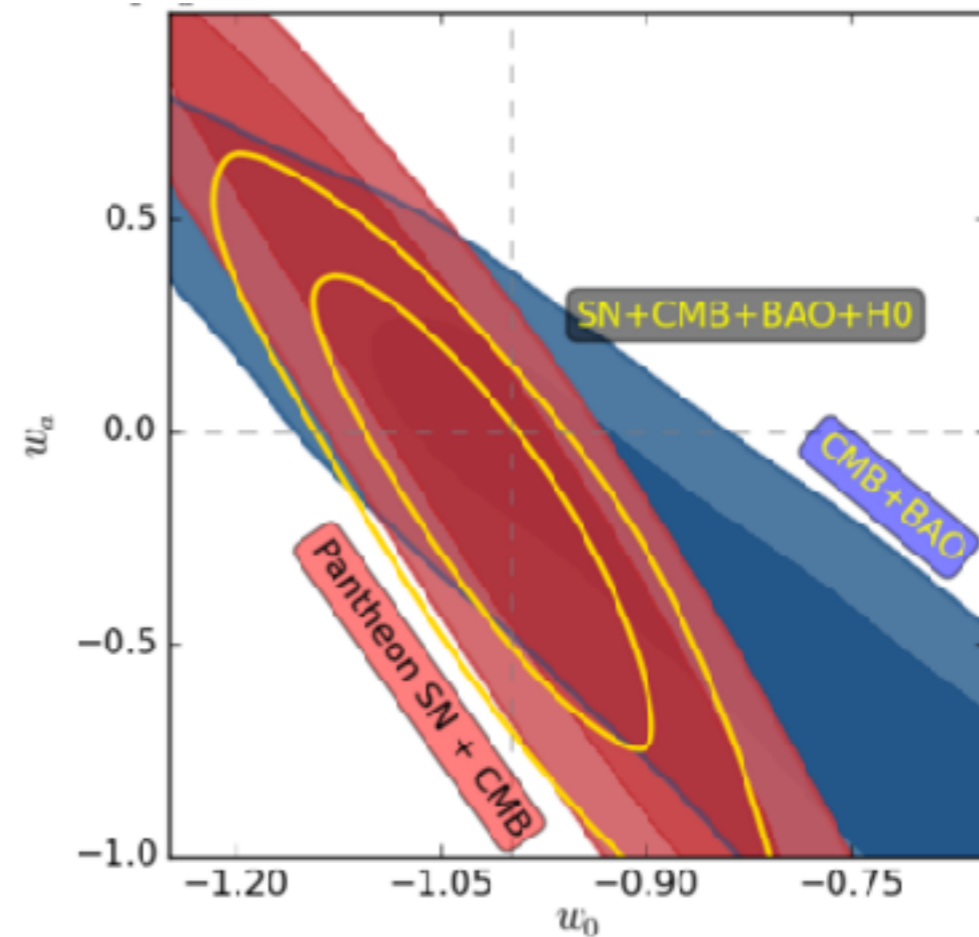
More than a theory, it is a consistent framework

Therefore, any theory should be in the UV equivalent to some String Theory

Observationally, even if  $\Lambda$  is consistent with all observations, other options are also possible



$w$  constant



$$w(z) = w_0 + w_a \frac{z}{1+z}$$

It's called a parametrization





Two approaches

# Cosmological Constant Problem

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To assume that some physics cancels completely  $\Lambda$   
because it is difficult to explain the value and because  
it is not included in String Theory

They are known as Dark Energy models

Or sometimes Modified Gravity



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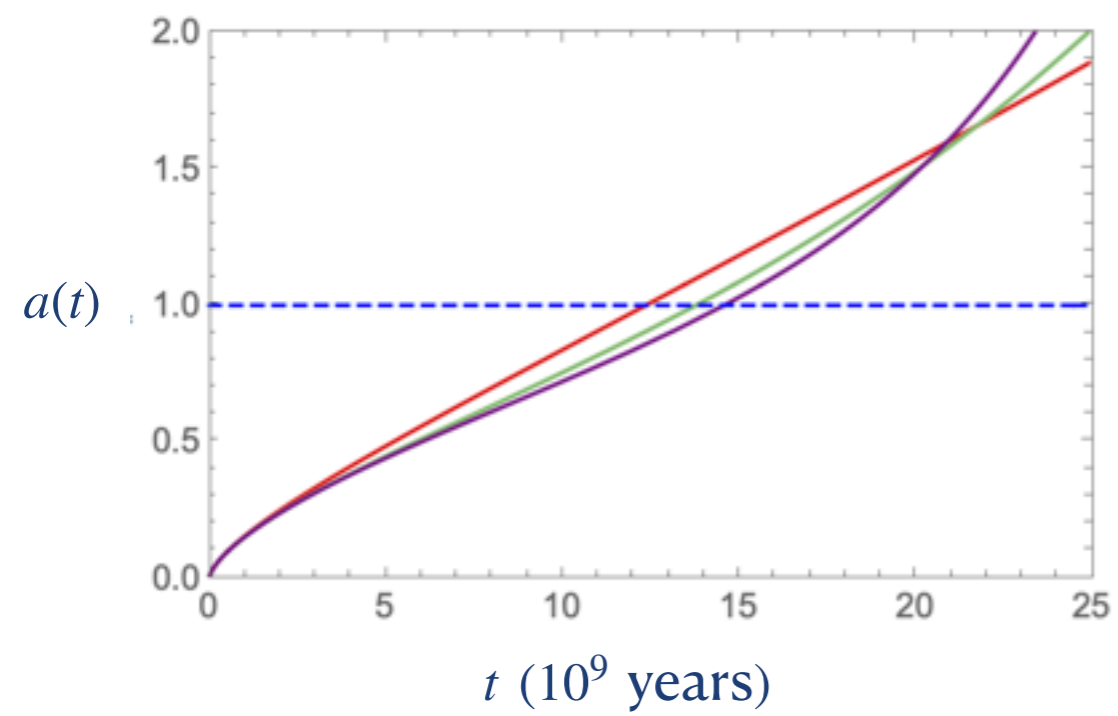
To try to find a mechanism which eliminates the vacuum energy for example a field which “eats” this vacuum in such a way that the cosmological constant doesn’t curve the spacetime

It is often part of what is called Modified Gravity

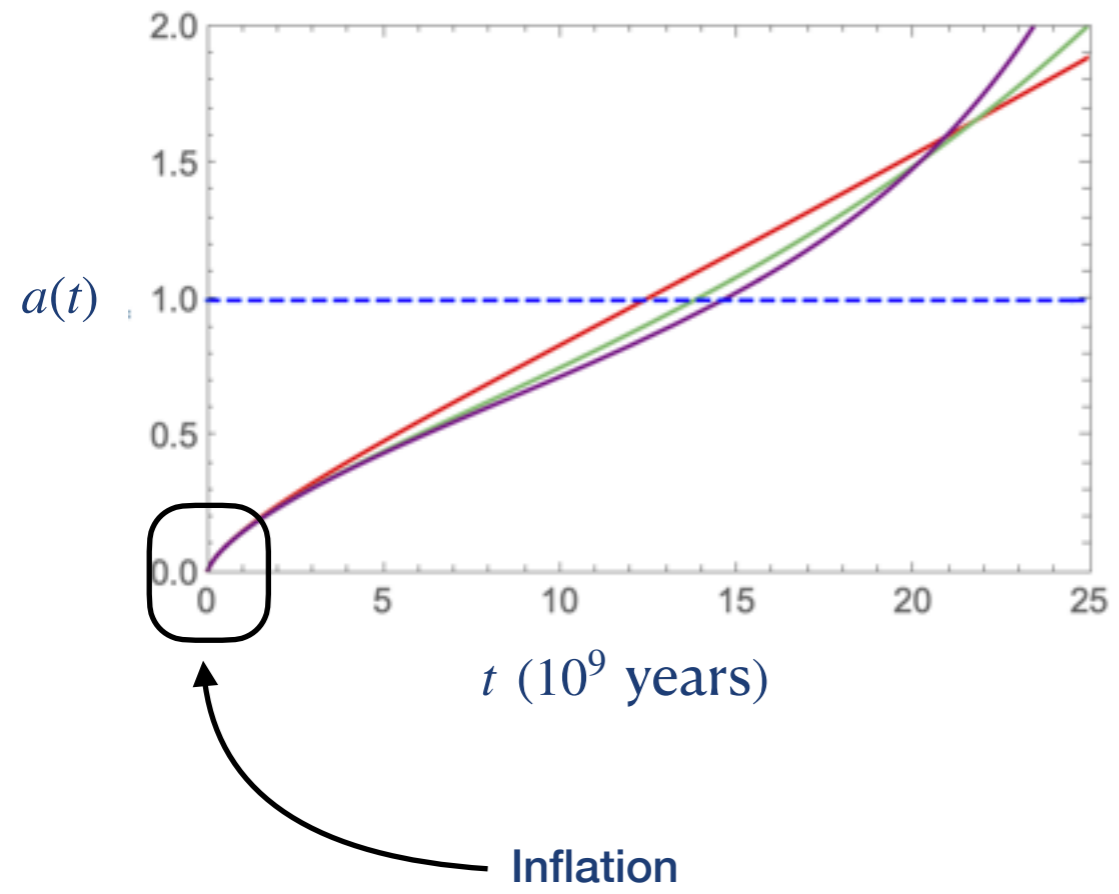


Two approaches

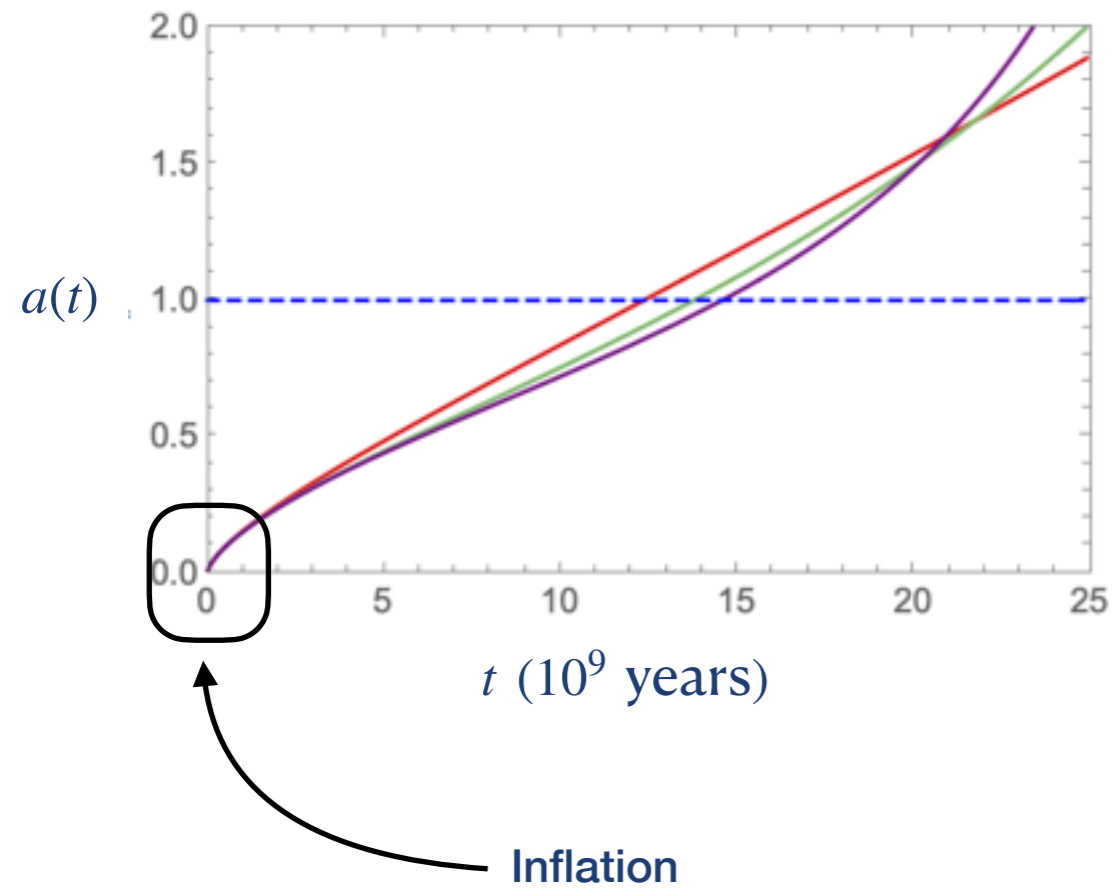
# Beyond $\Lambda$ CDM



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## Scale invariant problem

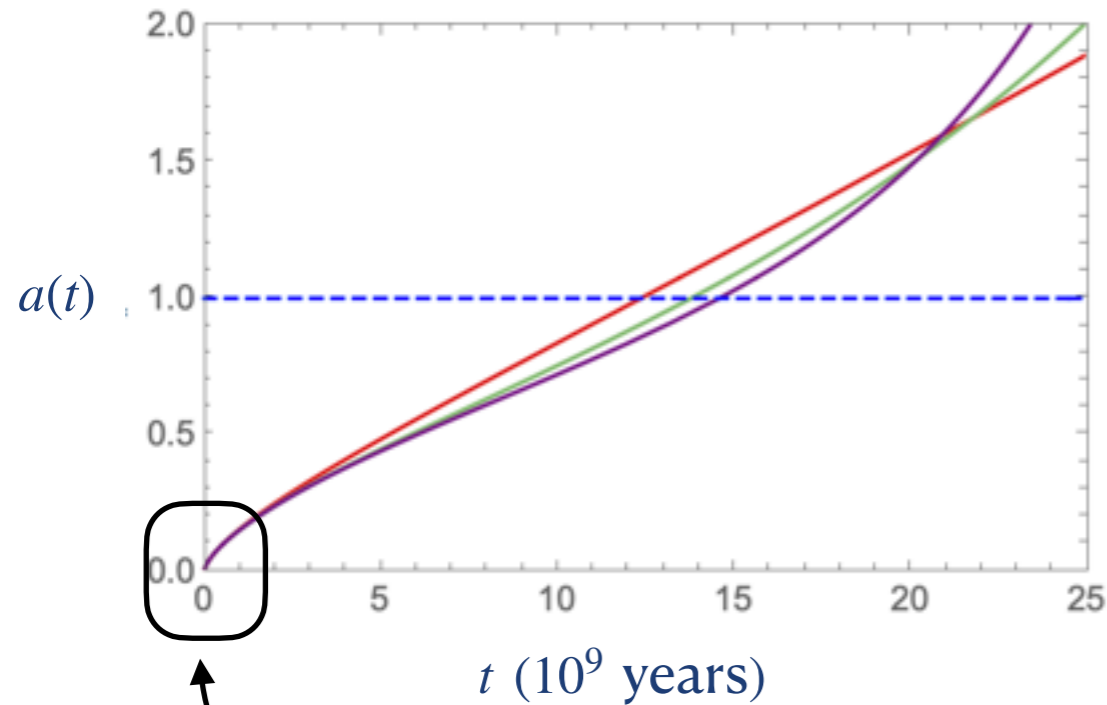
Amplitude of perturbations observed is approximately (4 %) the same at all scales ( $10^4$  Mpc to 10 Mpc)

Origin? Quasi de Sitter Universe

$$ds^2 = \frac{1}{t^2 H^2} (-dt^2 + d\vec{x}^2)$$

It is scale invariant

$$t \rightarrow \lambda t \quad \vec{x} \rightarrow \lambda \vec{x}$$



Inflation

During inflation, the Universe accelerated

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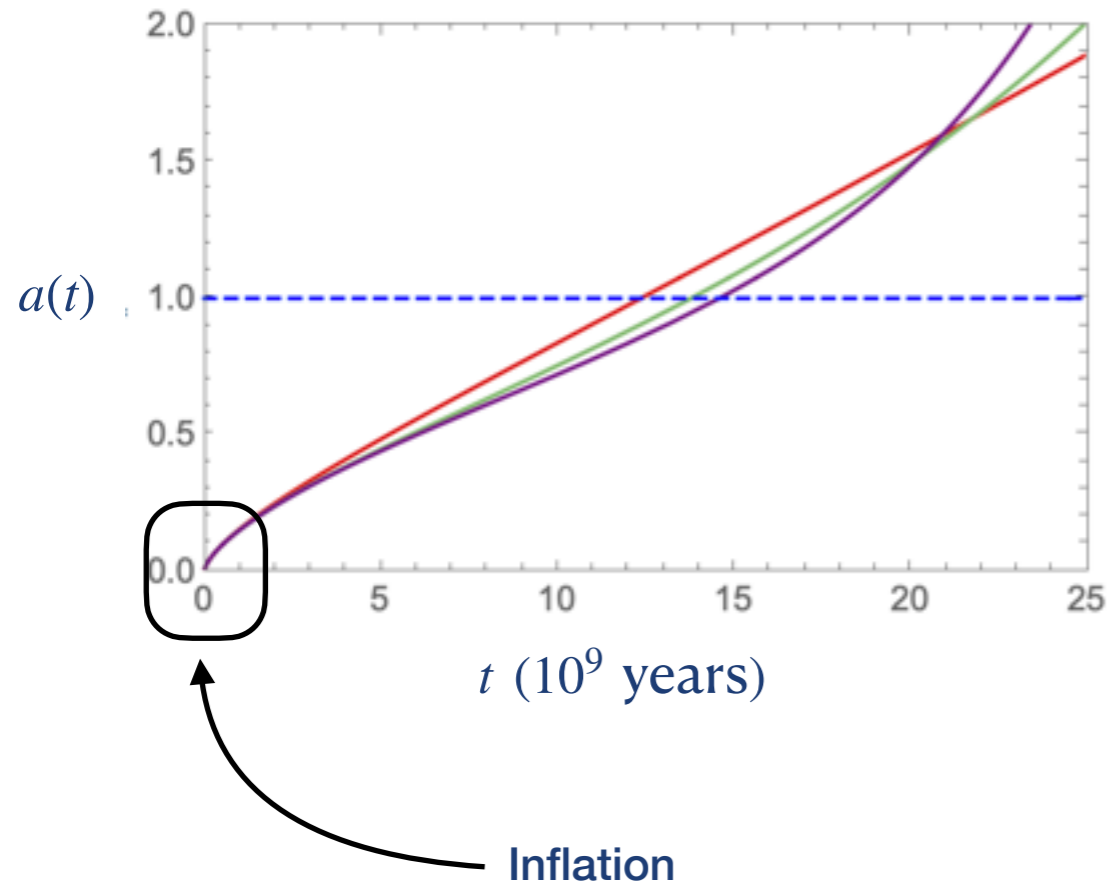
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Passing from  $a(t_i)$  to  $10^{26} a(t_i)$

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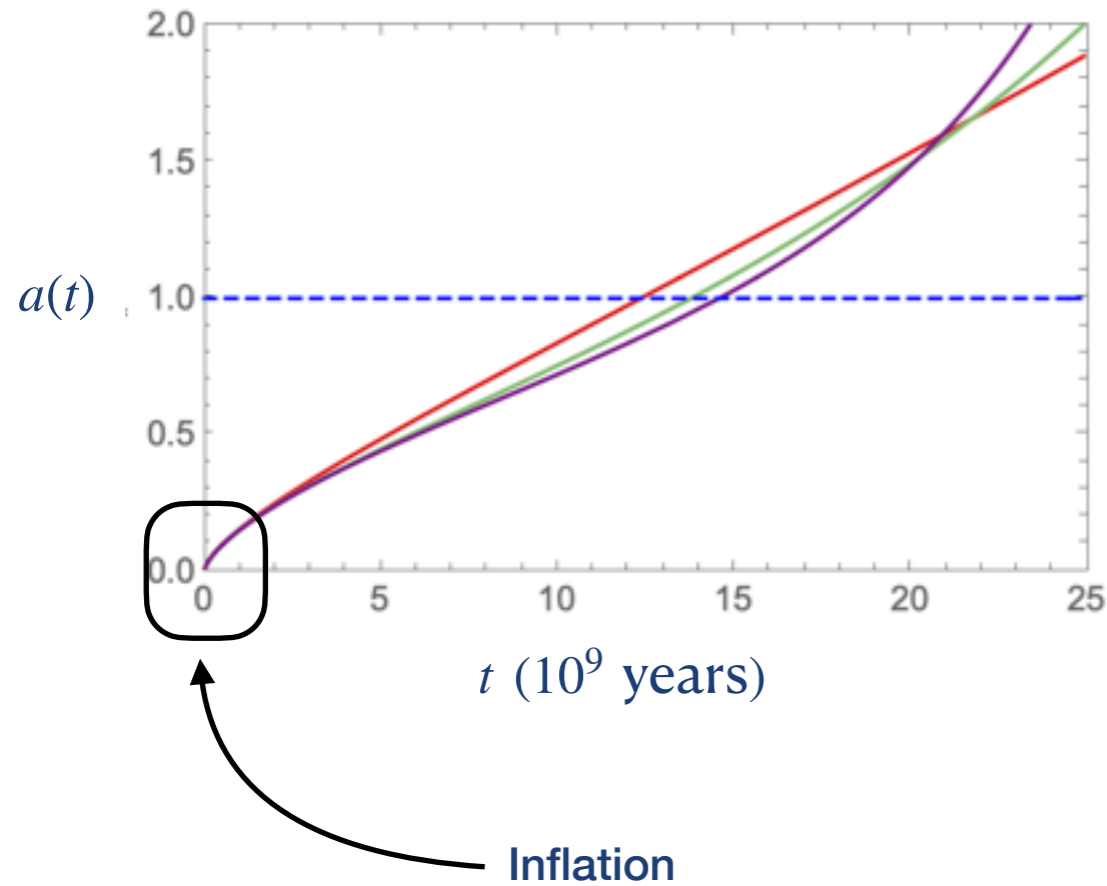
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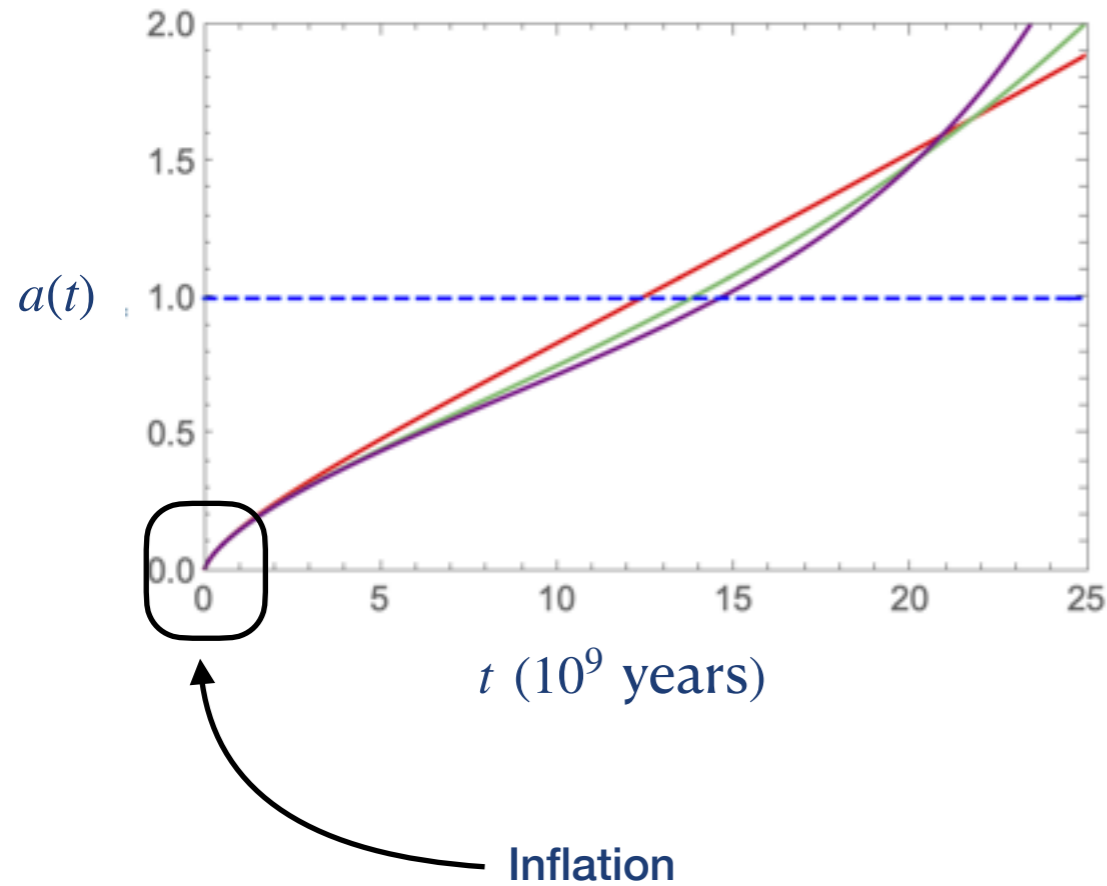
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Why not something similar today

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**It is not  $\Lambda$**

**because of inflation**

**because  $\Lambda$  is problematic**

As we said  $\Lambda$  is not a solution so we need something to accelerate the Universe

We need a fluid  $X$  with  $w < -1/3$  to get an acceleration so we need negative pressure

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Early Universe

$$\rho \simeq \sqrt{A}$$

Late Universe

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$$T_{\mu\nu} = \rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu)$$

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$$\frac{\ddot{a}}{a} = 12\pi GH\zeta - \frac{1+3w}{2}H^2$$

Acceleration if  $\zeta > 0$

## Quintessence

A scalar field  $\phi(t, x, y, z)$

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Pressure  $P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$w = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)}$$

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+ generalizations K-essence, Horndeski...

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Modified gravity modifies how matter and radiation curve spacetime

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f(R)-gravity

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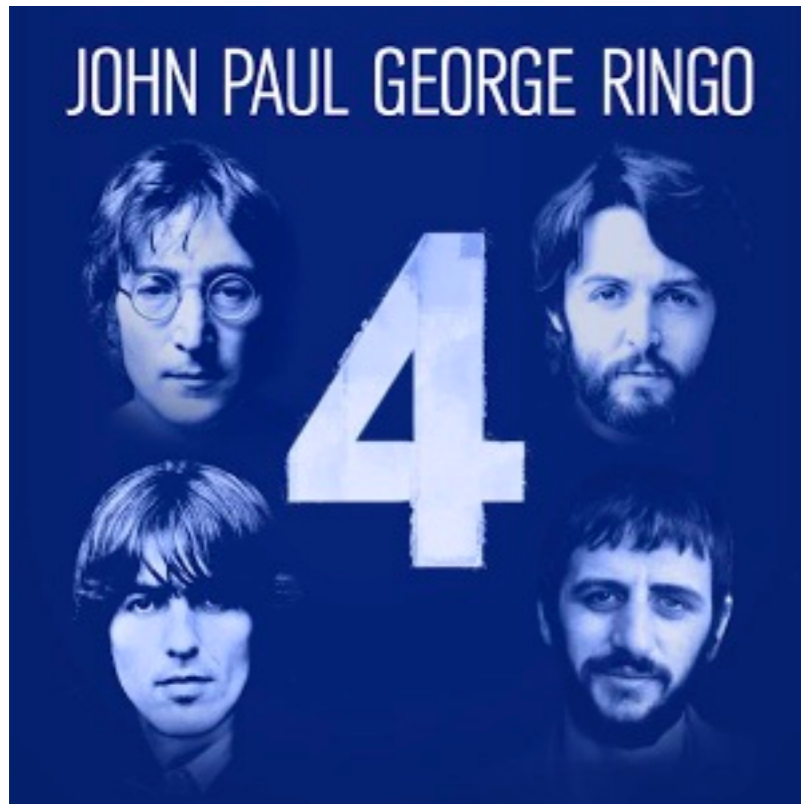
Starobinsky model produces inflation

Why not dark energy

## Fab Four

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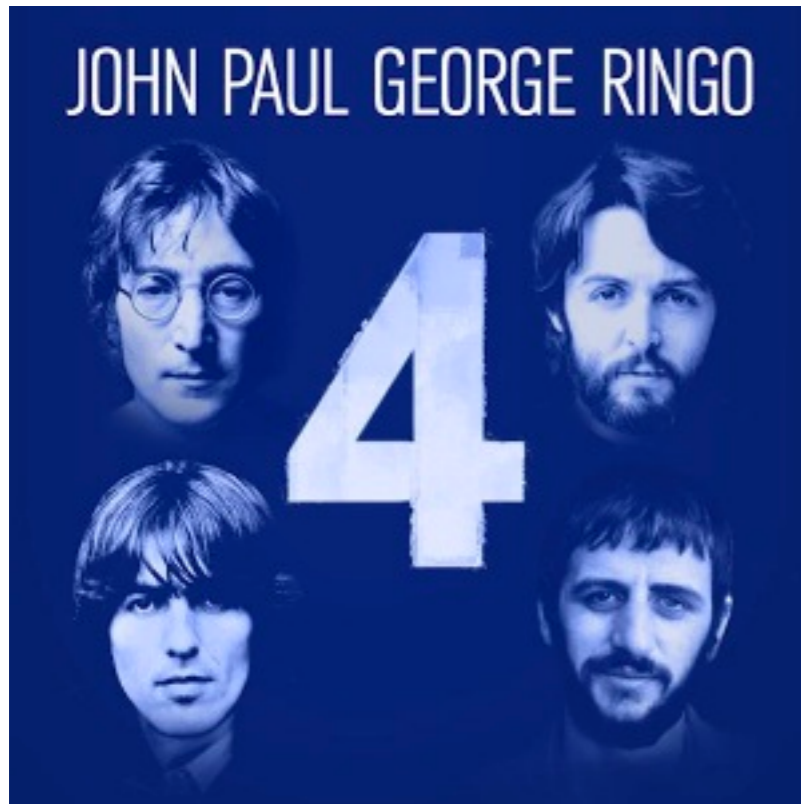


$$L_{John} = V_{John}(\phi)G^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi$$

$$L_{Paul} = V_{Paul}(\phi)P^{\mu\nu\alpha\beta}\partial_{\mu}\phi\partial_{\alpha}\phi\nabla_{\nu\beta}\phi$$

$$L_{Georges} = V_{Georges}(\phi)R$$

$$L_{Ringo} = V_{Ringo}(\phi)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right)$$



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## Self-tuning solution

Admit a Minkowski vacuum for any value of the cosmological constant

Remains true before and after any phase transition where the cosmological constant jumps instantaneously by a finite amount

# Degravitation

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Extra dimensions could be the solution

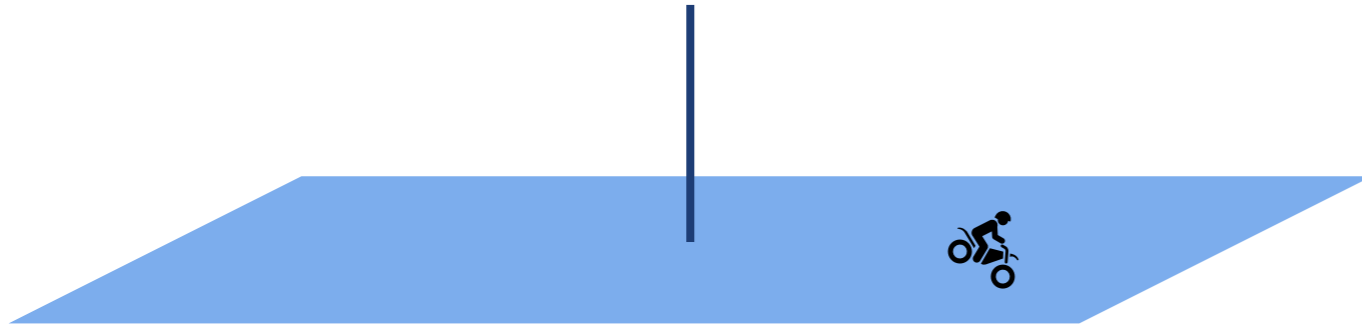


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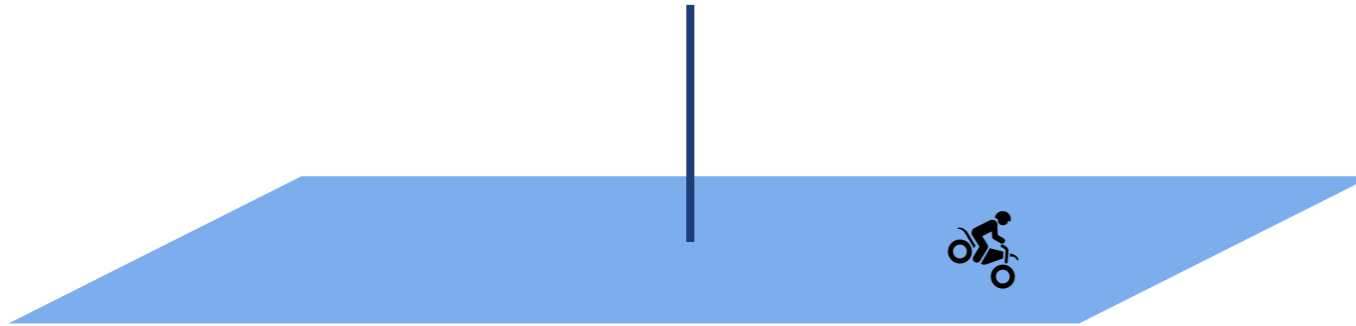


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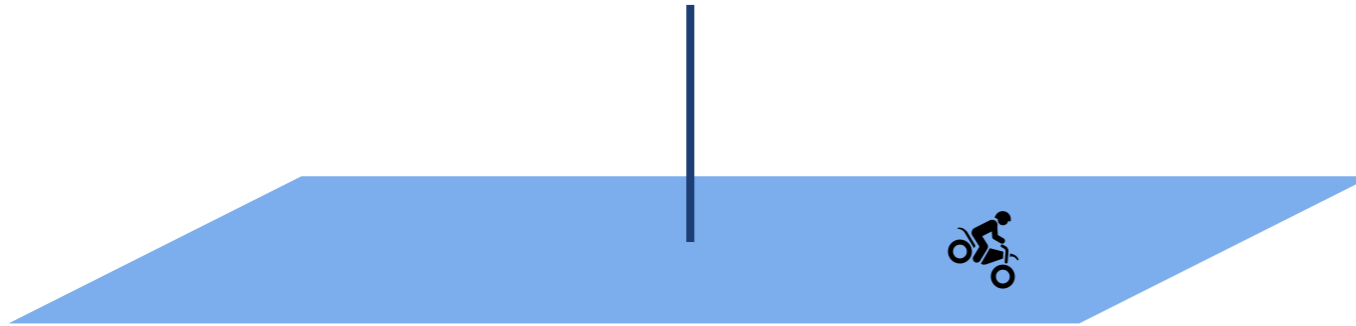


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With D dimensions

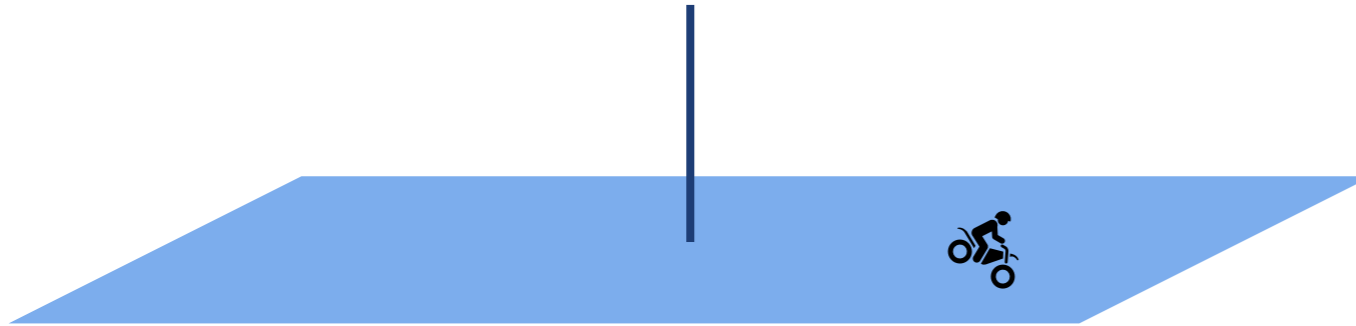
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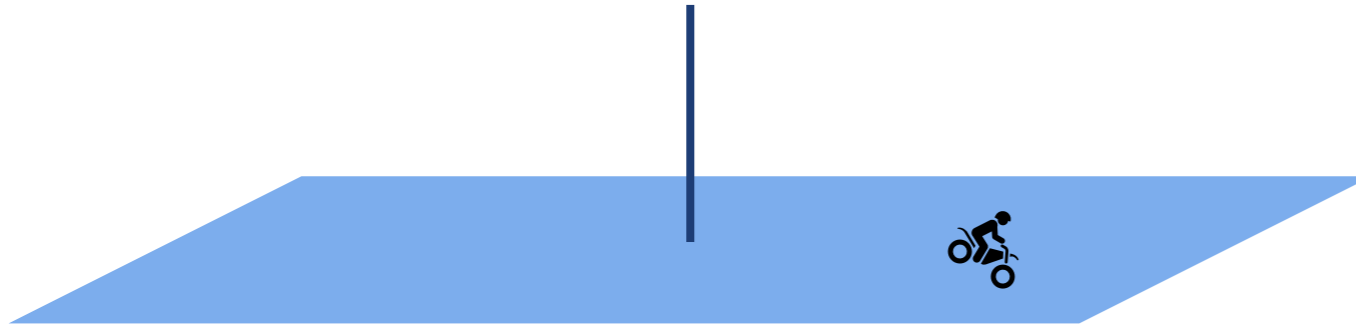
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That is because the 4D graviton acquires a mass

Let's consider the case of a uniform source in electromagnetism

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For gravity

$$m \vec{a} = \left( \frac{m\Lambda}{3} r - \frac{GMm}{r^2} \right) \vec{e}_r$$

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De-electrifying

It is similar to give a mass to the photons

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Stueckelberg

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Variation wrt  $\phi$

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Variation wrt  $B^\mu$

$$\begin{aligned} \partial_\mu F^\mu{}_\nu - m^2 B_\nu - m\partial_\nu\phi + J_\nu &= 0 \\ -\frac{m^2}{\square}\partial^\mu(\partial_\mu B_\nu) &\quad \frac{m^2}{\square}\partial^\mu(\partial_\nu B_\mu) \quad = -\frac{m^2}{\square}\partial^\mu F_{\mu\nu} \end{aligned}$$

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Variation wrt  $B^\mu$

$$\partial_\mu F^\mu{}_\nu - m^2 B_\nu - m\partial_\nu\phi + J_\nu = 0$$

$$-\frac{m^2}{\square}\partial^\mu(\partial_\mu B_\nu) \qquad \frac{m^2}{\square}\partial^\mu(\partial_\nu B_\mu) \qquad = -\frac{m^2}{\square}\partial^\mu F_{\mu\nu}$$

$$\left(1 - \frac{m^2}{\square}\right)\partial_\mu F^\mu{}_\nu = -J_\nu$$



# Degravitation

What about non-linearities

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + A^\mu J_\mu - \frac{\lambda}{4}(A_\mu A^\mu)^2$$

It does not spoil the de-electrification

$$\text{We have the solution } \vec{E} = \vec{0} \quad \text{with } A_\mu = a \delta_\mu^0 \quad m^2 a + \lambda a^3 = \Lambda$$

The non-linearities shift  $A_\mu$  without changing the electric field

We want the same for gravity which is a non-linear theory with a cosmological constant  $\Lambda$

We would like gravity to be ignorant of  $\Lambda$ : degravitation

For that we need a massive gravity theory

Let us look to the linear theory

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda g_{\mu\nu}$$

$$\square h_{\mu\nu} - \partial_\lambda \partial_\mu h^\lambda_\nu - \partial_\lambda \partial_\nu h^\lambda_\mu + \eta_{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h = \Lambda \eta_{\mu\nu}$$

Massive gravity

$$\square h_{\mu\nu} - \partial_\lambda \partial_\mu h^\lambda_\nu - \partial_\lambda \partial_\nu h^\lambda_\mu + \eta_{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h - m^2(h_{\mu\nu} - \eta_{\mu\nu} h) = \Lambda \eta_{\mu\nu}$$

# Degravitation

Massive gravity  $\square h_{\mu\nu} - \partial_\lambda \partial_\mu h^\lambda_\nu - \partial_\lambda \partial_\nu h^\lambda_\mu + \eta_{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h - m^2(h_{\mu\nu} - \eta_{\mu\nu} h) = \Lambda \eta_{\mu\nu}$

$$h_{\mu\nu} = \frac{\Lambda}{3m^2} \eta_{\mu\nu}$$

It shifts the usual  $h_{\mu\nu} = 0$  Minkowski vacuum to a new one but which is also flat

$$\square h_{\mu\nu} - \partial_\lambda \partial_\mu h^\lambda_\nu - \partial_\lambda \partial_\nu h^\lambda_\mu + \eta_{\mu\nu} \partial_\lambda \partial_\sigma h^{\lambda\sigma} + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h - m^2(h_{\mu\nu} - \eta_{\mu\nu} h) = \Lambda \eta_{\mu\nu}$$

$$\mathcal{O}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta}$$

$$\mathcal{O}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - m^2(h_{\mu\nu} - \eta_{\mu\nu} h) = \Lambda \eta_{\mu\nu}$$

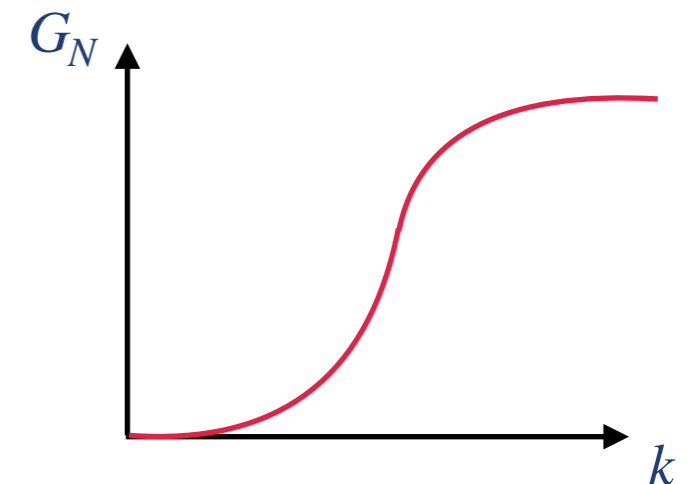
$$\left(1 - \frac{m^2}{\square}\right) \mathcal{O}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = \Lambda \eta_{\mu\nu}$$

We need a theory which gives this type of behavior in the linear regime

$$G_N^{-1} G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$G_N^{-1} \left(\frac{m^2}{\square}\right) G_{\mu\nu} = 8\pi T_{\mu\nu}$$

A gravitational constant which filters long wavelengths



# Anthropic Principle

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Copernican Principle: Humans do not occupy a privileged position in the Universe

B. Carter: *Although our situation is not necessarily central, it is inevitably privileged to some extent*

Very strong anthropic principle: everything in our universe has something to do with humankind

Very weak anthropic principle: takes the very existence of humankind as a piece of experimental data. For example, in order not to kill a person with the products of radio-decay, the life-time of a proton must be at least  $10^{16}$  years

Weak anthropic principle: there are many regions in the universe, a multiverse. In these regions physical laws are in different forms. It just happens that in the region we are dwelling all physical laws, physical constants and cosmological parameters are such that clusters of galaxies, galaxies and our solar system can form, and humankind can appear.

# Seesaw mechanism

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Used for example in neutrino physics to explain their small non-zero mass

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$$M = \begin{pmatrix} 0 & x \\ x & y \end{pmatrix} \quad \text{Eigenvalues} \quad \lambda_{\pm} = \frac{y \pm \sqrt{y^2 + 4x^2}}{2} \quad \lambda_+ \lambda_- = -x^2$$

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$$\text{If } y \gg x \quad \lambda_+ = y \quad \lambda_- = -\frac{x^2}{y}$$

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$$M_{\Lambda} = \rho_{\Lambda}^{1/4} \simeq 10^{-12} GeV$$

$$M_{Pl} \simeq 10^{19} GeV$$

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Let's consider 2 coupled Universes with 2 cosmological constants

$$\left( \dots - \Lambda_1 \right) |\Psi_1\rangle + \sqrt{\Lambda_1 \Lambda_2} |\Psi_2\rangle = 0$$

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$$\Lambda_1 \simeq (10 \text{ TeV})^4$$

$$\Lambda_- \simeq 10^{-46} \text{ GeV}^4$$

$$\Lambda_2 \simeq M_{Pl}^4$$



**Obrigado**