Dark Matter viewed as cosmic gluonic background
(Cosmogonic implications of de Sitter, Anti de Sitter and Poincaré symmetries)

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J.-P.G., Mass in de Sitter and Anti-de Sitter Universes with Regard to Dark Matter. Universe 2020, 6, 66;
G. Cohen-Tannoudji (U. Paris Saclay & CEA) and J.-P. G.
Cold Dark Matter : A Gluonic Bose-Einstein Condensate in Anti-de Sitter Space Time
Universe 2021,7,402 & arXiv :2111.01130v2 [gr-qc] and references therein

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PERSONAL REMEMBRANCE OF MEETINGS WITH RICHARD
GROUP 4 NIJMEGEN 1975: was our first participation in ICGTMP meetings.
We did not speak to each other.
Three maximal symmetries, Poincaré, dS, AdS

Dark matter from QCD : A relic of Quark Period

Λ CDM standard model

GROUP 10 CANTERBURY 1981 : we start to speak to each other and enjoy jokes!
Three maximal symmetries, Poincaré, dS, AdS

Dark matter from QCD: A relic of Quark Period

Complements: facts of $\Lambda$CDM standard model

GROUP 24 PARIS 2002: we organised it!
Three maximal symmetries, Poincaré, dS, AdS

Dark matter from QCD: A relic of Quark Period

Complements: facts of $\Lambda CDM$ standard model

Ubu Roi (Joan Miró, 1944)

BRAVO RICHARD FOR AN EXCELLENT BIRTHDAY

J.-P. Gazeau

The concept of an “elementary system” requires that all states of the system be obtainable from the relativistic transforms of any state by superpositions. In other words, there must be no relativistically invariant distinction between the various states of the system which would allow for the principle of superposition. This condition is often referred to as irreducibility condition ... The concept of an elementary system (...) is a description of a set of states which forms, in mathematical language, an irreducible representation space for the inhomogeneous Lorentz (≈ Poincaré) group

C. Fronsdal, Elementary Particles in a Curved Space, *Rev. Mod. Phys.* **37** 221 (1965) :

A physical theory that treats spacetime as Minkowskian flat must be obtainable as a well-defined limit of a more general physical theory, for which the assumption of flatness is not essential.


The presence of the action

\[
S_{grav} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}(R - 2\Lambda) .
\]  

leads to a metrical elasticity of space, i.e., to generalized forces which oppose the curving of space. Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved.
1. Three maximal symmetries, Poincaré, dS, AdS

2. Dark matter from QCD: A relic of Quark Period

3. Complements: facts of $\Lambda$CDM standard model
Einstein Equations in GR: three (equivalent or not?) standpoints

► Standpoint 1

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\kappa T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad \kappa = \frac{8\pi G}{c^4}. \]

geometrical content

Here, the fundamental state that contains the maximum number of symmetries is the Minkowskian geometry, and the cosmological term \( \Lambda g_{\mu\nu} \) may be interpreted as an extra pressure, named world matter by de Sitter in his debate with Einstein:

\( \Lambda > 0 \sim \text{“dark energy”} \quad \Lambda < 0 \sim \text{“dark matter”?} \)

► Standpoint 2

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa T_{\mu\nu}. \]

geometrical content

Here, the fundamental states that contain the maximum number of symmetries are the de-Sitter (dS) (\( \Lambda \equiv \Lambda_{dS} > 0 \)) and the Anti-de-Sitter (AdS) (\( \Lambda \equiv \Lambda_{AdS} < 0 \)) geometries.

► Standpoint 3

Actually the split between these two standpoints should not be considered as absolute, since we could as well model situations in a mixed way:

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda_L g_{\mu\nu} = -\kappa T_{\mu\nu} - \Lambda_R g_{\mu\nu}. \]

geometrical content

matter content
General remarks on the interest of dS/AdS studies

- dS and AdS are *maximally symmetric*  
  (in a metric space of dimension \( n \), the maximum number of metric preserving symmetries is \( n(n + 1)/2 \), here 10 since \( n = 4 \))

- Their symmetries are one-parameter deformations of Minkowskian symmetry with
  - negative curvature \( -\kappa_{\text{dS}} = -\sqrt{\Lambda_{\text{dS}}/3} \) (= \(-H/c\), \( H \): Hubble parameter)
  - positive curvature \( \kappa_{\text{AdS}} = \sqrt{|\Lambda_{\text{AdS}}|/3} \)
  respectively

- As soon as a constant curvature is present, we lose some of our so familiar conservation laws like energy-momentum conservation!

- Then what is the physical meaning of a scattering experiment (“space" in dS is like the sphere \( S^3 \), let alone the fact that time is ambiguous)?

- Which relevant “physical” quantities are going to be considered as (asymptotically ? contractively ?) experimentally available?¹

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With the requirements of kinematical rotation, parity, and time-reversal invariance, there exists only one way to “deform” the Poincaré group, namely, in endowing space-time with a certain curvature.

**Fig. 1.** The contraction scheme for the relativity groups.

**Relative-time groups:**  \((dS), \ (P'), \ (P), \ (C)\).

**Absolute-time groups:**  \((N), \ (G'), \ (G), \ (St)\).

**Relative-space groups:**  \((dS), \ (N), \ (P), \ (G)\).

**Absolute-space groups:**  \((P'), \ (G'), \ (C), \ (St)\).

**Cosmological groups:**  \((dS), \ (N), \ (P'), \ (G')\).

**Local groups:**  \((P), \ (G), \ (C), \ (St)\).

*Remark:* The effect of the symmetry \(S\) of Eq. (6) is equivalent to a symmetry of the cube with respect to the plane containing the vertices \((dS), \ (N), \ (C), \ and \ (St)\).
The eleven kinematics (Bacry & Levy-Leblond, JMP (1968))

**Theorem**

*Under the assumptions that:*

1. space is isotropic (rotation invariance);
2. parity and time-reversal are automorphisms of the kinematical groups;
3. inertial transformations in any given direction form a noncompact subgroup,

then there are eight types of Lie algebras for kinematical groups corresponding to eleven possible kinematics. These algebras are:

- **R1** The two de Sitter Lie algebras isomorphic, respectively, to the Lie algebras of $\text{SO}(4,1)$ and $\text{SO}(3,2)$;
- **R2** The Poincaré Lie algebra;
- **R3** Two "para-Poincaré" Lie algebras, of which one is isomorphic to the ordinary Poincaré Lie algebra but physically different and the other is the Lie algebra of an inhomogeneous $\text{SO}(4)$ group;
- **R4** The Carroll Lie algebra;
- **A1** The two “nonrelativistic cosmological” Lie algebras, Newton$_\pm$;
- **A2** The Galilei Lie algebra;
- **A3** The “para-Galilei” Lie algebra;
- **A4** The “static” Lie algebra.

*While the Lie algebras of class $R$ have no nontrivial central extensions by a one-parameter Lie algebra, those of class $A$ each have one class of such extensions.*
The eleven kinematics (Bacry & Levy-Leblond, JMP (1968))
The symmetries

- Respective invariance (in the relativity or kinematical sense) groups:
  - de Sitter group is the ten-parameter $SO_0(1, 4)$ (or its universal covering $Sp(2, 2)$)
  - Anti de Sitter group is the ten-parameter $SO_0(2, 3)$ (or its two-fold covering $Sp(4, \mathbb{R})$)

- Both are deformations of the proper orthochronous Poincaré group $\mathbb{R}^{1,3} \rtimes SO_0(1, 3)$ (or $\mathbb{R}^{1,3} \rtimes SL(2, \mathbb{C})$), the kinematical group of Minkowski spacetime
de Sitter space viewed as a hyperboloid embedded in a five-dimensional Minkowski space with metric $\eta_{\alpha \beta} = \text{diag}(1, -1, -1, -1, -1)$ (but keep in mind that all points are physically equivalent)

$$M_{dS} \equiv \left\{ x \in \mathbb{R}^5 ; x^2 = \eta_{\alpha \beta} x^\alpha x^\beta = -\frac{3}{\Lambda_{dS}} \right\} \simeq SO(1, 4)/SO(1, 3) \quad \alpha, \beta = 0, 1, 2, 3, 4,$$
Anti de Sitter geometry

- Anti de Sitter space viewed as a one-sheeted hyperboloid embedded in another five-dimensional space with metric \( \eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1, 1) \) (here too all points are physically equivalent):

\[
M_{\text{AdS}} \equiv \left\{ x \in \mathbb{R}^5; \ x^2 = \eta_{\alpha\beta} x^\alpha x^\beta = \frac{3}{|\Lambda_{\text{AdS}}|} \right\} \simeq \text{SO}_0(2, 3)/\text{SO}_0(1, 3), \quad \alpha, \beta = 0, 1, 2, 3, 5
\]
Compared classifications of Poincaré, dS and AdS UIR’s for quantum elementary systems

- In a given unitary irreducible representation (UIR) of dS and AdS groups, (∼ elementary system in Wigner’s sense) their respective generators map to self-adjoint operators in Hilbert spaces of spinor-tensor valued fields on dS and AdS respectively:

\[ K_{\alpha\beta} \mapsto L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta} , \]

with orbital part \( M_{\alpha\beta} = -i(x_\alpha \partial_\beta - x_\beta \partial_\alpha) \) and spinorial part \( S_{\alpha\beta} \) acting on the field components.

- The physically relevant UIR’s of the Poincaré, dS and AdS groups are denoted by \( \mathcal{P} > (m, s) \) (">" for positive energies), \( U_{dS}(\varsigma_{dS}, s) \), and \( U_{AdS}(\varsigma_{AdS}, s) \), respectively.

- These UIR’s are specified by the spectral values \( \langle \cdot \rangle \) of their quadratic and quartic Casimir operators.

- The latter define two invariants, the most basic ones being predicted by the relativity principle, namely proper mass \( m \) for Poincaré and \( \varsigma_{dS}, \varsigma_{AdS} \) for dS and AdS respectively, and spin \( s \) for the three cases.\(^2\)

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Two basic invariants for Poincaré, dS, and AdS

- For Poincaré the Casimir operators are fixed as

\[
Q_{\text{Poincaré}}^{(1)} = P_\mu P_\mu = P_0^2 - P_2 = m^2 c^2, \\
Q_{\text{Poincaré}}^{(2)} = W_\mu W_\mu = -m^2 c^2 s(s+1)h^2, \quad W_\mu := \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma.
\]

- For de Sitter,

\[
Q_{\text{dS}}^{(1)} = -\frac{1}{2} L_\alpha L_\beta L^{\alpha\beta} = \varsigma_{\text{dS}}^2 - \left( s - \frac{1}{2} \right)^2 + 2 \equiv \langle Q_{\text{dS}}^{(1)} \rangle, \\
Q_{\text{dS}}^{(2)} = -W_\alpha W_\alpha = \left( \varsigma_{\text{dS}}^2 + \frac{1}{4} \right) s(s+1), \quad W_\alpha := -\frac{1}{8} \epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta}.
\]

- For Anti-de Sitter,

\[
Q_{\text{AdS}}^{(1)} = -\frac{1}{2} L_\alpha L_\beta L^{\alpha\beta} = \varsigma_{\text{AdS}} (\varsigma_{\text{AdS}} - 3) + s(s+1) \equiv \langle Q_{\text{AdS}}^{(1)} \rangle, \\
Q_{\text{AdS}}^{(2)} = -W_\alpha W_\alpha = -\varsigma_{\text{AdS}} - 1)(\varsigma_{\text{AdS}} - 2)s(s+1), \quad W_\alpha := -\frac{1}{8} \epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta}.
\]
While proper mass $\equiv$ at rest energy $\equiv$ energy spectrum infimum of self-adjoint time translation generator $P_0$ in Minkowski, these two quantities come apart in de Sitterian/Anti-de Sitterian geometry.

They have to be devised from a flat-limit viewpoint, i.e. from the study of the contraction limit $\Lambda \to 0$ of these representations.

In this respect, a mass formula for dS has been established by Garidi (2003)\(^3\):

$$m_{dS}^2 \equiv \frac{\hbar^2 \Lambda_{dS}}{3c^2} (\langle Q_{dS}^{(1)} \rangle - 2) = \frac{\hbar^2 \Lambda_{dS}}{3c^2} \left( \varsigma_{dS}^2 + \left(s - \frac{1}{2}\right)^2 \right).$$

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Proper mass versus “at rest” energy in de Sitter : contraction dS $\rightarrow$ Poincaré

- This definition should be understood through the contraction limit of representations:

$$\text{dS UIR} \rightarrow \text{Poincaré UIR}.$$ 

- More precisely, with

$$\Lambda_{\text{dS}} \rightarrow 0 \quad \varsigma_{\text{dS}} \rightarrow \infty,$$

while fixing

$$\varsigma_{\text{dS}} \hbar \sqrt{\Lambda_{\text{dS}}}/\sqrt{3c} = m_{\text{Poincaré}} \equiv m.$$ 

we have

$$U_{\text{dS}}(\varsigma_{\text{dS}}, s) \rightarrow \begin{cases} c > P > (m, s) \oplus c < P < (m, s). \\
|\varsigma_{\text{dS}}| \sqrt{\Lambda_{\text{dS}}} / \sqrt{3c} = \frac{mc}{\hbar} 
\end{cases}$$

- Note the breaking of dS irreducibility into a direct sum of two Poincaré UIR’s with positive and negative energy respectively.

- To some extent the choice of the factors $c_-, c_+$, is left to a local tangent observer. The latter will naturally fix one of these factors to 1 and so the other one is forced to vanish.

- This crucial dS feature originates from the dS group symmetry mapping any point $(x^0, P) \in H_{\text{dS}}$ into its mirror image $(x^0, -P) \in H_{\text{dS}}$ with respect to the $x^0$-axis.

- Under such a symmetry the four dS generators $L_{0a}$, $a = 1, 2, 3, 4$, (and particularly $L_{04}$ which contracts to energy operator !) transform into their respective opposite $-L_{0a}$, whereas the six $L_{ab}$’s remain unchanged.
Proper mass versus at rest energy in de Anti de Sitter

- Concerning AdS a mass formula (JPG&Novello 2007)\(^4\) similar to that one for dS exists:

\[
m^2_{\text{AdS}} = \frac{\hbar^2 |\Lambda_{\text{AdS}}|}{3c^2} \left( \langle Q^{(1)}_{\text{AdS}} \rangle - \langle Q^{(1)}_{\text{AdS}} |_{s_{\text{AdS}}=s+1} \rangle \right) = \frac{\hbar^2 |\Lambda_{\text{AdS}}|}{3c^2} \left[ \left( s_{\text{AdS}} - \frac{3}{2} \right)^2 - \left( s - \frac{1}{2} \right)^2 \right].
\]

- One here deals with the AdS group representations \(U_{\text{AdS}}(s_{\text{AdS}}, s)\) with \(s_{\text{AdS}} \geq s + 1\) (discrete series and its lowest limit), and their contraction limit holds with no ambiguity

\[
U_{\text{AdS}}(s_{\text{AdS}}, s) \xrightarrow{\Lambda_{\text{AdS}} \to 0, s_{\text{AdS}} \to \infty} \mathcal{P}^>(m, s).
\]

- Now, contraction formulae for both dS and AdS give us the freedom to write

\[
m_{\text{dS}} = m_{\text{AdS}} = m,
\]

- This agrees with the Einstein position that the proper mass of an elementary system should be independent of the geometry of space-time, or equivalently it should not exist any difference between inertial and gravitational mass.

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Rest energy of a free particle in AdS versus dS and Poincaré

► Each **Anti-deSitterian quantum elementary system** (in the Wigner sense) has a discrete energy spectrum bounded below by its **rest energy**

\[
E_{\text{rest}}^{\text{AdS}} = \left[ m^2 c^4 + \hbar^2 c^2 \frac{|\Lambda_{\text{AdS}}|}{3} \left( s - \frac{1}{2} \right)^2 \right]^{1/2} + \frac{3}{2} \hbar \sqrt{\frac{|\Lambda_{\text{AdS}}|}{3}} c ,
\]

► Hence, to the order of \( \hbar \), a “massive” AdS elementary system is a **deformation of both a relativistic free particle with rest energy** \( mc^2 \) and a **3d isotropic quantum harmonic oscillator with ground state energy** \( 3/2 \hbar \sqrt{|\Lambda_{\text{AdS}}|/3} c \equiv 3/2 \hbar \omega_{\text{AdS}} \)

► In contrast to AdS, energy is ill-defined for dS. However a local tangent observer will naturally choose the invariant with positive sign in :

\[
E_{\text{rest}}^{\text{dS}} = \pm \left[ m^2 c^4 - \hbar^2 c^2 \frac{\Lambda_{\text{dS}}}{3} \left( s - \frac{1}{2} \right)^2 \right]^{1/2} , \quad m \geq \frac{\hbar}{c} \sqrt{\frac{\Lambda_{\text{dS}}}{3}} \left| s - \frac{1}{2} \right| .
\]

► Noticeable simplification in both AdS and dS for **fermions** \( s = 1/2 \) :

- for dS : \( E_{\text{dS}}^{\text{rest}} = mc^2 \),
- for AdS : \( E_{\text{AdS}}^{\text{rest}} = mc^2 + \frac{3}{2} \hbar \omega_{\text{AdS}} \).

---

In the massless case and spin $s$, we have

\[ E_{\text{rest}}^{\text{dS}} = \pm i\hbar \sqrt{\frac{\Lambda_{\text{dS}}}{3}} c \left( s - \frac{1}{2} \right), \]

\[ E_{\text{rest}}^{\text{AdS}} = \hbar \sqrt{\frac{|\Lambda_{\text{AdS}}|}{3}} c(s + 1). \]

Therefore, while for dS the energy at rest makes sense only for massless fermionic systems and is just zero, for AdS the energy at rest makes sense for any spin, and in particular for spin 1 massless bosons we get

\[ E_{\text{AdS}}^{\text{rest}} = 2\hbar \omega_{\text{AdS}}. \]

and for scalar massless bosons

\[ E_{\text{AdS}}^{\text{rest}} = \hbar \omega_{\text{AdS}}. \]
Cosmology chronology: (effective) de Sitter and Anti de Sitter phases.

- Today $t_0$
  - Life on Earth
  - Solar system
  - Quasars

- Galaxy formation
  - Epoch of gravitational collapse

- Recombination
  - Relic radiation decouples (CBR)

- Matter domination
  - Onset of gravitational instability

- Nucleosynthesis
  - Light elements created: D, He, Li

- Quark-hadron transition
  - Hadrons form - protons & neutrons

- Electroweak phase transition
  - Electromagnetic & weak nuclear forces become differentiated:
    $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)$

- Grand unification transition
  - $G \rightarrow SU(3)_C \times SU(2)_L \times U(1)$
  - Inflation, baryogenesis, monopoles, cosmic strings, etc.

- The Planck epoch
  - The quantum gravity barrier

- $t = 15$ billion years
  - $T = 3K$ (1 mV)

- $t = 400,000$ years
  - $T = 3000K$ (1 eV)

- $t = 3 minutes$

- $t = 1 second$
  - $T = 1 MeV$

- $t = 10^{-6}$ s
  - $T = 1 GeV$

- $t = 10^{-11}$ s
  - $T = 10^{-5}$ GeV

- $t = 10^{-35}$ s
  - $T = 10^{-5}$ GeV

- Anti de Sitter phase

- de Sitter phase
Cosmology chronology: Hubble radius $L(a) \equiv H^{-1}(a)$ ($c = 1$) is plotted versus the scale factor $a(t) \equiv R(t)$ in logarithmic scale.
Some (observational) facts about dark matter

- According to Planck 2015 analysis\(^6\) of CMB power spectrum, our Universe is spatially flat, accelerating, and composed of 5% baryonic matter, 27% cold dark matter (CDM, non-baryonic) and 68% dark energy (\(\Lambda\))\(^7\)

- (Cold) dark matter is observed by its gravitational influence on luminous, baryonic matter

- The dark matter mass halo and the total stellar mass are coupled through a function that varies smoothly with mass (with controversial exception(s)...)\(^\text{a}\)

- Up to now (?), all hypothetical particle models (WIMP, Axions, Neutrinos ...) failed direct or indirect detection tests

- Similarly, alternative theories (e.g. MOND) to dark matter are not fully successful (?) in explaining clusters and the observed pattern in the CMB

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Largest virtual universe ever simulated: Quantum cosmogony is a gedanken experiment that consists in bringing back into the tableau the visible matter that has been removed as a foreground possibly interfering with the CMB. A section of it on the right.

Our interpretation of the complex topology of the spacetime of the dark universe: a web of dark filaments that are tensionless dark strings freely moving in a void space (the white regions in the figure) with negative curvature (i.e. de Sitter) related to the (positive) cosmological constant, whereas the spacetime inside the filaments has a positive curvature (i.e. Anti de Sitter) related to an effective negative cosmological constant.
Theories predicting the existence of quark-gluon plasma were developed in the late 1970s and early 1980s (Satz, Rafelsky, Kapusta, Müller, Letessier...)

Quark-gluon plasma was detected for the first time at CERN (2000)

Lead and gold nuclei have been used for collisions yielding QGP at CERN SPS and BNL RHIC, respectively

The current estimate of the hadronization temperature for light quarks is

\[ T_{cf} = 156.5 \pm 1.5 \text{ MeV} \approx 1.8 \times 10^{12} \text{ K} \] ("chemical freeze-out temperature").  


Cold dark matter $\sim$ CGB: A parallel between dark matter, CMB, and CNB:

- **CMB** $\rightarrow$ photon decoupling, i.e. photons started to travel freely through space rather than constantly being scattered by electrons and protons in plasma (QED effect): *we see those photons!*

- **CNB** $\rightarrow$ is the relic of the neutrino decoupling when the rate of weak interactions between neutrinos and other forms of matter dropped below the rate of expansion of the universe, which produced a cosmic neutrino background of freely streaming neutrinos (electroweak effect): *so far indirect evidence, e.g. Big Bang nucleosynthesis predictions of the helium abundance, CMB anisotropies ...*

- **CGB**? Our scenario: Dark matter $\rightarrow$ gluonic component of the quark epoch (quark-gluon plasma) which freely subsists after hadronization within an effective AdS environment (QCD effect): *we do not observe those gluons but we see their gravitational effects and dark matter could be as well named cosmic gluonic background (CGB)....*
Cold dark matter: a QCD effect

As a matter of fact the contribution of the so-called di-gluons through what is called by Adler (1982)\textsuperscript{9} the *gluon pairing amplitude* to the QCD *trace anomaly* reads

\[ \langle T^\mu_\mu \rangle_0 = - \frac{1}{8} [11N_c - 2N_f] \left\langle \frac{\alpha_s}{\pi} (F^a_{\mu\nu}F^{a\mu\nu})^r \right\rangle_0 \propto \Lambda_{\text{AdS}} \]

\( N_c \) is the number (=3) of colors, and \( N_f \) the effective number of quark flavors (\( \sim 3 \)). As asserted by G. Cohen-Tannoudji\textsuperscript{10}: *The minus sign in the right hand side shows that when the factor \( (11N_c - 2N_f) \) is positive, all the QCD condensates contribute negatively to the energy density, which means that the QCD world-matter is globally an AdS world-matter (dominance of an AdS world-matter over a smaller dS world-matter), and so

- *the bosonic* (gluon) loops, proportional to \( N_c \), contribute to the AdS world matter
- *the fermionic* (quark) loops, proportional to \( N_f \), contribute to the normal dS world matter

Compare the ratio \( \frac{11}{2} \frac{N_c}{N_f} \sim 5.5 \) with the estimate \([\text{dark matter}]/[\text{visible matter}] \sim \frac{27}{5} = 5.4 \).

\textbf{References:}


First remind that in an AdS effective background \((\sqrt{|\Lambda_{\text{AdS}}|}/3c \equiv \omega_{\text{AdS}})\):

\[
E_{\text{AdS}}^{\text{rest}} = \left[ m^2c^4 + \hbar^2\omega_{\text{AdS}}^2 \left( s - \frac{1}{2} \right)^2 \right]^{1/2} + \frac{3}{2}\hbar\omega_{\text{AdS}}
\]

As an assembly of \(N_G\) non-interacting (i.e. colorless) scalar bosonic di-gluons with individual energies \(E_n = E_{\text{AdS}}^{\text{rest}} + n\hbar\omega_{\text{AdS}}\) and degeneracy \(g_n = (n + 1)(n + 3)/2\), those remnant components, analogous to isotropic harmonic oscillators in 3-space, are assumed to form a grand canonical Bose-Einstein ensemble whose the chemical potential \(\mu\) is, at temperature \(T\), fixed by the requirement that the sum over all occupation probabilities at temperature \(T\) yields

\[
N_G = \sum_{n=0}^{\infty} \frac{g_n}{\exp \left[ \frac{\hbar\omega_{\text{AdS}}}{k_BT} (n + \nu_0 - \mu) \right] - 1}, \quad \nu_0 := \frac{E_{\text{AdS}}^{\text{rest}}}{\hbar\omega_{\text{AdS}}}.
\]
Cold dark matter: Bose-Einstein condensation of (di-)gluons in effective Anti-de Sitter background

The number $N_G$ is very large and so the gas condensates at temperature

$$T_c \approx \frac{\hbar \omega_{\text{AdS}}}{k_B} \left( \frac{N_G}{\zeta(3)} \right)^{1/3} \zeta(3) \approx 1.2$$

(Riemann zeta function)

to become the currently observed dark matter

The above formula is standard for all isotropic harmonic traps. Actually there is no harmonic trap here, it is the AdS geometry **due to QCD trace anomaly** which originates the harmonic spectrum on the quantum level

To support this scenario it is known from ultra-cold atoms physics that Bose Einstein condensation can occur in non-condensed matter but also in gas, and that this phenomenon is not linked to *interactions* but rather to the *correlations implied by quantum statistics*. 
Although we do not precisely know at which stage beyond the hadronization phase transition does take place the gluonic Bose Einstein condensation, let us see if our estimate on $T_c$ yields reasonable orders of magnitude.

Take $T_c$ equal to the current CMB temperature, $T_c = 2.78$K, and $|\Lambda_{\text{AdS}}| \approx \frac{5.5}{6.5} \times \frac{11}{24} \times \Lambda_{\text{dS}} = 0.39 \times \Lambda_{\text{dS}}$ (an estimate based on the $\Lambda$CDM model, see complements), with $\Lambda_{\text{dS}} \equiv$ present $\Lambda = 1.1 \times 10^{-52}$m$^{-2}$. We then get the estimate on the number of di-gluons in the condensate:

$$N_G \approx 5 \times 10^{88}$$

This seems reasonable since the gluons are around $10^9$ times the number of baryons, and the latter is estimated to be around $10^{80}$. 

An estimate
We have tentatively explained dark matter by actually asking a simple question (!):

**what becomes the huge amount of gluons after the transition from QGP period to hadronization?**

Moreover, our approach lies at the crossroads of the two current standard models, the ΛCDM Model of cosmology and the Standard Model of particle physics.

THANK FOR YOUR ATTENTION!
A reminder about the cosmological formalism \((c = 1)\) : Robertson metric

- **In an isotropic and homogeneous** cosmology, the Einstein’s equation reads as

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu}
\]

where the stress energy momentum stands for a perfect fluid with density \(\rho\) and isotropic pressure \(P\), i.e.,

\[
T_{\mu\nu} = -P g_{\mu\nu} + (P + \rho) u_{\mu} u_{\nu}.
\]

- Its solution is the Robertson metric

\[
ds^2 = dt^2 - R^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right)
\]

- \(R(t)\) : the time-dependent radius of the universe. It is the cosmological scale factor (also noted \(a(t)\)) which determines proper distances in terms of the comoving coordinates.

- \(k\) : curvature index

- \(r\) is dimensionless
A reminder about the cosmological formalism ($c = 1$): Friedmann-Lemaître equations

- Physical quantities $R$, $\rho$, and $P$ obey the Friedmann-Lemaître equations of a perfect fluid modelling the material content of the universe.

\[
H^2 \equiv \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} \\
\frac{\dot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3} (\rho + 3P) \\
\dot{\rho} = -3H (\rho + P) \quad \text{(Conservation of the energy)}
\]

- Note that the cosmological term $\Lambda g_{\mu\nu}$ is taken to the right-hand side of the Einstein’s equation and may be interpreted as an extra pressure, named world matter by de Sitter in his debate with Einstein:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N (P + \rho) u_\mu u_\nu + (\Lambda - 8\pi G_N P) g_{\mu\nu}
\]

- According to the sign of this extra pressure one talks of a de Sitter world matter ($\Lambda$ positive, pressure negative) or an anti-de Sitter world matter ($\Lambda$ negative, pressure positive).
Cosmology chronology: Hubble radius $L(a) \equiv H^{-1}(a)$ ($c=1$) is plotted versus the scale factor $a(t) \equiv R(t)$ in logarithmic scale.
Flatness sum rule and its consequences

From the first FL equation at $\Lambda \approx 0$:

$$\frac{k}{R^2} = \frac{8\pi G_N}{3} \rho - H^2 \equiv \frac{8\pi G_N}{3} \rho - \frac{8\pi G_N}{3} \rho_c$$

The ($\sim$ observed) flatness rule $k = 0$ expresses the vanishing of the spatial curvature:

$$\rho - \rho_c \equiv \rho_{\text{vis}} + \rho_{\text{DM}} + \rho_{\text{DE}} - \rho_c = 0,$$

with

$$\rho_{\text{vis}} = \rho_{\text{bar}} + \rho_{\text{rad}}, \quad \rho_{\text{DE}} = \frac{\Lambda}{8\pi G_N}.$$

Hence the so-called critical density $\rho_c := \frac{3H^2}{8\pi G_N}$ is the energy density at the boundaries in the far past and in the far future of the Hubble horizon in the absence of any “integration constant” $\Lambda$ and any spatial curvature ($k = 0$).
Three maximal symmetries, Poincaré, dS, AdS

Dark matter from QCD: A relic of Quark Period

Complements: facts of $\Lambda$CDM standard model

68.3%, 95.4% and 99.7% confidence level contours on $\Omega_\Lambda \equiv \Omega_{DE}$ and $\Omega_M$ obtained from CMB, BAO and the Union SN set ($P/\rho \equiv w = -1$).
Inside the “confidence area” of the figure $\Omega_\Lambda \equiv \Omega_{DE}$ versus $\Omega_M$

- From the second FL equation
  \[
  \frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3} (\rho + 3P) \equiv -\frac{4\pi G_N}{3} (\rho - 2\rho_{DE} + 3P) \equiv -\frac{4\pi G_N}{3} (\rho_{\text{effective}} + 3P)
  \]
  one infers that at the inflection points $\ddot{R} = 0$ one has the “equation of state” (EoS)
  \[w_{\text{inflexion}} \equiv \frac{P}{\rho_{\text{effective}}} = -\frac{1}{3}\]

- Inside the “confidence area” of the figure $\Omega_\Lambda = \rho_{DE}/\rho_c$ versus $\Omega_M = \rho_m/\rho_c$ one finds the points
  - $(\Omega_{DM}, \Omega_{DE} + \Omega_{vis})$
  - $(\Omega_M = 1/3, \Omega_{DE} = 2/3)$

- The value $\Omega_{DE} = 2/3$ results from our assumption completing the flatness sum rule as which the total energy vanishes (from the Robertson metric):
  \[\rho_{\text{vis}} + \rho_{DM} + \rho_{DE} = \rho_c = \frac{3}{2} \rho_{DE}\]