Black Hole Shadows

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Agenda

1. General Information

- The Event Horizon Telescope has shown the images at the centre of M87*
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Figura 1: M87.

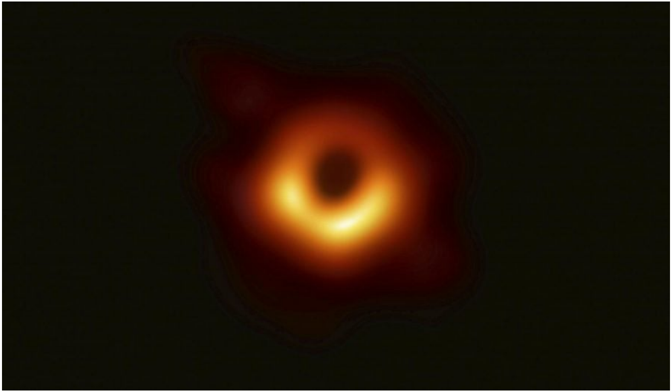
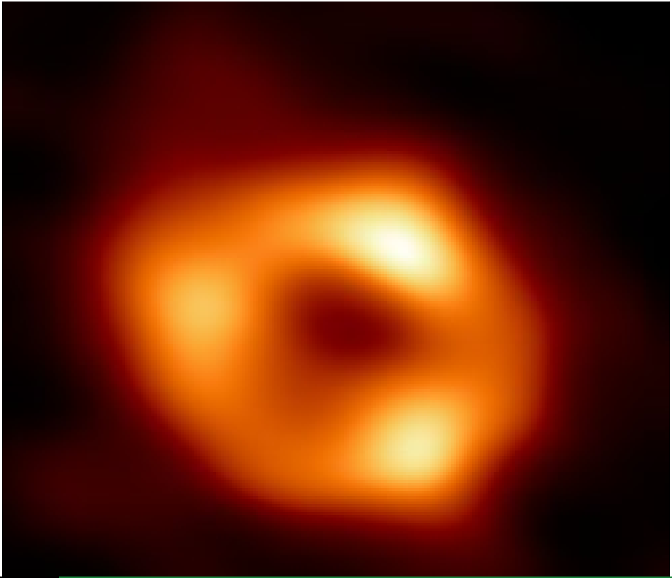
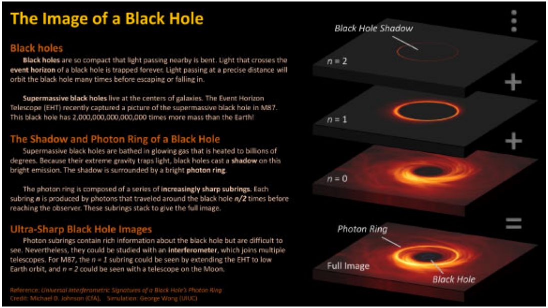


Figura 2: Sagittarius A.



Structure of the Shadow

- The Shadow is composed of a primary image and self similar subrings indexed by the number of photon orbits around the black hole.



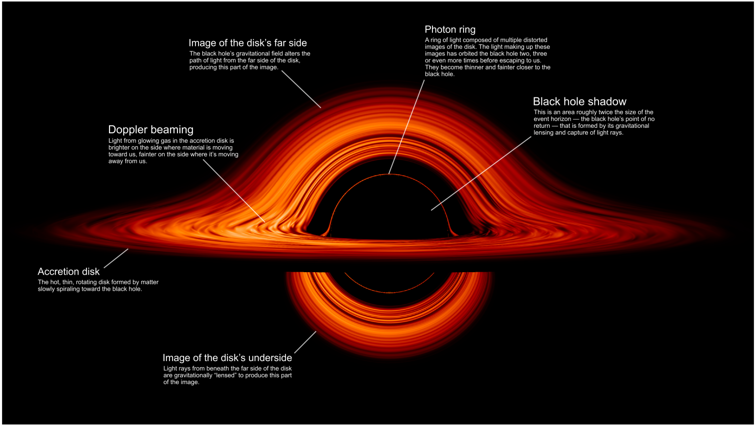


Image of the disk's far side
The black hole's gravitational field alters the path of light from the far side of the disk, producing this part of the image.
Photon ring
A ring of light composed of multiple distorted images of the disk. The light making up these images has orbited the black hole two, three or even more times before escaping to us. They become thinner and fainter closer to the black hole.
Black hole shadow
This is an area roughly twice the size of the event horizon — the black hole's point of no return — that is formed by its gravitational lensing and capture of light rays.
Doppler beaming
Light from glowing gas in the accretion disk is brighter on the side where material is moving toward us, fainter on the side where it's moving away from us.
Accretion disk
The hot, thin, rotating disk formed by matter slowly spiraling toward the black hole.
Image of the disk's underside
Light rays from beneath the far side of the disk are gravitationally "lensed" to produce this part of the image.

What type of geodesics are required?

$$\frac{\rho^2}{E}p^r = \pm\sqrt{R(r)}, \tag{1}$$

$$\frac{\rho^2}{E}p^\theta = \pm\sqrt{\Theta(\theta)}, \tag{2}$$

$$\frac{\rho^2}{E}p^\phi = \frac{(ar^2 + a^3 - a\Delta - a^2\lambda)}{\Delta} + \frac{\lambda}{\sin^2\theta}, \tag{3}$$

$$\frac{\rho^2}{E}p^t = \frac{(r^2 + a^2)(r^2 + a^2 - a\lambda)}{\Delta} + a\lambda - a^2\sin^2\theta, \tag{4}$$

Where

$$R(r) = (r^2 + a^2 - a\lambda)^2 - \Delta(\eta + (\lambda - a)^2), \tag{5}$$

$$\Theta(\theta) = \eta + a^2\cos^2\theta - \lambda^2\cot^2\theta. \tag{6}$$

Specifically the spherical photon orbits.

$R(r) = R'(r) = 0$

$$\eta = -\frac{3r^3\left(4a^2(-3M) + 3r(r-3M)^2\right)}{a^2(3M-3r)^2}, \tag{7}$$

$$\lambda = \frac{3a^2M + 3a^2r - 9Mr^2 + 3r^3}{a(3M-3r)}. \tag{8}$$

Examples of selected orbits.

Figura 3: Zero Angular Momentum Orbit, $\lambda = 0$

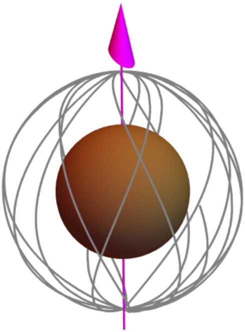


Figura 4: Equatorial circular prograde orbit

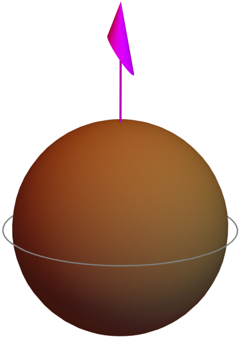
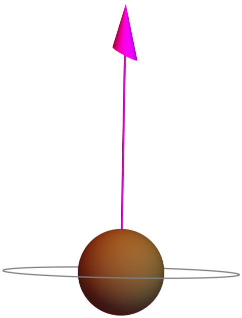


Figura 5: Equatorial circular retrograde orbit



Photon Region

$$r_{ph+} = 2M + 6\sqrt{\frac{M^2\,(1)}{(3)^2}}\cos\left(\frac{\tilde{\kappa}}{3} + \frac{4\pi}{3}\right), \tag{9}$$

$$r_{ph-} = 2M + 6\sqrt{\frac{M^2\,(1)}{(3)^2}}\cos\left(\frac{\tilde{\kappa}}{3}\right). \tag{10}$$

Celestial Coordinates

For distant Observers, we have that,

$$\alpha = \lim_{r_o \longrightarrow \infty} \left(-r_o^2 \sin\theta_o \frac{d\phi}{dr} \right), \tag{11}$$

$$\beta = \lim_{r_o \longrightarrow \infty} \left(r_o^2 \frac{d\theta}{dr} \right), \tag{12}$$

This yields

$$\alpha = -\lambda \csc\theta, \tag{13}$$

$$\beta = \pm\sqrt{\eta + a^2\cos^2\theta - \lambda^2\cot^2\theta}. \tag{14}$$

Figura 6: Critical curves at different angles of inclination

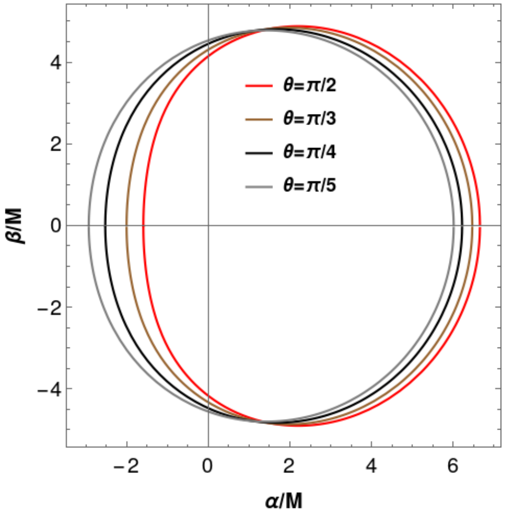
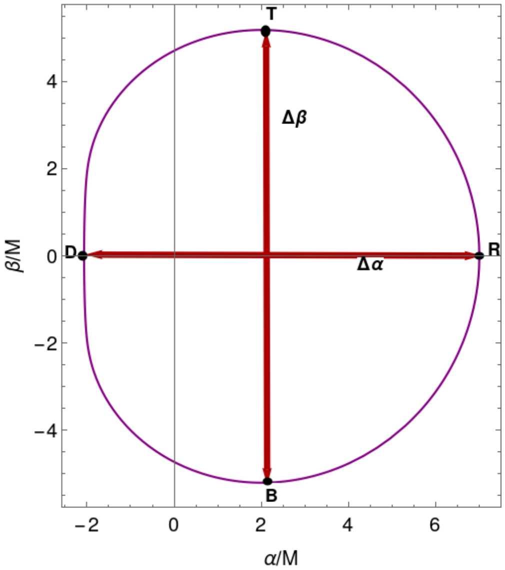


Figura 7: Characteristic points of the critical curve



Radius of Curvature

$$R_{curvature} = \left| \frac{(\alpha'(r)^2 + \beta'(r)^2)^{3/2}}{\alpha'(r)\beta''(r) - \beta'(r)\alpha''(r)} \right|. \tag{15}$$

What can we use this radius for?

Using EHT results, this radius can be used to constraint the black hole parameters.

My previous results

My previous results using radius of curvature

Spin $a/M > 0.812311$, and large angles of inclination ($\theta > 30.5107^{\circ}$) do not pass the M87* constraints,

$$4.31M \approx r_{sh,EHT-min} \leq \tilde{r}_{sh}, \quad r_{sh,A} \leq r_{sh,EHT-max} \approx 6.08M, \tag{16}$$

Conclusion

Black holes cast shadows.

The shadow is a collection of a primary image and self similar subrings.

To study the critical curve, we need spherical photon orbits.

Inorder to locate the critical curve, a set of coordinates are required, which are observer dependent.

From the critical curve, we can obtain various aspects of the shadow such as the radius of curvature.

The radius of curvature can be used to constraint black hole parameters.


OBRIGADA