FORMAÇÃO DE ESTRUTURAS NO UNIVERSO

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Inverno Astrofísico

Aula I

- Quais são as estruturas que estamos querendo entender? Galáxias e aglomerados de galáxias e sua distribuição em grande escala

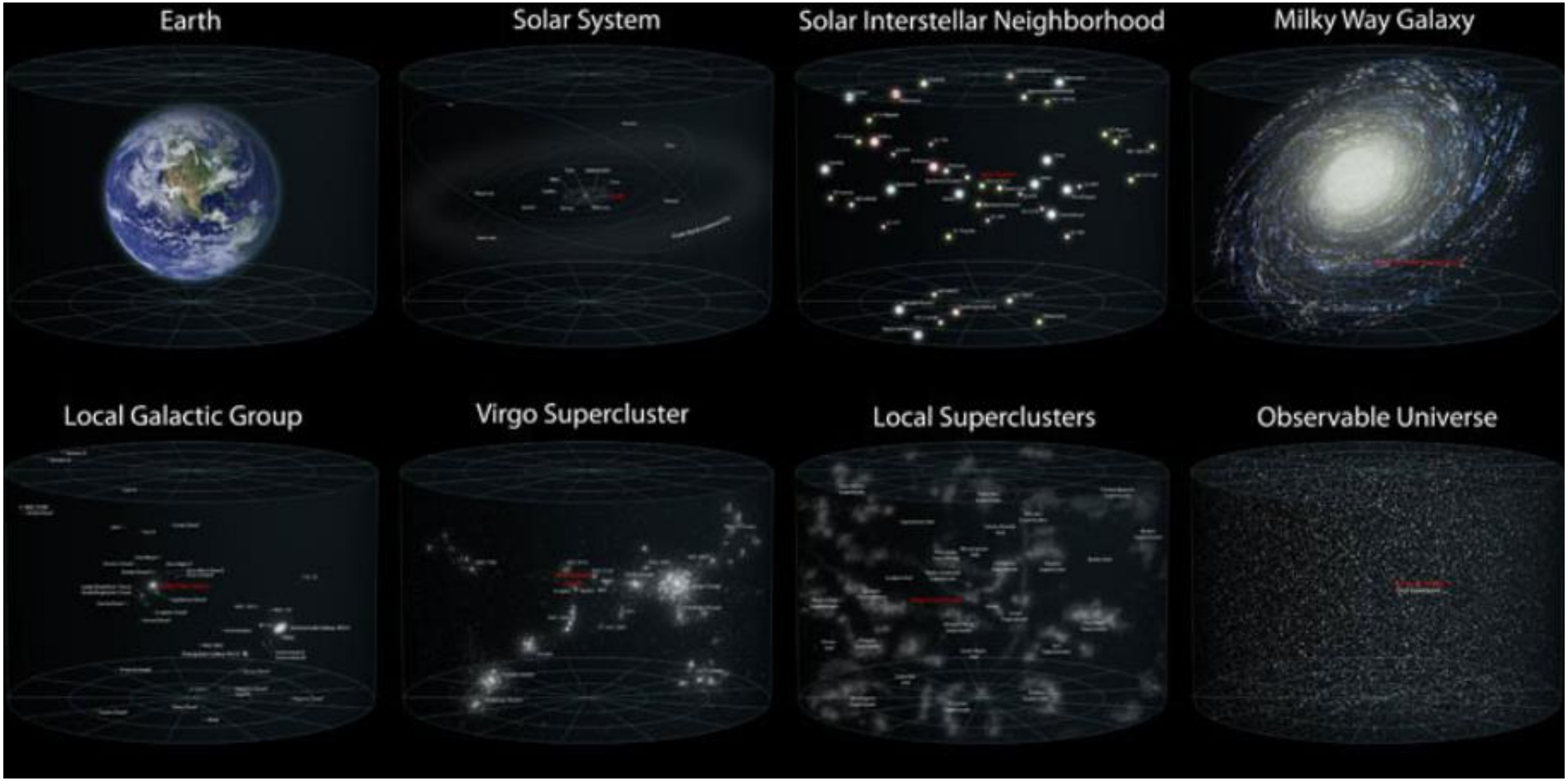
Aula II

- Entender como pequenas perturbações existentes na distribuição quase homogênea de densidade de matéria no universo primordial evoluem até o ponto de formar uma estrutura como conhecemos

Aula III

- Efeito Mészaros -> Bônus: Precisamos de matéria escura

- Estágio não-linear de formação de estruturas. Entender o porquê de um excesso de densidade na distribuição de matéria se desacopla do fluxo de Hubble e colapsa para formar um objeto astronômico.


Earth
Solar System
Solar Interstellar Neighborhood
Milky Way Galaxy
Local Galactic Group
Virgo Supercluster
Local Superclusters
Observable Universe

Princípio cosmológico
vs
Escala de homogeneidade do universo

A structure in the early universe at z $\sim$ 1.3 that exceeds the homogeneity scale of the R-W concordance cosmology

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Luis E. Campusano,[2] Ilona K. Söchting[3] and Matthew J. Graham[4]
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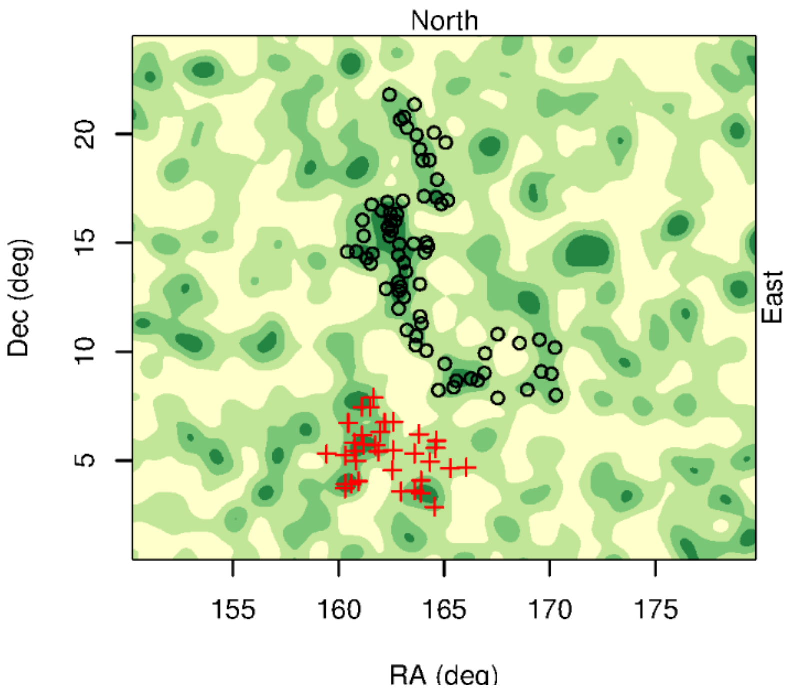
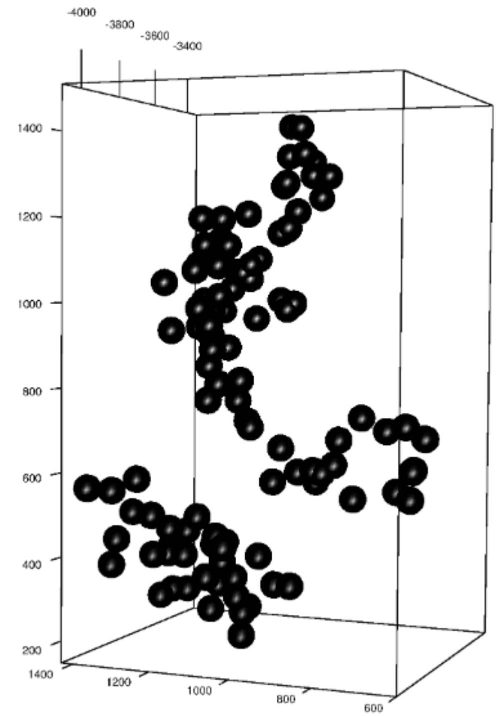
Clowes R. G., Harris K. A., Raghunathan S., Campusano L. E.,
Soechting I. K., Graham M. J., 2013, MNRAS, 429, 2910

ABSTRACT
A Large Quasar Group (LQG) of particularly large size and high membership has been identified in the DR7QSO catalogue of the Sloan Digital Sky Survey. It has characteristic size (volume$^{1/3}$) $\sim$ 500 Mpc (proper size, present epoch), longest dimension $\sim$ 1240 Mpc, membership of 73 quasars, and mean redshift $\bar{z} = 1.27$. In terms of both size and membership it is the most extreme LQG found in the DR7QSO catalogue for the redshift range $1.0 \leqslant z \leqslant 1.8$ of our current investigation. Its location on the sky is $\sim 8.8°$ north ($\sim$ 615 Mpc projected) of the Clowes & Campusano LQG at the same redshift, $\bar{z} = 1.28$, which is itself one of the more extreme examples. Their boundaries approach to within $\sim 2°$ ($\sim$ 140 Mpc projected). This new, huge LQG appears to be the largest structure currently known in the early universe. Its size suggests incompatibility with the Yadav et al. scale of homogeneity for the concordance cosmology, and thus challenges the assumption of the cosmological principle.





Mon.Not.Roy.Astr.Soc.434:398,2013

Seeing patterns in noise: Gigaparsec-scale 'structures' that do not violate homogeneity

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ABSTRACT
Clowes et al. (2013) have recently reported the discovery of a Large Quasar Group (LQG), dubbed the Huge-LQG, at redshift $z \sim 1.3$ in the Data Release 7 quasar catalogue of the Sloan Digital Sky Survey. On the basis of its characteristic size ~ 500 Mpc and longest dimension > 1 Gpc, it is claimed that this structure is incompatible with large-scale homogeneity and the cosmological principle. If true, this would represent a serious challenge to the standard cosmological model. However, the homogeneity scale is an average property which is not necessarily affected by the discovery of a single large structure. I clarify this point and provide the first fractal dimension analysis of the DR7 quasar catalogue to demonstrate that it is in fact homogeneous above scales of at most $130\,h^{-1}$ Mpc, which is much less than the upper limit for ΛCDM. In addition, I show that the algorithm used to identify the Huge-LQG regularly finds even larger clusters of points, extending over Gpc scales, in explicitly homogeneous simulations of a Poisson point process with the same density as the quasar catalogue. This provides a simple null test to be applied to any cluster thus found in a real catalogue, and suggests that the interpretation of LQGs as 'structures' is misleading.

3 TESTING HOMOGENEITY WITI CATALOGUES

3.1 Fractal analysis

The simplest test of homogeneity that cai set is based on the average of the numbe $N_i(< R)$ contained within a sphere of ra member of the point set, with the requiren lies within the distribution of points:

$$N(< R) = \frac{1}{M} \sum_{i=1}^{M} N_i(< R) \,,$$

where M is the number of sphere centres. For a homogeneous distribution $N(< R) \propto R^D$, where D is the number of dimensions, three in this case. The correlation dimension $D_2(R)$ is calculated as the derivative

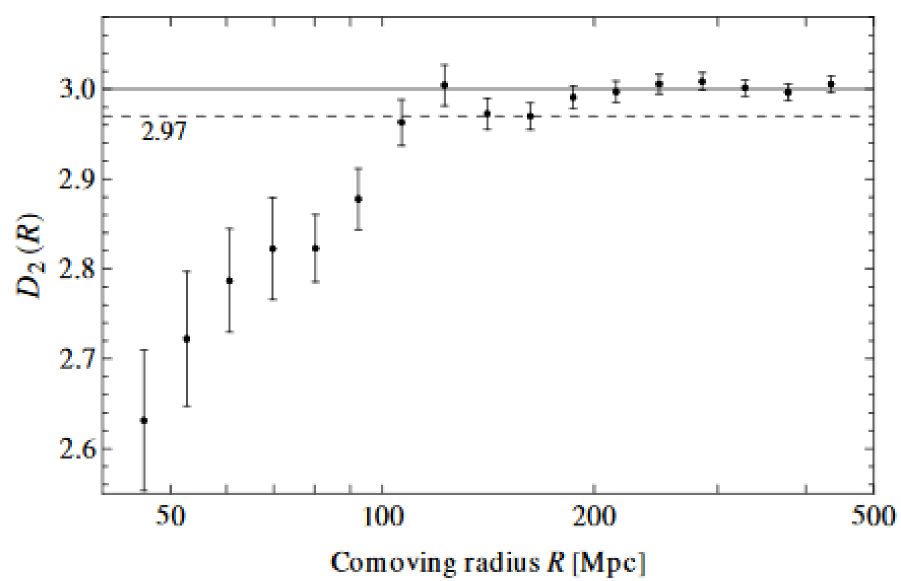
$$D_2(R) = \frac{d \ln N(< R)}{d \ln R} \,, \quad (2)$$

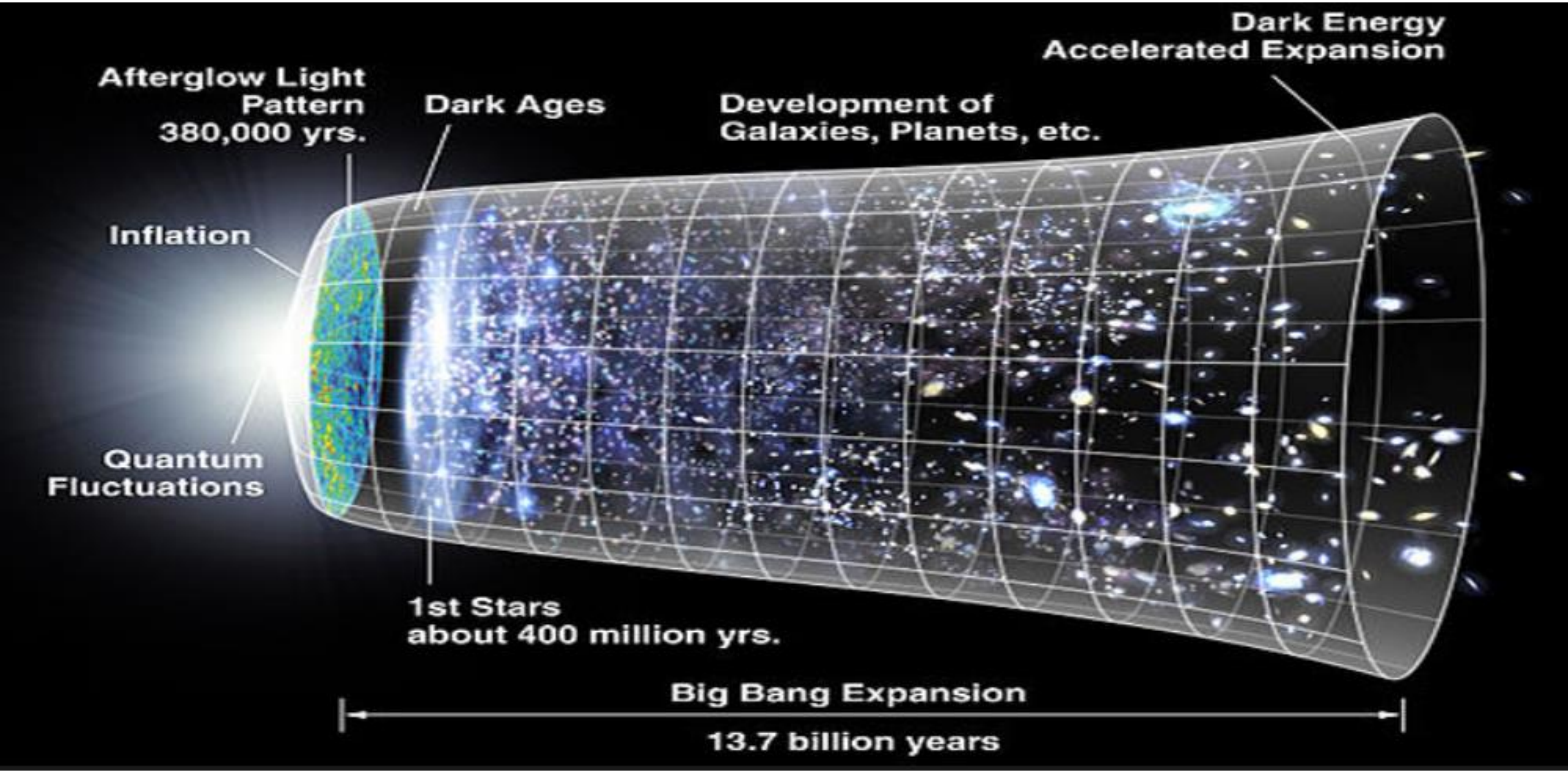
and quantifies the deviation from this homogeneous scaling.

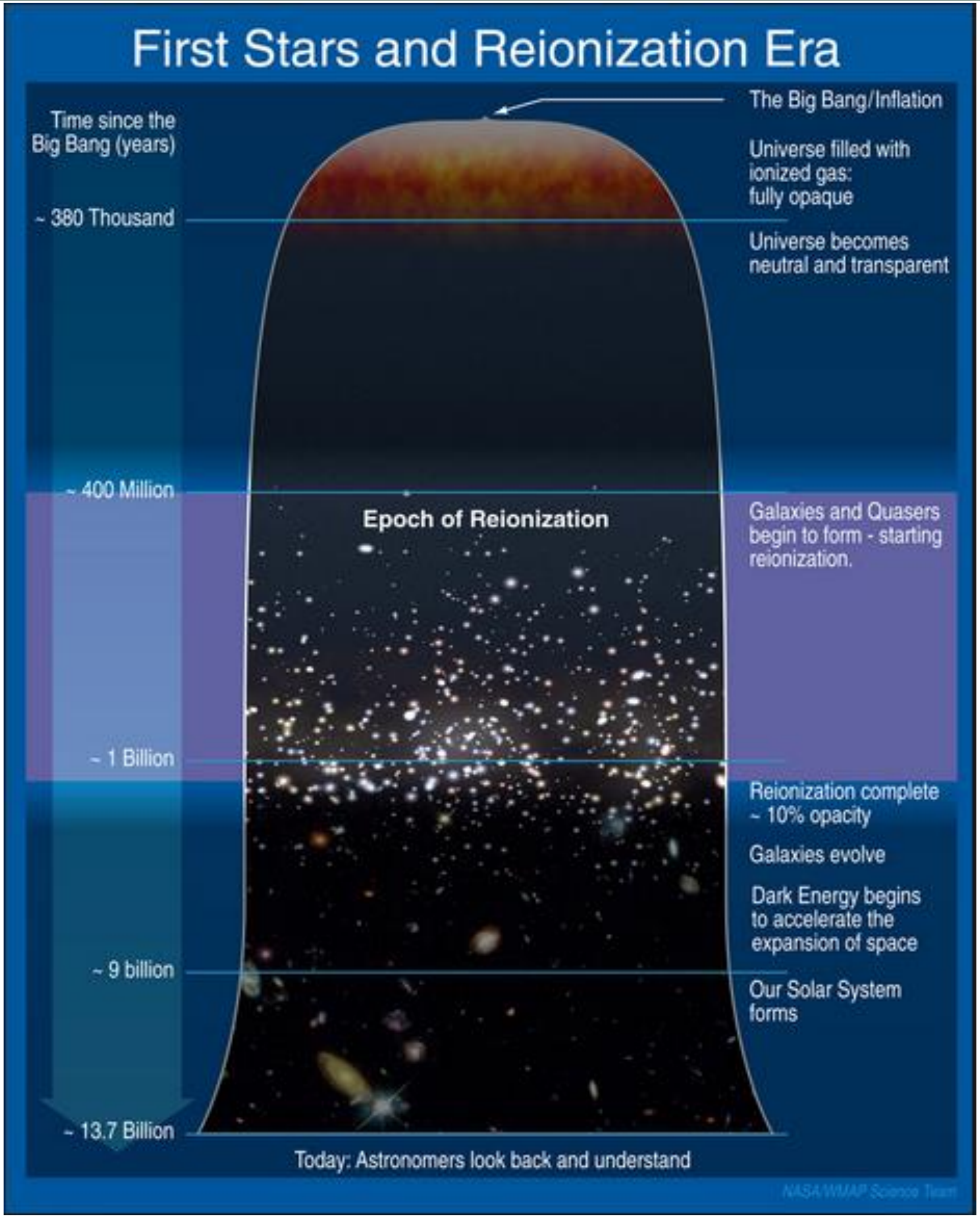
For any given catalogue of objects that trace the matter density of the Universe, $N(< R)$ can be related to the two-point correlation function $\xi(r)$ by

$$N(< R) = \bar{\rho} \int_0^R \left(1 + b^2 \xi(r)\right) 4\pi r^2 dr \,, \quad (3)$$

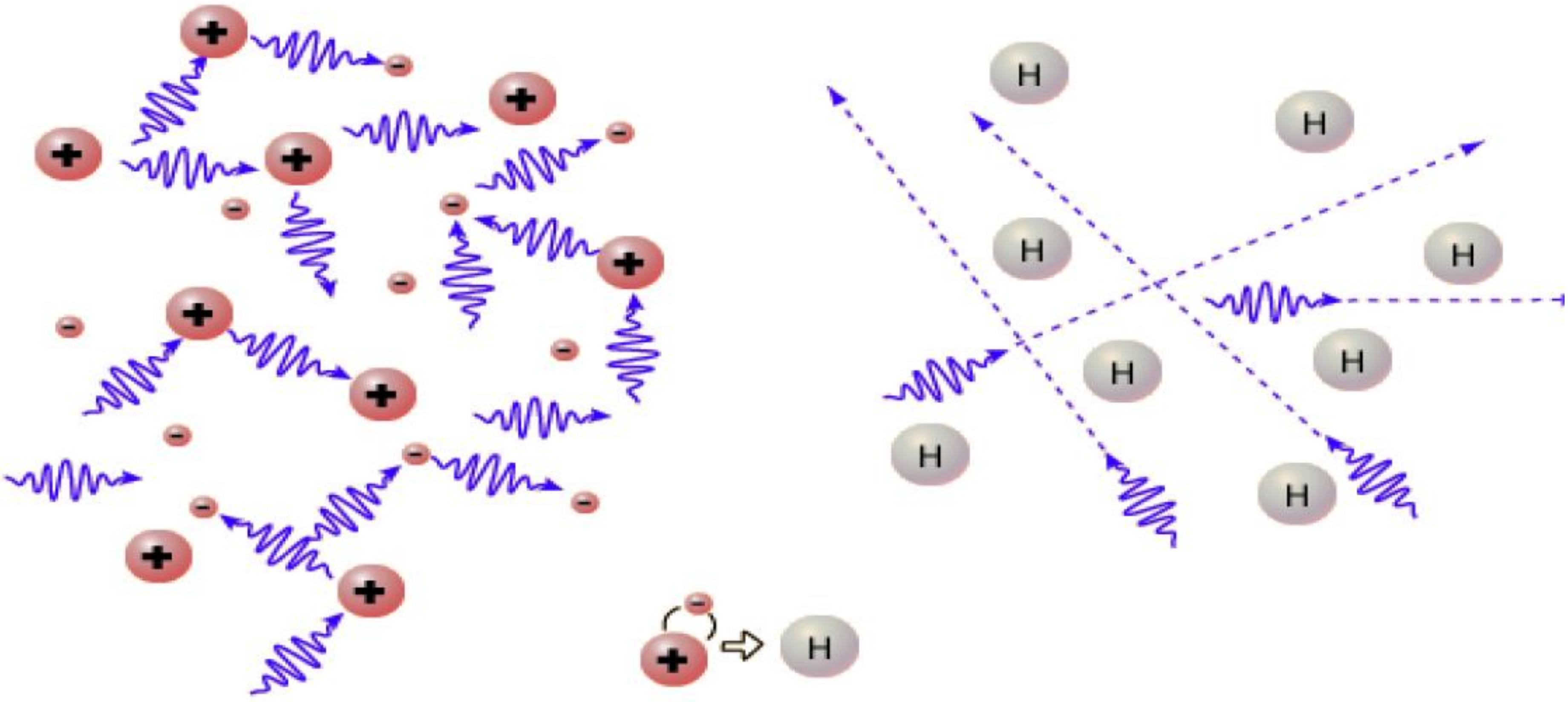
where $\bar{\rho}$ is the mean matter density and b is the bias of the tracer population. Note that the relationship in eq. (3) requires the as-




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years


First Stars and Reionization Era
Time since the Big Bang (years)
~ 380 Thousand
~ 400 Million
~ 1 Billion
~ 9 billion
~ 13.7 Billion
The Big Bang/Inflation
Universe filled with ionized gas: fully opaque
Universe becomes neutral and transparent
Epoch of Reionization
Galaxies and Quasers begin to form - starting reionization.
Reionization complete ~ 10% opacity
Galaxies evolve
Dark Energy begins to accelerate the expansion of space
Our Solar System forms
Today: Astronomers look back and understand

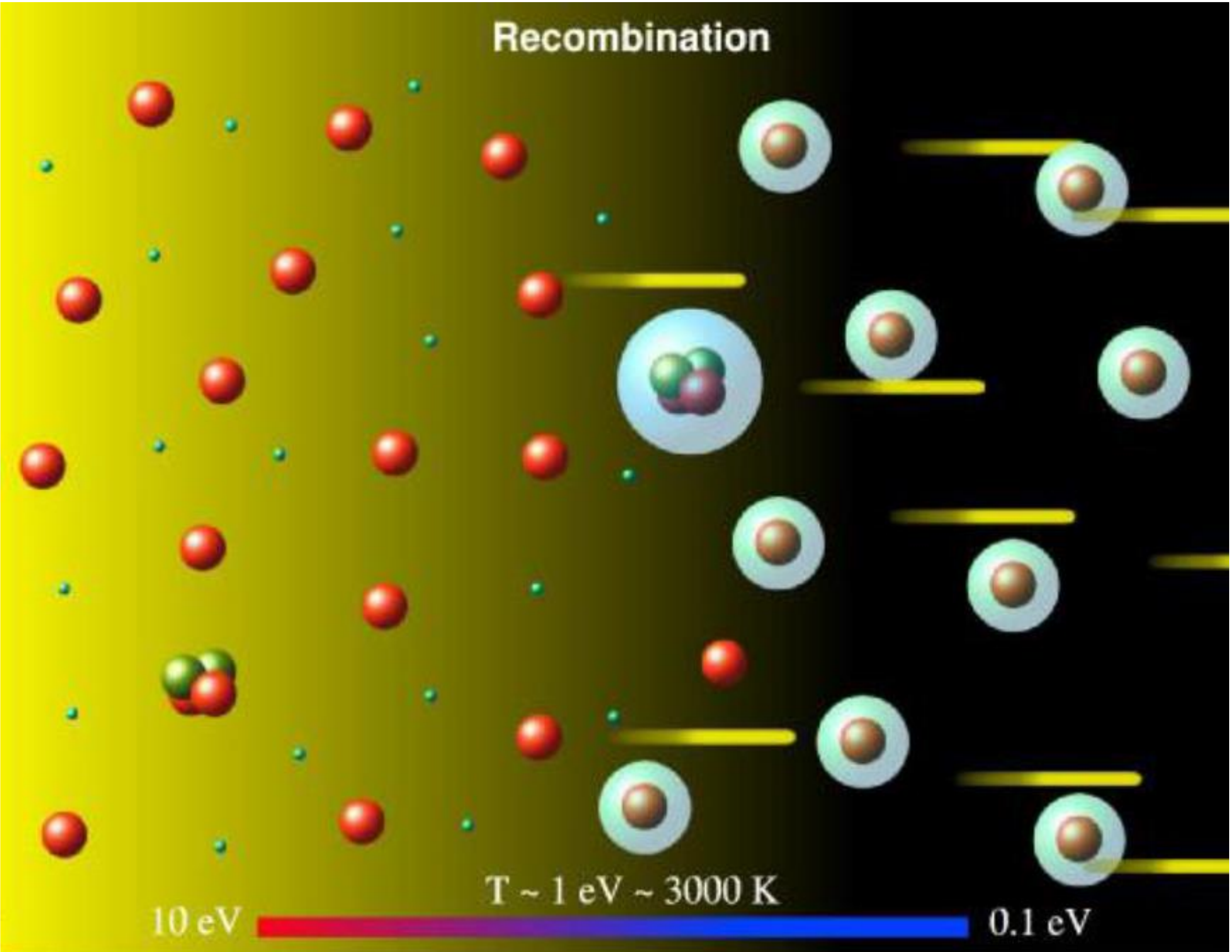
Superfície de último espalhamento

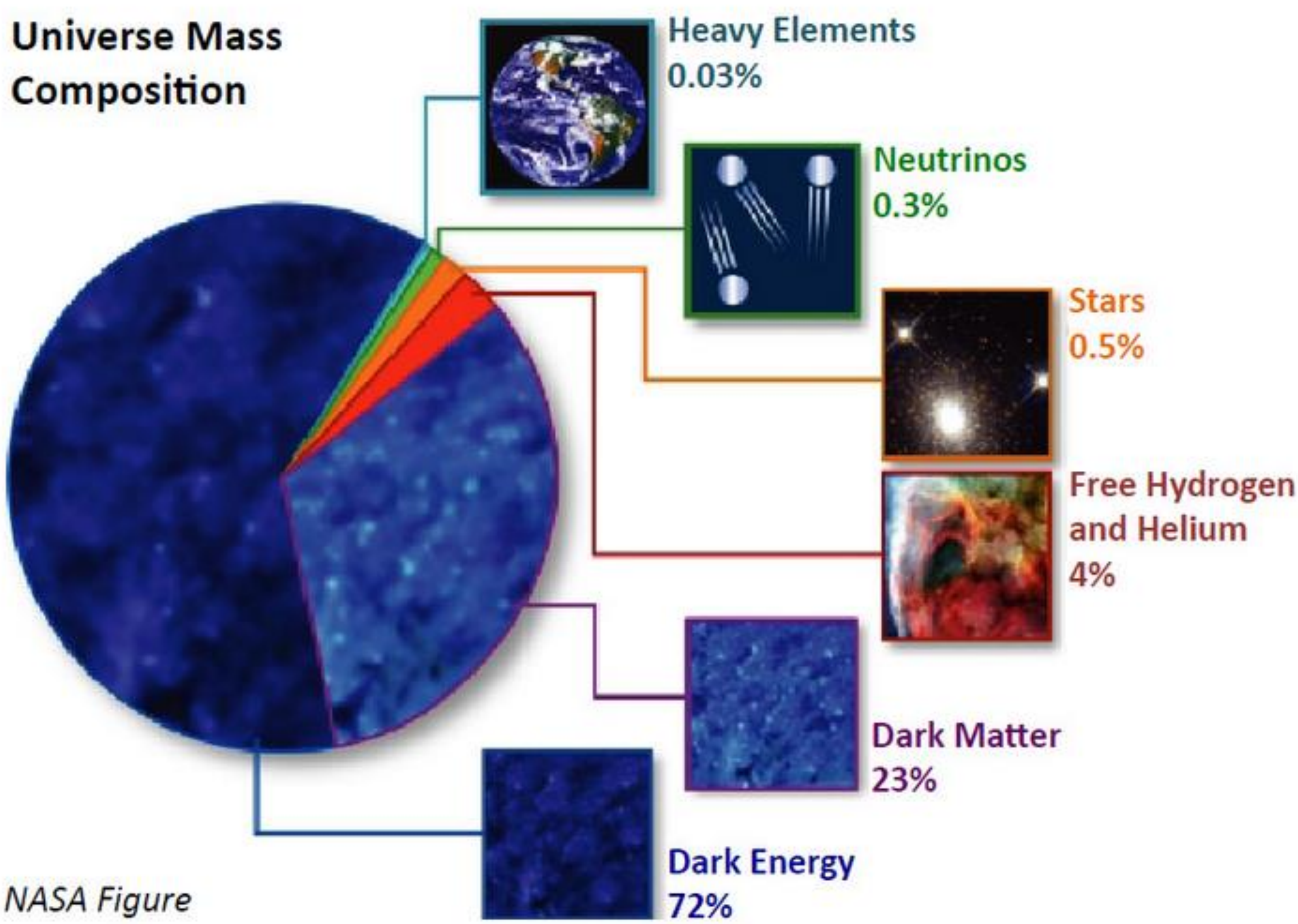


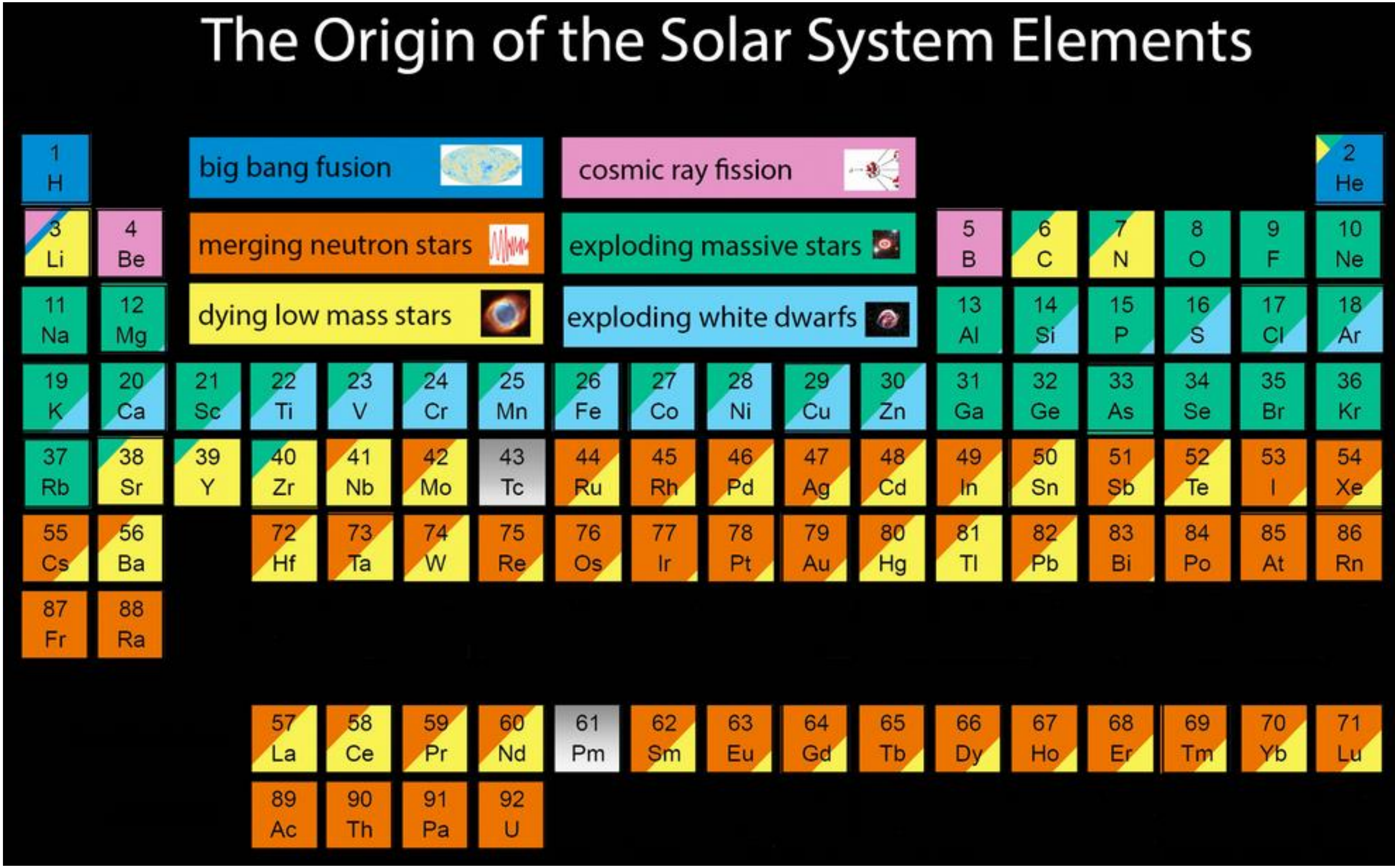
At high temperatures, a hot plasma of charged particles interacts strongly with the radiation. That effectively confines it in the interior of stars and in the early universe.

Below about 3000K, protons and electrons can combine into neutral hydrogen.

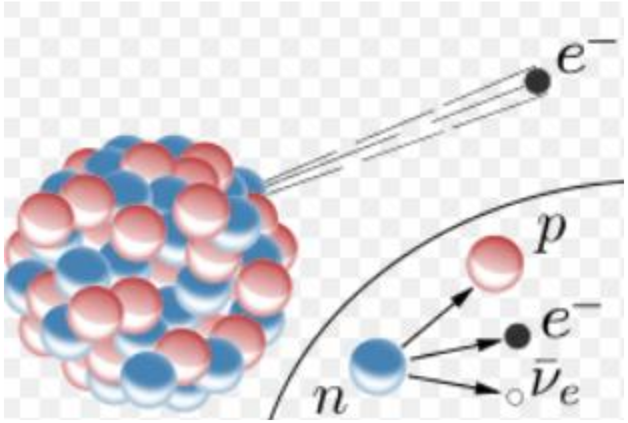
Photons can travel large distances in the neutral hydrogen, so the confinement is effectively broken. Photons can move freely throughout the space.

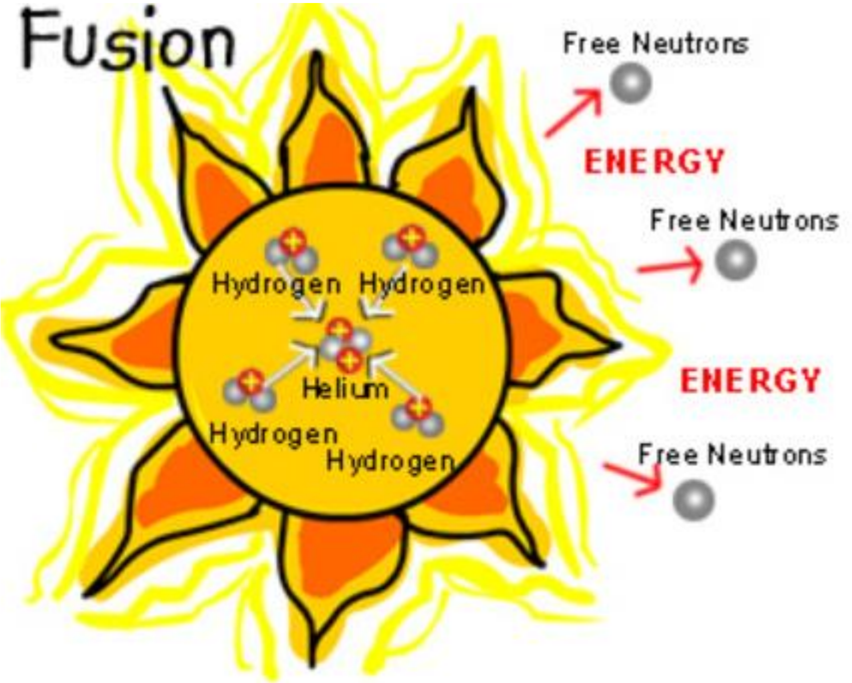

Recombination
T ~ 1 eV ~ 3000 K
10 eV
0.1 eV


Universe Mass
Composition
Heavy Elements
0.03%
Neutrinos
0.3%
Stars
0.5%
Free Hydrogen
and Helium
4%
Dark Matter
23%
Dark Energy
72%
NASA Figure


The Origin of the Solar System Elements
big bang fusion
cosmic ray fission
merging neutron stars
exploding massive stars
dying low mass stars
exploding white dwarfs
1 H
2 He
3 Li
4 Be
5 B
6 C
7 N
8 O
9 F
10 Ne
11 Na
12 Mg
13 Al
14 Si
15 P
16 S
17 Cl
18 Ar
19 K
20 Ca
21 Sc
22 Ti
23 V
24 Cr
25 Mn
26 Fe
27 Co
28 Ni
29 Cu
30 Zn
31 Ga
32 Ge
33 As
34 Se
35 Br
36 Kr
37 Rb
38 Sr
39 Y
40 Zr
41 Nb
42 Mo
43 Tc
44 Ru
45 Rh
46 Pd
47 Ag
48 Cd
49 In
50 Sn
51 Sb
52 Te
53 I
54 Xe
55 Cs
56 Ba
72 Hf
73 Ta
74 W
75 Re
76 Os
77 Ir
78 Pt
79 Au
80 Hg
81 Tl
82 Pb
83 Bi
84 Po
85 At
86 Rn
87 Fr
88 Ra
57 La
58 Ce
59 Pr
60 Nd
61 Pm
62 Sm
63 Eu
64 Gd
65 Tb
66 Dy
67 Ho
68 Er
69 Tm
70 Yb
71 Lu
89 Ac
90 Th
91 Pa
92 U

Partículas: Neutrinos





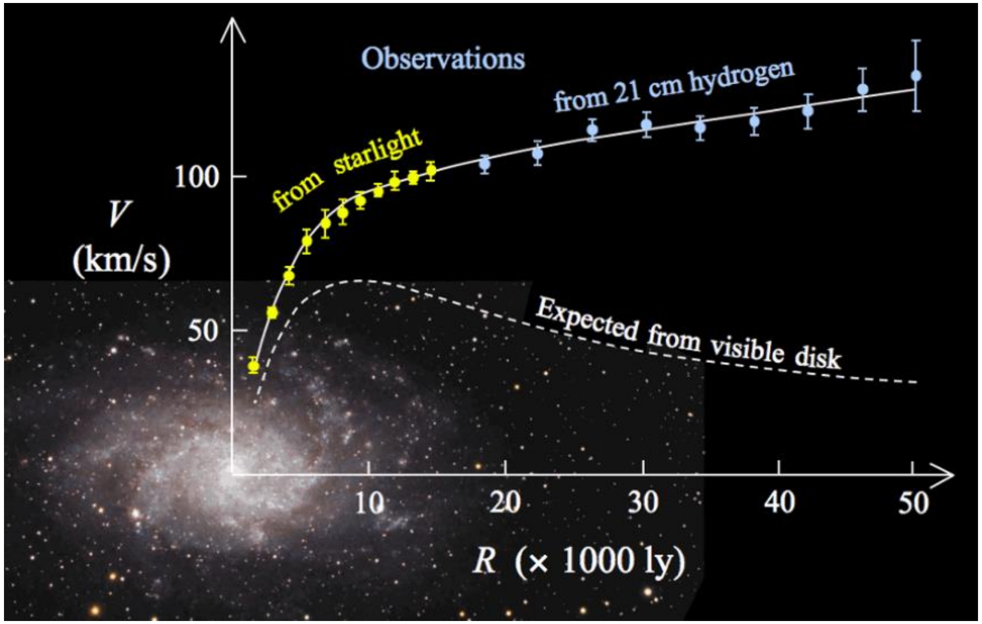
Partículas: Matéria Escura

Fritz Zwicky (1929)

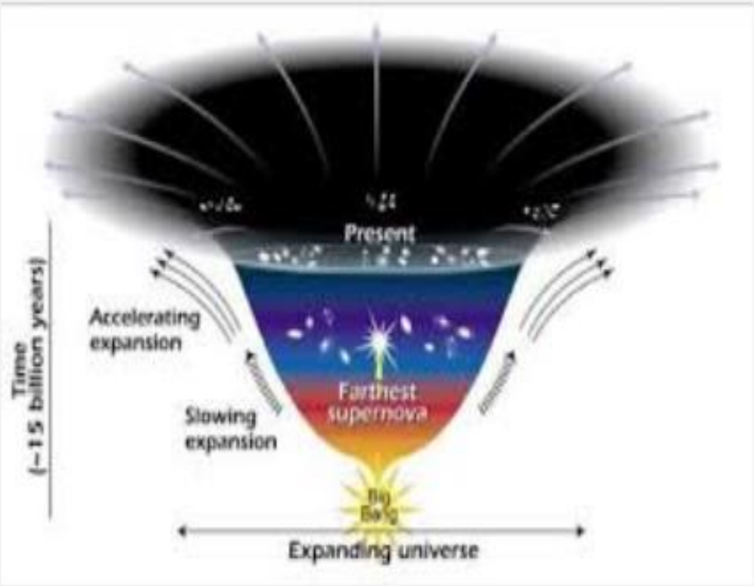
Aglomerados de Galáxias: Mais massa do que se podia ver

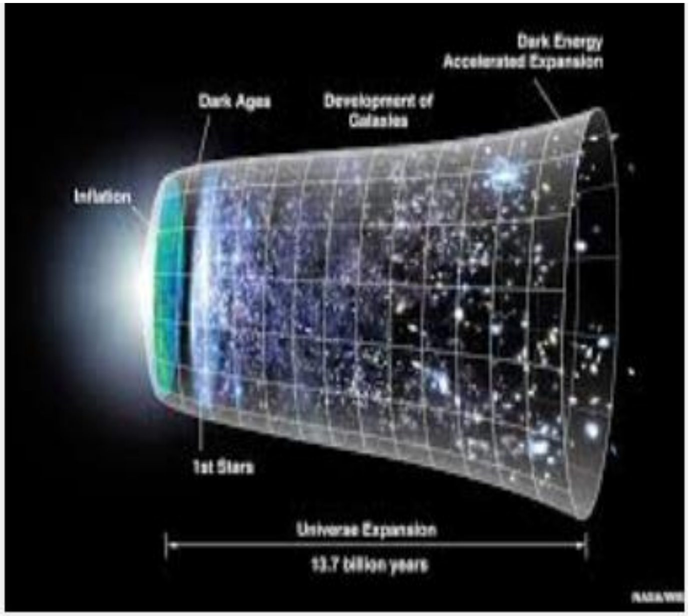
Anos de 1970

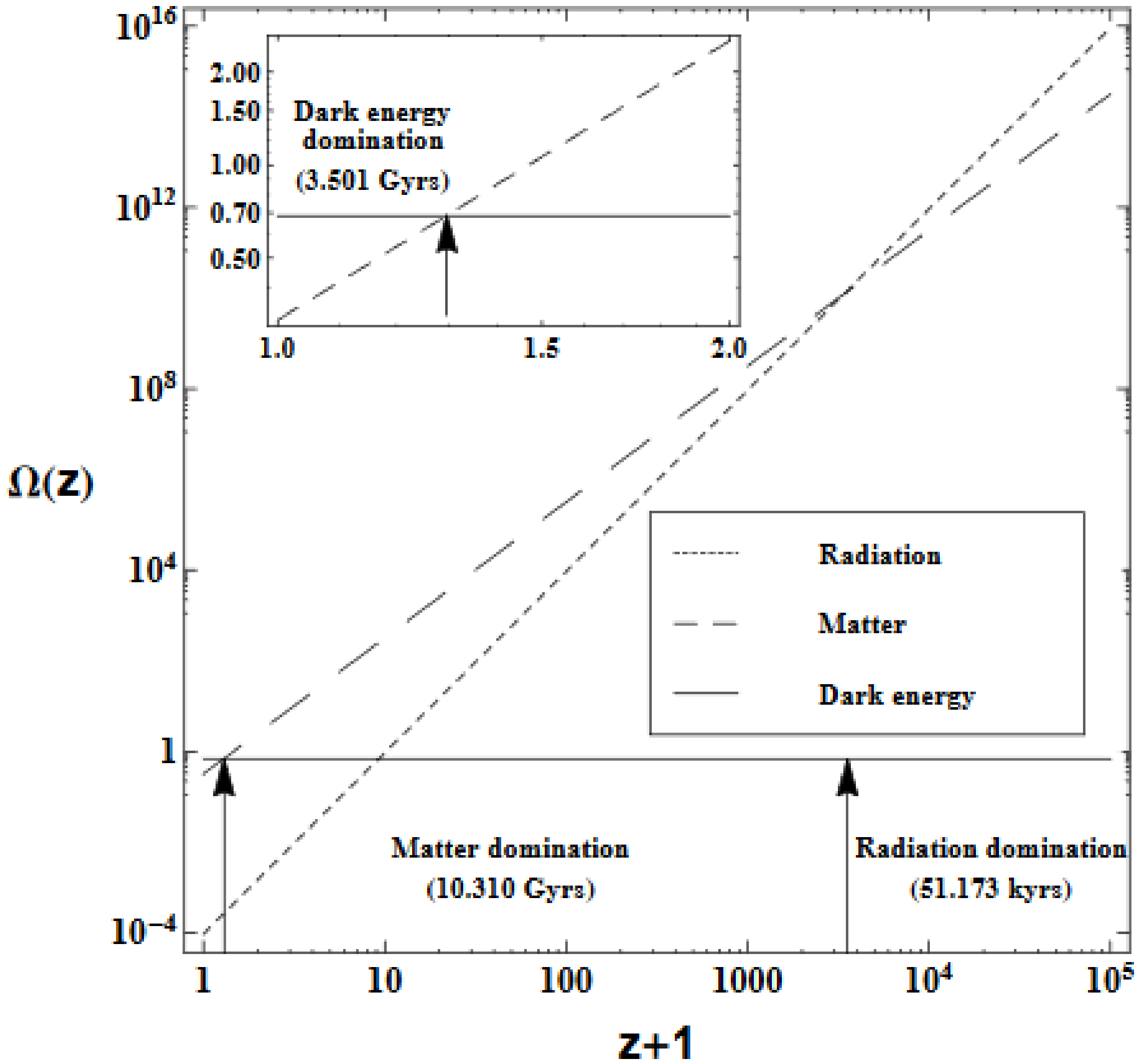
Rotação de Galáxias:



Energia: Energia escura







$$\Omega_i(z) = \frac{\rho_i(z)}{\rho_{c0}},$$

$$\rho_{c0} = \frac{3}{8\pi G} H_0^2,$$

FIG. 1: Evolution of fractionary energy densities Ω as a function of redshift.

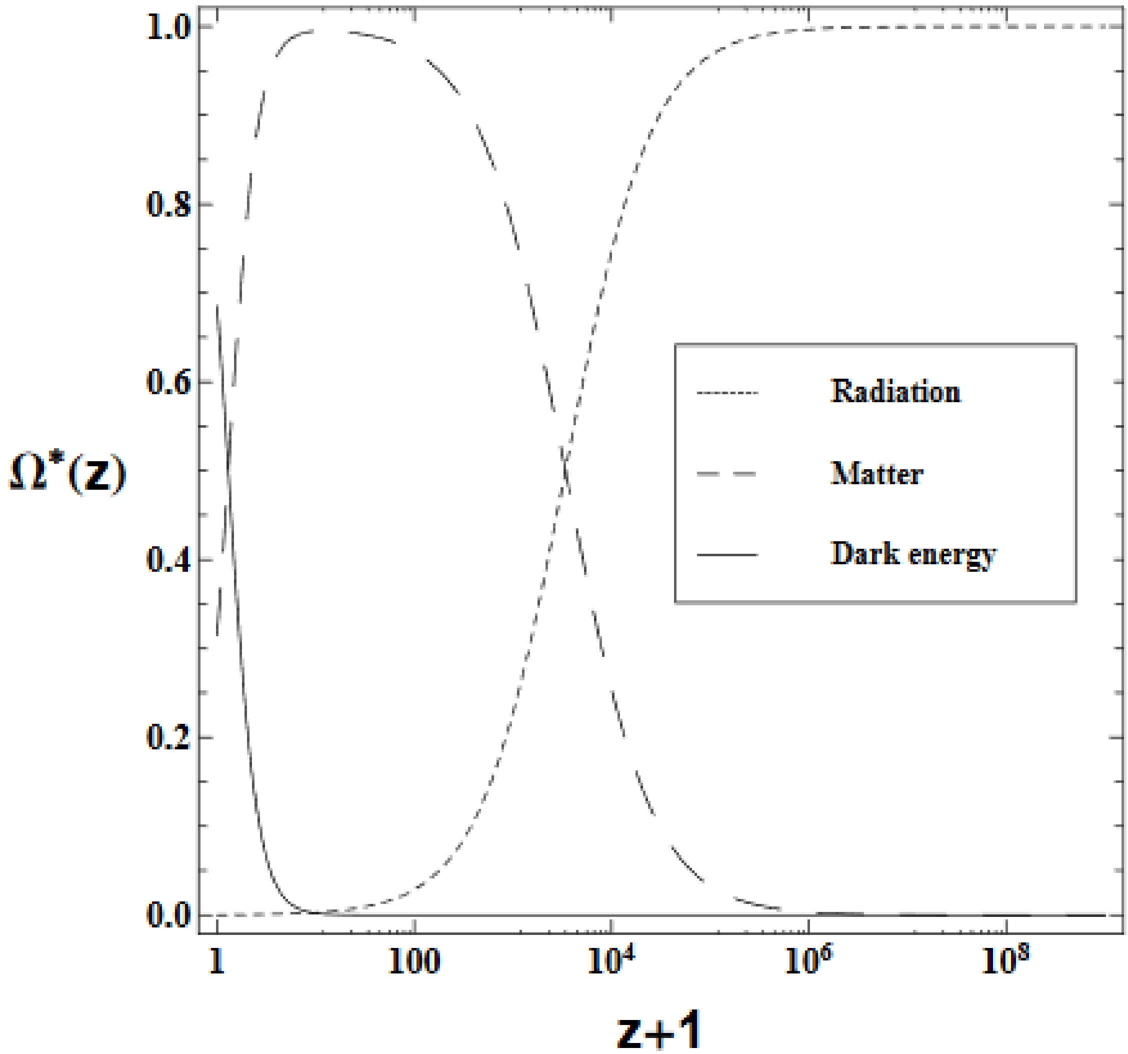


FIG. 2: Evolution of fractionary energy densities $\Omega^{\star}$ as a function of redshift.

$$\Omega_i^{\star}(z) = \frac{\rho_i(z)}{\rho_c(z)},$$

$$\rho_c(z) = \frac{3}{8\pi G} H^2(z)$$

$$\Omega_r^{\star}(z) + \Omega_m^{\star}(z) + \Omega_k^{\star}(z) + \Omega_\Lambda^{\star}(z) = 1.$$

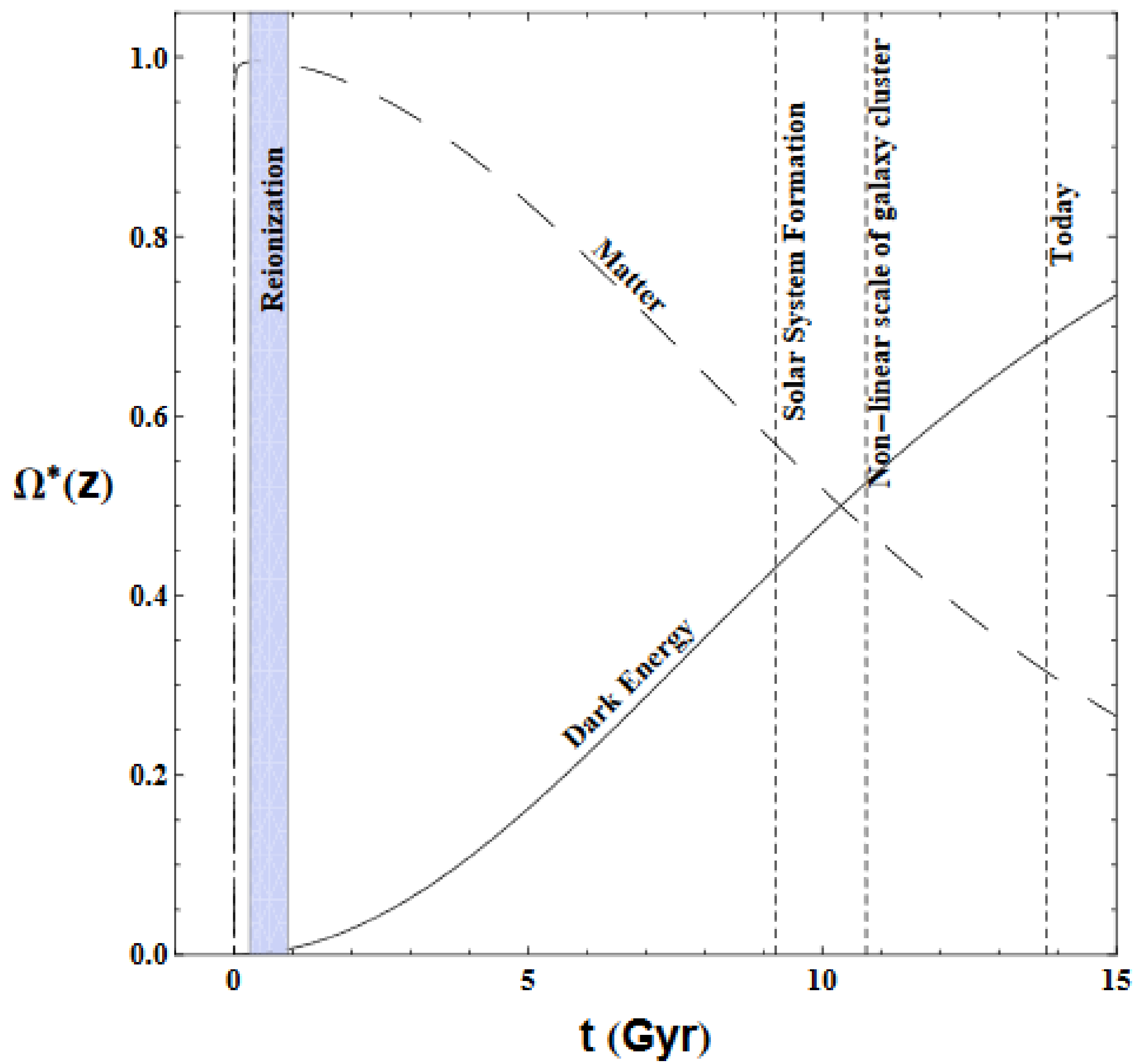


FIG. 3: Evolution of fractionary energy densities as a function of the cosmic time.

$$\int_0^t dt = \int_0^a \frac{da'}{a' H(a')} \, .$$

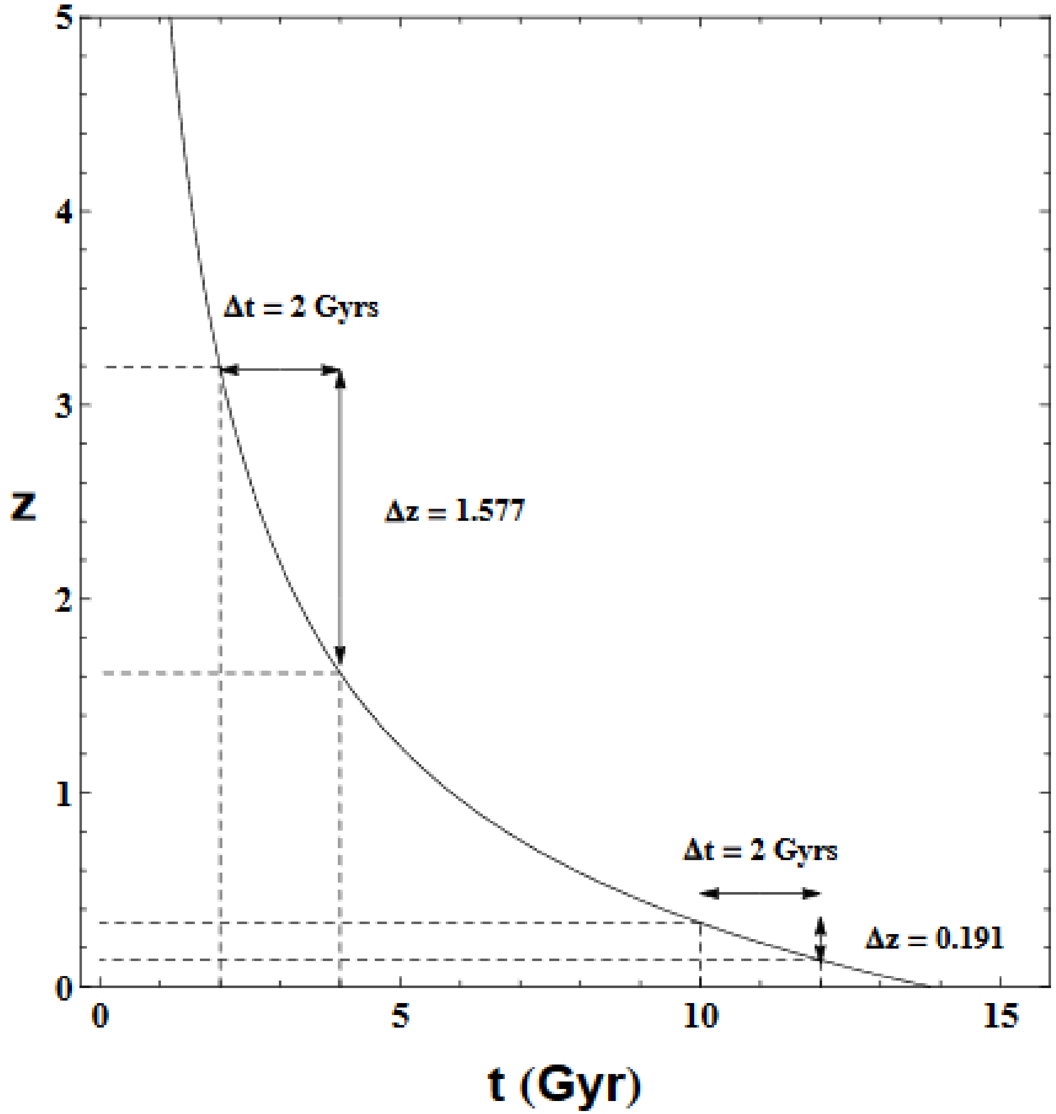


FIG. 4: Variation of the redshift as a function of the cosmic time.

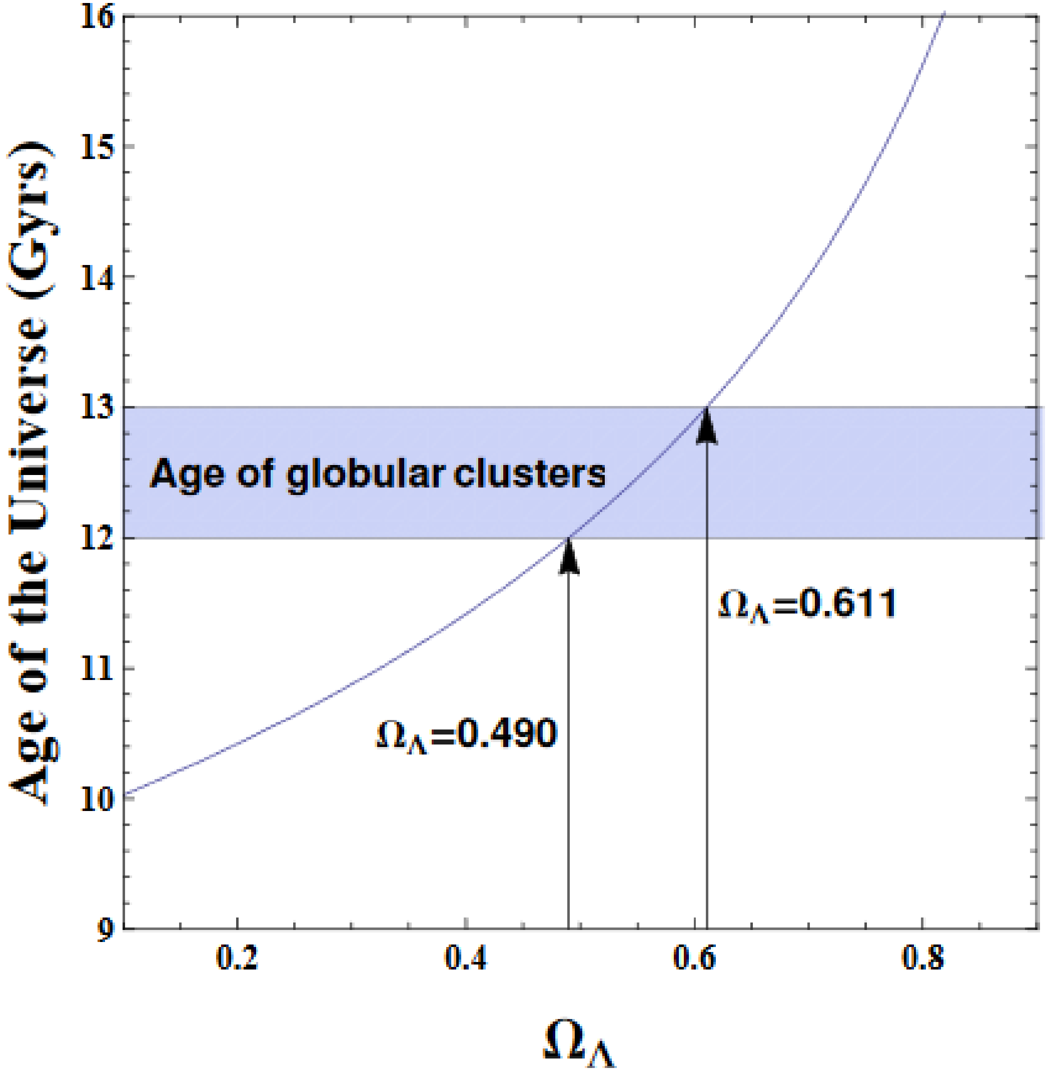
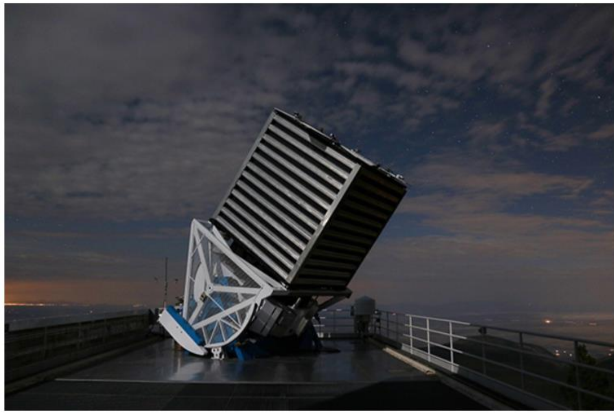


FIG. 6: Age of the Universe as a function of Ω_Λ. The horizontal stripe corresponds to the age of some known globular clusters.

Fontes observacionais

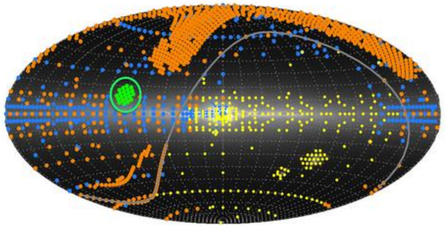
SDSS

The Sloan Digital Sky Survey: Mapping the Universe



The Sloan Digital Sky Survey has created the most detailed three-dimensional maps of the Universe ever made, with deep multi-color images of one third of the sky, and spectra for more than three million astronomical objects.

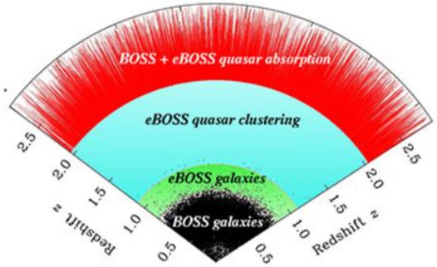
APOGEE-2



A stellar spectroscopic survey of the Milky Way, with two major components: a northern survey using the bright time at APO (APOGEE-2N), and a southern survey using the 2.5m du Pont Telescope at Las Campanas (APOGEE-2S).

Explore APOGEE-2

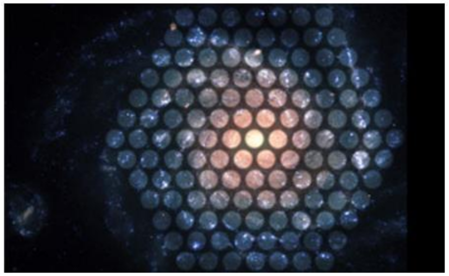
eBOSS



A cosmological survey of quasars and galaxies, also encompassing subprograms to survey variable objects (TDSS) and X-Ray sources (SPIDERS).

Explore eBOSS

MaNGA



The galaxy survey for people who love galaxies! MaNGA (Mapping Nearby Galaxies at Apache Point Observatory) will explore the detailed internal structure of nearly 10,000 nearby galaxies using spatially resolved spectroscopy.

Explore MaNGA

ESPAÑOL ENGLISH

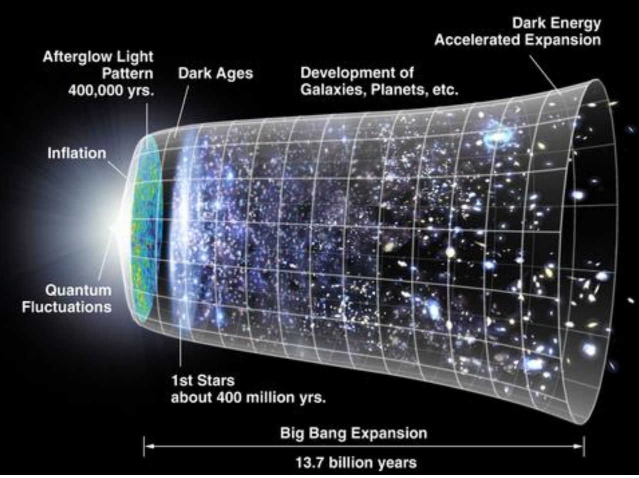
THE DARK ENERGY SURVEY

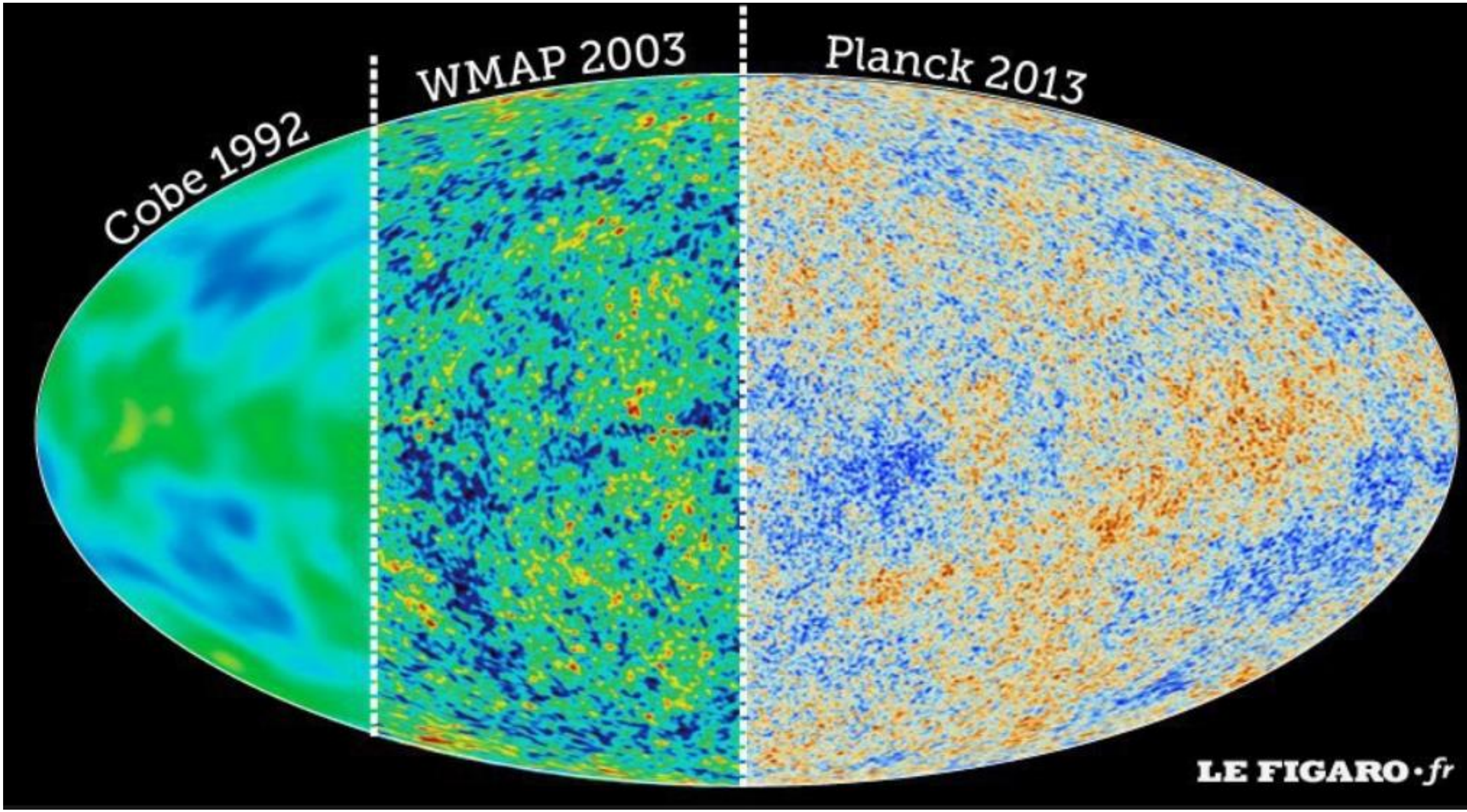
THE DES PROJECT | NEWS AND RESULTS | DATA ACCESS | MULTIMEDIA | EDUCATION | CONTACT US | SEARCH HERE

Overview

The Dark Energy Survey (DES) is an international, collaborative effort to map hundreds of millions of galaxies, detect thousands of supernovae, and find patterns of cosmic structure that will reveal the nature of the mysterious dark energy that is accelerating the expansion of our Universe. DES began searching the Southern skies on August 31, 2013.

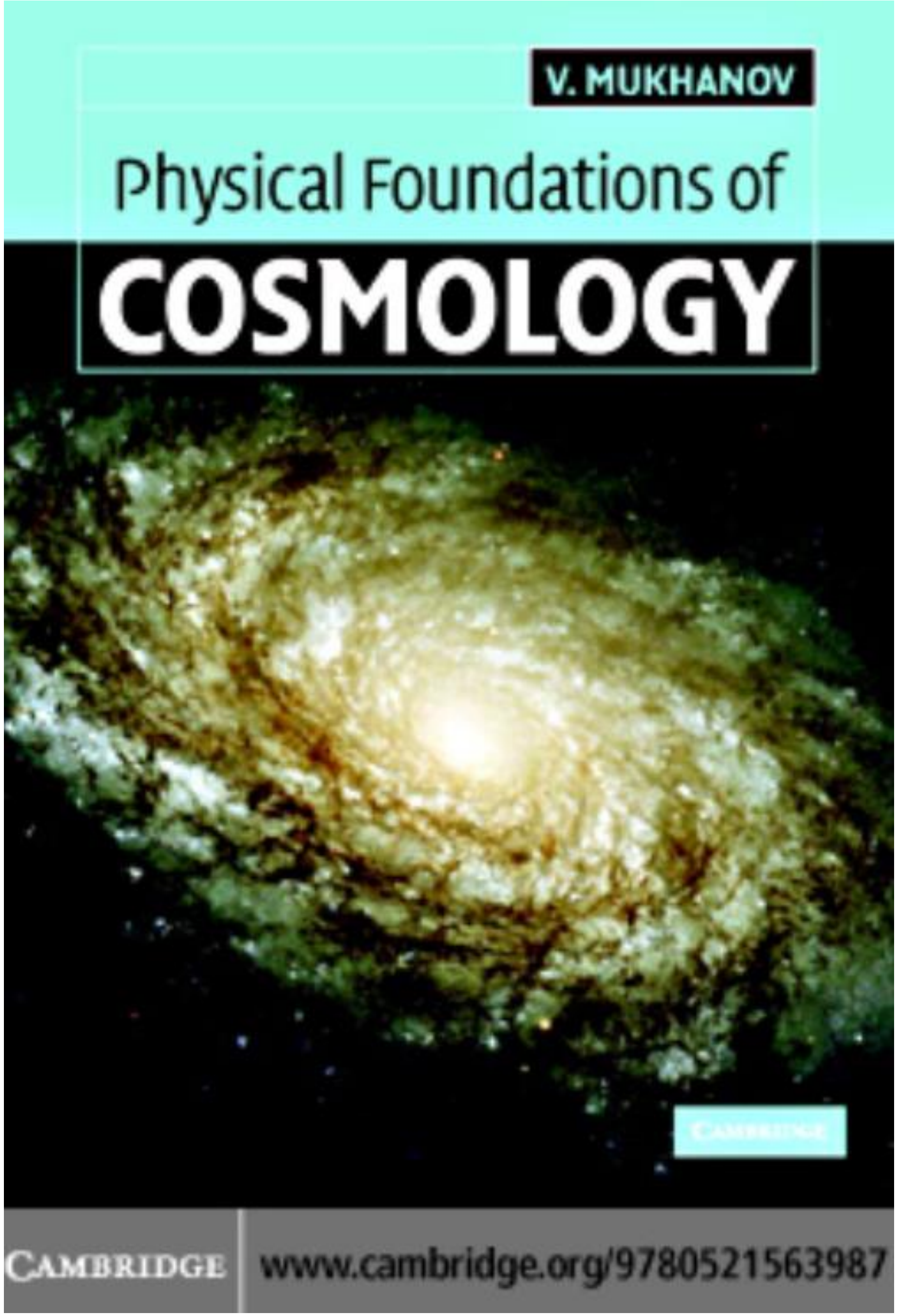
According to Einstein's theory of General Relativity, gravity should lead to a slowing of the cosmic expansion. Yet, in 1998, two teams of astronomers studying distant supernovae made the remarkable discovery that the expansion of the universe is speeding up. To explain cosmic acceleration, cosmologists are faced with two possibilities: either 70% of the universe exists in an exotic form, now called dark energy, that exhibits a gravitational force opposite to the attractive gravity of ordinary matter, or General Relativity must be replaced by a new theory of gravity on cosmic scales.




Cobe 1992
WMAP 2003
Planck 2013
LE FIGARO·fr

Aula II

- Entender como pequenas perturbações existentes na distribuição quase homogênea de densidade de matéria no universo primordial evoluem até o ponto de formar uma estrutura como conhecemos


V. MUKHANOV
Physical Foundations of
COSMOLOGY
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Perturbações cosmológicas

- 1) Perturbações Newtonianas
- 1.a) Modos escalares
- 1.b) Modos vetoriais

- 2) Perturbações relativísticas
- 2.a) Modos escalares
- 2.b) Modos vetoriais
- 2.c) Modos tensoriais -> Ondas gravitacionais

Continuity equation If we consider a *fixed volume element* ΔV in Euler (noncomoving) coordinates $\mathbf{x}$, then the rate of change of its mass can be written as

$$\frac{dM(t)}{dt} = \int_{\Delta V} \frac{\partial \varepsilon(\mathbf{x}, t)}{\partial t} dV. \tag{6.1}$$

On the other hand, this rate is entirely determined by the flux of matter through the surface surrounding the volume:

$$\frac{dM(t)}{dt} = -\oint \varepsilon \mathbf{V} \cdot d\boldsymbol{\sigma} = -\int_{\Delta V} \nabla(\varepsilon \mathbf{V})\, dV. \tag{6.2}$$

These two expressions are consistent only if

$$\frac{\partial \varepsilon}{\partial t} + \nabla(\varepsilon \mathbf{V}) = 0. \tag{6.3}$$

Euler equations The acceleration $\mathbf{g}$ of a small *matter element* of mass ΔM is determined by the gravitational force

$$\mathbf{F}_{gr} = -\Delta M \cdot \nabla\phi, \tag{6.4}$$

where ϕ is the gravitational potential, and by the pressure p:

$$\mathbf{F}_{pr} = -\oint p \cdot d\boldsymbol{\sigma} = -\int_{\Delta V} \nabla p dV \simeq -\nabla p \cdot \Delta V. \tag{6.5}$$

With

$$\mathbf{g} \equiv \frac{d\mathbf{V}(\mathbf{x}(t)\,,t)}{dt} = \left(\frac{\partial \mathbf{V}}{\partial t}\right)_x + \frac{dx^i(t)}{dt}\left(\frac{\partial \mathbf{V}}{\partial x^i}\right) = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \boldsymbol{\nabla})\,\mathbf{V}, \tag{6.6}$$

Newton's force law

$$\Delta M \cdot \mathbf{g} = \mathbf{F}_{gr} + \mathbf{F}_{pr} \tag{6.7}$$

becomes the Euler equations

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \boldsymbol{\nabla})\,\mathbf{V} + \frac{\nabla p}{\varepsilon} + \nabla\phi = 0. \tag{6.8}$$

Conservation of entropy Neglecting dissipation, the entropy of a *matter element* is conserved:

$$\frac{dS(\mathbf{x}(t),t)}{dt} = \frac{\partial S}{\partial t} + (\mathbf{V}\cdot\boldsymbol{\nabla})\,S = 0. \tag{6.9}$$

Poisson equation Finally, the equation which determines the gravitational potential is the well known Poisson equation,

$$\Delta\phi = 4\pi G\varepsilon. \tag{6.10}$$

Equations (6.3), (6.8)–(6.10), taken together with the equation of state

$$p = p(\varepsilon, S)\,, \tag{6.11}$$

form a complete set of seven equations which, in principle, allows us to determine the seven unknown functions ε, $\mathbf{V}$, S, ϕ, p. Note that only the first five equations contain first time derivatives. Hence the most general solution of these equations should depend on five constants of integration which in our case are five arbitrary functions of the spatial coordinates $\mathbf{x}$. The hydrodynamical equations are nonlinear and in general it is not easy to find their solutions. However, to study the behavior of *small* perturbations around a homogeneous, isotropic background, it is appropriate to linearize them.

6.2 Jeans theory

Let us first consider a static nonexpanding universe, assuming the homogeneous, isotropic background with constant, time-independent matter density: $\varepsilon_0(t, \mathbf{x}) =$ const. This assumption is in obvious contradiction with the hydrodynamical equations. In fact, the energy density remains unchanged only if the matter is at rest and the gravitational force, $F \propto \nabla\phi$, vanishes. But then the Poisson equation $\Delta\phi = 4\pi G\varepsilon_0$ is not satisfied. This inconsistency can, in principle, be avoided if we consider a static Einstein universe, where the gravitational force of the matter is compensated by the "antigravitational" force of an appropriately chosen cosmological constant.

Slightly disturbing the matter distribution, we have:

$$\begin{aligned} \varepsilon(\mathbf{x}, t) &= \varepsilon_0 + \delta\varepsilon(\mathbf{x}, t), \quad \mathbf{V}(\mathbf{x}, \mathbf{t}) = \mathbf{V}_0 + \delta\mathbf{v} = \delta\mathbf{v}(\mathbf{x}, \mathbf{t}), \\ \phi(\mathbf{x}, t) &= \phi_0 + \delta\phi(\mathbf{x}, t), \quad S(\mathbf{x}, t) = S_0 + \delta S(\mathbf{x}, t), \end{aligned} \tag{6.12}$$

where $\delta\varepsilon \ll \varepsilon_0$, etc. The pressure is equal to

$$p(\mathbf{x}, t) = p(\varepsilon_0 + \delta\varepsilon, S_0 + \delta S) = p_0 + \delta p(\mathbf{x}, t), \tag{6.13}$$

and in linear approximation its perturbation δp can be expressed in terms of the energy density and entropy perturbations as

$$\delta p = c_s^2 \delta\varepsilon + \sigma \delta S. \tag{6.14}$$

Here $c_s^2 \equiv (\partial p/\partial\varepsilon)_S$ is the square of the speed of sound and $\sigma \equiv (\partial p/\partial S)_\varepsilon$. For nonrelativistic matter ($p \ll \varepsilon$), the speed of sound as well as the velocities $\delta\mathbf{v}$ are much less than the speed of light.

Substituting (6.12) and (6.14) into (6.3), (6.8)–(6.10) and keeping only the terms which are linear in the perturbations, we obtain:

$$\frac{\partial\delta\varepsilon}{\partial t} + \varepsilon_0 \nabla(\delta\mathbf{v}) = 0, \tag{6.15}$$

$$\frac{\partial\delta\mathbf{v}}{\partial t} + \frac{c_s^2}{\varepsilon_0}\nabla\delta\varepsilon + \frac{\sigma}{\varepsilon_0}\nabla\delta S + \nabla\delta\phi = 0, \tag{6.16}$$

$$\frac{\partial\delta S}{\partial t} = 0, \tag{6.17}$$

$$\Delta\delta\phi = 4\pi G\delta\varepsilon. \tag{6.18}$$

Equation (6.17) has a simple general solution

$$\delta S(\mathbf{x}, t) = \delta S(\mathbf{x}), \tag{6.19}$$

which states that the entropy is an arbitrary time-independent function of the spatial coordinates.

Taking the divergence of (6.16) and using the continuity and Poisson equations to express $\nabla \delta \mathbf{v}$ and $\Delta \delta \phi$ in terms of $\delta \varepsilon$, we obtain

$$\frac{\partial^2 \delta \varepsilon}{\partial t^2} - c_s^2 \Delta \delta \varepsilon - 4\pi G \varepsilon_0 \delta \varepsilon = \sigma \Delta \delta S(\mathbf{x}). \tag{6.20}$$

This is a closed, linear equation for $\delta \varepsilon$, where the entropy perturbation serves as a given source.

6.2.1 Adiabatic perturbations

First we will assume that entropy perturbations are absent, that is, $\delta S = 0$. The coefficients in (6.20) do not depend on the spatial coordinates, so upon taking the Fourier transform,

$$\delta \varepsilon(\mathbf{x}, t) = \int \delta \varepsilon_{\mathbf{k}}(t) \exp(i\mathbf{k}\mathbf{x}) \frac{d^3 k}{(2\pi)^{3/2}}, \tag{6.21}$$

we obtain a set of independent ordinary differential equations for the time-dependent Fourier coefficients $\delta \varepsilon_{\mathbf{k}}(t)$:

$$\delta \ddot{\varepsilon}_{\mathbf{k}} + \left(k^2 c_s^2 - 4\pi G \varepsilon_0\right) \delta \varepsilon_{\mathbf{k}} = 0, \tag{6.22}$$

where a dot denotes the derivative with respect to time t and $k = |\mathbf{k}|$.

Equation (6.22) has two independent solutions

$$\delta\varepsilon_{\mathbf{k}} \propto \exp(\pm i\omega(k)\,t), \tag{6.23}$$

where

$$\omega(k) = \sqrt{k^2 c_s^2 - 4\pi G\varepsilon_0}.$$

The behavior of these so-called adiabatic perturbations depends crucially on the sign of the expression under the square root. Defining the Jeans length as

$$\lambda_J = \frac{2\pi}{k_J} = c_s\left(\frac{\pi}{G\varepsilon_0}\right)^{1/2}, \tag{6.24}$$

so that $\omega(k_J) = 0$, we conclude that if $\lambda < \lambda_J$, the solutions describe sound waves

$$\delta\varepsilon \propto \sin(\omega t + \mathbf{kx} + \alpha), \tag{6.25}$$

propagating with phase velocity

$$c_{phase} = \frac{\omega}{k} = c_s\sqrt{1 - \frac{k_J^2}{k^2}}. \tag{6.26}$$

In the limit $k \gg k_J$, or on very small scales $(\lambda \ll \lambda_J)$ where gravity is negligible compared to the pressure, we have $c_{phase} \to c_s$, as it should be.

On large scales gravity dominates and if $\lambda > \lambda_J$, we have

$$\delta\varepsilon_{\mathbf{k}} \propto \exp(\pm\,|\omega|\,t)\,. \tag{6.27}$$

One of these solutions describes the exponentially fast growth of inhomogeneities, while the other corresponds to a decaying mode. When $k \to 0$, $|\omega|\,t \to t/t_{gr}$, where $t_{gr} \equiv (4\pi G\varepsilon_0)^{-1/2}$. We interpret t_{gr} as the characteristic collapse time for a region with initial density ε_0.

The Jeans length $\lambda_J \sim c_s t_{gr}$ is the "sound communication" scale over which the pressure can still react to changes in the energy density due to gravitational instability. Gravitational instability is very efficient in a static universe. Even if the adiabatic perturbation is initially extremely small, say 10^{-100}, gravity needs only a short time $t \sim 230 t_{gr}$ to amplify it to order unity.

6.3 Instability in an expanding universe

Background In an expanding homogeneous and isotropic universe, the background energy density is a function of time, and the background velocities obey the Hubble law:

$$\varepsilon = \varepsilon_0(t), \quad \mathbf{V} = \mathbf{V}_0 = H(t) \cdot \mathbf{x}. \tag{6.32}$$

Substituting these expressions into (6.3), we obtain the familiar equation

$$\dot{\varepsilon}_0 + 3H\varepsilon_0 = 0, \tag{6.33}$$

which states that the total mass of nonrelativistic matter is conserved. The divergence of the Euler equations (6.8) together with the Poisson equation (6.10) leads to the Friedmann equation:

$$\dot{H} + H^2 = -\frac{4\pi G}{3}\varepsilon_0. \tag{6.34}$$

Perturbations Ignoring entropy perturbations and substituting the expressions

$$\begin{aligned} &\varepsilon = \varepsilon_0 + \delta\varepsilon(\mathbf{x}, t), \quad \mathbf{V} = \mathbf{V}_0 + \delta\mathbf{v}, \quad \phi = \phi_0 + \delta\phi, \\ &p = p_0 + \delta p = p_0 + c_s^2 \delta\varepsilon, \end{aligned} \tag{6.35}$$

into (6.3), (6.8), (6.10), we derive the following set of linearized equations for small perturbations:

$$\frac{\partial \delta\varepsilon}{\partial t} + \varepsilon_0 \nabla \delta\mathbf{v} + \boldsymbol{\nabla}(\delta\varepsilon \cdot \mathbf{V}_0) = 0, \tag{6.36}$$

$$\frac{\partial \delta\mathbf{v}}{\partial t} + (\mathbf{V}_0 \cdot \nabla)\delta\mathbf{v} + (\delta\mathbf{v} \cdot \nabla)\mathbf{V}_0 + \frac{c_s^2}{\varepsilon_0}\nabla\delta\varepsilon + \nabla\delta\phi = 0, \tag{6.37}$$

$$\Delta\delta\phi = 4\pi G \delta\varepsilon. \tag{6.38}$$

The Hubble velocity $\mathbf{V}_0$ depends explicitly on $\mathbf{x}$ and therefore the Fourier transform with respect to the Eulerian coordinates $\mathbf{x}$ does not reduce these equations to a decoupled set of ordinary differential equations. This is why it is more convenient to use the Lagrangian (*comoving with the Hubble flow*) coordinates $\mathbf{q}$, which are related to the Eulerian coordinates via

$$\mathbf{x} = a(t)\,\mathbf{q}, \tag{6.39}$$

where $a(t)$ is the scale factor. The partial derivative with respect to time taken at constant $\mathbf{x}$ is different from the partial derivative taken at constant $\mathbf{q}$. For a general function $f(\mathbf{x}, t)$ we have

$$\left(\frac{\partial f(\mathbf{x} = a\mathbf{q}, t)}{\partial t}\right)_{\mathbf{q}} = \left(\frac{\partial f}{\partial t}\right)_{\mathbf{x}} + \dot{a}q^i\left(\frac{\partial f}{\partial x^i}\right)_t \tag{6.40}$$

and therefore

$$\left(\frac{\partial}{\partial t}\right)_{\mathbf{x}} = \left(\frac{\partial}{\partial t}\right)_{\mathbf{q}} - (\mathbf{V}_0 \cdot \nabla_{\mathbf{x}}). \tag{6.41}$$

The spatial derivatives are more simply related:

$$\nabla_{\mathbf{x}} = \frac{1}{a}\nabla_{\mathbf{q}}. \tag{6.42}$$

Replacing the derivatives in (6.36)–(6.38) and introducing the fractional amplitude of the density perturbations $\delta \equiv \delta\varepsilon/\varepsilon_0$, we finally obtain

$$\left(\frac{\partial \delta}{\partial t}\right) + \frac{1}{a}\nabla \delta \mathbf{v} = 0, \tag{6.43}$$

$$\left(\frac{\partial \delta \mathbf{v}}{\partial t}\right) + H\delta\mathbf{v} + \frac{c_s^2}{a}\nabla\delta + \frac{1}{a}\nabla\delta\phi = 0, \tag{6.44}$$

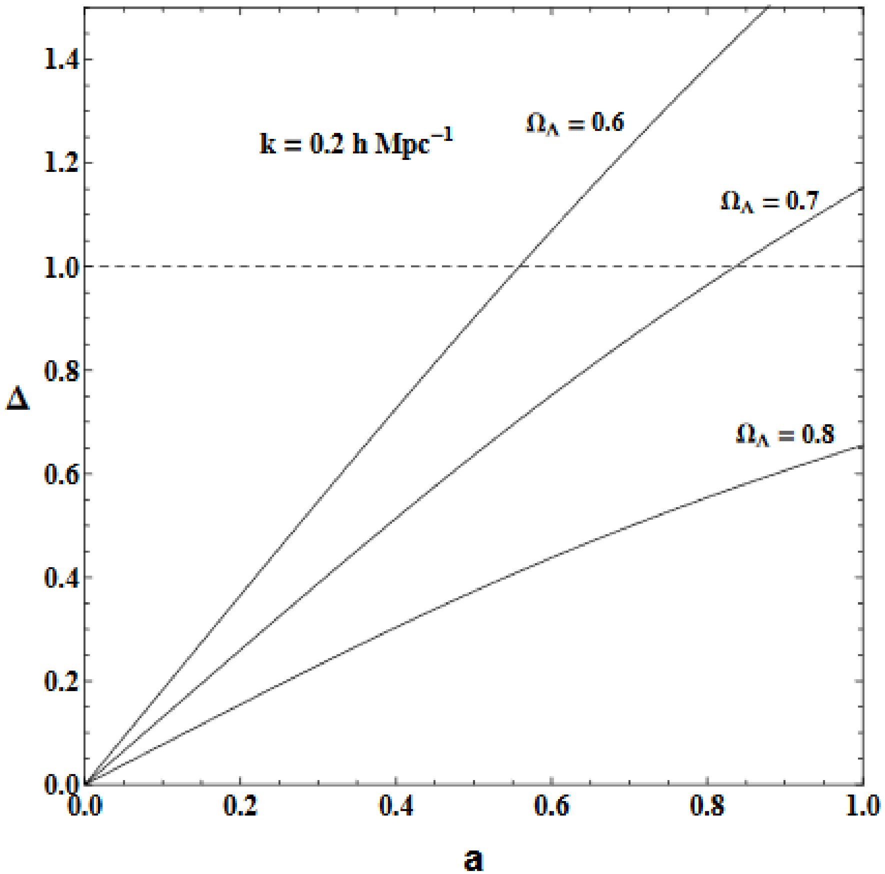
$$\Delta\delta\phi = 4\pi G a^2 \varepsilon_0 \delta, \tag{6.45}$$

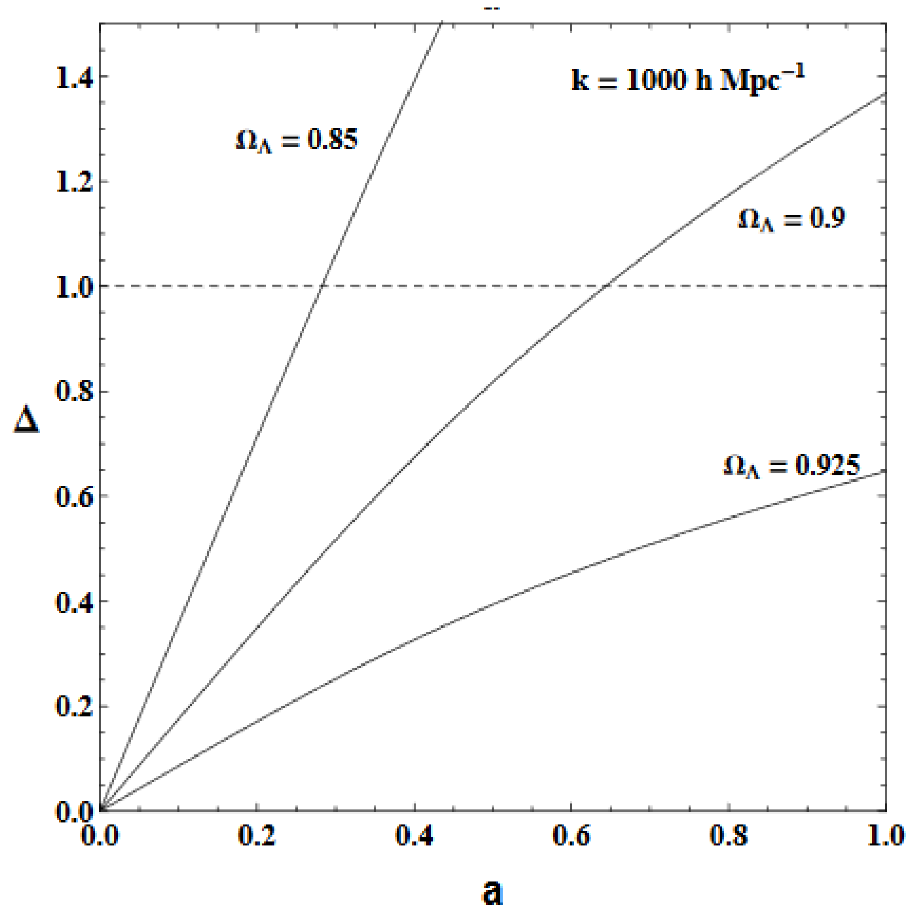
where $\nabla \equiv \nabla_{\mathbf{q}}$ and Δ are now the derivatives with respect to the Lagrangian coordinates $\mathbf{q}$ and the time derivatives are taken at constant $\mathbf{q}$. In deriving (6.43) we have used (6.33) for the background and noted that $\nabla_{\mathbf{x}}\mathbf{V}_0 = 3H$ and $(\delta\mathbf{v} \cdot \nabla_{\mathbf{x}})\mathbf{V}_0 = H\delta\mathbf{v}$. Taking the divergence of (6.44) and using the continuity and Poisson equations to express $\nabla\delta\mathbf{v}$ and $\Delta\delta\phi$ in terms of δ, we derive the closed form equation

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\Delta\delta - 4\pi G\varepsilon_0\delta = 0, \tag{6.46}$$

which describes gravitational instability in an expanding universe.

$$\ddot{\Delta} + 2H\dot{\Delta} + \left(\frac{c_s^2 k^2}{a^2} - 4\pi G\rho\right)\Delta = 0,$$





Time

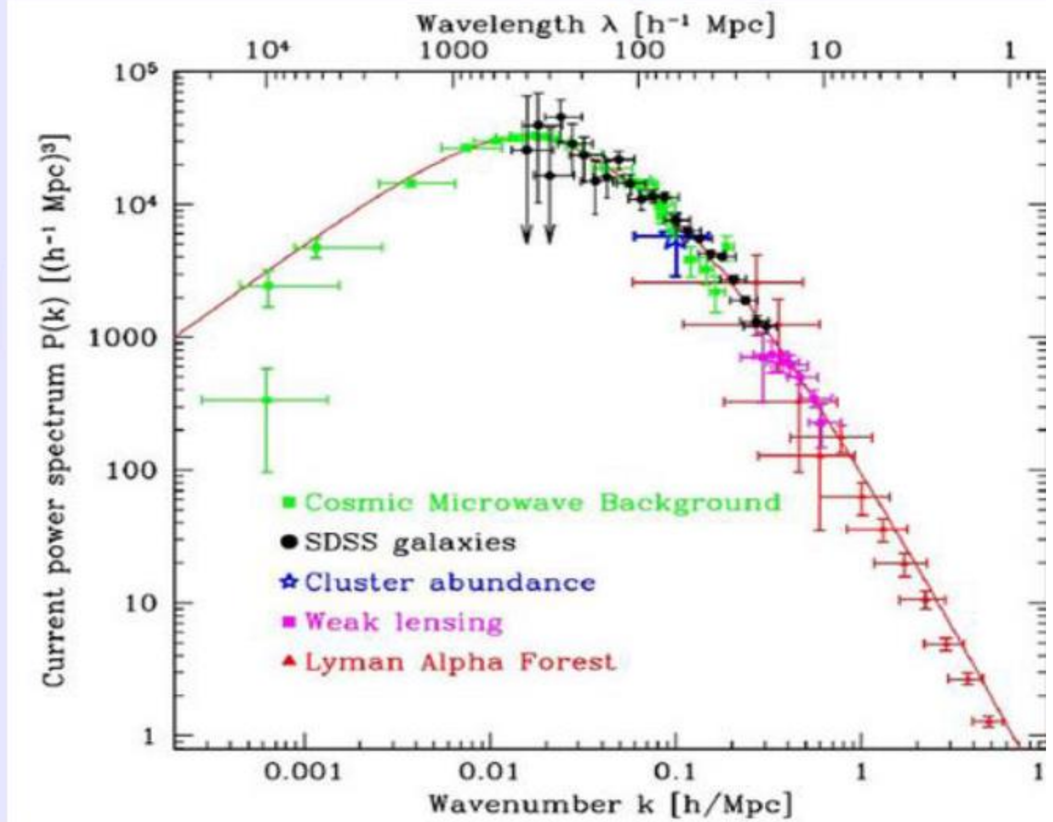
Just after Inflation

z=0 (Today)

The Initial Power Spectrum

$$P(k) = |\Delta_k|^2 \propto k^n$$

The Harrison–Zeldovich power spectrum has $n = 1$



T(k) modifies the shape of P(k).

$$\Delta_k(z = 0) = T(k)\; f(z)\; \Delta_k(z)$$

Aula III

- Efeito Mészaros -> Bônus: Precisamos de matéria escura

- Estágio não-linear de formação de estruturas. Entender o porquê de um excesso de densidade na distribuição de matéria se desacopla do fluxo de Hubble e colapsa para formar um objeto astronômico.

Evolução das perturbações na matéria escura e bárions

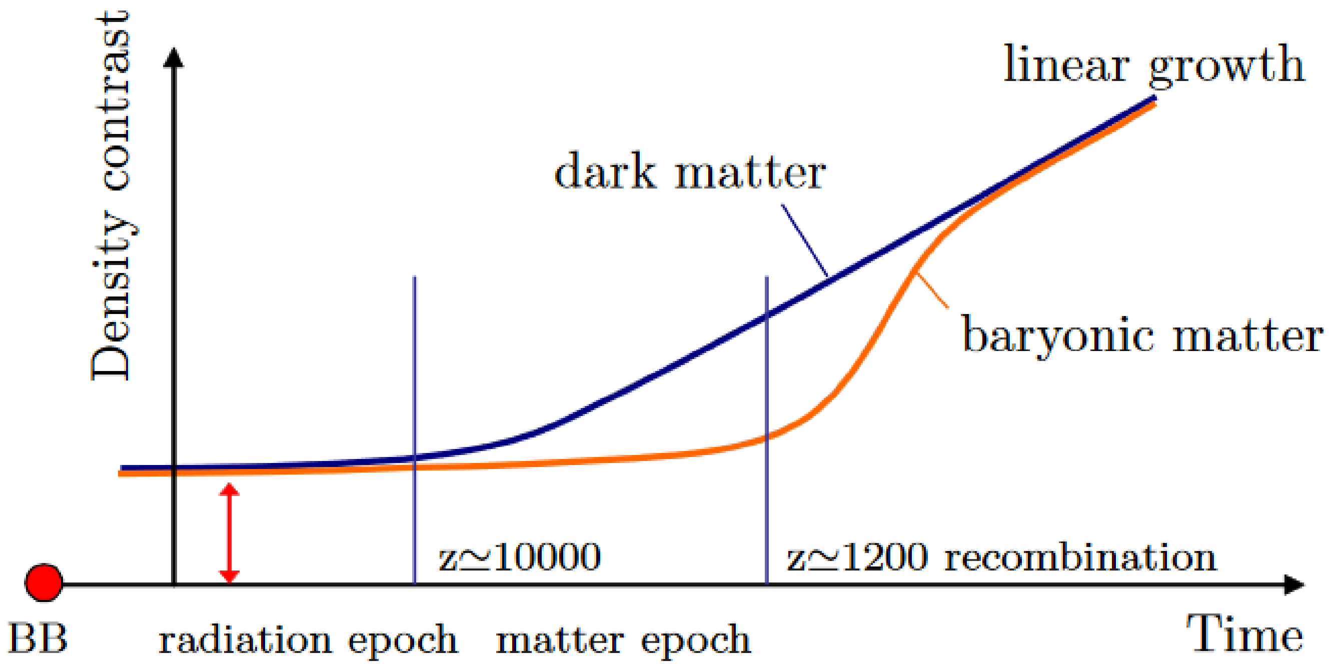
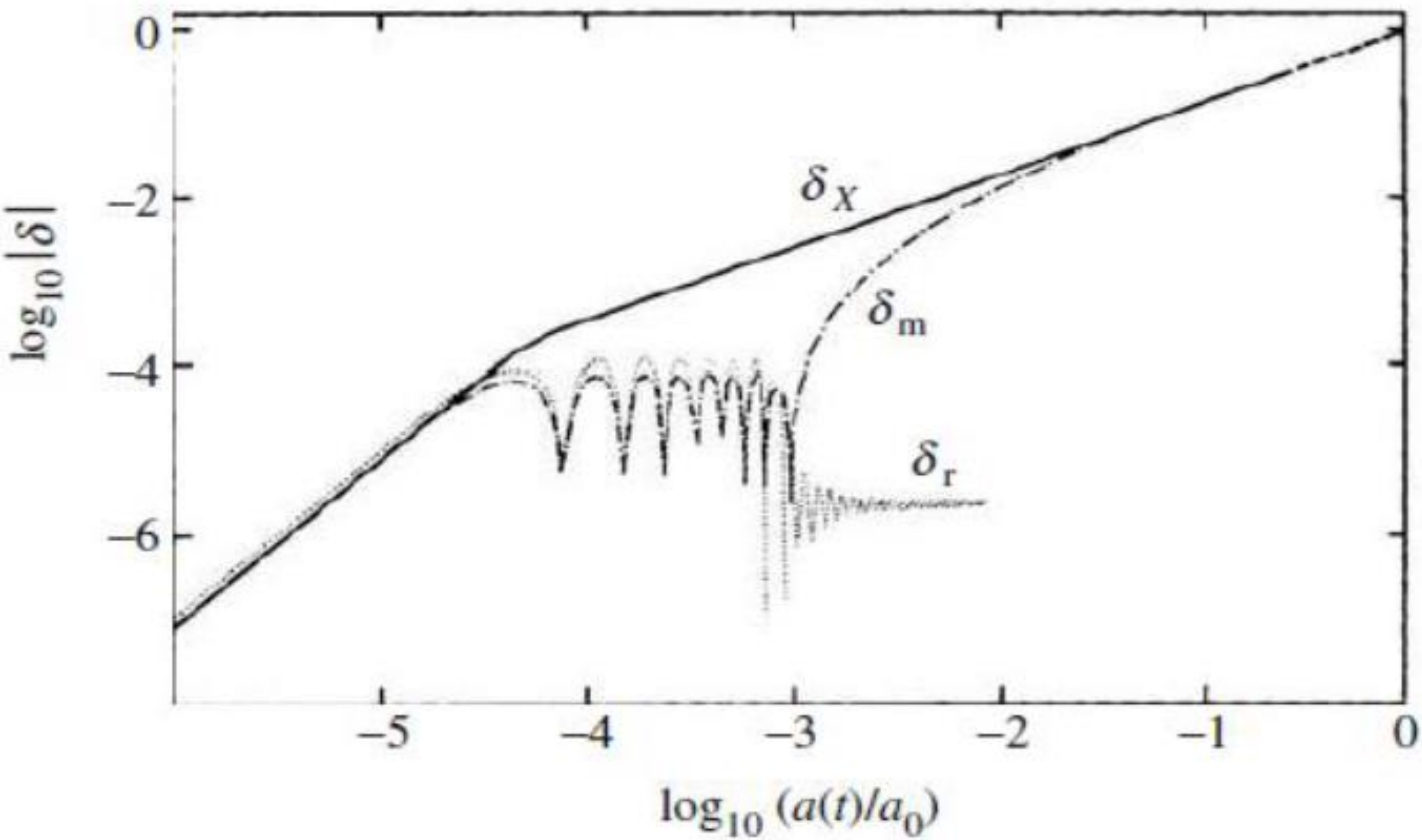


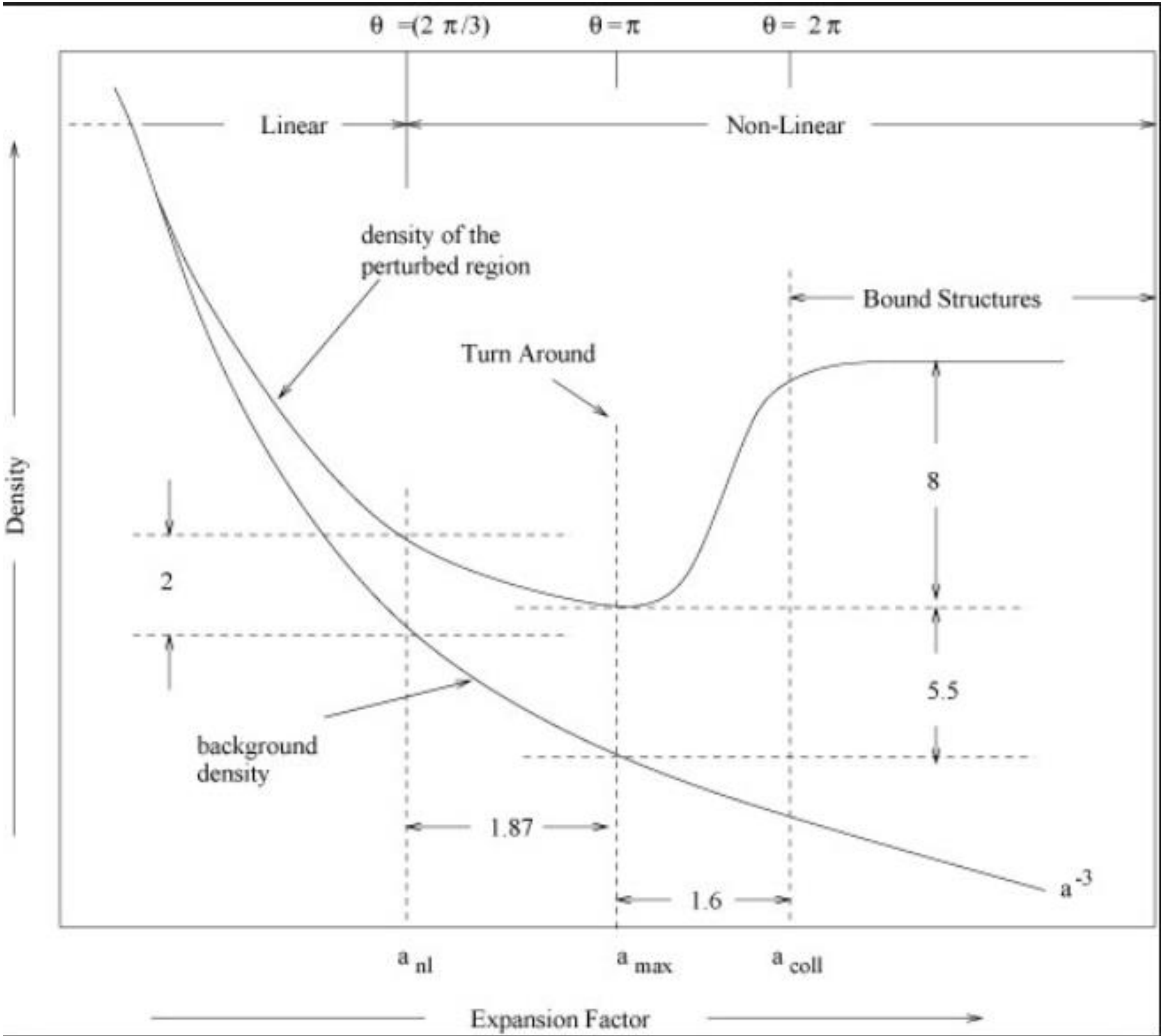
Figure 2.4: Growth of matter density contrast. After the recombination the baryonic is free to evolve and fall into the dark mater halos which formed earlier. Afterward both matter grow together.



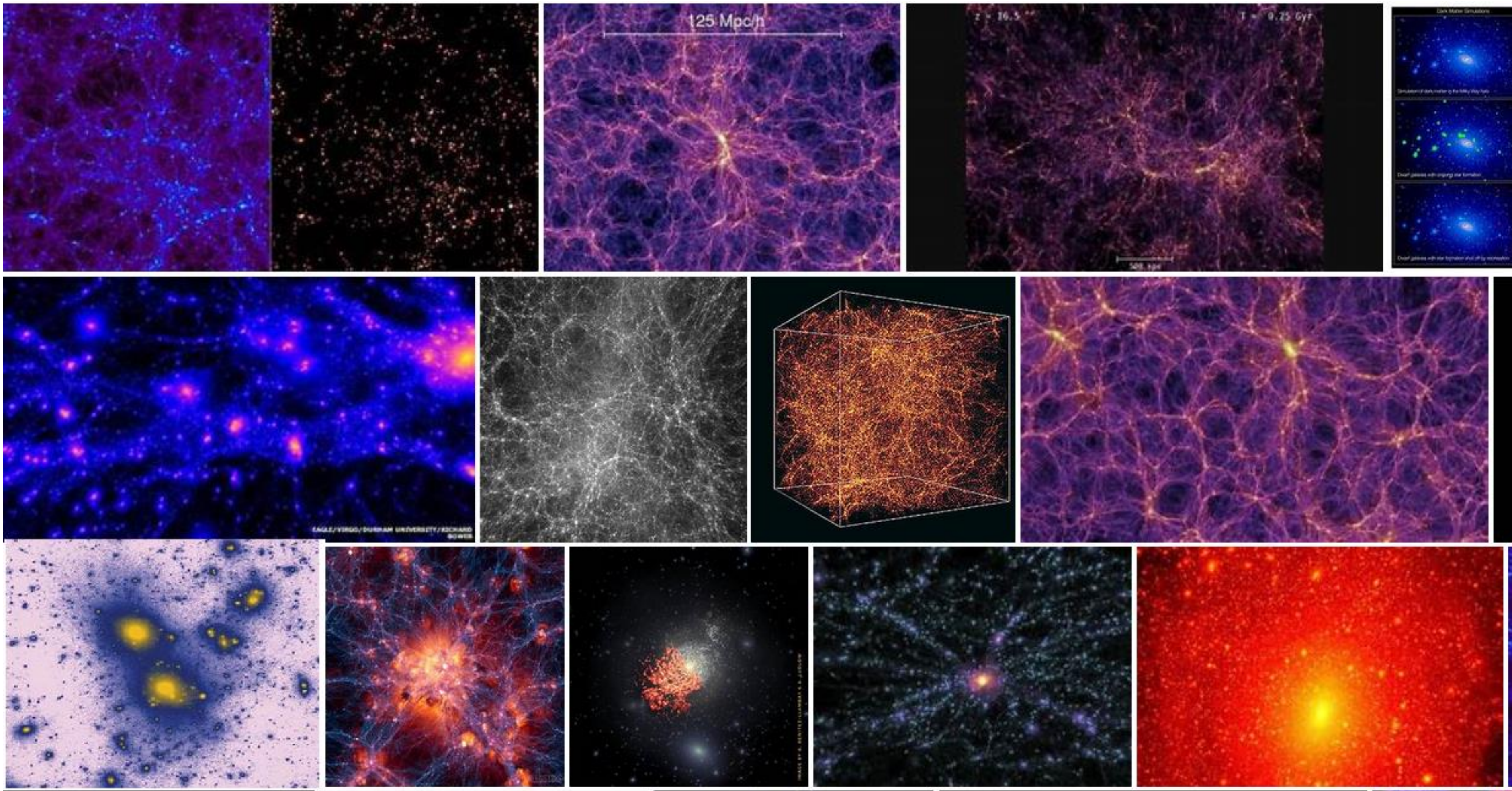
0
−2
−4
−6
$\log_{10}|\delta|$
δ_X
δ_m
δ_r
−5
−4
−3
−2
−1
0
$\log_{10}(a(t)/a_0)$

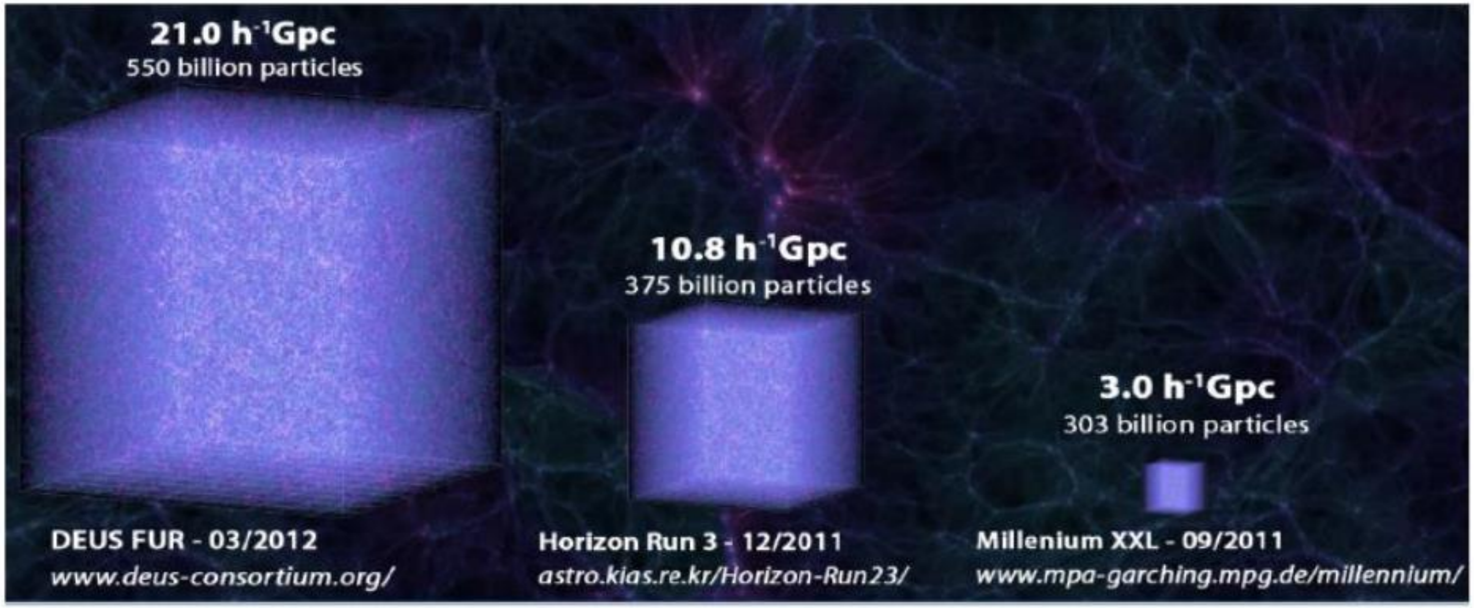
As perturbações lineares finalmente chegaram ao estágio não-linear.

E agora?


θ =(2 π/3)
θ = π
θ = 2 π
Linear
Non-Linear
density of the
perturbed region
Bound Structures
Turn Around
Density
8
2
5.5
background
density
1.87
1.6
a -3
a nl
a max
a coll
Expansion Factor

Simulações numéricas





Comparison of some N-Body simulations in terms of particles and volume sizes.

Mean Field approximation: Vlasov-Poisson equations and N-body system

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v}.\frac{\partial f}{\partial \vec{r}} - \frac{\partial \phi}{\partial \vec{r}}.\frac{\partial f}{\partial \vec{v}} = 0,$$

$$\nabla^2 \phi(\vec{r}, t) = 4\pi G \rho(\vec{r}, t),$$

where the density of finding a particle is

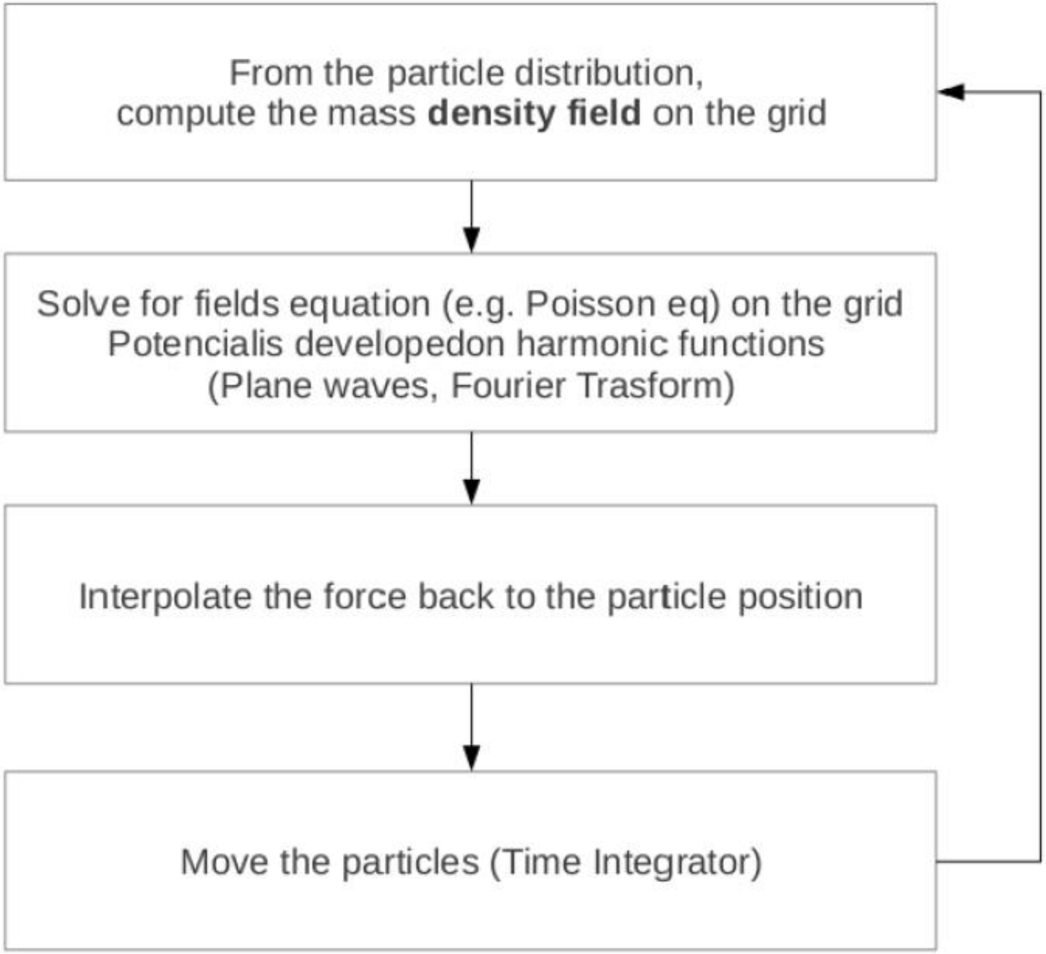
$$\rho(\vec{r}, t) = \int f(\vec{r}, \vec{v}, t) d\vec{v}.$$

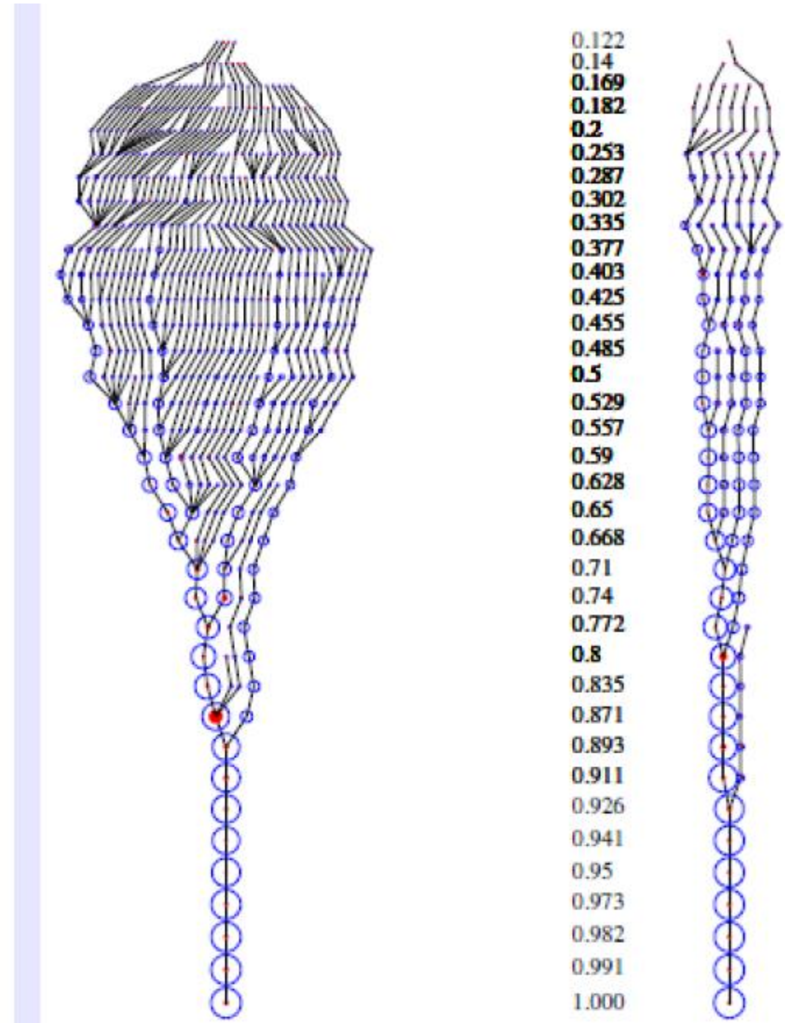
velocity/acceleration equation

$$\frac{d\vec{r}}{dt} = \vec{v} \qquad \rightarrow \qquad \frac{d\vec{v}}{dt} = -\nabla \phi.$$

The potential ϕ is given by,

$$\phi(\vec{r}) = -\sum_{j \neq i} G \frac{m_j}{\sqrt{|\vec{r} - \vec{r}_j|^2 + \epsilon^2}} \qquad \text{or} \qquad \nabla^2 \phi = 4\pi G \rho_T$$


From the particle distribution,
compute the mass **density field** on the grid
Solve for fields equation (e.g. Poisson eq) on the grid
Potencialis developedon harmonic functions
(Plane waves, Fourier Trasform)
Interpolate the force back to the particle position
Move the particles (Time Integrator)


0.122
0.14
0.169
0.182
0.2
0.253
0.287
0.302
0.335
0.377
0.403
0.425
0.455
0.485
0.5
0.529
0.557
0.59
0.628
0.65
0.668
0.71
0.74
0.772
0.8
0.835
0.871
0.893
0.911
0.926
0.941
0.95
0.973
0.982
0.991
1.000

O que as simulações revelam? Existem problemas com nosso modelo padrão?

- Distribuição da matéria ao longo do halo: Core/cusp
- Muitos satélites preditos! Mas não vemos todos eles.

Cold dark matter: controversies on small scales

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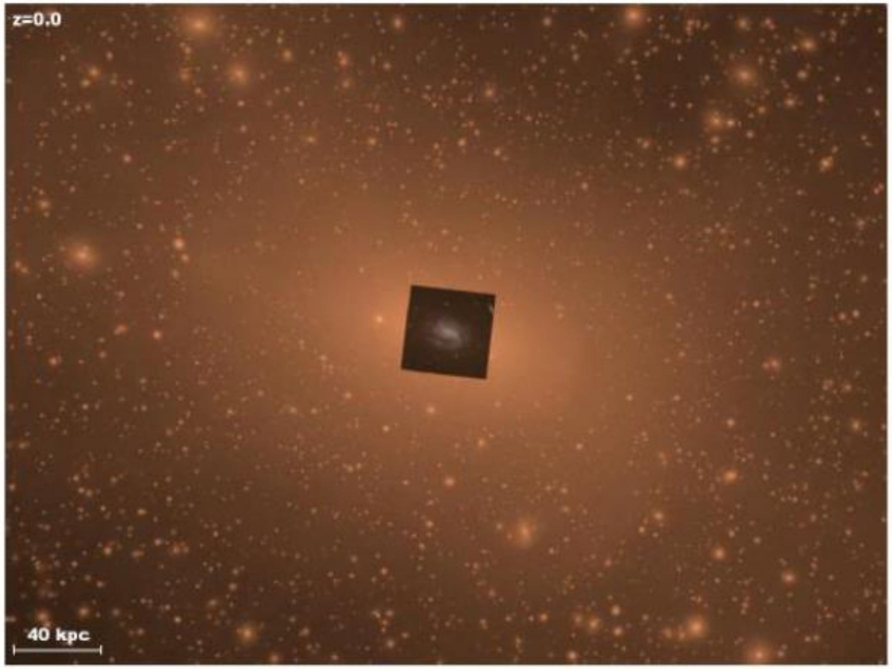
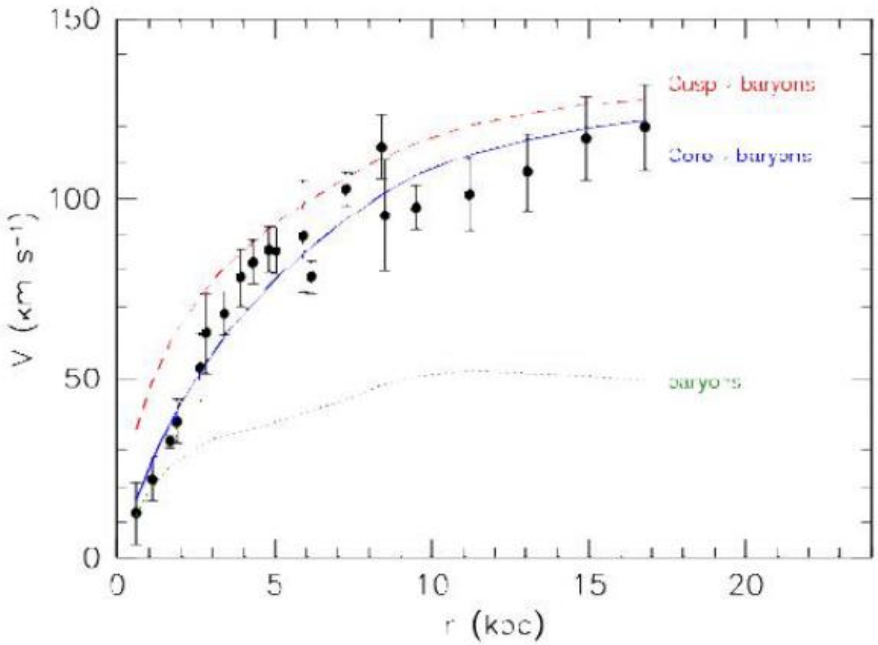
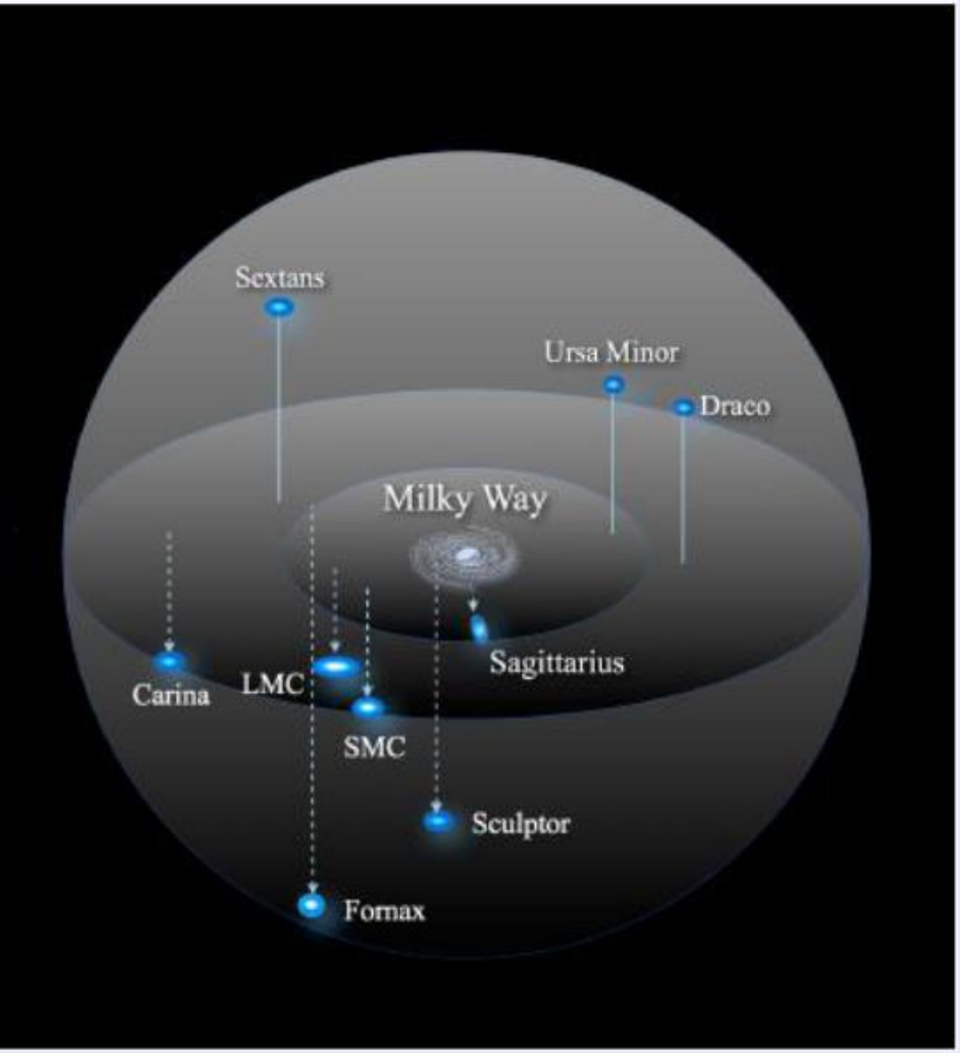
Submitted to Proceedings of the National Academy of Sciences of the United States of America

1) The CUSP-CORE Problem





2) The Missing Satellites Problem
2.1) Too Big to Fail Problem



Fim. Obrigado.