Lensing in McVittie metric

http://arxiv.org/abs/1508.04763

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Outline

References

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Non-cosmological manifestation of Λ

- Eddington, 1923. Upper limit $\Lambda \lesssim 10^{-42}$ cm^{-2} in order to avoid a detectable correction to the Mercury perihelion precession.
- Pioneer anomaly, 1970. Now agreed to be a thermal recoil force effect (Turyshev, 2012).
- Islam, 1983. No influence of Λ on the bending of light because Λ does not enter the orbital equation for photons.
- Ishak and Rindler, 2007. Λ does influence the bending of light via the metric, which has to be used for computing the bending angle.

Effect of Λ on the bending of light

Ishak and Rindler, 2007 (2010)

Kottler metric, 1918.

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \ ,$$

where

$$f(r) \equiv 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} \ .$$

This metric has two horizons, $r \approx 2m$ and $r \approx \sqrt{3/\Lambda}$, and it is not asymptotically flat.

Geometry of Kottler solution

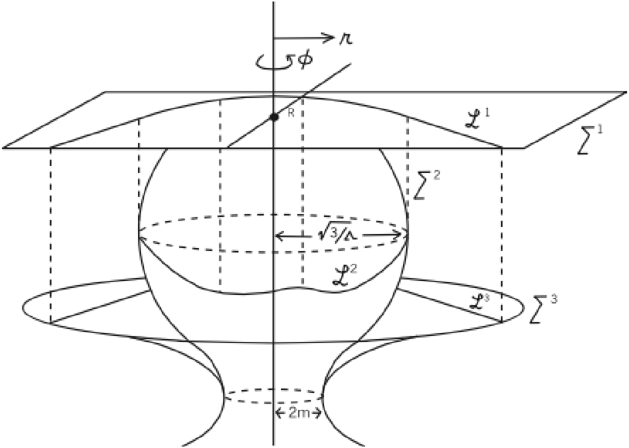


Fig. 1 Schwarzschild and Schwarzschild-de Sitter geometries. Σ^3 is the Flamm paraboloid representation of a central coordinate plane in Schwarzschild; Σ^2 is the corresponding surface in Schwarzschild-de Sitter; Σ^1 is an auxiliary plane with an r, ϕ graph, $\mathcal{L}^1$, of the orbit equation (6). The curves $\mathcal{L}^2$ and $\mathcal{L}^3$ are the vertical projections of $\mathcal{L}^1$ onto Σ^2 and Σ^3, and represent the true spatial curvature of the orbits.

Orbital equation for photons

For $\theta = \pi/2$ and at first-order in m/r:

$$\frac{1}{r} = \frac{\sin\phi}{R} + \frac{3m}{2R^2}\left(1 + \frac{1}{3}\cos 2\phi\right) ,$$

where, for $\phi = \pi/2$:

$$\frac{1}{r_0} = \frac{1}{R} + \frac{m}{R^2} ,$$

and r_0 is the closest approach distance and R is the distance of the zeroth-order solution (a straight line) from the centre.

The above equations hold true both for Schwarzschild and Kottler metrics. No Λ appears.

Trajectory of photons

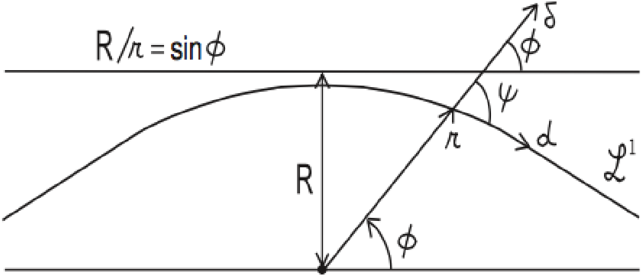


Fig. 2 The orbital map. This is a plane graph of the orbit equation (6) and coincides with Σ^1 in Figure 1. The one-sided deflection angle is $\psi - \phi \equiv \epsilon$.

Calculating the bending angle

In Schwarzschild metric, one takes $r \to \infty$ and, since the space is asymptotically flat, the coordinate angle ϕ is also the measured angle.

The same is not true for Kottler space. Ishak and Rindler propose then to use:

$$\cos\psi = \frac{g_{ij}\delta^i d^j}{(g_{ij}\delta^i\delta^j)^{1/2}(g_{ij}d^i d^j)^{1/2}} \ .$$

This is where Λ comes into play. Another form:

$$\tan\psi = \frac{\sqrt{g_{\phi\phi}}}{\sqrt{g_{rr}}}\left|\frac{d\phi}{dr}\right| = r\sqrt{f(r)}\left|\frac{d\phi}{dr}\right| \ .$$

Results

Assuming small angles and $2m/R \ll 1$ and $\Lambda r^2 \ll 1$.
For a generic lens system:

$$\psi = \phi + \frac{2m}{R} - \frac{\Lambda R^3}{6(2m + \phi R)} \ .$$

For an Einstein's ring ($\phi = 0$):

$$\psi = \frac{2m}{R} - \frac{\Lambda R^3}{12m} \ .$$

The total bending angle is:

$$\delta = 2(\psi - \phi) \ .$$

Other results

- Schücker, 2009. He uses a self-contained method, avoiding the lens equation. Analysing the lensing cluster of SDSS J1004+4112, he finds:

$$\Lambda = (2.1 \pm 1.5) \times 10^{-52}\ \mathrm{m}^{-2}\ ;$$

- Biressa and Pacheco, 2011. They find for the bending angle:

$$\delta \simeq \frac{4M}{b} - Mb\left(\frac{1}{r_S^2} + \frac{1}{r_{\rm obs}^2}\right) + \frac{2Mb\Lambda}{3} - \frac{b\Lambda}{6}(r_S + r_{\rm obs})\ ,$$

  and determine corrections of 2% in the mass estimates. Masses are slightly lower if a cored density profile is used and slightly higher if an isothermal density profile is adopted.

Criticisms

Park, 2008

The main criticism is that the results presented above are based on Kottler metric, which is static, and therefore does not take into account the relative motion of source, lens and observer.

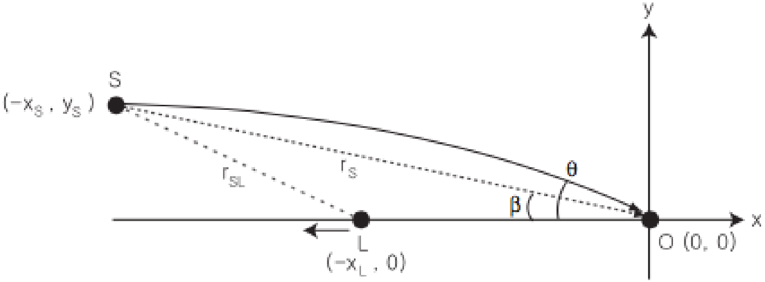


FIG. 1. Lensing schematics.

Result found by Park

Park found for the following lens equation:

$$\theta = \beta + \frac{2md_{\rm SL}}{\beta d_{\rm S} d_{\rm L}} \left[1 + \mathcal{O}(H^3) + \mathcal{O}(\beta^2)\right] + \mathcal{O}(m^2) \ ,$$

in contradiction with the results based on Kottler metric, which assert that there should be a $\mathcal{O}(\Lambda) \sim \mathcal{O}(H^2)$ correction to the conventional lensing analysis.

Ishak, Rindler and Dossett, 2010, questioned the final result of Park since other terms including $H^2 = \Lambda/3$ terms were apparently dropped out the calculation at some point, leading to the conclusion that Λ does not contribute to lensing except via the angular diameter distances.

McVittie metric

McVittie, 1933

McVittie metric has the following form:

$$ds^2 = -\left(\frac{1-\mu}{1+\mu}\right)^2 dt^2 + (1+\mu)^4 a(t)^2 (d\rho^2 + \rho^2 d\Omega^2) \ ,$$

where $a(t)$ is the scale factor and

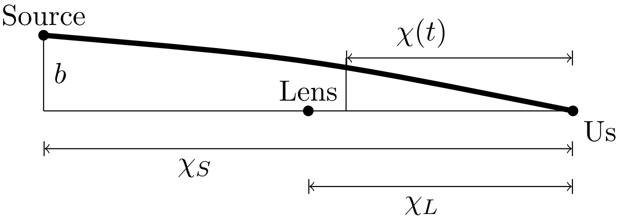
$$\mu \equiv \frac{M}{2a(t)\rho} \ ,$$

where M is the mass of the point-like lens. When $\mu \ll 1$, McVittie metric can be approximated as

$$ds^2 = -(1-4\mu)\, dt^2 + (1+4\mu) a(t)^2 (d\rho^2 + \rho^2 d\Omega^2) \ ,$$

which is the usual perturbed FLRW metric in the Newtonian gauge; 2μ is the gravitational potential.

Scheme of lensing



The relation between χ and the background expansion is the usual one for the FLRW metric:

$$\frac{d\chi}{dt} = -\frac{1}{a} \, ,$$

and the comoving distances of the source and of the lens, χ_S and χ_L respectively, do not change.

Null geodesics equations

For the transversal displacement l^i:

$$\frac{a}{p}\frac{1-\mu}{1+\mu}\frac{d}{d\chi}\left(\frac{p}{a}\frac{1+\mu}{1-\mu}\frac{dl^i}{d\chi}\right)=\frac{2(1-\mu)}{(1+\mu)^7}\delta^{il}\partial_l\mu$$

$$+2Ha\left[1+\frac{2\partial_t\mu}{(1+\mu)H}\right]\frac{dl^i}{d\chi}$$

$$-\frac{2}{1+\mu}\left(\delta^i_j\partial_k\mu+\delta^i_k\partial_j\mu-\delta_{jk}\delta^{il}\partial_l\mu\right)\frac{dl^j}{d\chi}\frac{dl^k}{d\chi}\ ,$$

The equation describing the evolution of the proper momentum p is the following:

$$\frac{1}{p}\frac{dp}{dt}=-H-\frac{2}{1+\mu}\partial_t\mu+2\frac{P^i\partial_i\mu}{p(1+\mu)^2}\ .$$

Approximations

We consider $\mu \ll 1$ and small displacements $l^i \ll \chi$.

$$\frac{d^2 l^i}{d\chi^2} = 4\partial_i \mu \ .$$

Using $\mu \equiv M/2a\rho$ in the equation above, one gets:

$$\frac{d^2 l}{d\chi^2} = -\frac{2Ml}{a(\chi)\left[(\chi-\chi_L)^2 + l^2\right]^{3/2}} \ .$$

With the following definitions:

$$x \equiv \chi/\chi_L \ , \qquad \alpha \equiv 2M/\chi_L \ , \qquad y \equiv l/\chi_L \ ,$$

the displacement equation becomes:

$$\frac{d^2 y}{dx^2} = -\alpha \frac{y}{a(x)\left[(x-1)^2 + y^2\right]^{3/2}} \ .$$

Note that a vanishing α implies that $a(x)$ has no effect on the trajectory.

Solving the equation

Considering small α:

$$y = y^{(0)} + \alpha y^{(1)} + \alpha^2 y^{(2)} + \dots \ ,$$

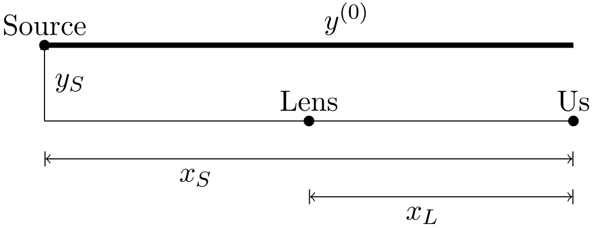
and initial conditions:

$$y(x_S) = y_S \ , \qquad y(0) = 0 \ ,$$

the zero-order solution is a straight line:

$$y^{(0)} = C_1 x + C_2 \ . \tag{1}$$

Zeroth-order solution



We choose the two integration constants so that $y^{(0)} = y_S$, i.e. the trajectory is a straight, horizontal line.

First-order equation

The first-order equation is the following:

$$\frac{d^2y^{(1)}}{dx^2} = -\frac{y_S}{a(x)\left[(x-1)^2 + y_S^2\right]^{3/2}} \, ,$$

for which we must choose the following initial conditions:

$$y^{(1)}(x_S) = 0 \, , \qquad y^{(1)}(0) = -y_S/\alpha \, .$$

For a constant Hubble factor $H = H_0$:

$$\chi = \int_0^z \frac{dz'}{H(z')} = \frac{z}{H_0} \equiv \frac{1}{H_0}\left(\frac{1}{a} - 1\right) \, ,$$

so that:

$$\frac{d^2y^{(1)}}{dx^2} = -\frac{y_S\left(1 + H_0\chi_L x\right)}{\left[(x-1)^2 + y_S^2\right]^{3/2}} \, .$$

The bending angle

In the limit $y_S \ll 1$ the deviation angle

$$\delta \equiv \left.\frac{dy}{dx}\right|_{x=0} - \left.\frac{dy}{dx}\right|_{x=x_S} ,$$

is the following:

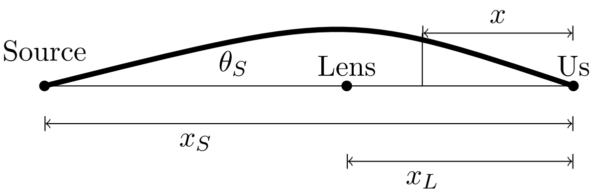
$$\delta = \frac{2\alpha(1+\chi_L H_0)}{y_S} + \mathcal{O}(y_S) .$$

Recalling that $\alpha \equiv 2M/\chi_L$ and $y_S = b/\chi_L$:

$$\delta = \frac{4M(1+\chi_L H_0)}{b} + \mathcal{O}(b/\chi_L) .$$

The mass has been increased by a relative amount of $H_0\chi_L = z_L$, the redshift of the lens.

Einstein's ring systems



The zeroth-order trajectory is now:

$$y^{(0)} = \theta_S(x_S - x) \ ,$$

where $\theta_S \ll 1$. In order for the trajectory to reach us, we must choose the initial condition $y^{(1)}(0) = -\theta_S x_S/\alpha$.
Computing again the deflection angle, we get:

$$\delta = \frac{4M(1+\chi_L H_0)}{\theta_S(\chi_S - \chi_L)} + \frac{\chi_L}{2(\chi_S - \chi_L)} + \mathcal{O}(\theta_S) \ .$$

Einstein radius

Introducing the angular diameter distance one has:

$$\theta_S(\chi_S - \chi_L) = \theta_S D_{LS} \frac{a_L}{a_S} = \theta_E D_L \frac{1+z_S}{1+z_L} \ .$$

From the lens equation, one has for the Einstein radius:

$$\theta_E = \sqrt{4M \frac{(1+z_L)^2}{1+z_S} \frac{D_{LS}}{D_L D_S}} \ .$$

Writing the angular diameter distances as functions of the redshift:

$$\theta_E = \sqrt{4MH_0 \frac{(1+z_L)^4 (z_S - z_L)}{(1+z_S) z_S z_L}} \ .$$

In the above formula, the new contribution is one of the four powers of $1+z_L$ in the square root.

Mass estimates

We apply our formula to some Einstein ring systems observed by the CASTLES Survey,[1] assuming $H_0 = 70$ km s^{-1} Mpc^{-1}.

Object	z_S	z_L	θ_E	$M/M_\odot$	$(1+z_L)^{-1}$
Q0047-2808	3.60	0.48	1.35	5.0×10^{11}	0.68
PMNJ0134-0931	2.216	0.77	0.365	2.7×10^{10}	0.56
B0218+357	0.96	0.68	0.17	8.6×10^{9}	0.60
CFRS03.1077	2.941	0.938	1.05	2.2×10^{11}	0.52
MG0751+2716	3.20	0.35	0.35	3.2×10^{10}	0.74
HST15433+5352	2.092	0.497	0.59	7.2×10^{10}	0.67
MG1549+3047	1.17	0.11	0.9	7.3×10^{10}	0.90
MG1654+1346	1.74	0.25	1.05	1.9×10^{11}	0.80
PKS1830-211	2.51	0.89	0.5	4.9×10^{10}	0.53
B1938+666	2.059	0.881	0.5	4.8×10^{10}	0.53

[1] https://www.cfa.harvard.edu/castles/

Caveat

Considering the standard ΛCDM model Friedmann equation

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_{\rm m}(1+z)^3 \; ,$$

H is approximately constant only as long as $\Omega_\Lambda \gg \Omega_{\rm m}(1+z)^3$.

Using the observed values for the density parameters, approximately $\Omega_\Lambda = 0.7$ and $\Omega_{\rm m} = 0.3$, the above condition amounts to state that $z_L \ll 0.3$. Therefore, a reliable correction on the bending angle is at most of 30%.

For the mass estimate, $(1+z_L)^{-1} \gg 0.77$.

Conclusions and perspectives

Adopting McVittie metric as description of the geometry of a point-like lens in the expanding universe:

1. There is an important $1 + z_L$ contribution to the bending angle:

$$M \to M(1 + z_L) \ ,$$

   i.e. a new $1/(1 + z_L)$ correction to the mass;
2. This contribution has been calculated assuming a constant $H = H_0$;
3. We have to consider the standard ΛCDM model:

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_\mathrm{m}(1 + z)^3$$

4. Calculation of the delay time;

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Thank you!

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