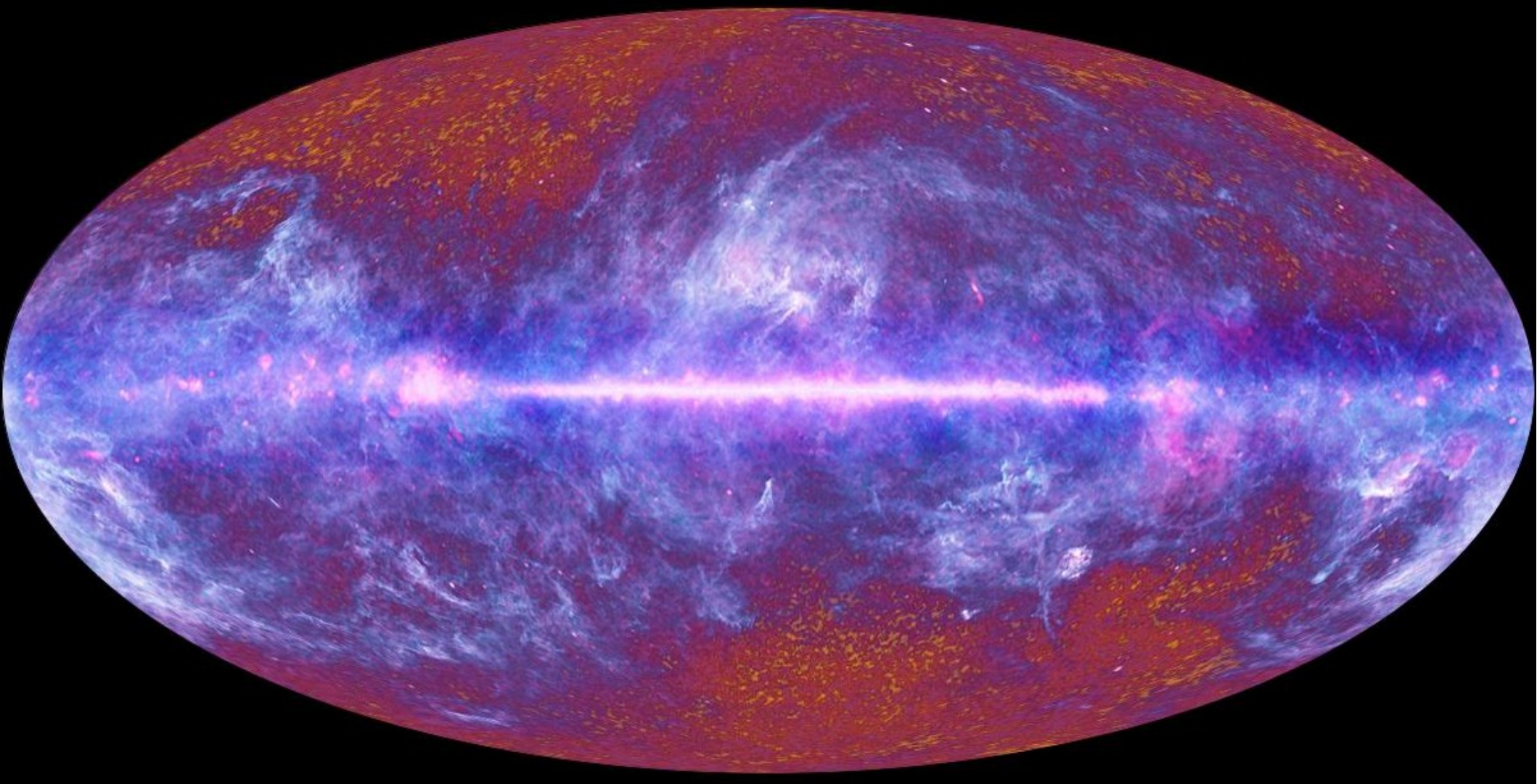


Introduction to the CMB physics



Grupo de Gravitação e Cosmologia, Vitória, UFES
Hermano Velten
23/09/2013

Let's start remembering Hubble's work

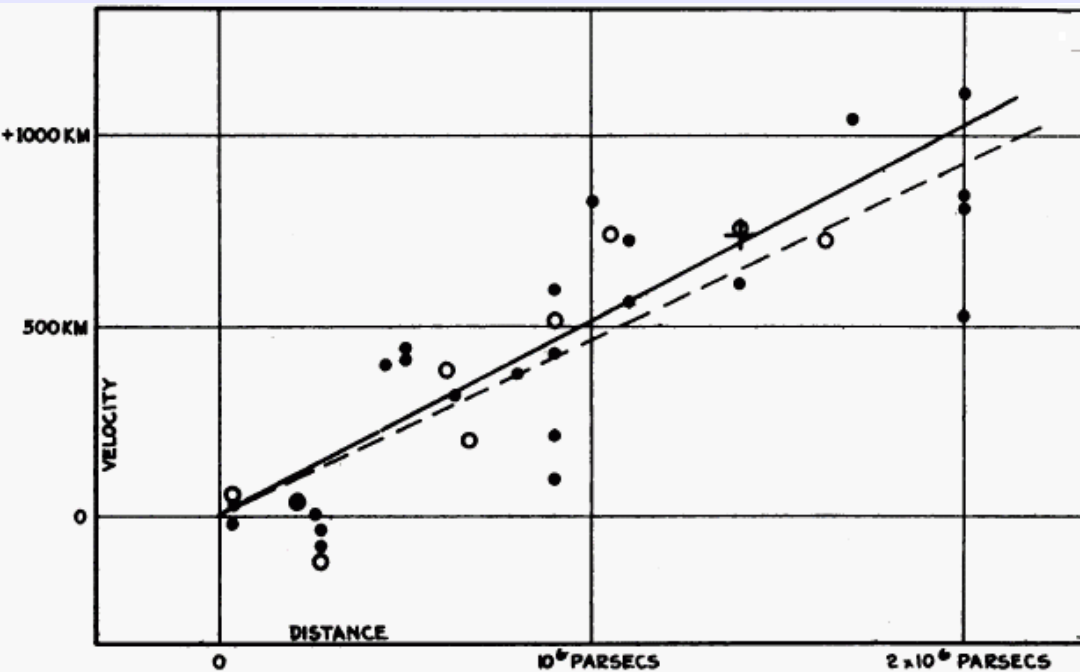
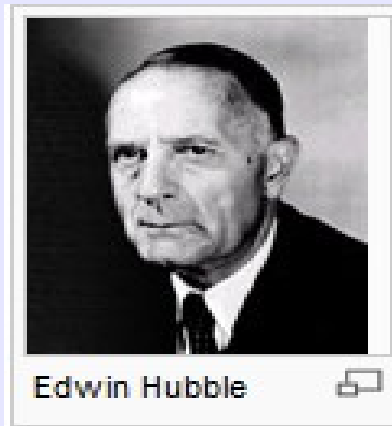


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

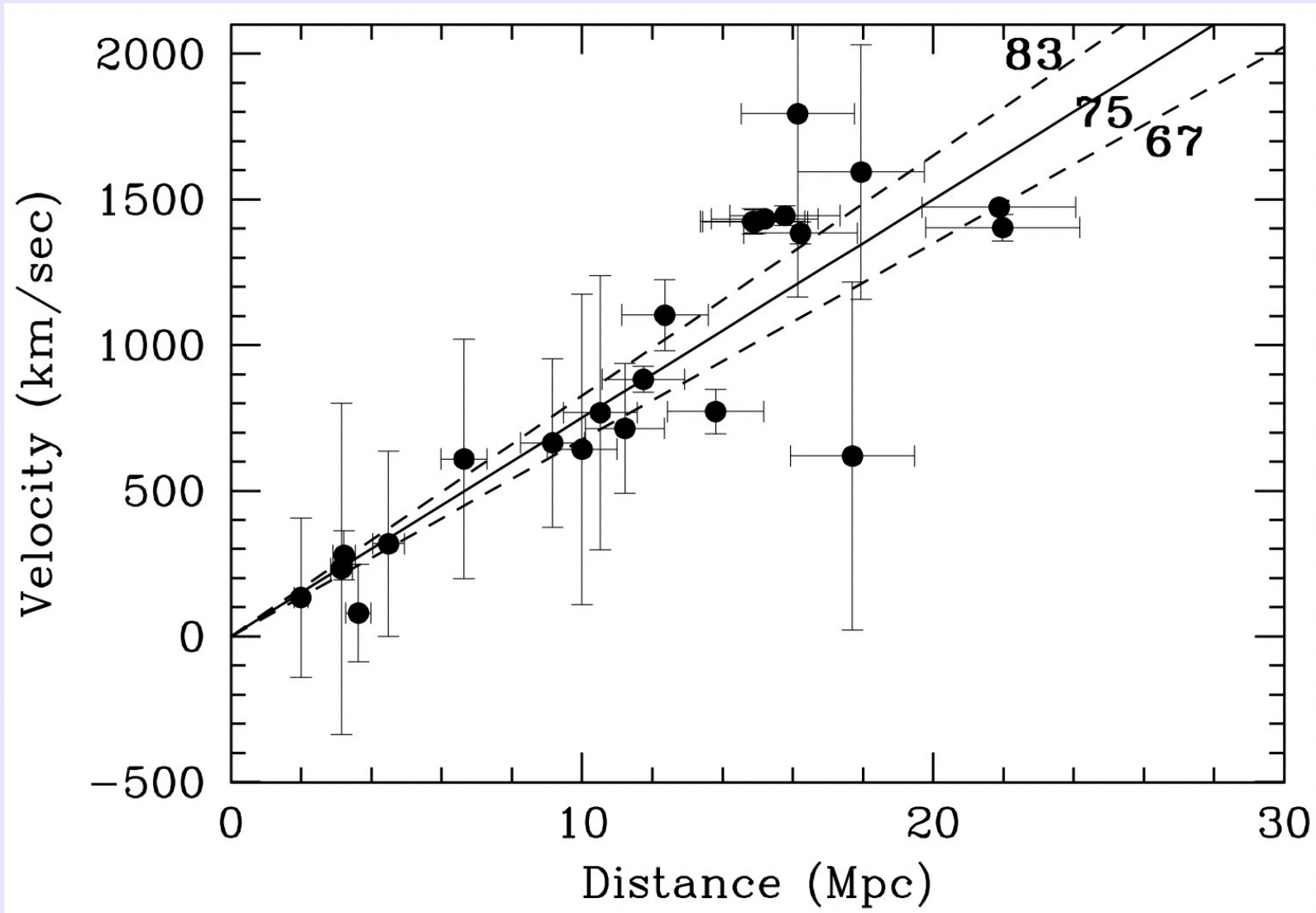
For each galaxy he could identify a known pattern of atomic spectral lines (from their relative intensities and spacings) which all exhibited a common redward frequency shift by a factor $1+z$. His fundamental discovery was that the velocities of distant galaxies he had studied increased linearly with distance

$$v = H_0 r$$

This is the Hubble's law
(1929)

Main message: The Universe is expanding following the Hubble's flow

Hubble law (recent data)



Let's continue remembering basic definitions

The redshift (z) can be seen as a measure of time or distance. It is a new variable related to the expansion parameter (a), the scale factor.

$$1 + z = \frac{a_0}{a}$$

- The emitted wavelength of the radiation from a distance source (a galaxy, for example) is shifted when it is observed (today)

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} .$$

- The wavelength from Andromeda M31 is blue-shifted!

Some astronomical numbers

- Distance is usually measured in Parsecs: $1 \text{ parsec} = 3.09 \times 10^{16} \text{ m}$
- One light year = $9.46 \times 10^{15} \text{ m}$

Distance to the nearest star (α Centauri) 1.3 pc

Diameters of globular clusters 5–30 pc

Thickness of our Galaxy, the 'Milky Way' 0.3 kpc

Distance to our galactic centre 8 kpc

Radius of our Galaxy, the 'Milky Way' 12.5 kpc

Distance to the nearest galaxy (Large Magellanic Cloud) 55 kpc

Distance to the Andromeda nebula (M31) 770 kpc

Size of galaxy groups $1/h$ Mpc

Thickness of filament clusters $5/h$ Mpc

Distance to the Local Supercluster centre (in Virgo) 17 Mpc

Distance to the 'Great Attractor' $44/h$ Mpc

Size of superclusters $50/h$ Mpc

Size of large voids $60/h$ Mpc

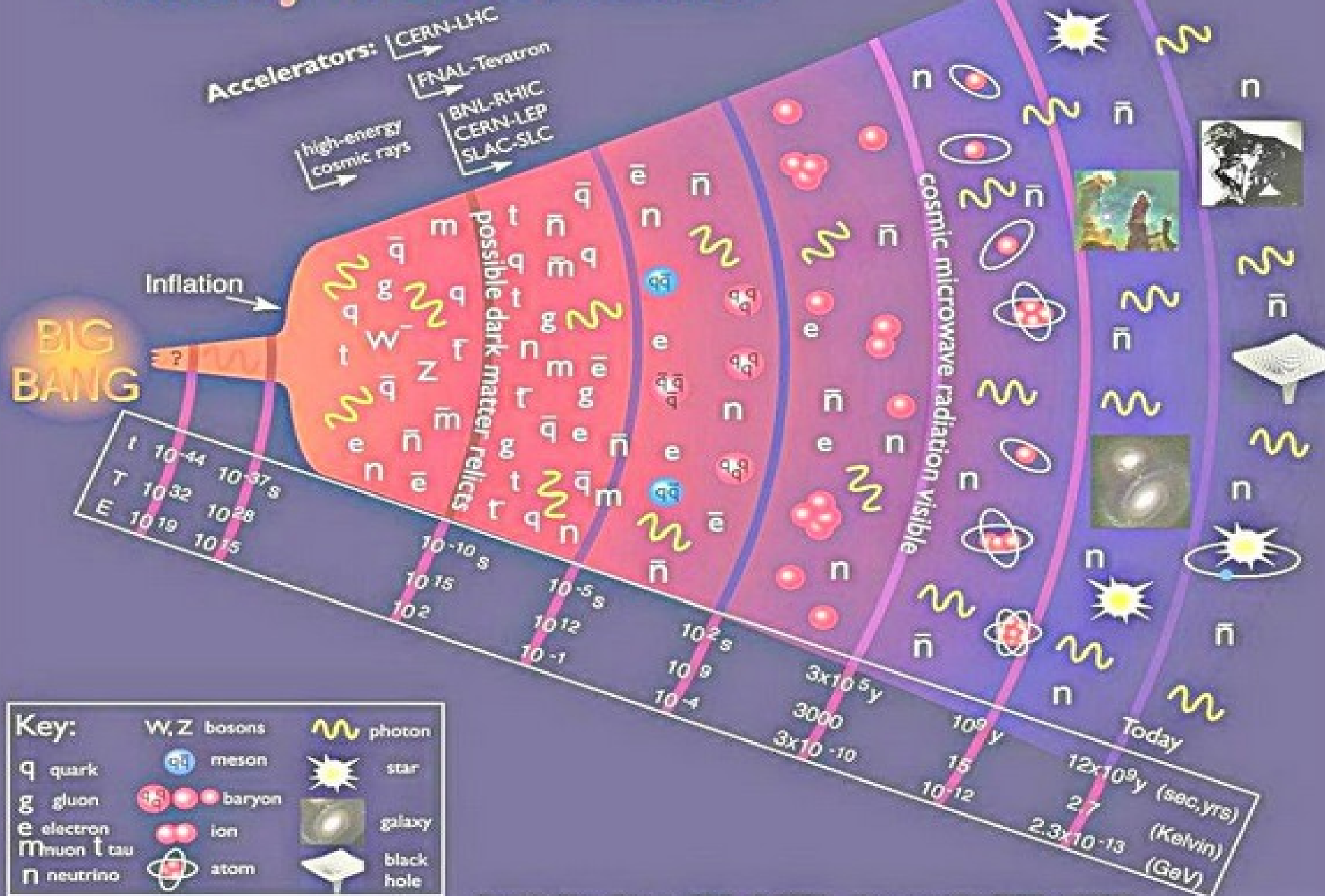
Distance to the Coma cluster $100/h$ Mpc

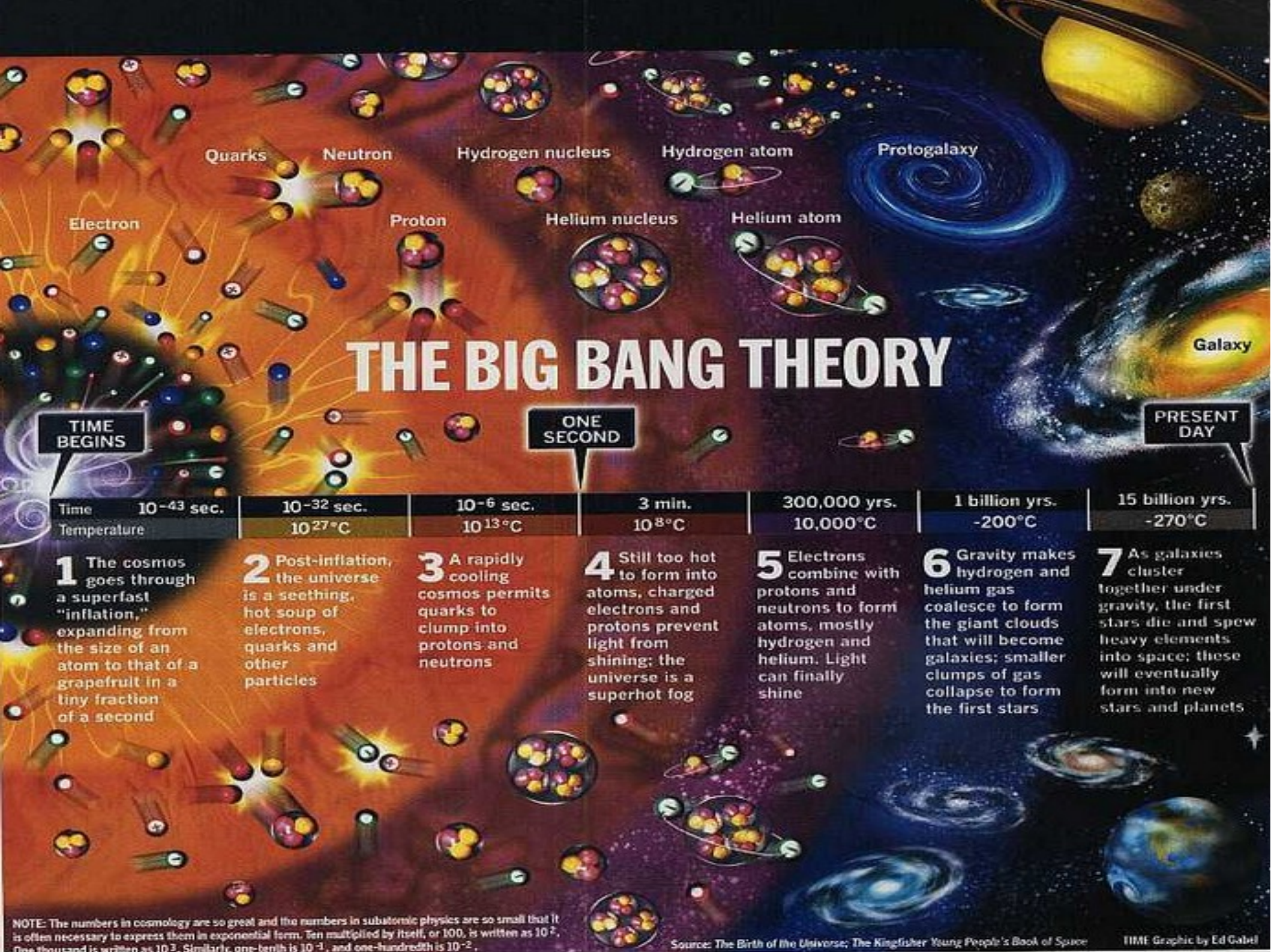
Length of filament clusters $100/h$ Mpc

Size of the 'Great Wall' $> 60 \times 170/h^2 \text{ Mpc}^2$

Hubble radius $3000/h$ Mpc

History of the Universe





Quarks

Neutron

Hydrogen nucleus

Hydrogen atom

Protogalaxy

Electron

Proton

Helium nucleus

Helium atom

Galaxy

THE BIG BANG THEORY

TIME
BEGINS

ONE
SECOND

PRESENT
DAY

Time	10 ⁻⁴³ sec.	10 ⁻³² sec.	10 ⁻⁶ sec.	3 min.	300,000 yrs.	1 billion yrs.	15 billion yrs.
Temperature		10 ²⁷ °C	10 ¹³ °C	10 ⁸ °C	10,000°C	-200°C	-270°C

1 The cosmos goes through a superfast "inflation," expanding from the size of an atom to that of a grapefruit in a tiny fraction of a second

2 Post-inflation, the universe is a seething, hot soup of electrons, quarks and other particles

3 A rapidly cooling cosmos permits quarks to clump into protons and neutrons

4 Still too hot to form into atoms, charged electrons and protons prevent light from shining; the universe is a superhot fog

5 Electrons combine with protons and neutrons to form atoms, mostly hydrogen and helium. Light can finally shine

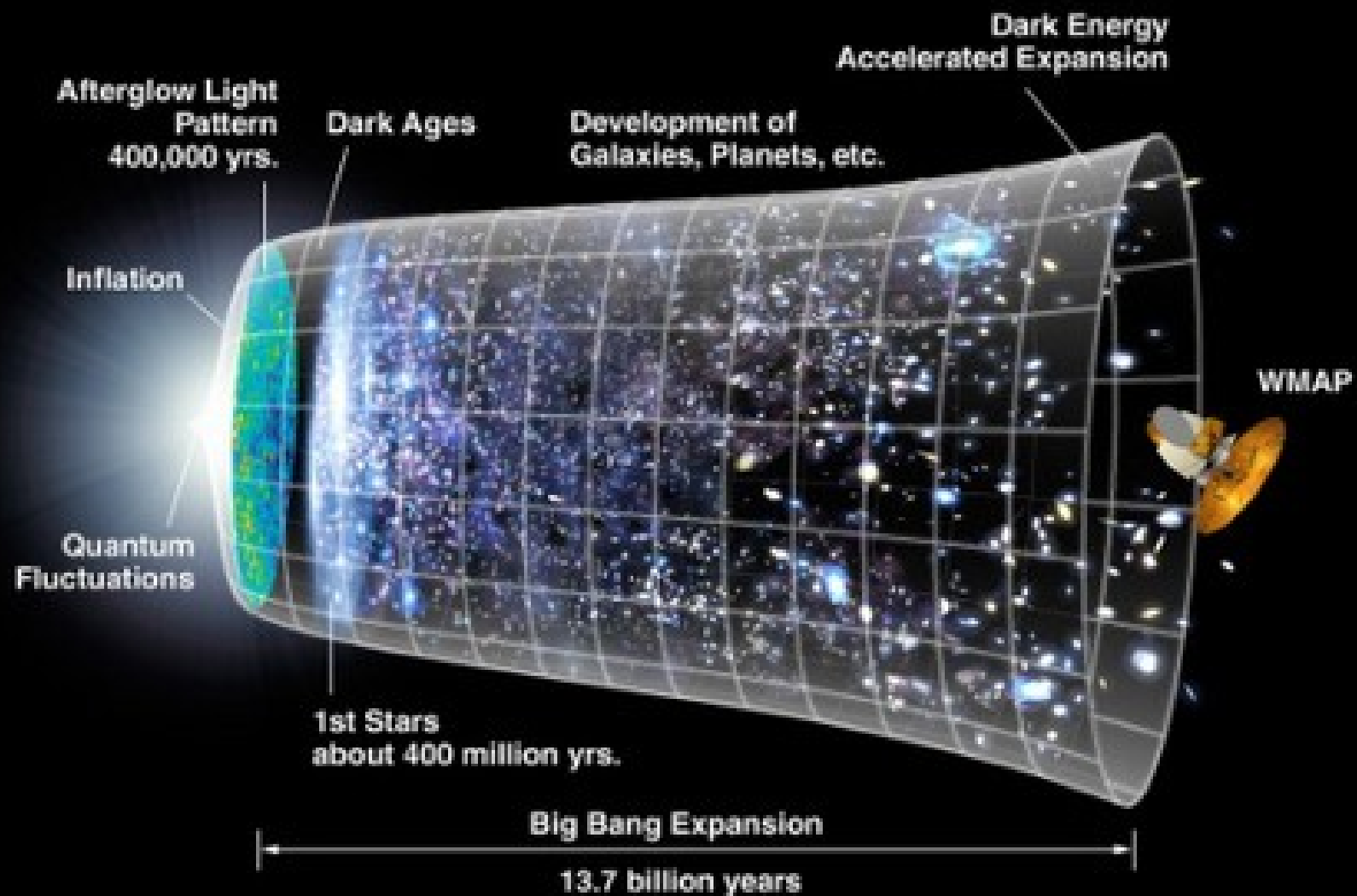
6 Gravity makes hydrogen and helium gas coalesce to form the giant clouds that will become galaxies; smaller clumps of gas collapse to form the first stars

7 As galaxies cluster together under gravity, the first stars die and spew heavy elements into space; these will eventually form into new stars and planets

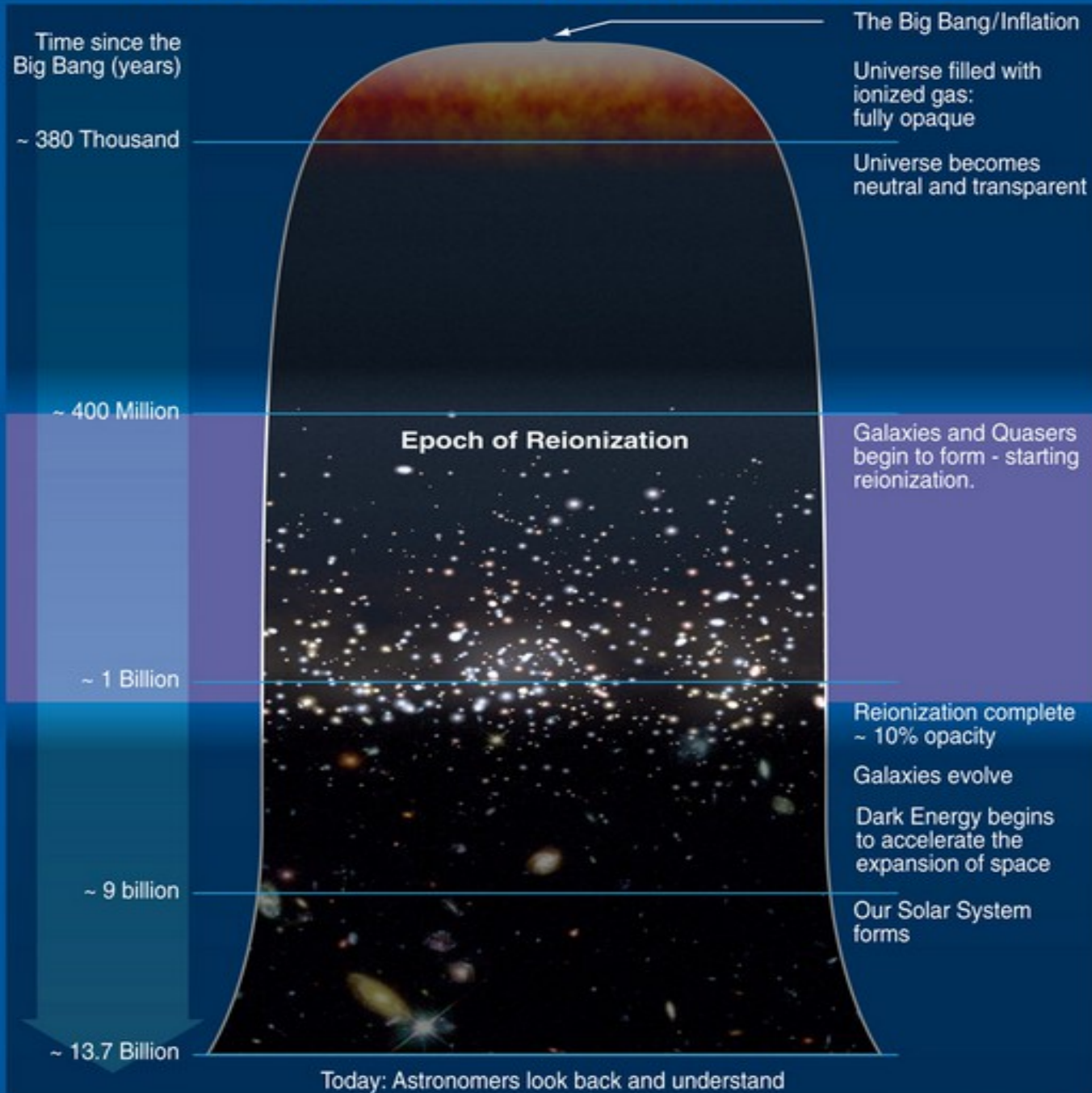
NOTE: The numbers in cosmology are so great and the numbers in subatomic physics are so small that it is often necessary to express them in exponential form. Ten multiplied by itself, or 100, is written as 10². One thousand is written as 10³. Similarly, one-billionth is 10⁻⁹, and one-hundredth is 10⁻².

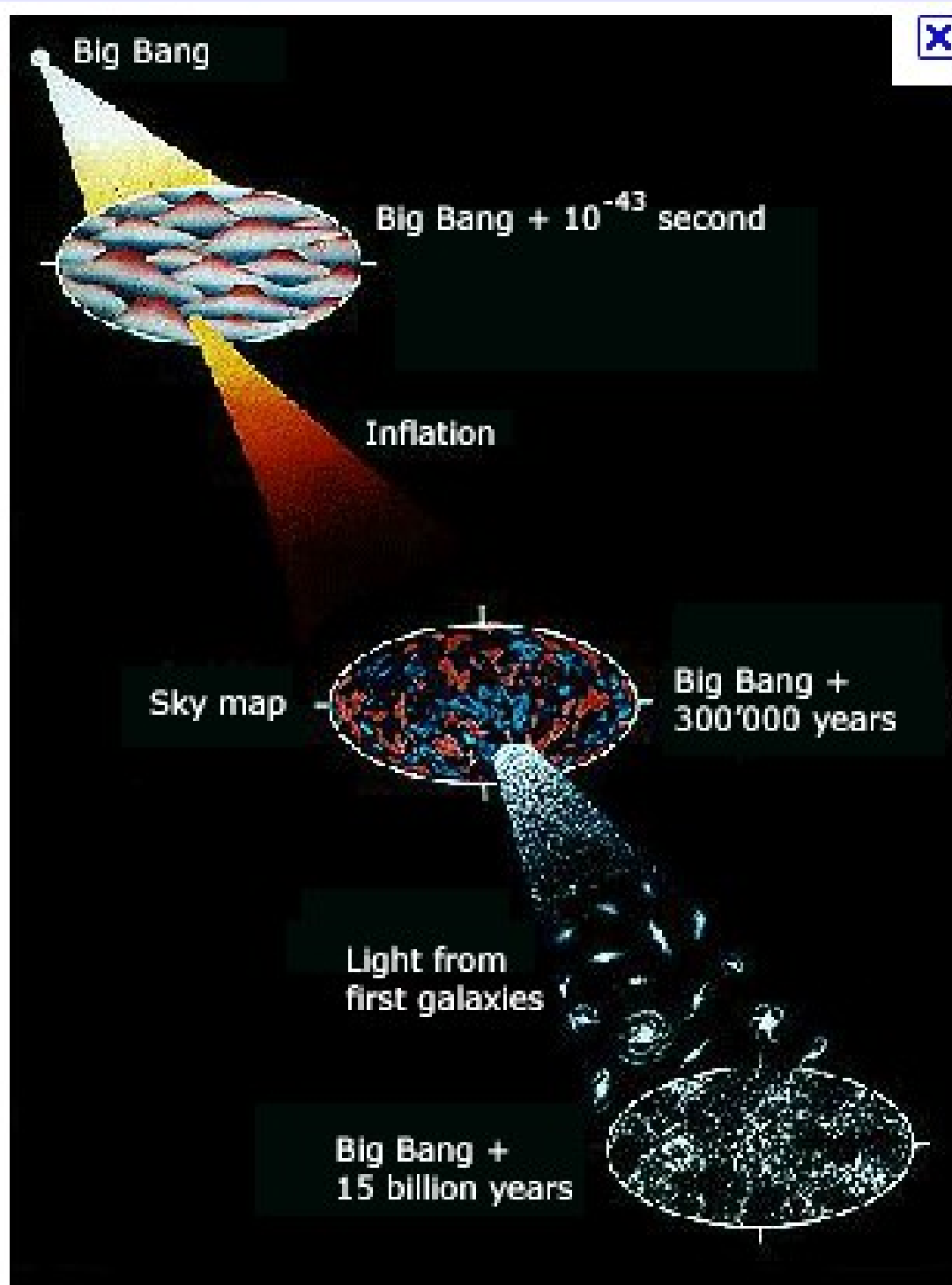
Sources: The Birth of the Universe; The Kingfisher Young People's Book of Space

TIME Graphic by Ed Galt



First Stars and Reionization Era





Quantum fluctuations present at the primordial matter/fields distribution. Seeds for galaxy formation

One has Inflation during ~ 60 e-folds
Microscopic fluctuations became macroscopic.

Matter-Radiation equality: $z \sim 3000$

The LSS ($z \sim 1000$): level of inhomogeneity is around 0.00001!!

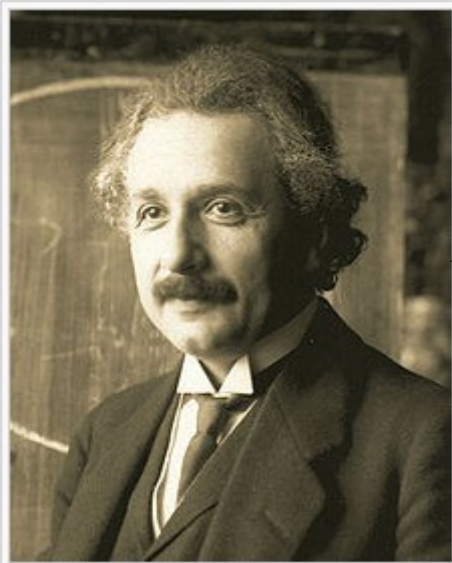
Structure formation Epoch: from 200,000 yrs until today!

Reionization at 1 Gy or $z \sim 15$. End of the Dark Ages.

Observable range for Supernovae $z < 1.5$

Let's continue remembering basic definitions

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad \text{FLRW metric}$$



Albert Einstein, 1921

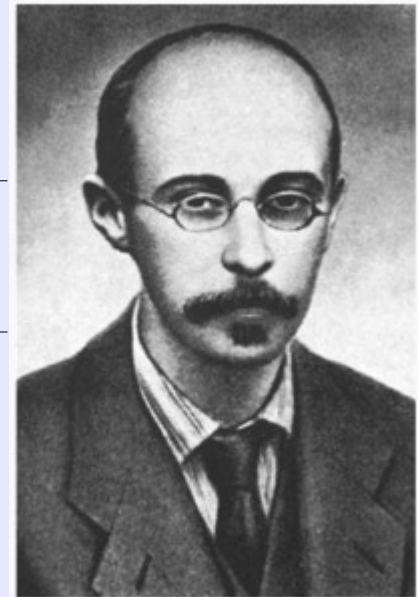
1915,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0$$

1922

Alexander Friedman



A. Friedman

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}$$

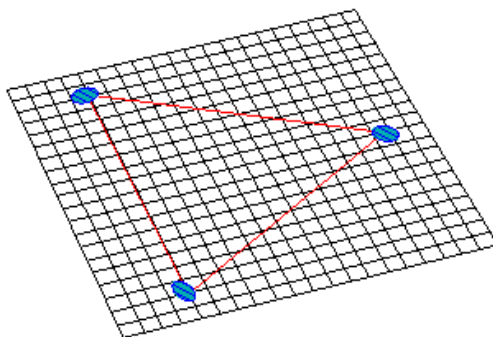
$$\rho_c = \frac{3H^2}{8\pi G}$$

$$\Omega \equiv \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3H^2}$$

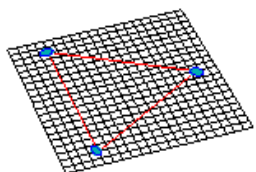
$$\frac{H^2}{H_0^2} = \Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$$

$$\dot{\rho}_r + 4 \frac{\dot{a}}{\rho_r} = 0 \Rightarrow \rho_r = \rho_{r0} a^{-4},$$

$$\dot{\rho}_m + 3 \frac{\dot{a}}{\rho_m} = 0 \Rightarrow \rho_m = \rho_{m0} a^{-3}$$

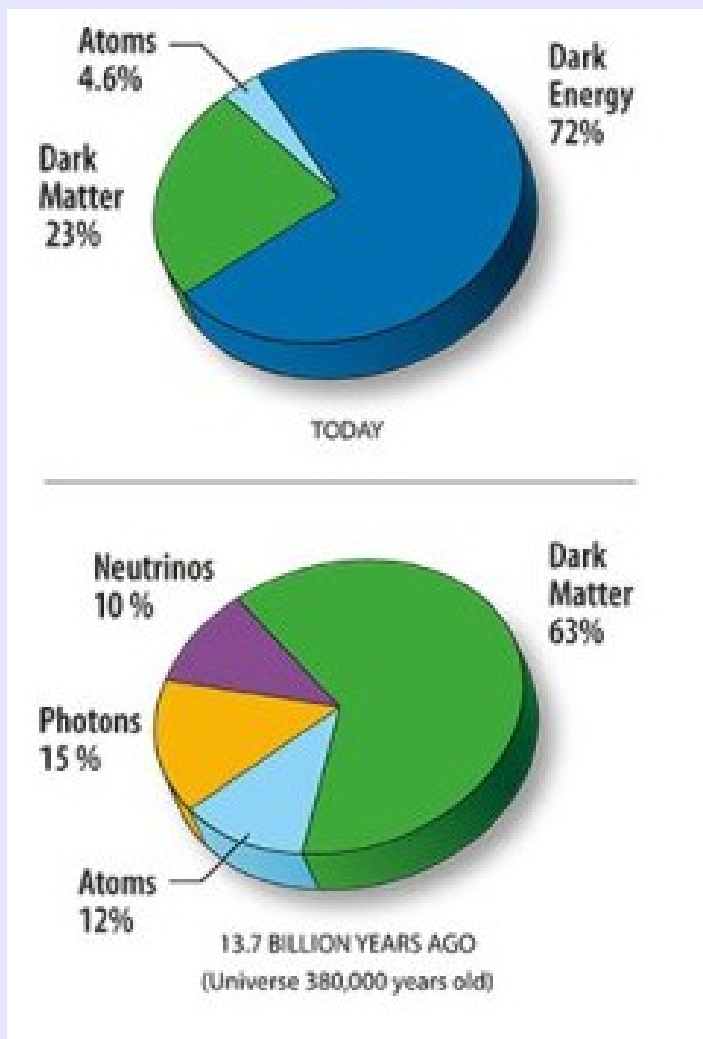


$t = t_0$
 $R = 1$



$R = 0.5$

Matter content of the Universe

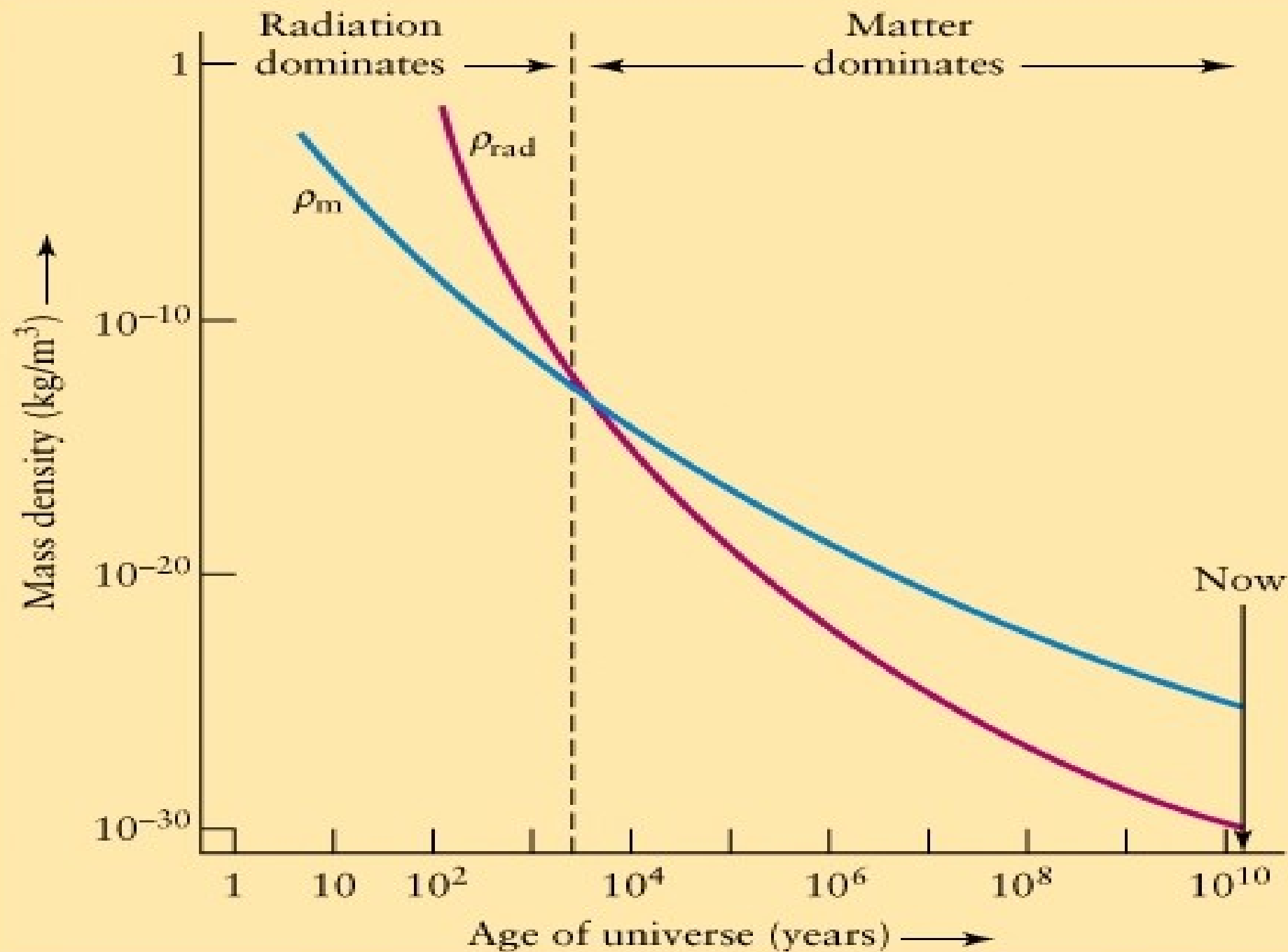


Standard description:

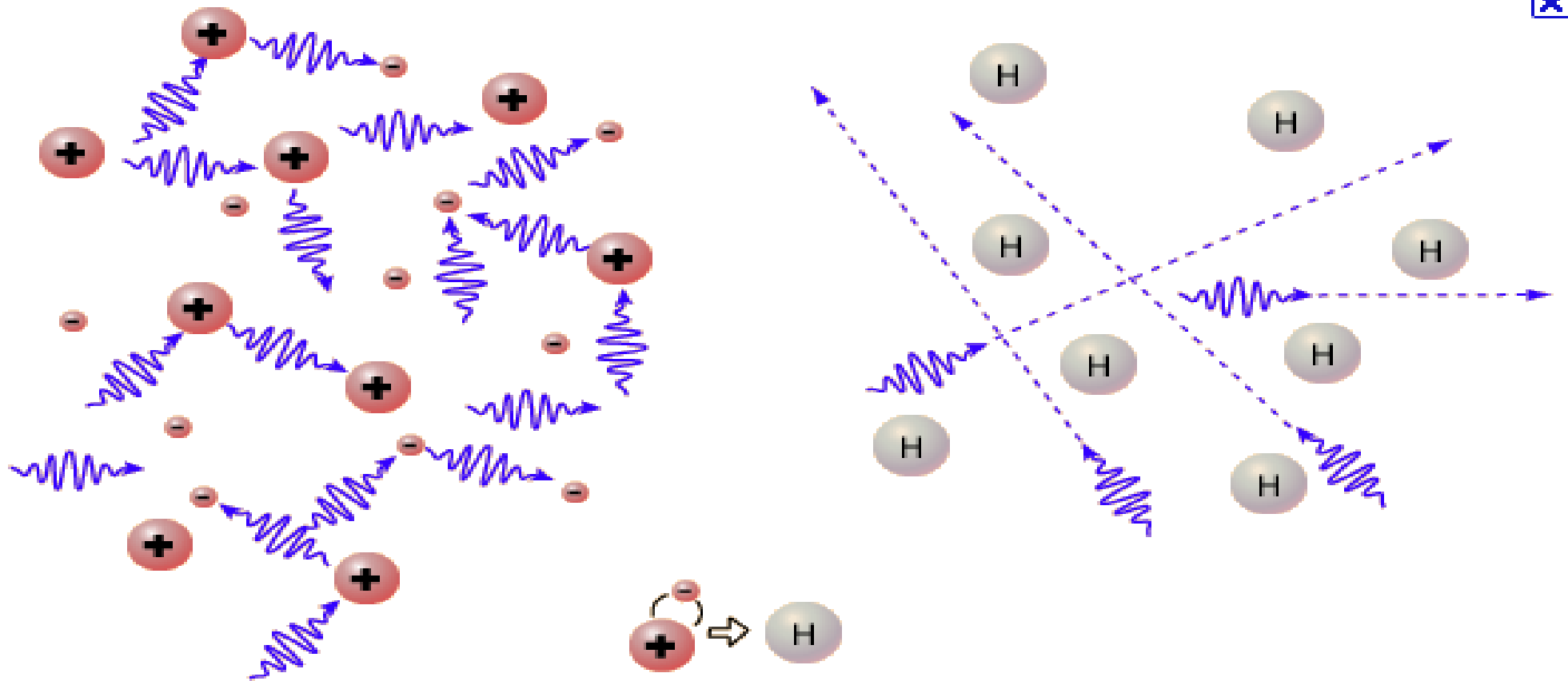
- Baryoninc Matter ($\sim 5\%$)
- Neutrinos & Photons ($\sim 0.08\%$)
- Cold Dark Matter ($p=0$)($\sim 25\%$)
- Dark Energy (Lambda?)($\sim 70\%$)

$$H^2 = H_0^2 \{ \Omega_{Bar} + \Omega_{Pho} + \Omega_{Neu} + \Omega_{DM} + \Omega_{DE} \}$$

$$H^2 = H_0^2 \left[\frac{\Omega_{b0} + \Omega_{dm0}}{a^3} + \frac{\Omega_{r0}}{a^4} + \Omega_{\Lambda} \right]$$



What happens at the Last Scattering Surface (LSS) Recombination and Decoupling (the origin of the CMB)

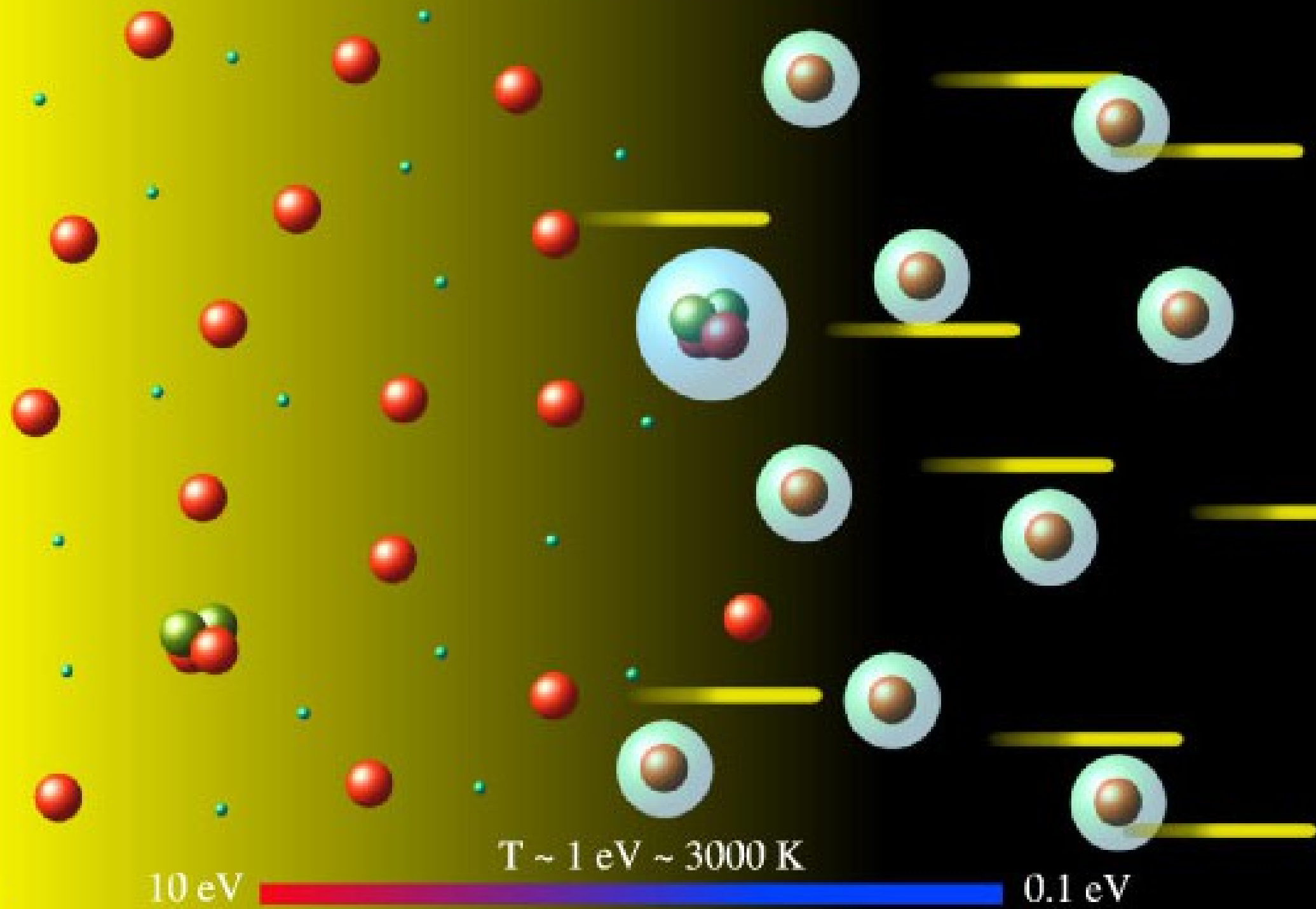


At high temperatures, a hot plasma of charged particles interacts strongly with the radiation. That effectively confines it in the interior of stars and in the early universe.

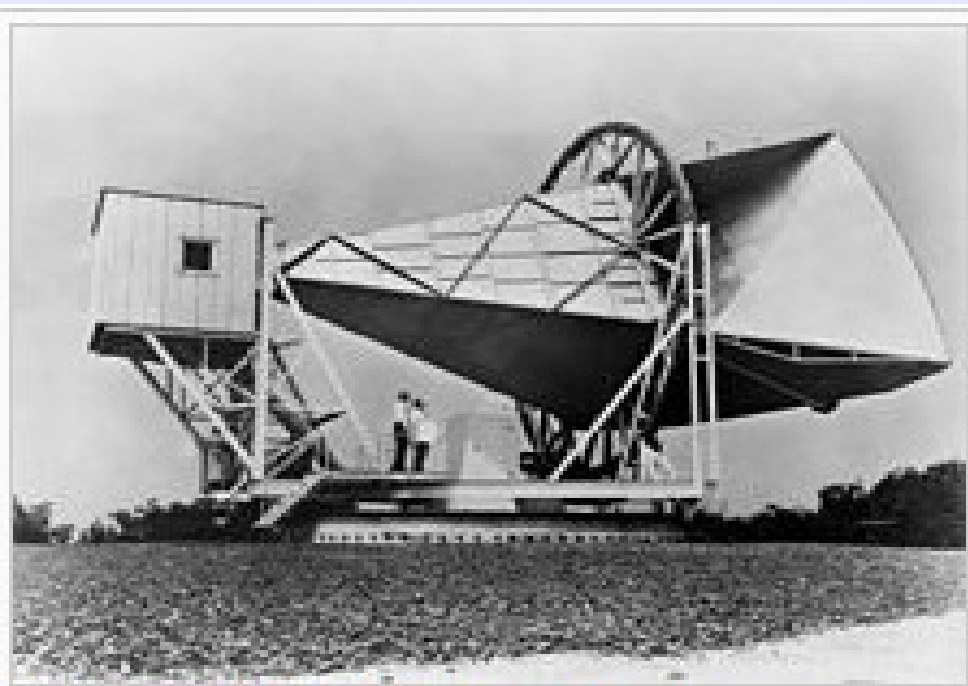
Below about 3000K, protons and electrons can combine into neutral hydrogen.

Photons can travel large distances in the neutral hydrogen, so the confinement is effectively broken. Photons can move freely throughout the space.

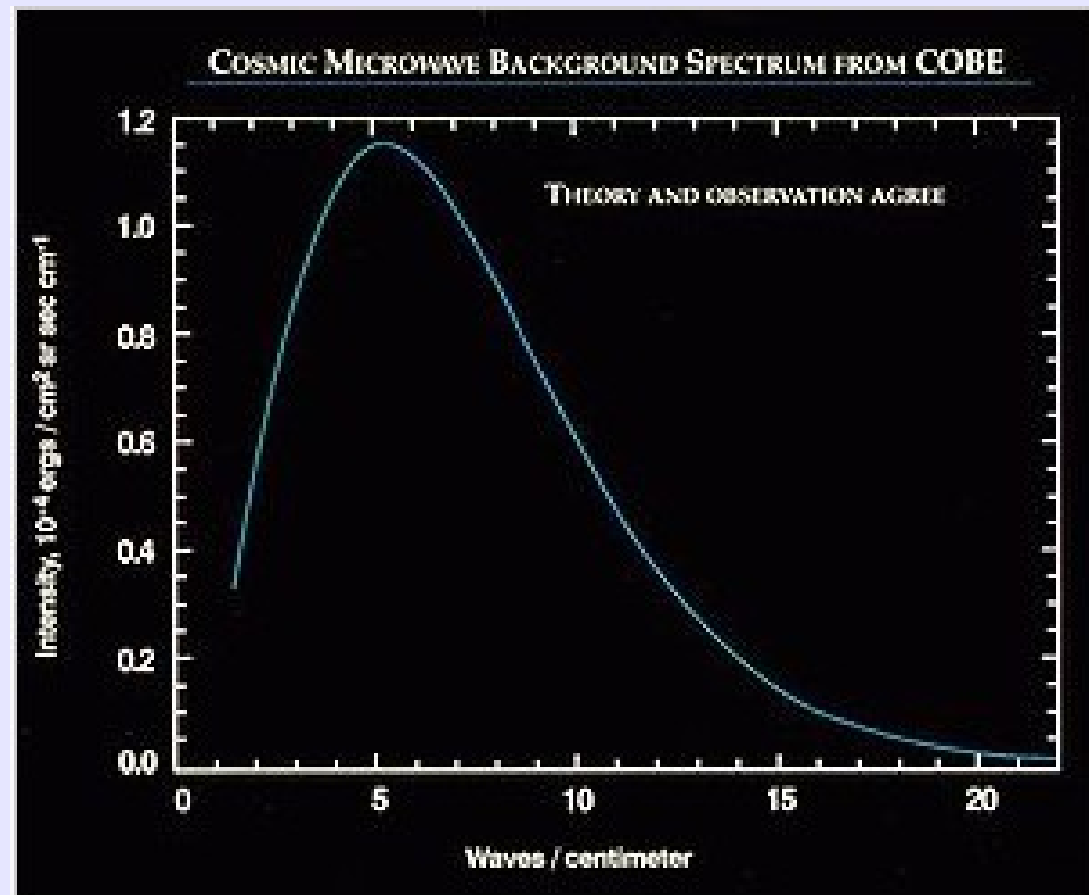
Recombination



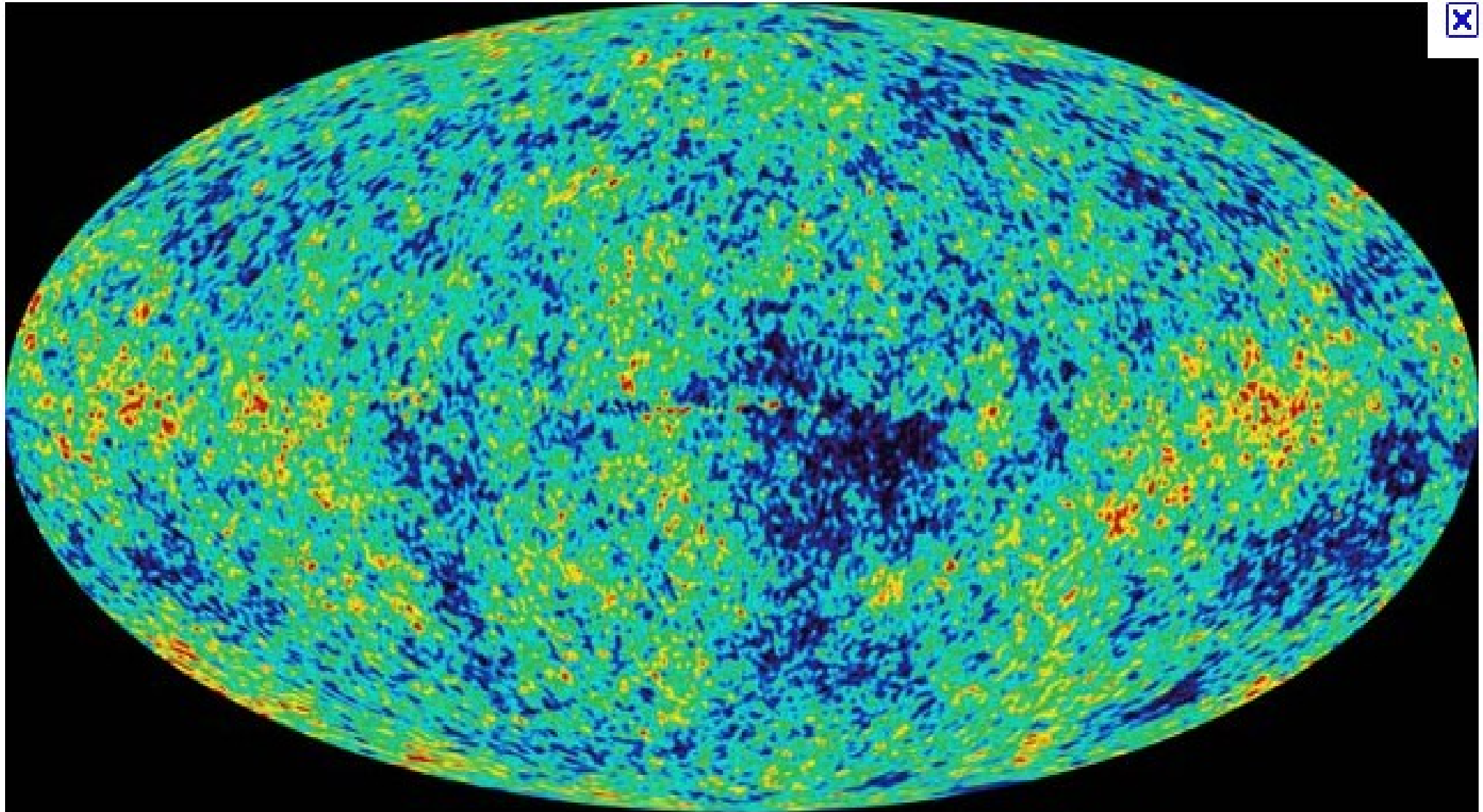
The cosmic microwave background (CMB) radiation



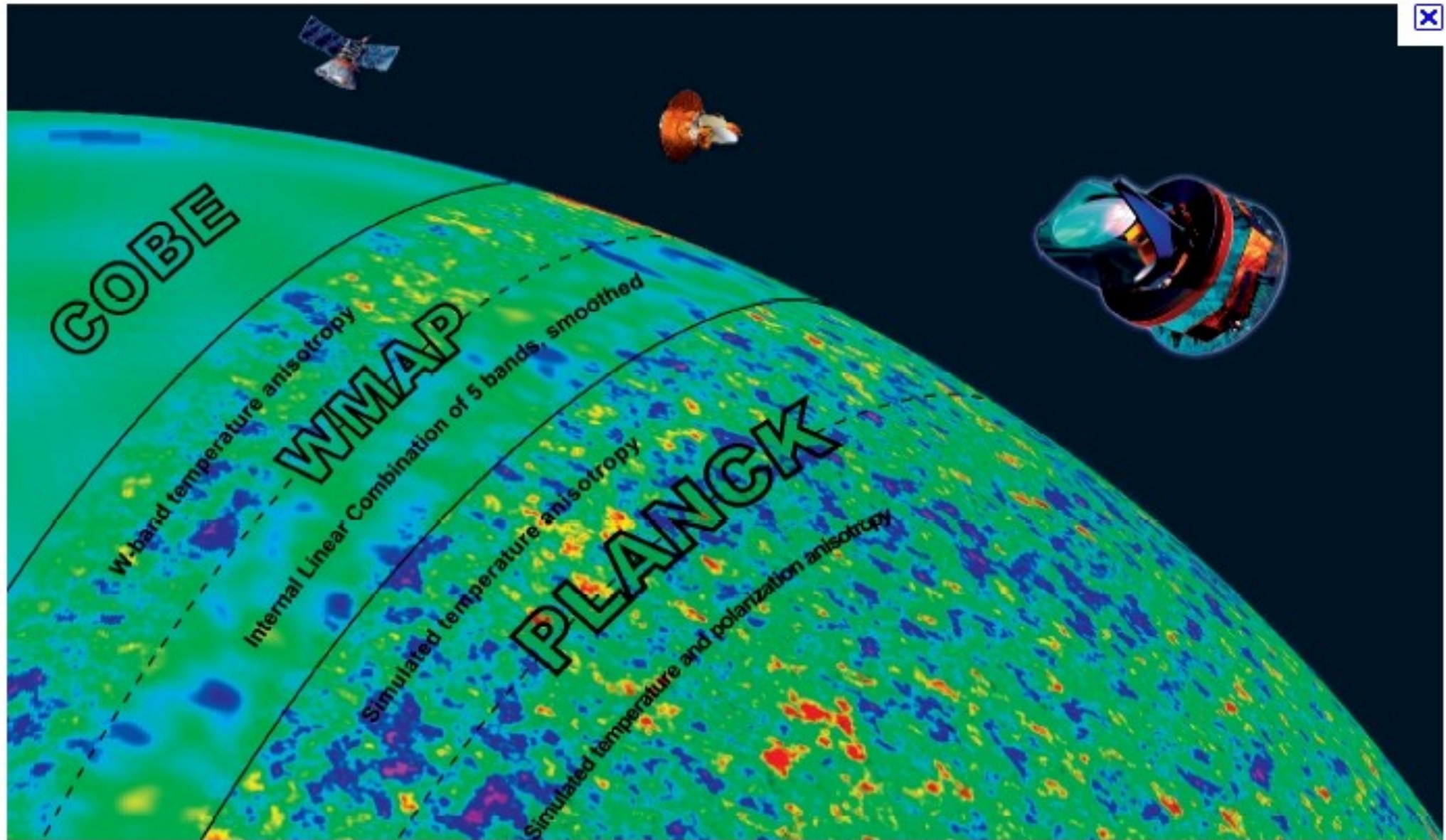
The [Holmdel Horn Antenna](#) on which Penzias and Wilson discovered the cosmic microwave background.



*A “thermal picture” of the radiation that fills the Universe
Fluctuations of order ~ 0.00001 K*

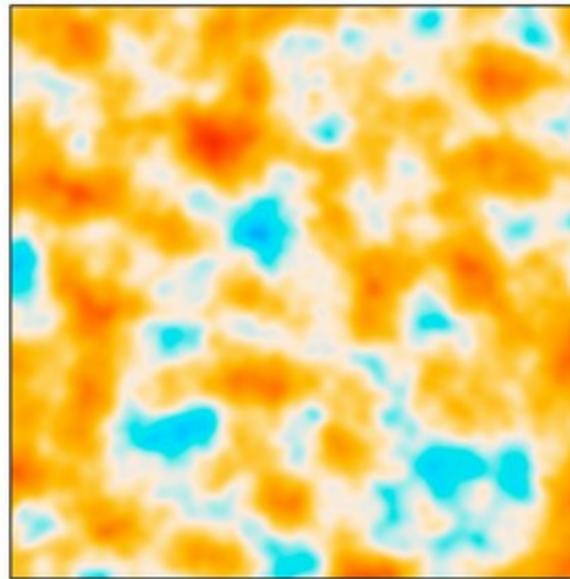
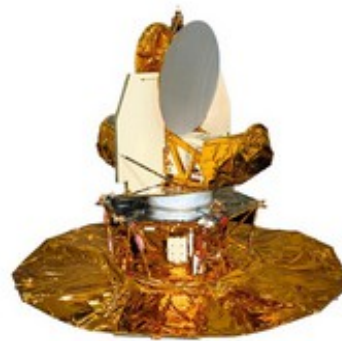


Evolution of the observations 1990ths, 2000ths, 2010ths

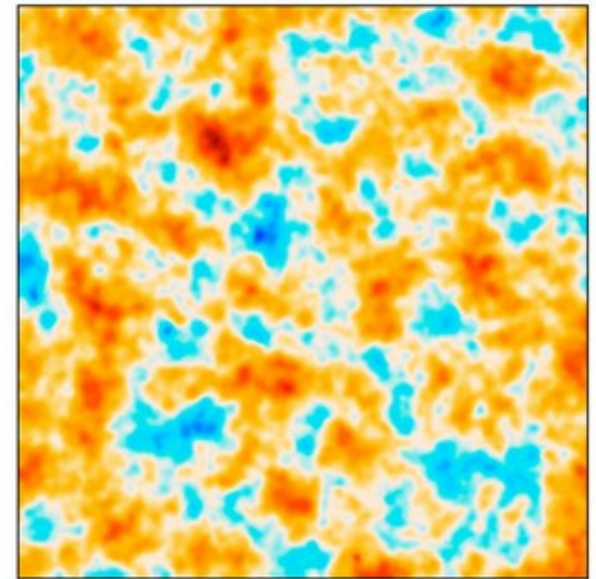




COBE



WMAP



Planck

Planck satellite – European Spatial Agency

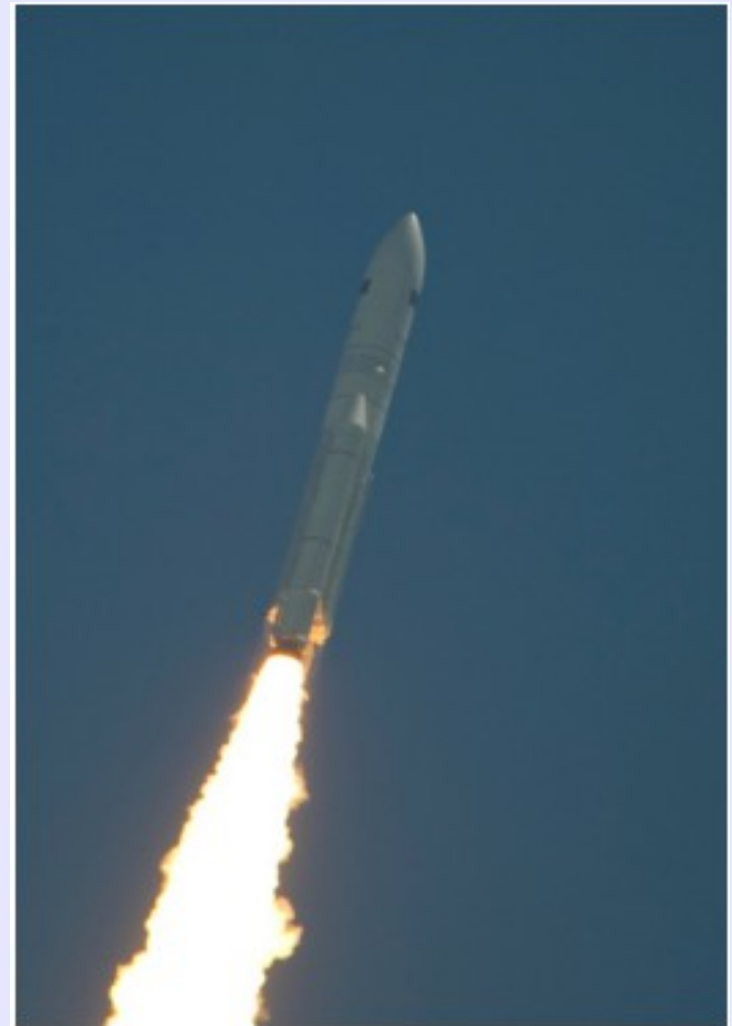


Flight to Kourou, French Guiana

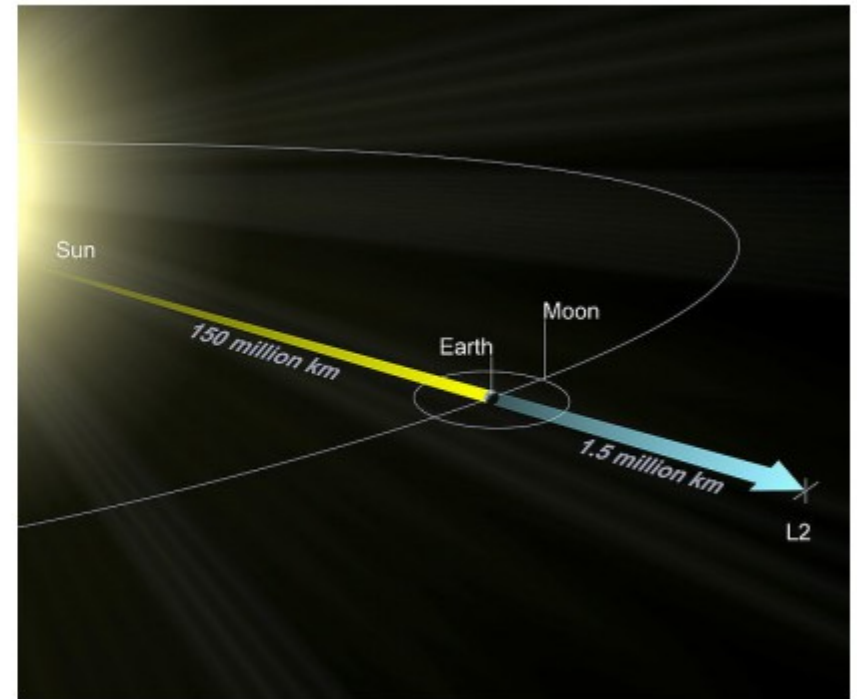
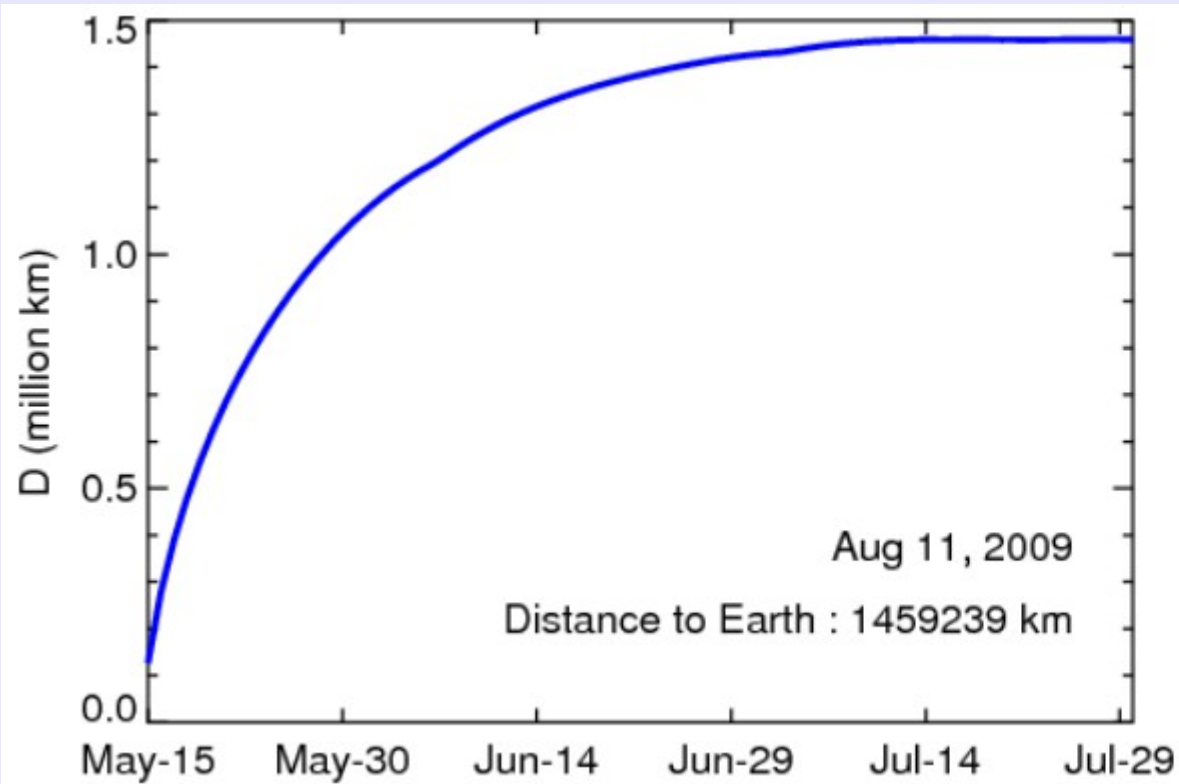
CSL (Liege) 18th February 2009



Launch day: Kourou, French Guiana, 14th May 2009.

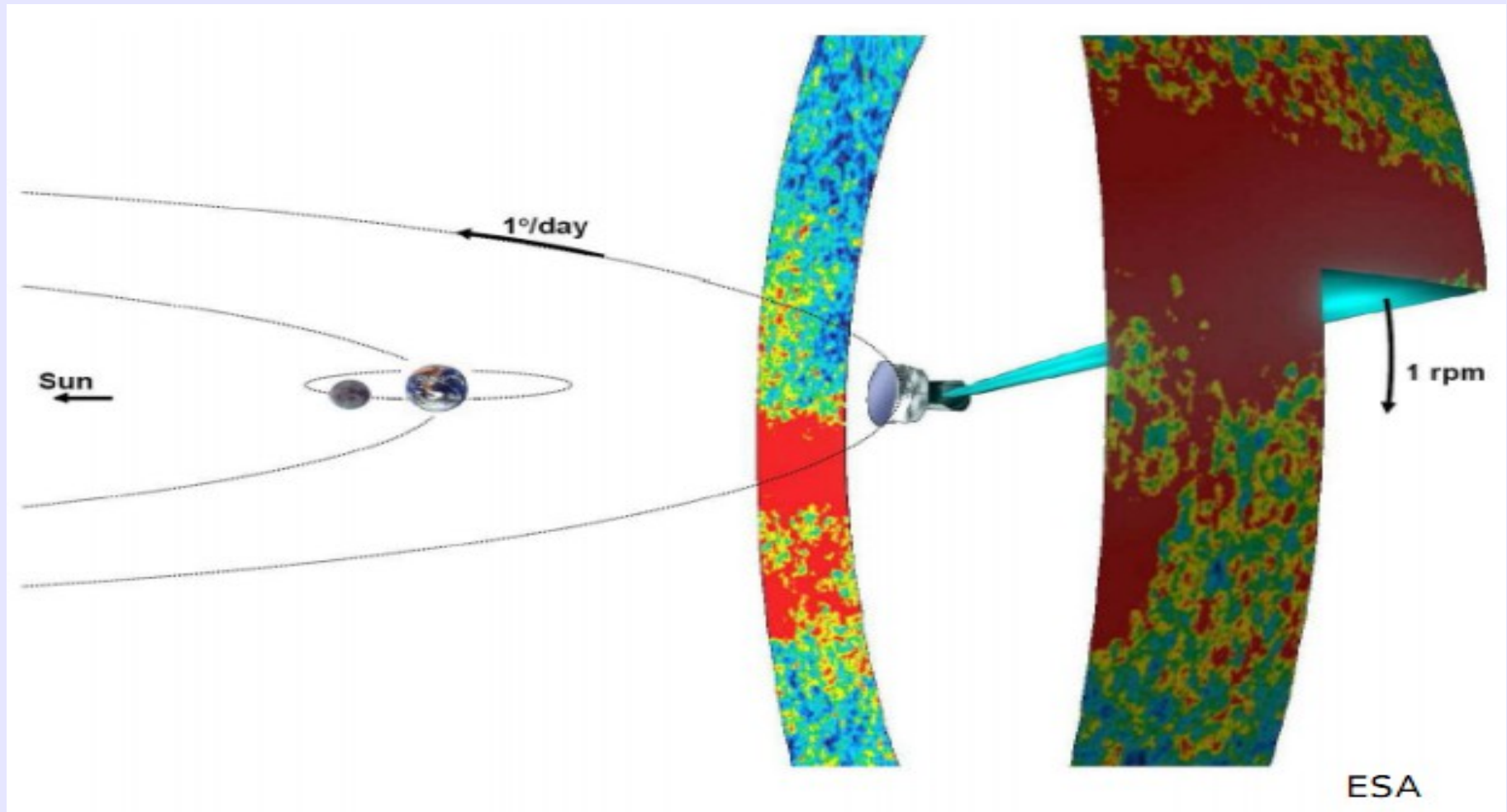


The travel

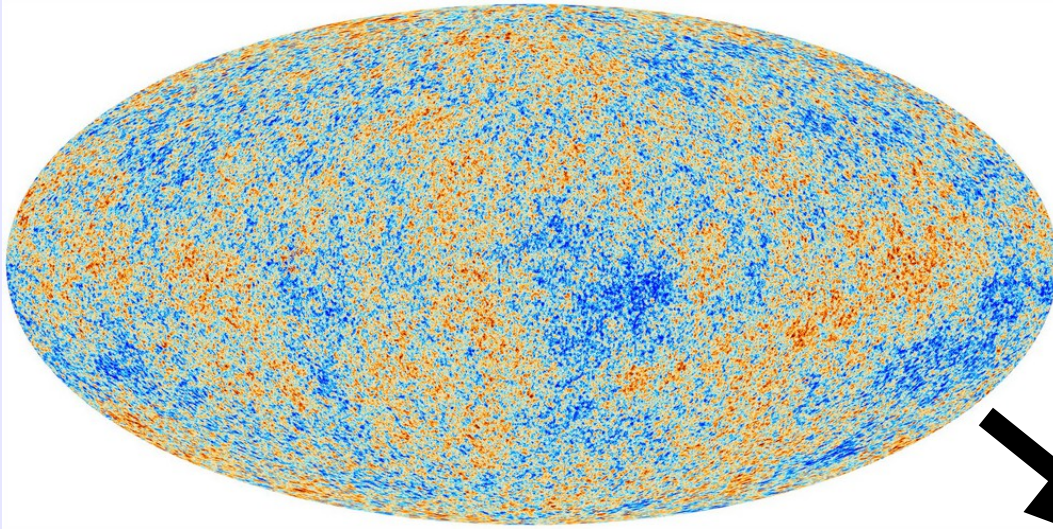


Scanning strategy

- Planck builds up a map from a series of “rings” – spinning at 1 rpm.
- One survey of the sky roughly every six months.

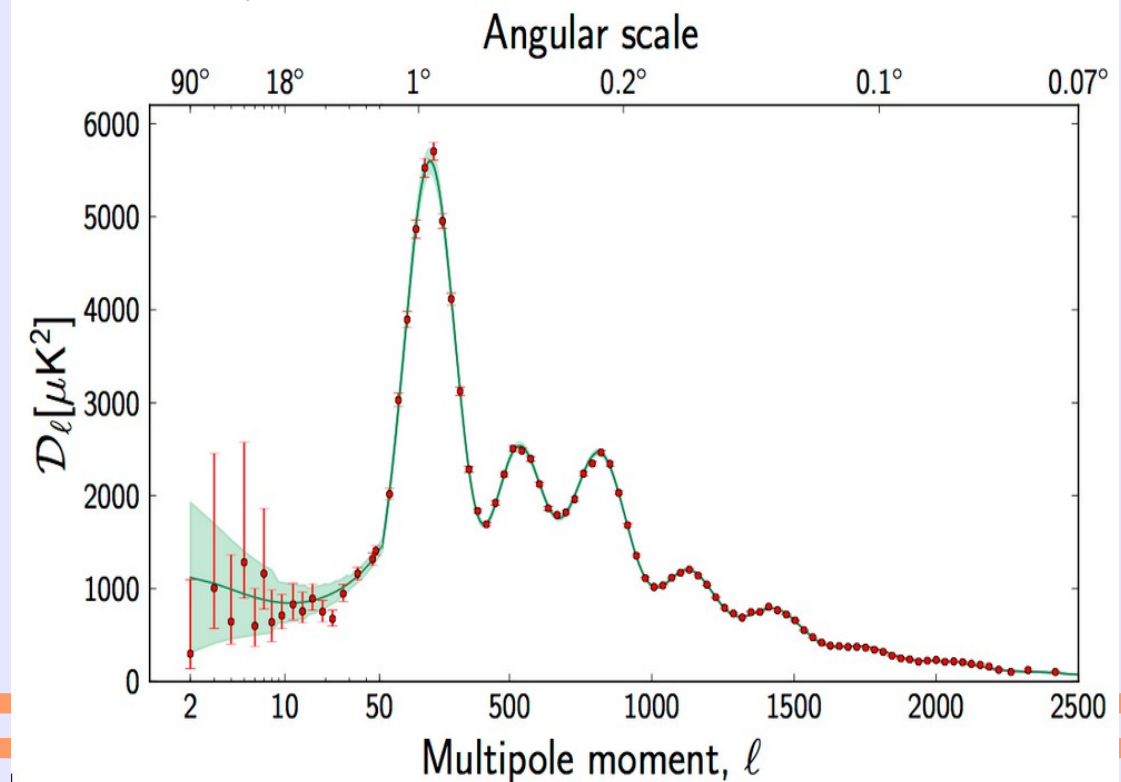


Temperature anisotropies



The information of photons coming from all directions is projected using spherical harmonics

The TT power spectrum



***Thus we have a map of the temperature anisotropies!
How to compare this observation with theory?***

The angular Power Spectrum: to expand the distribution of temperature (T) as a sum over spherical harmonics.

$$\frac{\Delta T(\theta, \phi)}{T} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} a_{lm} Y_{lm}(\theta, \phi)$$

$$C_l \equiv \langle |a_{lm}|^2 \rangle$$

$$\vartheta \simeq 60^\circ / l$$

$$C(\vartheta) = \left\langle \frac{\Delta T}{T}(\hat{\mathbf{n}}_1) \frac{\Delta T}{T}(\hat{\mathbf{n}}_2) \right\rangle$$

$$C(\vartheta) = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\cos \vartheta)$$

The theory of structure formation in more details

The growth of structures

The Jeans Mechanism (1902) – small fluctuations in a static background. There was no evidence for a expanding Universe! Usefull to understand formation of planets and stars. It is easy to implement the effects of Expansion.

Perturbations in Newtonian Cosmology – hydrodynamical approximation. It provides the main qualitative aspects of the structure formation process.

Perturbations in General Relativity – the full treatment. Scalar, Vektorial and Tensorial perturbations (the latter occurs only in the relativistic context) .

Newtonian theory of cosmological perturbations

The following set of equations defines the Newtonian Cosmology

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0,$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} = -\vec{\nabla} \Psi - \frac{\vec{\nabla} p}{\rho},$$

$$\nabla^2 \Psi = 4\pi G \rho,$$

The solutions are:

$$\rho = \frac{\rho_0}{a^3}, \quad \vec{u} = H \vec{r}, \quad \vec{g} = -\frac{4}{3}\pi G \rho \vec{r}.$$

We will introduce small fluctuations around the background quantities

$$\rho = \rho_0(t) [1 + \delta(\vec{r}, t)]$$

$$p = p_0(t) + \delta p(\vec{r}, t)$$

$$\Psi = \Psi_0(\vec{r}, t) + \varphi(\vec{r}, t)$$

$$\vec{u} = \vec{u}_0(\vec{r}, t) + \vec{v}(\vec{r}, t)$$

Perturbed equations

We Fourier-transform the first order quantities as

$$\delta f(\vec{r}, t) = \delta f(t) e^{-\frac{i\vec{k} \cdot \vec{r}}{a}}$$

where k is the wavelenght of the perturbation.

$$\begin{aligned}\dot{\delta} &= -\frac{i\vec{k} \cdot \vec{v}}{a} \\ \dot{\vec{v}} + \frac{\dot{a}}{a}\vec{v} &= -\frac{i\vec{k}}{a}\varphi - v_s^2 \frac{i\vec{k}}{a}\delta \\ -\frac{k^2}{a^2}\varphi &= 4\pi G\rho\delta,\end{aligned}$$

Combining the above equations we end up with

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left\{ \frac{k^2 v_s^2}{a^2} - 4\pi G\rho \right\} \delta = 0,$$

Cosmological Perturbation Theory in the Synchronous vs. Conformal Newtonian Gauge

Chung-Pei Ma and Edmund Bertschinger

Synchronous gauge —

$$ds^2 = a^2(\tau) \{ -d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \} .$$

$$h_{ij} = h\delta_{ij}/3 + h_{ij}^{\parallel} + h_{ij}^{\perp} + h_{ij}^T .$$

$$h \equiv h_{ii}$$

We will be working in the Fourier space k in this paper. We introduce two fields $h(\vec{k}, \tau)$ and $\eta(\vec{k}, \tau)$ in k -space and write the scalar mode of h_{ij} as a Fourier integral

$$h_{ij}(\vec{x}, \tau) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \left\{ \hat{k}_i \hat{k}_j h(\vec{k}, \tau) + \left(\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) 6\eta(\vec{k}, \tau) \right\} , \quad \vec{k} = k \hat{k} .$$

Note that h is used to denote the trace of h_{ij} in both the real space and the Fourier space.

Conformal Newtonian gauge —

$$ds^2 = a^2(\tau) \left\{ -(1 + 2\psi) d\tau^2 + (1 - 2\phi) dx^i dx_i \right\} .$$

Einstein Equations and Energy-Momentum Conservation

Synchronous gauge —

$$\begin{aligned}k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} &= 4\pi Ga^2\delta T^0_0(\text{Syn}), \\k^2\dot{\eta} &= 4\pi Ga^2(\bar{\rho} + \bar{P})\theta(\text{Syn}), \\\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta &= -8\pi Ga^2\delta T^i_i(\text{Syn}), \\\ddot{h} + 6\ddot{\eta} + 2\frac{\dot{a}}{a}(\dot{h} + 6\dot{\eta}) - 2k^2\eta &= -24\pi Ga^2(\bar{\rho} + \bar{P})\Theta(\text{Syn}).\end{aligned}$$

Conformal Newtonian gauge —

$$\begin{aligned}k^2\phi + 3\frac{\dot{a}}{a}\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) &= 4\pi Ga^2\delta T^0_0(\text{Con}), \\k^2\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) &= 4\pi Ga^2(\bar{\rho} + \bar{P})\theta(\text{Con}), \\\ddot{\phi} + \frac{\dot{a}}{a}(\dot{\psi} + 2\dot{\phi}) + \left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\psi + \frac{k^2}{3}(\phi - \psi) &= \frac{4\pi}{3}Ga^2\delta T^i_i(\text{Con}), \\k^2(\phi - \psi) &= 12\pi Ga^2(\bar{\rho} + \bar{P})\Theta(\text{Con}),\end{aligned}$$

The conformal Newtonian potentials ϕ and ψ are related to the synchronous potentials h and η in k -space by

$$\begin{aligned}\psi(\vec{k}, \tau) &= \frac{1}{2k^2} \left\{ \ddot{h}(\vec{k}, \tau) + 6\ddot{\eta}(\vec{k}, \tau) + \frac{\dot{a}}{a} \left[\dot{h}(\vec{k}, \tau) + 6\dot{\eta}(\vec{k}, \tau) \right] \right\} , \\ \phi(\vec{k}, \tau) &= \eta(\vec{k}, \tau) - \frac{1}{2k^2} \frac{\dot{a}}{a} \left[\dot{h}(\vec{k}, \tau) + 6\dot{\eta}(\vec{k}, \tau) \right] .\end{aligned}$$

The label “Syn” is used to distinguish the components of the energy-momentum tensor in the synchronous gauge from those in the conformal Newtonian gauge. The variables θ and Θ are defined as

$$(\bar{\rho} + \bar{P})\theta \equiv ik^j \delta T_j^0 , \quad (\bar{\rho} + \bar{P})\Theta \equiv -(\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij})\Sigma_j^i ,$$

and $\Sigma_j^i \equiv T_j^i - \delta_j^i T^k_k/3$ denotes the traceless component of T_j^i . When the different components of matter and radiation (i.e., CDM, HDM, baryons, photons, and massless neutrinos) are treated separately, $(\bar{\rho} + \bar{P})\theta = \sum_i (\bar{\rho}_i + \bar{P}_i)\theta_i$ and $(\bar{\rho} + \bar{P})\Theta = \sum_i (\bar{\rho}_i + \bar{P}_i)\Theta_i$, where the index i runs over the particle species.

The conservation of energy-momentum is a consequence of the Einstein equations. Let $w \equiv P/\rho$ describe the equation of state. Then the perturbed part of energy-momentum conservation equations

$$T^{\mu\nu}{}_{;\mu} = \partial_\mu T^{\mu\nu} + \Gamma^\nu_{\alpha\beta} T^{\alpha\beta} + \Gamma^\alpha_{\alpha\beta} T^{\nu\beta} = 0$$

Synchronous gauge —

$$\begin{aligned}\dot{\delta} &= -(1+w) \left(\theta + \frac{\dot{h}}{2} \right) - 3 \frac{\dot{a}}{a} \left(\frac{\delta P}{\delta \rho} - w \right) \delta, \\ \dot{\theta} &= -\frac{\dot{a}}{a} (1-3w) \theta - \frac{\dot{w}}{1+w} \theta + \frac{\delta P / \delta \rho}{1+w} k^2 \delta - k^2 \Theta,\end{aligned}$$

Conformal Newtonian gauge —

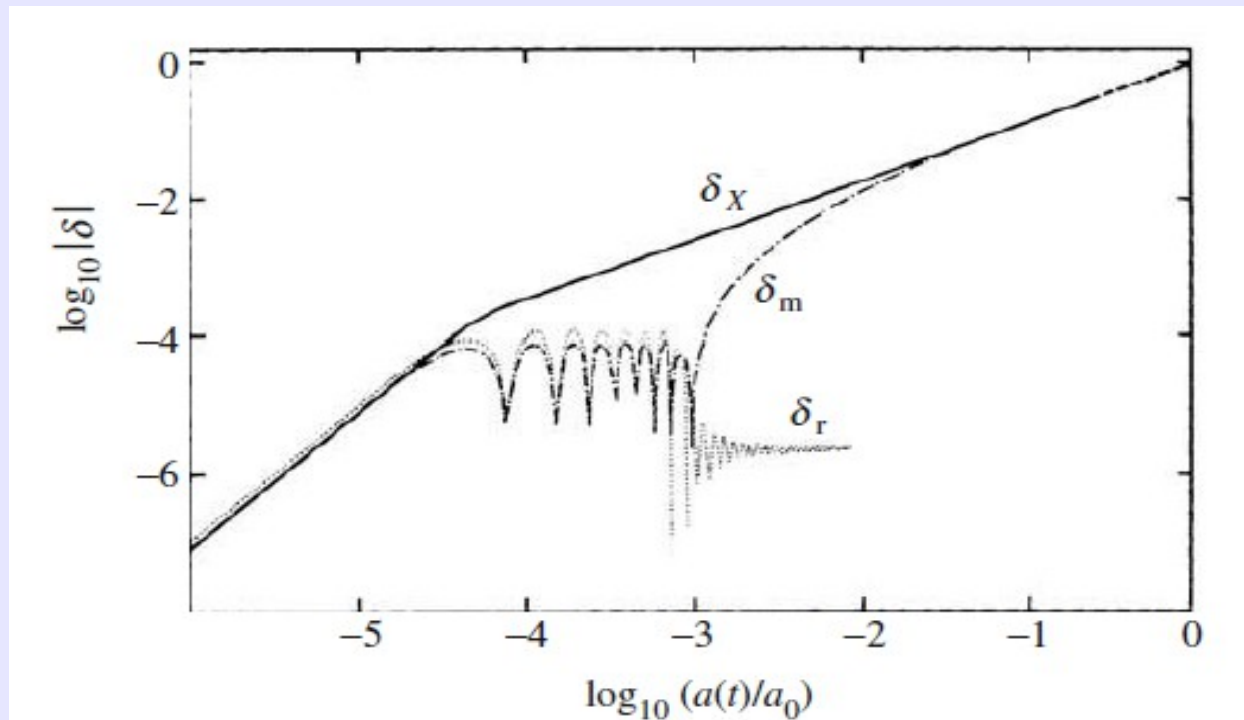
$$\begin{aligned}\dot{\delta} &= -(1+w) \left(\theta - 3\dot{\phi} \right) - 3 \frac{\dot{a}}{a} \left(\frac{\delta P}{\delta \rho} - w \right) \delta, \\ \dot{\theta} &= -\frac{\dot{a}}{a} (1-3w) \theta - \frac{\dot{w}}{1+w} \theta + \frac{\delta P / \delta \rho}{1+w} k^2 \delta - k^2 \Theta + k^2 \psi.\end{aligned}$$

The Meszaros effect

At the matter-radiation equality ($z \sim 3000$) Dark Matter structures start to grow linearly w.r.t scale factor. Before this, their growth is suppressed (Actually they grow logarithmically).

However, baryons (known kind of matter) are coupled to radiation until $z \sim 1000$.

After the decoupling ($z \sim 1000$), baryons follow the dark matter gravitational potential wells (dark halos) to successfully form structures we observe today.



Power spectrum of the matter fluctuations

Transfer Functions $T(k)$

Just after Inflation

$z=0$ (Today)

Time

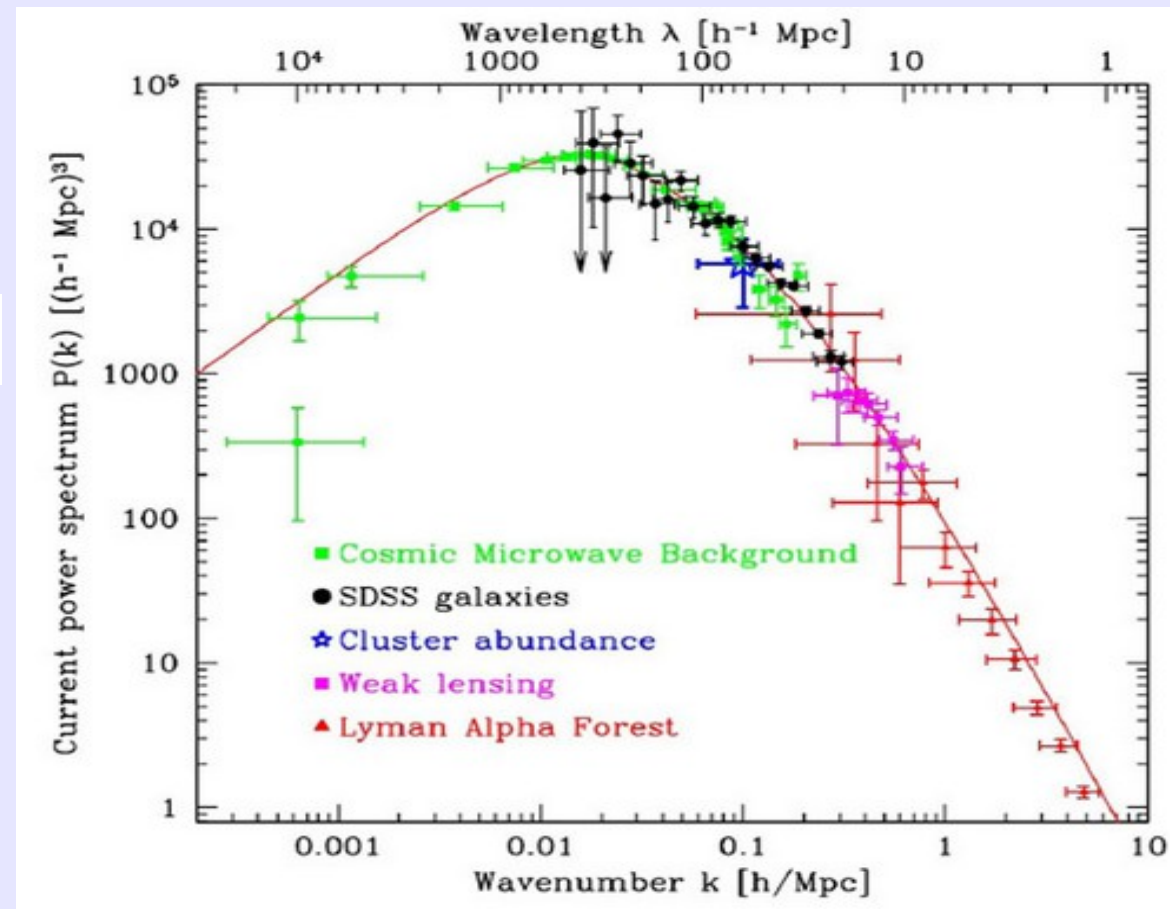


The Initial Power Spectrum

$$P(k) = |\Delta_k|^2 \propto k^n$$

The Harrison–Zeldovich power spectrum has $n = 1$

$T(k)$ modifies the shape of $P(k)$.



$$\Delta_k(z=0) = T(k) f(z) \Delta_k(z)$$

The Sachs-Wolfe formula

$$\frac{\delta T^{esc}}{T}(\vec{e}) = \left[\frac{\delta T_\gamma}{T} + \phi - \vec{e} \cdot \vec{v}_\gamma \right]_{dec} + \int_{\eta_{dec}}^{\eta_0} d\eta \frac{\partial}{\partial \eta} (\phi + \psi)$$

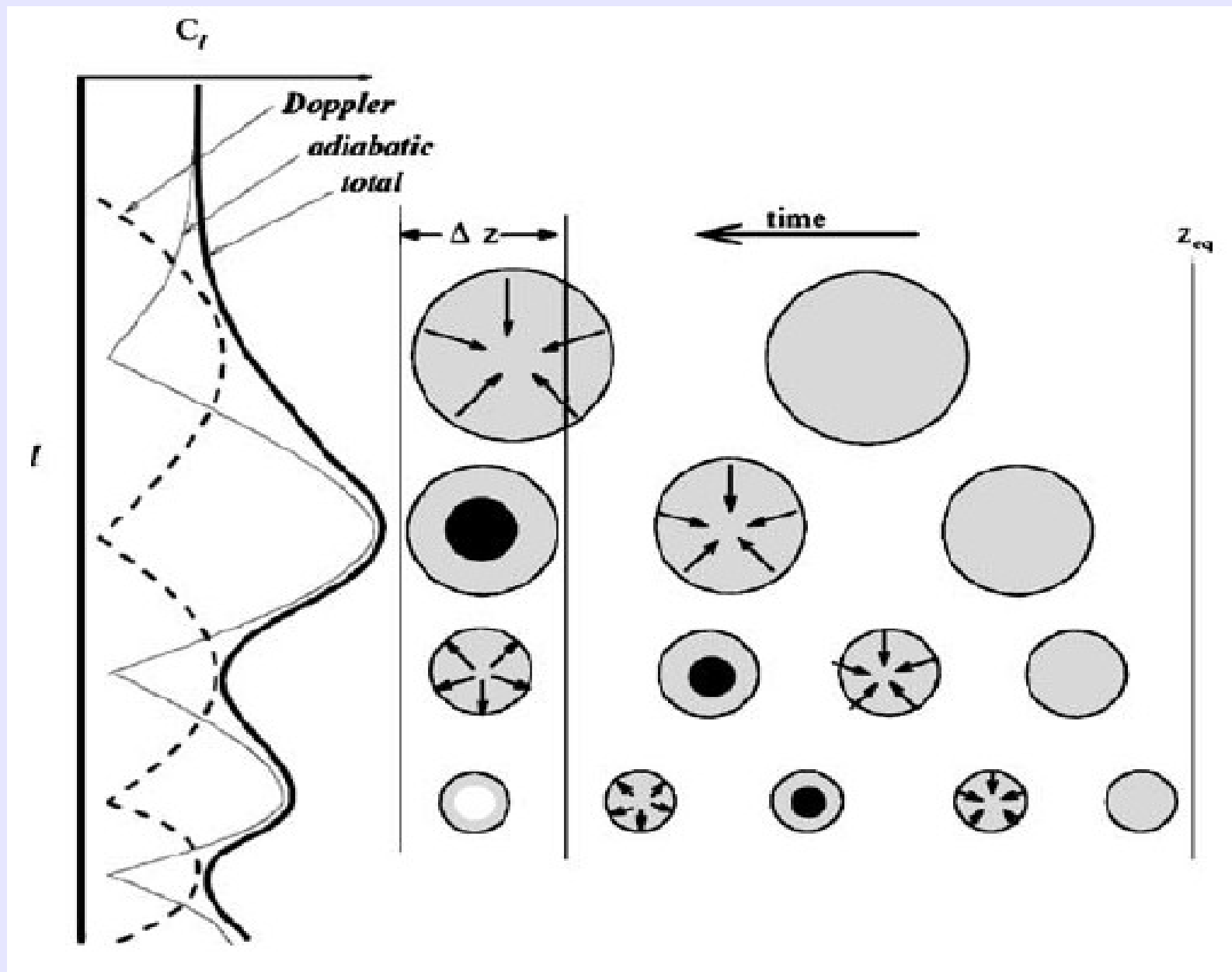
$$\frac{\delta T^{ten}}{T}(\vec{e}) = -\frac{1}{2} e^i e^j \int_{\eta_{dec}}^{\eta_0} d\eta \frac{\partial}{\partial \eta} h_{ij}$$

Thus, the observation of a hot spot might be due to:

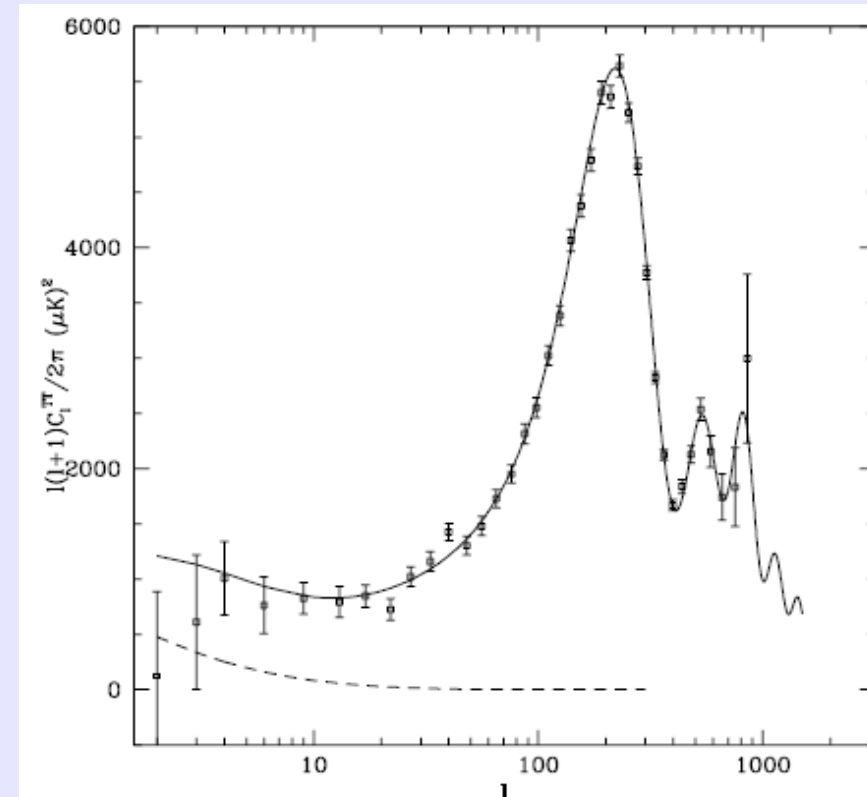
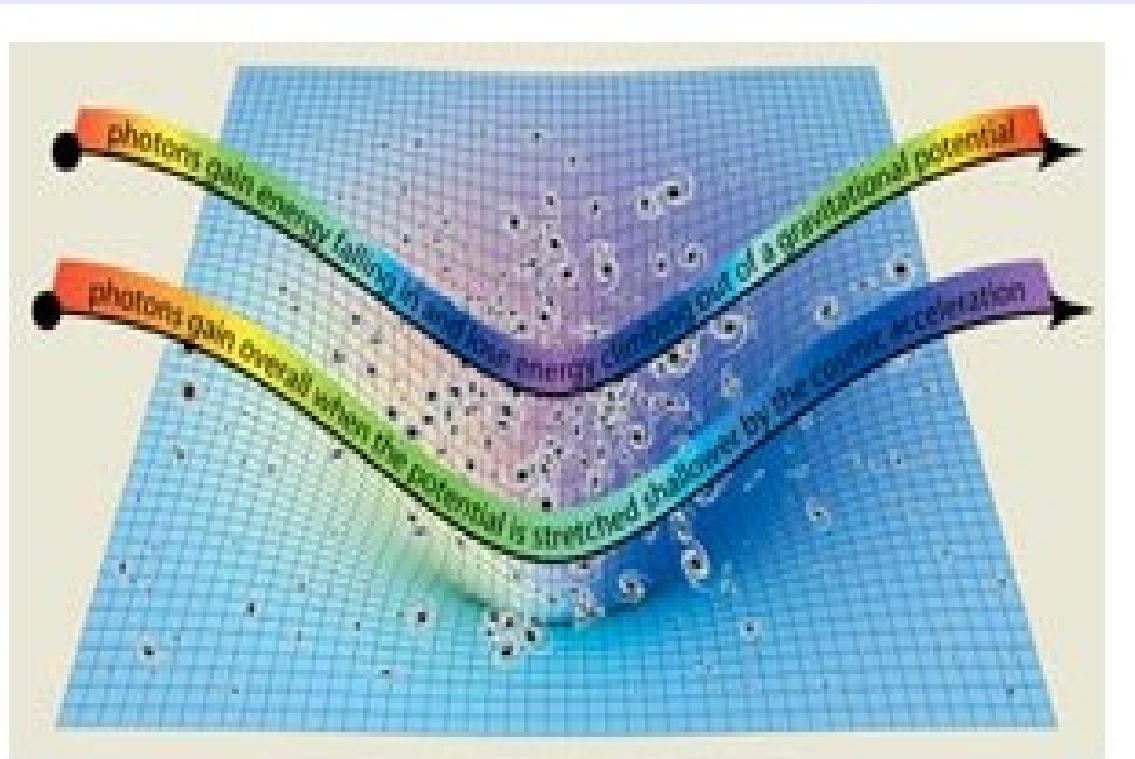
- i) a higher temperature at the moment of the last scattering;
- ii) a higher gravitational potential (on super horizon scales ϕ is negative);
- iii) or, due to a relative motion of the last scattering surface toward the observer

The last term is the ISW effect.

The main contribution to the CMB spectrum comes from the gravitational potential



The integrated Sachs-Wolfe effect



$$S_i = \left(\frac{\Delta T}{T} \right)_{ISW} = 2 \int_{\eta_r}^{\eta_0} d\eta \frac{\partial \psi_i}{\partial \eta} [(\eta_0 - \eta) \hat{\eta}, \eta],$$

➔ Gravitational potential

➔ Conformal time

Overview: The structure formation process. What should we learn?

- 1) Small (quantum) fluctuations at the primordial stages are the seeds of cosmic structures.
- 2) General relativity is "weak" to form baryonic structures. We need to consider DM potential wells at the decoupling to successfully form galaxies.
- 3) Cosmic microwave background gives a picture of the situation at the time of decoupling. A lot of physics involved on CMB.