Introduction to 2D dilaton gravity

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Why dilaton?

General model and particular cases

Even more general: Poisson sigma models

All classical solutions

AdS_2/CFT_1

Review: Grumiller, Kummer and D.V, Phys.Rept. (2002).

Support: CNPq, FAPESP

The Einstein-Hilbert action in 2D

$$\int d^2x \sqrt{-g} R$$

has vanishing local variations and thus describes no dynamics. A remedy: include a scalar field X. For example:

$$S = \int d^2x \sqrt{-g}\, X (R - 2\Lambda).$$

This is the first 2D dilaton gravity - the Jackiw-Teitelboim (JT) model (1984). Classical solutions to this model are locally dS or AdS spacetimes.

Let us add to the action a kinetic term for X and a potential [Banks, ..., I. Shapiro, ..., around 1990-1992]:

$$L = \int d^2x \sqrt{-g}\left(-\tfrac{1}{2}RX - \tfrac{1}{2}U(X)(\nabla X)^2 + V(X)\right)$$

Important particular cases:

Spherical reduction from D dimensions:

$$V = -(D-2)(D-3)X^{\frac{D-4}{D-2}} \qquad U = -\frac{D-3}{(D-2)X}.$$

String gravity (CGHS):

$$V = -2X \qquad U = -\frac{1}{X}.$$

The action above is equivalent to

$$L = \int [X^a D e_a + X d\omega + \ V(X_a X^a; X)] ;$$

where ω is the spin-connection, e_a is zweibein, $a = \ ,$ $De_a = de_a + a \omega \wedge e_a$, $\ $ is the volume 2-form, X^a are two auxiliary fields that generate torsion constraints.

$$V = \tfrac{1}{2} U(X) X^a X_a + V(X)$$

To prove the equivalence: solve the torsion constraints to express through e. The resulting action will depend on the metric rather than on the zweibein.

These are 2D sigma models with a target space being a Poisson manifold with local coordinates X^I (scalars from the 2D point of view), gauge fields A_I (one-forms from the 2D point of view) and a Poisson tensor $P^{IJ}(X)$ satisfying the Jacobi identity

$$P^{IJ}\partial_J P^{KL} + \text{cycl}(IKL) = 0$$

The action [Schaller & Strobl, 1994]

$$L = \int \left[dX^I \wedge A_I + \tfrac{1}{2} P^{IJ} A_J \wedge A_I \right]$$

is invariant under the gauge transformations

$$\delta X^I = P^{IJ} \varepsilon_J$$
$$\delta A_I = -d\varepsilon_I - (\partial_I P^{JK}) \varepsilon_K A_J$$

With the choice $X^I = (X; X^a)$, $A_I = (!;\ e_a)$ and

$$P^{ab} = V\ {}^{ab}; \qquad P^{aX} = X^b\ {}_b{}^a$$

one recovers 2D dilaton gravities in the 1st order formalism. The gauge parameters^a correspond to di eomorphisms, while^X describes local Lorentz rotations.

[Bergamin, Grumiller, Kummer, DV, 2005]
Consider a 1st order Euclidean dilaton gravity in complex variables

$$\int \left(Y \bar{D} e + \bar{Y} D \bar{e} + X d\omega + \epsilon V(2Y\bar{Y}; X) \right)$$

where

$$Y = (X^1 + iX^2)/\sqrt{2}; \qquad e = (e^1 + ie^2)/\sqrt{2}$$

Though the bar means a complex conjugation, it is convenient to view all fields as independent complex variables.
Equations of motion:

$$dX - iY\bar{e} + i\bar{Y}e = 0; \qquad DY + ieV = 0$$

$$d\omega + \epsilon \frac{\partial V}{\partial X} = 0; \qquad De + \epsilon \frac{\partial V}{\partial \bar{Y}} = 0$$

By combining equations on the 1st line, we obtain

$$d(YY) + VdX = 0$$

that is integrated to

$$dC = 0; \qquad C = w(X) + e^{Q(X)} Y Y$$

with

$$Q(X) = \int^X U(z)dz; \qquad w(X) = \int^X e^{Q(z)} V(z)$$

This allows to express YY through X.
The existence and the form of absolutely conserved quantity follows also from the Poisson sigma model arguments.

Similarly,

$$d(e{=}Y) = \; dX \wedge (e{=}Y)U(X);$$

which yields

$$d(e^{\;Q}{=}Y) = \; 0$$

So that

$$e^{\;Q}{=}Y \; = \; df$$

with some complex zero-form f. That is it,

$$e = \; Ye^{Q}df\,; \qquad e = \; i\frac{dX}{Y} + \; Y e^{Q} df$$

$$! \; = \; Ve^{Q}df \quad i\frac{dY}{Y}$$

By imposing the reality condition one obtains a general classical solution depending of 3 arbitrary functions (gauge degrees of freedom) and one integration constant C (mass of the solution).

According to the AdS/CFT conjecture gravity theories in asymptotically AdS spaces correspond to conformal theories at the asymptotic boundary. To implement this conjecture in 2D at the classical level, one has to

- de ne asymptotic conditions asymptotic symmetry algebras
- de ne asymptotic degrees of freedom
- compute the action for these degrees of freedom

Let us see, how the rst step can be done [Grumiller, Salzer, DV, 2015].

Let us take the JT gravity, denote by the radial coordinate, s.t. $\to \infty$ corresponds to the asymptotic region of AdS_2. Let ' be a second coordinate. We require that at the asymptotics all elds behave as listed below plus subleading terms

$$X^0 = X_R e \quad X_L e \qquad e_0 = \tfrac{1}{2} e \quad \tfrac{1}{2} M e$$

$$X^1 = X^1(') \qquad e_1 = 1$$

$$X = X_R e + X_L e \qquad \omega_{'} = \tfrac{1}{2} e \quad \tfrac{1}{2} M e$$

Other elds vanish. X_R, X_L and M are some functions of '. This comes from partially xing the gauge, partially solving the equations of motion and by an educated guess. By equations of motion, $X_{R;L}$ and X^1 may be expressed through M.

The asymptotic symmetry algebra is generated by bulk gauge transformations that

- Preserve the form of asymptotic conditions
- Change M.

Parameters of these transformations depend on a single arbitrary function (')

$$
\begin{aligned}
{}^{0} &= \tfrac{1}{2}\ e \qquad \tfrac{1}{2}(M \quad + 2 \ {}^{00})e \\
{}^{1} &= \quad {}^{0} \\
{}^{X} &= \quad \tfrac{1}{2}\ e \qquad \tfrac{1}{2}(M \quad + 2 \ {}^{00})e
\end{aligned}
$$

M is transformed as

$$
M = \quad M^{0} \qquad 2M \ {}^{0} \quad 2 \ {}^{000}
$$

This is the Virasoro algebra with a non-zero central charge.

Q: How can one make sure that $\mathcal{M}$ represents a true asymptotic degree of freedom?
A: It is enough to compute canonical boundary charges that have to depend on asymptotic degrees of freedom and be finite.

Q: How can one obtain an action for asymptotic degrees of freedom?
A: By substituting the solutions in JT action with a suitable boundary term. Depending on details, the resulting model may be a de Alfaro-Fubini-Furlan conformal quantum mechanics or a Schwarzian action [Maldacena et al, 2016].

By imposing looser asymptotic conditions in the JT model one can get larger asymptotic symmetry algebras, as, e.g., the loop algebra of $sl(2)$, Virasoro plus an $u(1)_k$, see [Grumiller et al, 2017].

There are higher spin and supersymmetric extensions.

Extensions to generic dilaton gravities: work in progress.