Functional renormalization group approach and background field formalism

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Ii is well-known fact that the gauge symmetry of an initial action is broken on quantum level because of the gauge fixing procedure in process of quantization.

Generating functional of vertex functions (effective action) being main quantity in quantum field theory depends on gauges [Jackiw, (1974); Dolan, Jackiw, (1974); Nielsen (1975); Fukuda, Kugo, (1976)] . This dependence has a special form and disappears on-shell [PML, Tyutin, (1981) ; Voronov, PML, Tyutin, (1982)]. In its turn it allows to have a physical interpretation of results obtained on quantum level.

Loss of gauge invariance in conventional formulation of quantum field theory leads to serious technical problems in explicit calculations of quantum effects. Fortunately there exists an approch known as the background-field formalism which helps to solve this problem.

The background field method [De Witt, (1967) ; Arefeva, Faddeev, Slavnov (1974); Abbott, (1981).] presents a reformulation of quantization procedure for gauge theories allowing to work with the effective action invariant under the gauge transformations of background fields and to reproduce all usual physical results by choosing a special background field condition.

Application of the background field method simplifies essentially calculations of Feynman diagrams in gauge theories (among recent applications of this approach in Yang-Mills theories see, for example, [Barvinsky, Blas, Herrero-Valea, Sibiryakov, Steinwachs, (2018); . Frenkel, Taylor, (2018) ; Batalin, PML, Tyutin (2018); Brandt, Frenkel, McKeon, (2019)].

Usual functional formulation of Quantum Field Theory is closely based on the perturbation theory when there exists strong proofs of basic properties of functional integrals [Slavnov (1974)].

There are a few of attempts to formulate Quantum Field Theory beyond the perturbation theory, among them the Gribov-Zwanziger theory [Gribov (1978); Zwanziger (1989)] and the functional renormalization group approach [Wetterich (1991,1993)].

Standard formulations of these approaches meet with the gauge dependence problems analyzied in our studies [PML, Lechtenfeld (2013); PML, Shapiro (2013)]. It was shown that in both cases the effective action depends on gauge even on-shell and it was proposed some solutions of the problems.

There is a growing interest in the use of the background field method for formulation of the FRG approach to have advantiges of gauge invariance of average effective action [Becker, Reuter (2014); Percacci, Vacca (2017); Wetterich (2018)].

Because the standard formulation of the FRG approach meets problems of gauge dependence of the average effective action [PML, Shapiro (2013); PML, Merzlikin (2015)] we are going to study the gauge invariance and gauge dependence problems for the FRG approach formulated in the background field method.

For a better understanding of the gauge invariance and gauge dependence problems in the FRG approach it is useful to remind the main features of the Faddeev-Popov method [Faddeev, Popov (1967)] for Yang-Mills theory fields within the background field formalism.

Yang-Mills action

$$\mathcal{S}_{YM}(A) = \int dx\Big(-\frac{1}{4}G^{\alpha}_{\mu\nu}(A(x))G^{\alpha}_{\mu\nu}(A(x))\Big),$$
$$G^{\alpha}_{\mu\nu}(A(x)) = \partial_\mu A^{\alpha}_{\nu}(x) - \partial_\nu A^{\alpha}_{\mu}(x) + gf^{\alpha\beta\gamma}A^{\beta}_{\mu}(x)A^{\gamma}_{\nu}(x),$$

Gauge invariance

$$\delta_\omega \mathcal{S}_{YM}(A) = 0,$$
$$\delta_\omega A^{\alpha}_{\mu}(x) = \big(\partial_\mu \delta_{\alpha\beta} + gf^{\alpha\sigma\beta}A^{\sigma}_{\mu}(x)\big)\omega_\beta(x) =$$
$$= D^{\alpha\beta}_{\mu}(A(x))\omega_\beta(x).$$

Yang-Mills theories in background field formalism

In the background field formalism [B.S. De Witt, (1967) ; I.Ya. Arefeva, L.D. Faddeev, A.A. Slavnov (1974); L.F. Abbott, (1981)] the gauge field $A^{\alpha}_{\mu}(x)$ appearing in classical action is replaced by $A^{\alpha}_{\mu}(x) + \mathcal{B}^{\alpha}_{\mu}(x)$,

$$\mathcal{S}_{YM}(A) \ \rightarrow \ \mathcal{S}_{YM}(A + \mathcal{B}),$$

where $\mathcal{B}^{\alpha}_{\mu}(x)$ is considered as an external field.
Gauge invariance

$$\delta_{\omega}\mathcal{S}_{YM}(A + \mathcal{B}) = 0, \quad \delta_{\omega} A^{\alpha}_{\mu} = D^{\alpha\beta}_{\mu}(A + \mathcal{B})\omega_{\beta}.$$

The corresponding Faddeev-Popov action $S_{FP} = S_{FP}(\phi, \mathcal{B})$ has the form

$$S_{FP} = \mathcal{S}_{YM}(A+\mathcal{B}) + \int dx \Big[\overline{C}^{\alpha} G^{\mu}_{\alpha\beta}(A,\mathcal{B}) D^{\beta\gamma}_{\mu}(A+\mathcal{B}) C^{\gamma} + B^{\alpha}\chi_{\alpha}(A,\mathcal{B}) \Big],$$

where

$$G^{\mu}_{\alpha\beta}(A,\mathcal{B}) = \frac{\delta\chi_{\alpha}(A,\mathcal{B})}{\delta A^{\beta}_{\mu}},$$

$\chi_{\alpha}(A,\mathcal{B})$ are functions lifting the degeneracy of the Yang-Mills action, The action S_{FP} is invariant under global supersymmetry (BRST symmetry)

$$\delta_B A^{\alpha}_{\mu} = D^{\alpha\beta}_{\mu}(A+\mathcal{B}) C^{\beta}\mu, \quad \delta_B C^{\alpha} = \frac{g}{2} f^{\alpha\beta\gamma} C^{\beta} C^{\gamma}\mu,$$
$$\delta_B \overline{C}^{\alpha} = B^{\alpha}\mu, \quad \delta_B B^{\alpha} = 0,$$

where μ is a constant anti-commuting parameter

Introducing the gauge fixing functional $\Psi = \Psi(\phi, \mathcal{B})$,

$$\Psi = \int dx \, \overline{C}^{\alpha} \chi_{\alpha}(A, \mathcal{B}),$$

the action rewrites in the form

$$S_{FP}(\phi, \mathcal{B}) = \mathcal{S}_{YM}(A + \mathcal{B}) + \Psi(\phi, \mathcal{B})\hat{R}(\phi, \mathcal{B}),$$

where

$$\hat{R}(\phi, \mathcal{B}) = \int dx \, \frac{\overleftarrow{\delta}}{\delta \phi^i} R^i(\phi, \mathcal{B}), \quad \hat{R}^2(\phi, \mathcal{B}) = 0,$$

is the generator of BRST transformations.

$$\mathcal{S}_{YM}(A + \mathcal{B})\hat{R}(\phi, \mathcal{B}) = 0, \quad S_{FP}(\phi, \mathcal{B})\hat{R}(\phi, \mathcal{B}) = 0,$$

The generating functional of Green functions in the background field method is defined in the form of functional integral

$$Z(J,\mathcal{B}) = \int d\phi\ \exp\Big\{\frac{i}{\hbar}\big[S_{FP}(\phi,\mathcal{B}) + J\phi\big]\Big\} = \exp\Big\{\frac{i}{\hbar}W(J,\mathcal{B})\Big\},$$

where $W(J,\mathcal{B})$ is the generating functional of connected Green functions. Here the notations

$$J\phi = \int dx J_i(x)\phi^i(x), \quad J_i(x) = (J^{\alpha}_{\mu}(x), J^{(B)}_{\alpha}(x), \overline{J}_{\alpha}(x), J_{\alpha})(x)$$

are used and $J_i(x)$ $\big(\varepsilon(J_i(x)) = \varepsilon_i,\ \mathrm{gh}(J_i(x)) = \mathrm{gh}(\phi^i(x))\big)$ are external sources to fields $\phi^i(x)$.

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Let $Z_\Psi(\mathcal{B})$ be the vacuum functional which corresponds to the choice of gauge fixing functional in the presence of external fields $\mathcal{B}$,

$$Z_\Psi(\mathcal{B}) = \int d\phi \ \exp\Big\{\frac{i}{\hbar}\big[\mathcal{S}_{YM}(A+\mathcal{B}) + \Psi(\phi,\mathcal{B})\hat{R}(\phi,\mathcal{B})\big]\Big\}.$$

In turn, let $Z_{\Psi+\delta\Psi}$ be the vacuum functional corresponding to a gauge fixing functional $\Psi(\phi,\mathcal{B}) + \delta\Psi(\phi,\mathcal{B})$,

$$Z_{\Psi+\delta\Psi}(\mathcal{B}) = \int d\phi \ \exp\Big\{\frac{i}{\hbar}\big[S_{FP}(\phi,\mathcal{B}) + \delta\Psi(\phi,\mathcal{B})\hat{R}(\phi,\mathcal{B})\big]\Big\}.$$

Here, $\delta\Psi(\phi,\mathcal{B})$ is an arbitrary infinitesimal odd functional which may, in general, has a form differing on background gauge fixing.

Making use of the change of variables ϕ^i in the form of BRST transformations but with replacement of the constant parameter μ by the following functional

$$\mu = \mu(\phi, \mathcal{B}) = \frac{i}{\hbar}\delta\Psi(\phi, \mathcal{B}),$$

and taking into account that the Jacobian of transformations is equal to

$$J = \exp\{-\mu(\phi, \mathcal{B})\hat{R}(\phi, \mathcal{B})\},$$

we find the gauge independence of the vacuum functional

$$Z_{\Psi}(\mathcal{B}) = Z_{\Psi+\delta\Psi}(\mathcal{B}).$$

The vacuum functional $Z(\mathcal{B}) = Z(J=0,\mathcal{B})$ obeys the very important property of gauge invariance with respect to gauge transformations of external fields,

$$\delta_\omega Z(\mathcal{B}) = 0, \quad \delta_\omega \mathcal{B}^\alpha_\mu = D^{\alpha\beta}_\mu(\mathcal{B})\omega_\beta.$$

The proof is based on using the change of variables in the functional integral of the following form

$$\delta_\omega A^\alpha_\mu = gf^{\alpha\gamma\beta}A^\gamma_\mu\omega_\beta, \quad \delta_\omega C^\alpha = gf^{\alpha\gamma\beta}C^\gamma\omega_\beta,$$
$$\delta_\omega \overline{C}^\alpha = gf^{\alpha\gamma\beta}\overline{C}^\gamma\omega_\beta, \quad \delta_\omega B^\alpha = gf^{\alpha\gamma\beta}B^\gamma\omega_\beta$$

assuming the transformation law of gauge fixing functions χ_α according to

$$\delta_\omega \chi_\alpha(A,\mathcal{B}) = gf^{\alpha\gamma\beta}\chi_\gamma(A,\mathcal{B})\omega_\beta.$$

In particular, it can be argued the invariance of $S_{FP}(\phi,\mathcal{B})$ under combined gauge transformations

$$\delta_\omega S_{FP}(\phi,\mathcal{B}) = 0.$$

Yang-Mills theories in background field formalism

The invariance of S_{FP} means that the functional $Z(J,\mathcal{B})$ is invariant

$$Z(J,\mathcal{B})\int dx \frac{\overleftarrow{\delta}}{\delta\mathcal{B}^{\alpha}_{\mu}} D^{\alpha\beta}_{\mu}(\mathcal{B})\omega_{\beta} =$$
$$= gf^{\alpha\gamma\beta}\omega_{\beta}\int dx\Big(J^{\alpha}_{\mu}\frac{\delta}{\delta J^{\gamma}_{\mu}} + J_{\alpha}\frac{\delta}{\delta J_{\gamma}} + \overline{J}_{\alpha}\frac{\delta}{\delta\overline{J}_{\gamma}} + J^{(B)}_{\alpha}\frac{\delta}{\delta J^{(B)}_{\gamma}}\Big)Z(J,\mathcal{B}),$$

under the gauge transformations of the background vector field $\mathcal{B}$ and simultaneously the tensor transformations of sources

$$\delta_{\omega}J^{\alpha}_{\mu} = gf^{\alpha\gamma\beta}J^{\gamma}_{\mu}\omega_{\beta},\ \delta_{\omega}\overline{J}_{\alpha} = gf^{\alpha\gamma\beta}\overline{J}_{\gamma}\omega_{\beta},$$
$$\delta_{\omega}J_{\alpha} = gf^{\alpha\gamma\beta}J_{\gamma}\omega_{\beta},\ \delta_{\omega}J^{(B)}_{\alpha} = gf^{\alpha\gamma\beta}J^{(B)}_{\gamma}\omega_{\beta}.$$

In terms of the functional $\Gamma(\Phi,\mathcal{B})$ the invariance of Z reads

$$\Gamma(\Phi,\mathcal{B})\int dx\frac{\overleftarrow{\delta}}{\delta\mathcal{B}^{\alpha}_{\mu}}D^{\alpha\beta}_{\mu}(\mathcal{B})\omega_{\beta}=$$
$$-\Gamma(\Phi,\mathcal{B})\int dx\Big(\frac{\overleftarrow{\delta}}{\delta\mathcal{A}^{\alpha}_{\mu}}\mathcal{A}^{\gamma}_{\mu}+\frac{\overleftarrow{\delta}}{\delta\mathcal{C}^{\alpha}}\mathcal{C}^{\gamma}+\frac{\overleftarrow{\delta}}{\delta\overline{\mathcal{C}}^{\alpha}}\overline{\mathcal{C}}^{\gamma}+\frac{\overleftarrow{\delta}}{\delta\Phi^{\alpha}_{(B)}}\Phi^{\gamma}_{(B)}\Big)gf^{\alpha\gamma\beta}\omega_{\beta}.$$

The relation proves the invariance of $\Gamma(\Phi,\mathcal{B})$ under the gauge transformation of external vector field $\mathcal{B}$ accompanied by the tensor transformations of fields $\mathcal{A},\mathcal{C},\overline{\mathcal{C}},\Phi_{(B)}$,

$$\delta_\omega\mathcal{A}^{\alpha}_{\mu}=gf^{\alpha\gamma\beta}\mathcal{A}^{\gamma}_{\mu}\omega_{\beta},\quad \delta_\omega\mathcal{C}^{\alpha}=gf^{\alpha\gamma\beta}\mathcal{C}^{\gamma}\omega_{\beta},$$
$$\delta_\omega\overline{\mathcal{C}}^{\alpha}=gf^{\alpha\gamma\beta}\overline{\mathcal{C}}^{\gamma}\omega_{\beta},\quad \delta_\omega\Phi^{\alpha}_{(B)}=gf^{\alpha\gamma\beta}\Phi^{\gamma}_{(B)}\omega_{\beta}.$$

Then we have the main property of functional $\Gamma(\mathcal{B}) = \Gamma(\Phi, \mathcal{B})|_{\Phi=0}$ in the background field formalism

$$\Gamma(\mathcal{B}) \int dx \frac{\overleftarrow{\delta}}{\delta \mathcal{B}^{\alpha}_{\mu}} D^{\alpha\beta}_{\mu}(\mathcal{B}) \omega_{\beta} = 0,$$

i.e. the gauge invariance of background effective action.

The relations between the standard generating functionals and the analogous quantities in the background field formalism are established with modification of gauge functions likes to $\chi_{\alpha}(A, \mathcal{B}) \to \chi_{\alpha}(A, \mathcal{B}) - \partial_{\mu} \mathcal{B}^{\alpha}_{\mu}$ ([L.F. Abbott, (1981).]).

Now we discuss the gauge invariance of average effective action for the FRG [Wetterich (1991,1993)] in the background field formalism. Of course this issue is not new (see, for example, [Freire, Litim, Pavlowski (2000),Wetterich (2018)], but we are going to demonstrate that requirement of gauge invariance of the average effective action restricts a tensor structure of regulator functions being essential objects of the approach.

One of main ideas of the FRG approach was to modify behavior of propagators of vector and ghost fields in IR and UV regions with the help of addition of a scale-dependent regulator action being quadratic in the fields.

$$S_{FP}(\phi,\mathcal{B}) \ \rightarrow \ S_k(\phi,\mathcal{B}) = S_{FP}(\phi,\mathcal{B}) + S_k(\phi),$$

The scale-dependent regulator action

$$S_k(\phi) = \int dx \Big[\frac{1}{2} A^{\alpha}_{\mu}(x) R^{(1)\mu\nu}_{k\ \alpha\beta}(x) A^{\beta}_{\nu}(x) + \overline{C}^{\alpha}(x) R^{(2)}_{k\ \alpha\beta}(x) C^{\beta}(x)\Big]$$

is defined by regulator functions $R^{(1)\mu\nu}_{k\ \alpha\beta}(x), R^{(2)}_{k\ \alpha\beta}(x)$ which are independent of fields. The regulator functions obey the properties

$$R^{(1)\mu\nu}_{k\ \alpha\beta} = R^{(1)\nu\mu}_{k\ \beta\alpha},$$

$$\lim_{k\rightarrow 0} R^{(1)\mu\nu}_{k\ \alpha\beta} = 0, \quad \lim_{k\rightarrow 0} R^{(2)}_{k\ \alpha\beta} = 0.$$

Let us require the invariance of $S_k(\phi)$ under backgroud gauge transformations

$$\delta_\omega S_k(\phi) = 0.$$

It leads to the equations

$$f^{\alpha\beta\sigma} R^{(1)\mu\nu}_{k\ \sigma\gamma} + R^{(1)\mu\nu}_{k\ \alpha\sigma} f^{\sigma\gamma\beta} = 0, \quad f^{\alpha\beta\sigma} R^{(2)}_{k\ \sigma\gamma} + R^{(2)}_{k\ \alpha\sigma} f^{\sigma\gamma\beta} = 0,$$

which can be presented in terms of Lie group generators $(t^\alpha)_{\beta\gamma} = f^{\beta\alpha\gamma}$ as

$$[t^\beta, R^{(1)\mu\nu}_k]_{\alpha\gamma} = 0, \quad [t^\beta, R^{(2)}_k]_{\alpha\gamma} = 0.$$

Due to the Schur's lemma it follows that

$$R^{(1)\mu\nu}_{k\ \alpha\beta} = \delta_{\alpha\beta} R^{(1)\mu\nu}_k, \quad R^{(2)}_{k\ \alpha\beta} = \delta_{\alpha\beta} R^{(2)}_k,$$

Therefore the regulator action should be of the form

$$S_k(\phi) = \int dx \Big[\frac{1}{2} A^{\alpha}_{\mu}(x) R^{(1)\mu\nu}_{k}(x) A^{\alpha}_{\nu}(x) + \overline{C}^{\alpha}(x) R^{(2)}_{k}(x) C^{\alpha}(x)\Big]$$

to retain the invariance. In this case the full action

$$S_k(\phi, \mathcal{B}) = S_{FP}(\phi, \mathcal{B}) + S_k(\phi),$$

is invariant under background gauge transformations,

$$\delta_{\omega} S_k(\phi, \mathcal{B}) = 0.$$

The invariance allows to extend all previous result concerning the gauge invariance problem on quantum level and to state the gauge invariance of the vacuum functional $Z_k(\mathcal{B}) = Z_k(0, \mathcal{B})$ and average effective action $\Gamma_k(\mathcal{B}) = \Gamma_k(\Phi, \mathcal{B})|_{\Phi=0}$

$$\delta_{\omega} Z_k(\mathcal{B}) = 0, \quad \delta_{\omega} \mathcal{B}^{\alpha}_{\mu} = D^{\alpha\beta}_{\mu}(\mathcal{B})\omega_{\beta}, \quad \delta_{\omega}\Gamma_k(\mathcal{B}) = 0.$$

Gauge dependence of average effective action

Now we are going to investigate the gauge dependence problem for the FRG approach in the background field formalism. Standard formulation of this method being applied to gauge theories leads to ill defined the average effective action and the corresponding flow equation which still remain gauge dependent even on-shell [PML, Shapiro (2013), PML, Merzlikin (2015)].

Let us consider the generating functionals of Green functions supplied with label "Ψ"

$$Z_{k\Psi}(J,\mathcal{B}) = \int d\phi \; \exp\Big\{\frac{i}{\hbar}\big[S_k(\phi,\mathcal{B}) + J\phi\big]\Big\},$$

For another choice of the gauge fixing functional $\Psi \to \Psi + \delta\Psi$

$$Z_{k\Psi+\delta\Psi}(J,\mathcal{B}) = \int d\phi \; \exp\Big\{\frac{i}{\hbar}\big[S_k(\phi,\mathcal{B}) + \delta\Psi(\phi,\mathcal{B})\hat{R}(\phi,\mathcal{B}) + J\phi\big]\Big\}$$

Now we are trying to compensate additional term $\delta\Psi\hat{R}$ using the changes of variables in the functional integral related closely to the symmetry of actions $S_{FP}(\phi,\mathcal{B})$ and $S_k(\phi,\mathcal{B})$. In the functional integral we make first a change of variables in the form of the BRST transformations but trading the constant parameter μ to a functional $\Lambda=\Lambda(\phi,\mathcal{B})$. The action $S_{FP}(\phi,\mathcal{B})$ is invariant under such change of variables but the action $S_k(\phi)$ is not invariant, with the following variation

$$\begin{aligned}\delta S_k(\phi) \;&=\; \int dx\Big(A^{\alpha}_{\mu}R^{(1)\mu\nu}D^{\alpha\beta}_{\nu}(A+\mathcal{B})C^{\beta}+\frac{1}{2}\overline{C}^{\alpha}R^{(2)}_{k}f^{\alpha\beta\gamma}C^{\beta}C^{\gamma}-\\ &\quad -B^{\alpha}R^{(2)}_{k}C^{\alpha}\Big)\Lambda.\end{aligned}$$

The corresponding Jacobian J_1 reads

$$J_1=\exp\Big\{-\int dx\Big(\frac{\delta\Lambda}{\delta A^{\alpha}_{\mu}}D^{\alpha\beta}_{\mu}(A+\mathcal{B})C^{\beta}+\frac{1}{2}f^{\alpha\beta\gamma}C^{\beta}C^{\gamma}\frac{\delta\Lambda}{\delta C^{\alpha}}+\frac{\delta\Lambda}{\delta\overline{C}^{\alpha}}B^{\alpha}\Big)\Big\}.$$

We make additionally a change of variables related to gauge transformations but using instead of parameters $\omega_\alpha(x)$ functions $\Omega_\alpha(x) = \Omega_\alpha(x, \phi(x), \mathcal{B}(x))$. The action $S_k(\phi, \mathcal{B})$ is invariant under these transformations but the relevant Jacobian, J_2 is not trivial,

$$J_2 = \exp\Big\{ g f^{\alpha\beta\gamma} \int dx \Big(A^\beta_\mu(x) \frac{\partial \Omega_\gamma(x)}{\partial A^\alpha_\mu(x)} - C^\beta(x) \frac{\partial \Omega_\gamma(x)}{\partial C^\alpha(x)} - \overline{C}^\beta(x) \frac{\partial \Omega_\gamma(x)}{\partial \overline{C}^\alpha(x)} \Big) \Big\}.$$

If the condition,

$$J_1 J_2 \exp\Big\{ \frac{i}{\hbar} \int dx \big[\delta\Psi(\phi, \mathcal{B}) \hat{R}(\phi, \mathcal{B}) + \delta S_k(\phi) \big] \Big\} = 1,$$

is satisfied then the functional $Z_{k\Psi}(\mathcal{B})$ does not depend on gauge fixing functional Ψ.

Having in mind the ghost numbers and Grassmann parities of functional Λ and functions $\Omega_\alpha(x)$

$$\mathrm{gh}(\Lambda) = -1, \quad \mathrm{gh}(\Omega_\alpha(x)) = 0, \quad \varepsilon(\Lambda) = 1, \quad \varepsilon(\Omega_\alpha(x)) = 0,$$

we have the following presentation in the lower power of ghost fields,

$$\Lambda = \Lambda^{(1)} + \Lambda^{(3)}, \quad \Omega_\alpha(x) = \Omega_\alpha^{(0)}(x) + \Omega_\alpha^{(2)}(x),$$
$$\Lambda^{(1)} = \int dx \overline{C}^\alpha(x) \lambda_\alpha^{(1)}(x, A(x), \mathcal{B}(x)),$$
$$\Lambda^{(3)} = \int dx \frac{1}{2} \overline{C}^\alpha(x) \overline{C}^\beta(x) \lambda_{\alpha\beta\gamma}^{(3)}(x, A(x), \mathcal{B}(x)) C^\gamma(x),$$
$$\Omega_\alpha^{(0)}(x) = \Omega_\alpha^{(0)}(x, A(x), \mathcal{B}(x)),$$
$$\Omega_\alpha^{(2)}(x, A(x), \mathcal{B}(x)) = \overline{C}^\beta(x) \omega_{\alpha\beta\gamma}^{(2)}(x, A(x), \mathcal{B}(x)) C^\gamma(x).$$

Vanishing terms which don't depend on ghost fields $C, \overline{C}$ and auxiliary field B leads to the condition

$$\Omega^{(0)}_{\alpha}(x, A(x), \mathcal{B}(x)) = 0.$$

Consider terms linear in B then we obtain

$$\lambda^{(1)}_{\alpha}(x, A(x), \mathcal{B}(x)) = \frac{i}{\hbar}\delta\chi_{\alpha}(x, A(x), \mathcal{B}(x)).$$

In turn analyzing the structures $B\overline{C}C$ we find the expression for $\lambda^{(3)}_{\alpha\beta\gamma}$,

$$\lambda^{(3)}_{\alpha\beta\gamma}(x, A, \mathcal{B}) = R^{(2)}(x)\big(\delta_{\beta\gamma}\lambda^{(1)}_{\alpha}(A, \mathcal{B}) - \delta_{\alpha\gamma}\lambda^{(1)}_{\beta}(A, \mathcal{B})\big),$$
$$\lambda^{(1)}_{\alpha}(A, \mathcal{B}) = \int dx\lambda^{(1)}_{\alpha}(x, A(x), \mathcal{B}(x)).$$

Vanishing structures $\overline{C}C$ leads to algebraic equations for $\omega^{(2)}_{\alpha\beta\gamma}$,

$$f^{\gamma\alpha\sigma}\omega^{(2)}_{\sigma\beta\gamma}(x,A(x),\mathcal{B}(x))+f^{\gamma\beta\sigma}\omega^{(2)}_{\sigma\gamma\alpha}(x,A(x),\mathcal{B}(x))=$$
$$=\frac{i}{g\hbar}D^{\gamma\alpha}_{\nu}(A+\mathcal{B})\big(A^{\gamma}_{\mu}(x)R^{(1)\mu\nu}_{k}(x)\big)\lambda^{(1)}_{\beta}(A,\mathcal{B}).$$

Therefore, we can reduce to zero all terms of the lowest order in fields $C,\overline{C},B$. Unfortunately, in its turn the $\lambda^{(3)}_{\alpha\beta\gamma}$ creates the non-local term of structure $B\overline{C}\,\overline{C}\,C\,C$ which cannot be eliminated in a proposed scheme. It is necessary to add for functional Λ and functions Ω_{α} new terms of higher orders in ghost fields up to infinity. This situation looks unsatisfactory in terms of conventional quantum field theory and we are forced to restrict ourself by the case when $\Omega_{\alpha}=0$ and $\Lambda=\Lambda^{(1)}$.

Then we have

$$Z_{k\Psi+\delta\Psi}(\mathcal{B}) = \int d\phi \ \exp\Big\{\frac{i}{\hbar}\big[S_k(\phi,\mathcal{B}) + \delta S_k(\phi)\big]\Big\},$$
$$Z_{k\Psi}(\mathcal{B}) \neq Z_{k\Psi+\delta\Psi}(\mathcal{B}).$$

Vacuum functional in the FRG approach within the background field formalism remains gauge dependent similar to the standard formulation [PML, Shapiro (2013), PML, Merzlikin (2015)]. The same statement is valid for elements of S-matrix due to the equivalence theorem [Kallosh, Tyutin (1973)]. There are no problems deriving a modified Ward identity which is a consequence of BRST invariance of action $S_{FP}(\phi,\mathcal{B})$ and identities which follow from gauge invariance of the action $S_k(\phi,\mathcal{B})$ as well as to study gauge dependence of average effective action on-shell. We omit all these issues of the FRG approach because they do not help to solve the gauge dependence problem of results which are obtained within this method.

Conclusions

- Investigation of gauge invariance and gauge dependence problems for Yang-Mills theories in the FRG apprpach formulated in the background field method was given.
- It was shown the gauge invariance of average effective action can be achived by using special tenzor structure of regulator functions involving in the FRG approach.
- It was proven that the average effectiva action being gauge invariant still remains dependent on gauge fixing functions even on-shell. It means that S-matrix depends on gauge. and, in particular, vacuum expectation values of gauge invariant operators such as $F^a_{\mu\nu}F^{a\mu\nu}$ do depend on gauge.

Thank you for attention!