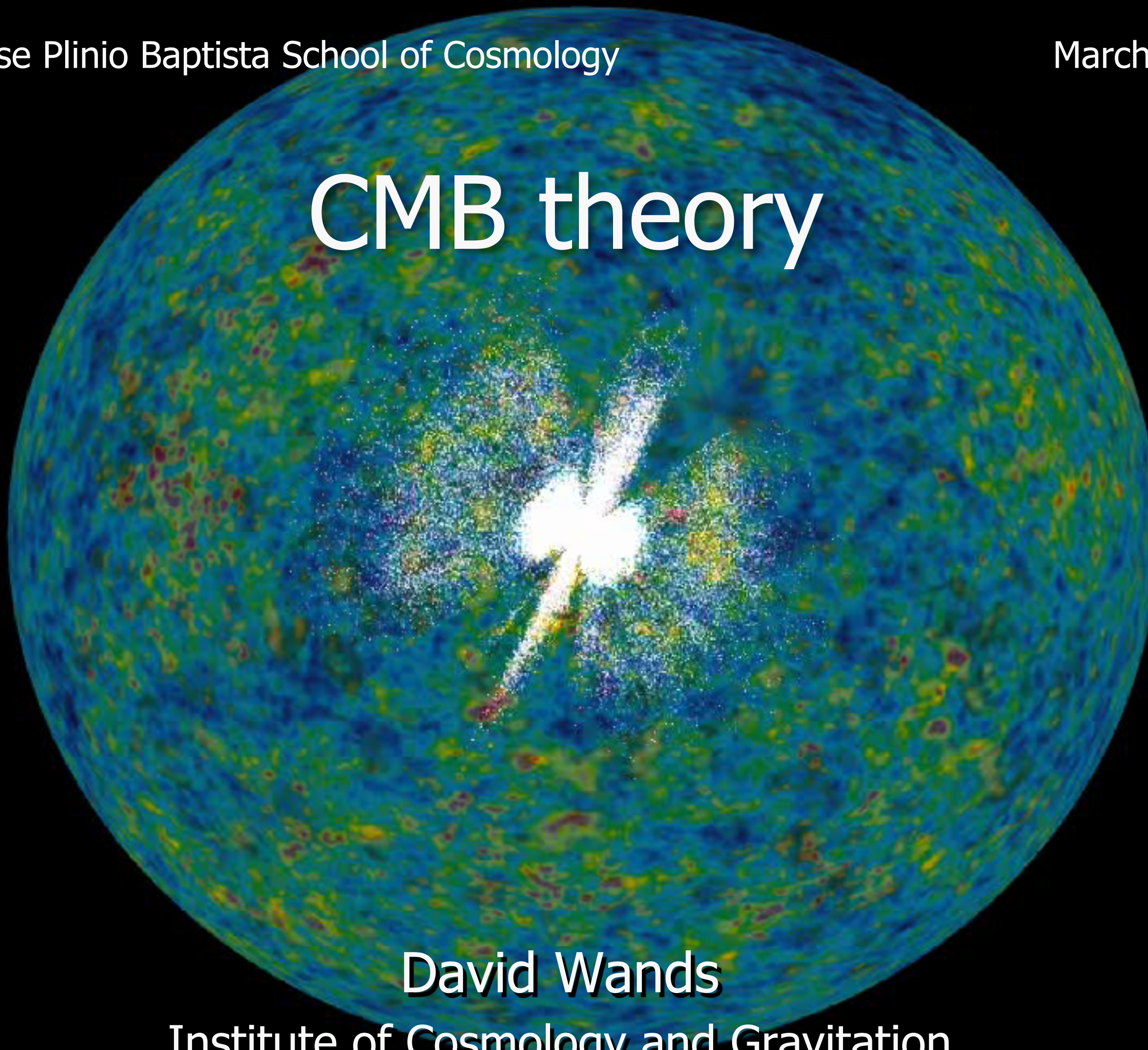

CMB theory

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Part 3: the angular power spectrum

- Perturbed FLRW geometry
  - large scale anisotropies
- Acoustic oscillations in plasma
  - Baryon loading
  - Diffusion damping
  - Reionisation
- Parameter constraints from angular power spectrum

Initial conditions on large scales:

- Curvature and entropy perturbation:

$$\zeta = \frac{4\rho_\gamma\zeta_\gamma + 4\rho_m\zeta_m}{4\rho_\gamma + 3\rho_m} \quad , \quad S_m = 3(\zeta_m - \zeta_\gamma)$$

  - Adiabatic initial conditions:

$$\zeta = \zeta_\gamma = \zeta_m = \text{constant} \quad , \quad S_m = 0$$

  - Isocurvature initial conditions:

$$\zeta_\gamma = 0 \quad , \quad S_m = 3\zeta_m = \text{constant}$$

Intrinsic Sachs-Wolfe effect (on large scales)

$$\frac{\delta T}{T} = \frac{1}{4}\delta_{\gamma *} + \Psi_* = \zeta_\gamma + 2\Psi_*$$

Newtonian potential:

- Barotropic fluid with constant (isotropic) equation of state $P = w\rho$
- Equation of motion for $\Psi = -\Phi$

$$\Psi'' + 3(1+w)\mathcal{H}\Psi' + w\nabla^2\Psi = 0$$

Initial conditions (large-scale)

$$\Psi \to \Psi_0 = -\frac{3(1+w)}{5+3w}\zeta$$

- Growing-mode solutions:
  - Radiation era ($w = 1/3$):

$$\begin{aligned} \Psi_k(\eta) &= \zeta_\gamma \left[\frac{6}{(k\eta)^2} \cos\left(\frac{k\eta}{\sqrt{3}}\right) - \frac{6\sqrt{3}}{(k\eta)^3} \sin\left(\frac{k\eta}{\sqrt{3}}\right) \right] \\ &\to -\frac{2}{3}\zeta_\gamma \quad \text{as } k\eta \to 0 \\ &\to 0 \quad \text{as } k\eta \to \infty \end{aligned}$$

Potential decays in radiation era

  - Matter era ($w = 0$):

$$\Psi_k(\eta) = -\frac{3}{5}\zeta_m$$

Potential constant in matter era

Intrinsic Sachs-Wolfe effect (on large scales)

$$\frac{\delta T}{T} = \frac{1}{4}\delta_{\gamma *} + \Psi_* = \zeta_\gamma + 2\Psi_* \to \zeta_\gamma - \frac{6}{5}\zeta_m$$

– Adiabatic perturbations ($S_m = 0,\ \zeta_\gamma = \zeta_m$)

$$\left.\frac{\delta T}{T}\right|_{ad} = -\frac{1}{5}\zeta_\gamma = \frac{1}{3}\Psi_*$$

– Isocurvature perturbations ($\zeta_\gamma = 0,\ S_m = 3\zeta_m$)

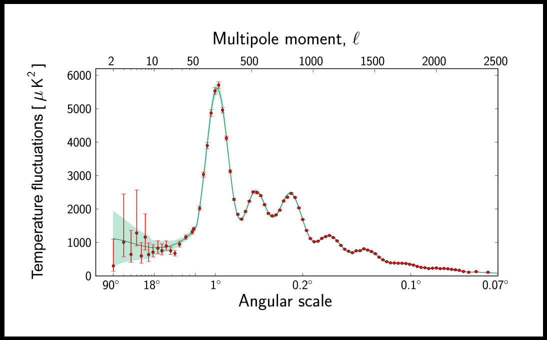
$$\left.\frac{\delta T}{T}\right|_{iso} = -\frac{2}{5}S_m = 2\Psi_*$$

– hence angular power spectrum on large scales

$$\frac{\ell(\ell+1)}{2\pi}C_\ell = \frac{1}{25}\mathcal{P}_\zeta(\ell/r_*) + \frac{4}{25}\mathcal{P}_S(\ell/r_*) + \frac{2}{25}\mathcal{C}_{\zeta S}(\ell/r_*)$$

Part 3: the angular power spectrum

- Perturbed FLRW geometry
  - large scale anisotropies



- Acoustic oscillations in plasma
  - Baryon loading
  - Diffusion damping
  - Reionisation

- Parameter constraints from angular power spectrum

Acoustic oscillations (neglecting baryons)

- Coupled energy and momentum conservation equations for radiation in conformal Newtonian gauge

$$\frac{1}{4}\delta'_\gamma = -\frac{1}{3}\vec{\nabla}.\vec{V}_\gamma - \Phi'$$

$$\vec{V}'_\gamma = -\frac{1}{4}\vec{\nabla}\delta_\gamma - \vec{\nabla}\Psi$$

eliminate $\vec{V}$ yields oscillator equation

$$\left(\frac{1}{4}\delta_\gamma + \Psi\right)'' - \frac{1}{3}\nabla^2\left(\frac{1}{4}\delta_\gamma + \Psi\right) = (\Psi - \Phi)''$$

- Matter era

$$\Psi = -\Phi = -\frac{3}{5}\zeta_m \quad \Rightarrow \quad (\Psi - \Phi)'' = 0$$

at recombination, for adiabatic perturbations

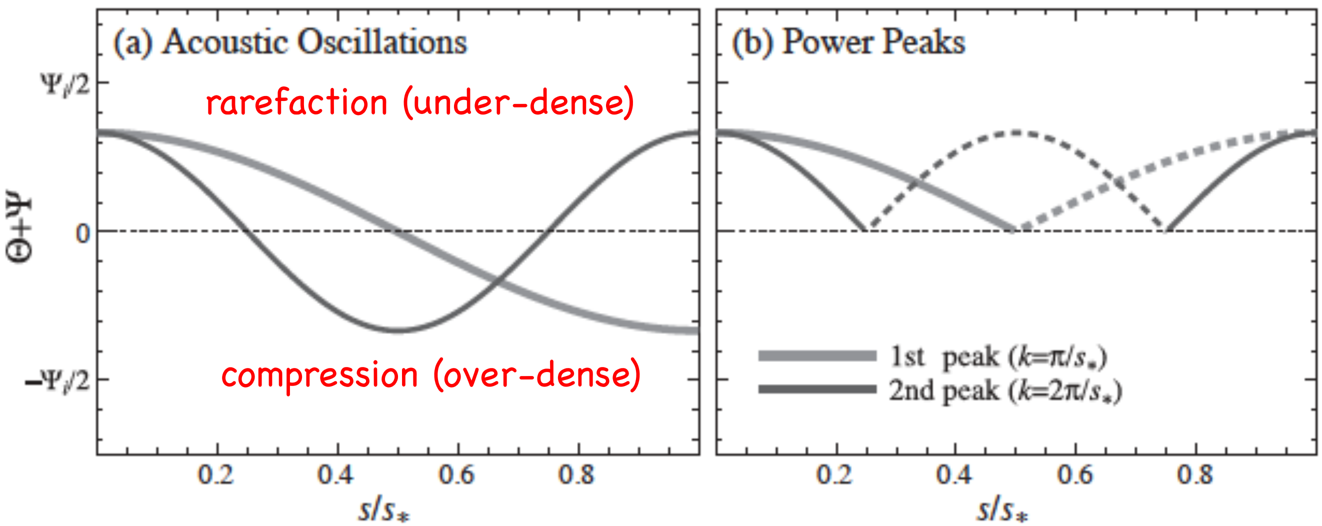
$$\Rightarrow \left(\frac{1}{4}\delta_\gamma + \Psi\right) = \left(\frac{1}{4}\delta_\gamma + \Psi\right)_0 \cos(ks)$$

$$\left(\frac{1}{4}\delta_\gamma + \Psi\right)_* = -\frac{1}{5}\zeta_\gamma \cos(ks_*) \quad \text{for } k \ll k_{eq}$$

sound horizon

$$s = \int c_s d\eta = \frac{1}{3}\eta$$

Acoustic oscillations (neglecting baryons)



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sound horizon

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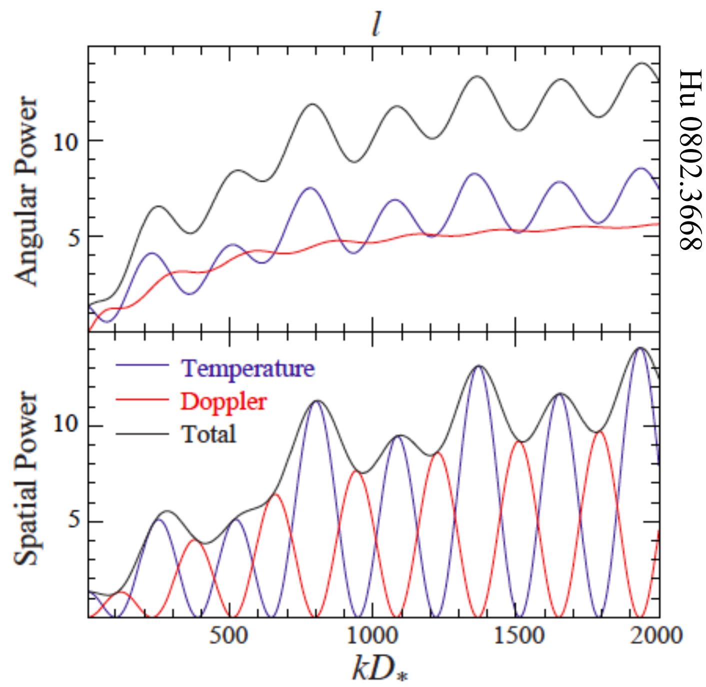
Acoustic oscillations



- Acoustic scale:
  - Wavenumber $$k_A s_* = \frac{k_A \eta_*}{\sqrt{3}} = \pi$$
  - Length scale $$\lambda_A = \frac{2\eta_*}{\sqrt{3}}$$
  - Angular scale $$\theta_A = \frac{\lambda_A}{\eta_0 - \eta_*}$$
  - in matter-dominated universe $$\theta_A \approx \frac{\eta_*}{\eta_0} \approx z_*^{-1/2} \approx 2^\circ$$

Acoustic oscillations (neglecting baryons)

- Coupled energy and momentum conservation equations for radiation in conformal Newtonian gauge

$$\frac{1}{4}\delta_\gamma' = -\frac{1}{3}\vec{\nabla}.\vec{V}_\gamma - \Phi'$$

$$\vec{V}_\gamma' = -\frac{1}{4}\vec{\nabla}\delta_\gamma - \vec{\nabla}\Psi$$

eliminate $\vec{V}$ yields oscillator equation

$$\left(\frac{1}{4}\delta_\gamma + \Psi\right)'' - \frac{1}{3}\nabla^2\left(\frac{1}{4}\delta_\gamma + \Psi\right) = (\Psi - \Phi)''$$

- Radiation era

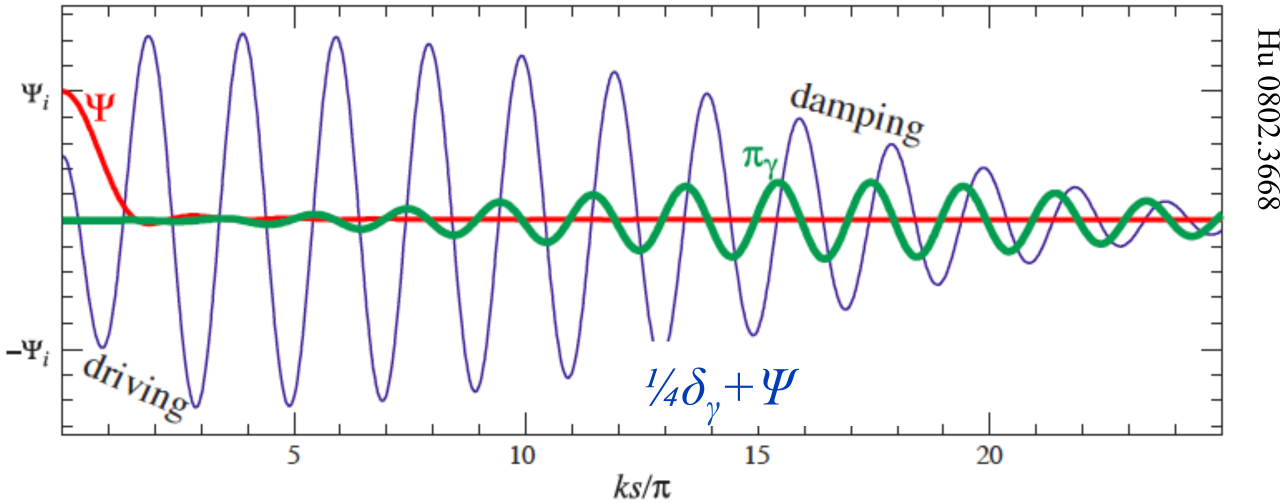
$$\Psi = -\Phi \to 0 \quad \text{as } k\eta \to \infty$$

$$\Rightarrow \left(\frac{1}{4}\delta_\gamma + \Psi\right) \approx \frac{1}{4}\delta_\gamma \propto \cos(ks) \quad \text{for } ks \gg 1$$

at recombination

$$\boxed{\left(\frac{1}{4}\delta_\gamma + \Psi\right)_* \approx -\zeta_\gamma \cos(ks_*) \quad \text{for } k \gg k_{eq}}$$

Acoustic oscillations (neglecting baryons)



- Radiation era

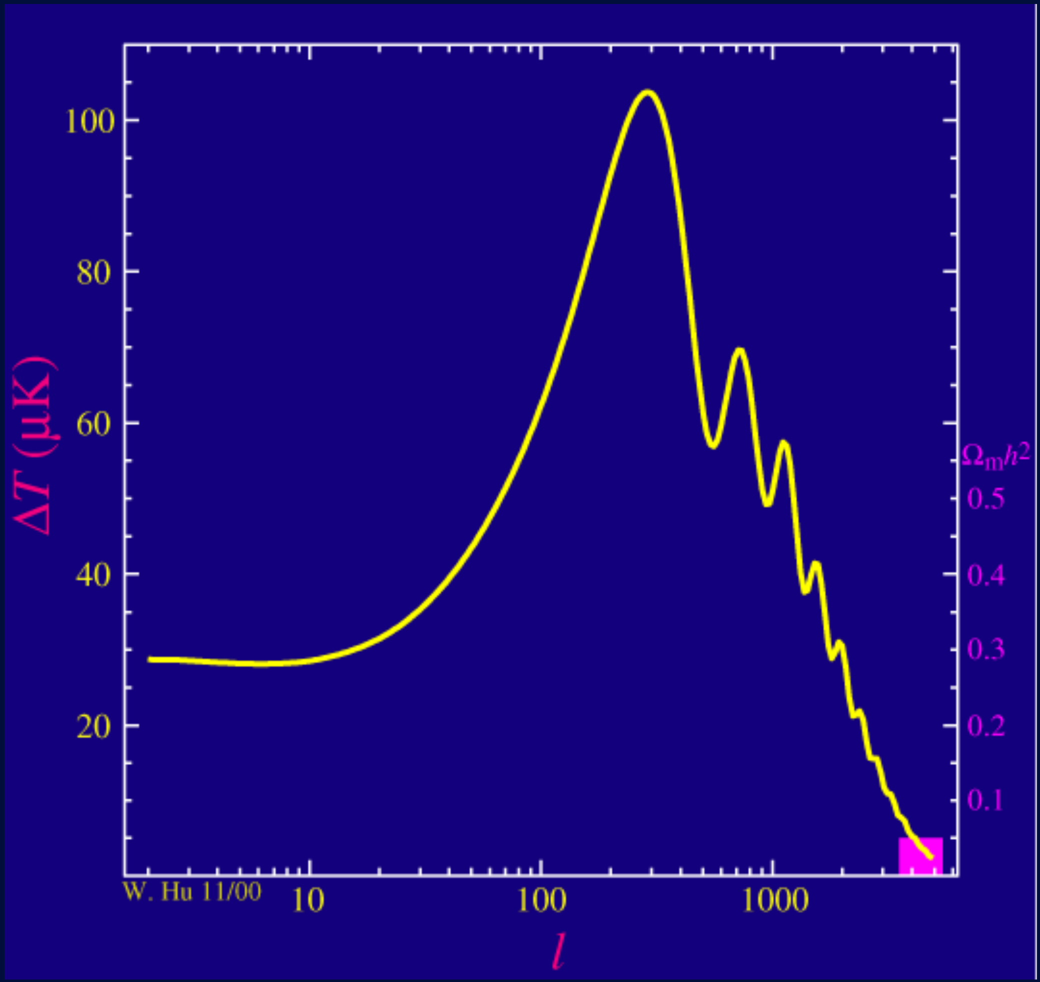
$$\Psi = -\Phi \to 0 \quad \text{as } k\eta \to \infty$$

$$\Rightarrow \left(\frac{1}{4}\delta_\gamma + \Psi\right) \approx \frac{1}{4}\delta_\gamma \propto \cos(ks) \quad \text{for } ks \gg 1$$

at recombination

$$\left(\frac{1}{4}\delta_\gamma + \Psi\right)_* \approx -\zeta_\gamma \cos(ks_*) \quad \text{for } k \gg k_{eq}$$

Effect of matter density



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Acoustic oscillations (including baryons)

Adding baryons

$$(\rho_\gamma + P_\gamma)V_\gamma + (\rho_b + P_b)V_b = (1+R)(\rho_\gamma + P_\gamma)V_{b\gamma}$$

where

$$R \equiv \frac{\rho_b + P_b}{\rho_\gamma + P_\gamma} = \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$$

Tight-coupling

$$V_\gamma = V_b = V_{b\gamma}$$

- Coupled energy and momentum conservation equations

$$\frac{1}{4}\delta'_\gamma = -\frac{1}{3}\vec{\nabla}.\vec{V}_{b\gamma} - \Phi'$$

$$\left((1+R)\vec{V}_{b\gamma}\right)' = -\frac{1}{4}\vec{\nabla}\delta_\gamma - (1+R)\vec{\nabla}\Psi$$

- eliminate $\vec{V}_{b\gamma}$, and assuming $\Psi' \approx R' \approx 0$, yields oscillator equation

$$\left(\frac{1}{4}\delta_\gamma + (1+R)\Psi\right)'' - \frac{1}{3(1+R)}\nabla^2\left(\frac{1}{4}\delta_\gamma + (1+R)\Psi\right) \approx 0$$

baryon-loading reduces speed of sound in photon-baryon plasma

$$\Rightarrow \left(\frac{1}{4}\delta_\gamma + (1+R)\Psi\right) = \left(\frac{1}{4}\delta_\gamma + (1+R)\Psi\right)_0 \cos(ks)$$

sound horizon

$$s \equiv \int c_s d\eta = \int \frac{d\eta}{3(1+R)}$$

Acoustic oscillations (including baryons)

sound horizon

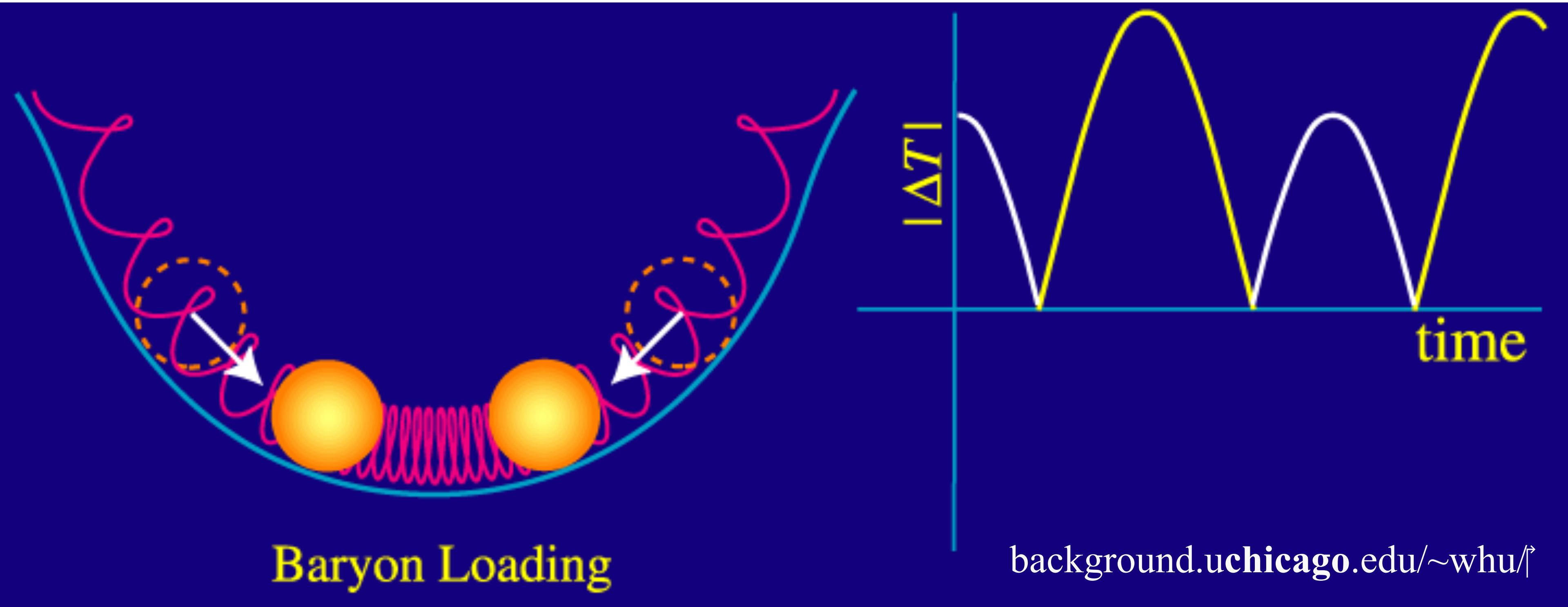
$$s \equiv \int c_s d\eta = \int \frac{d\eta}{3(1+R)}$$

at recombination in matter era

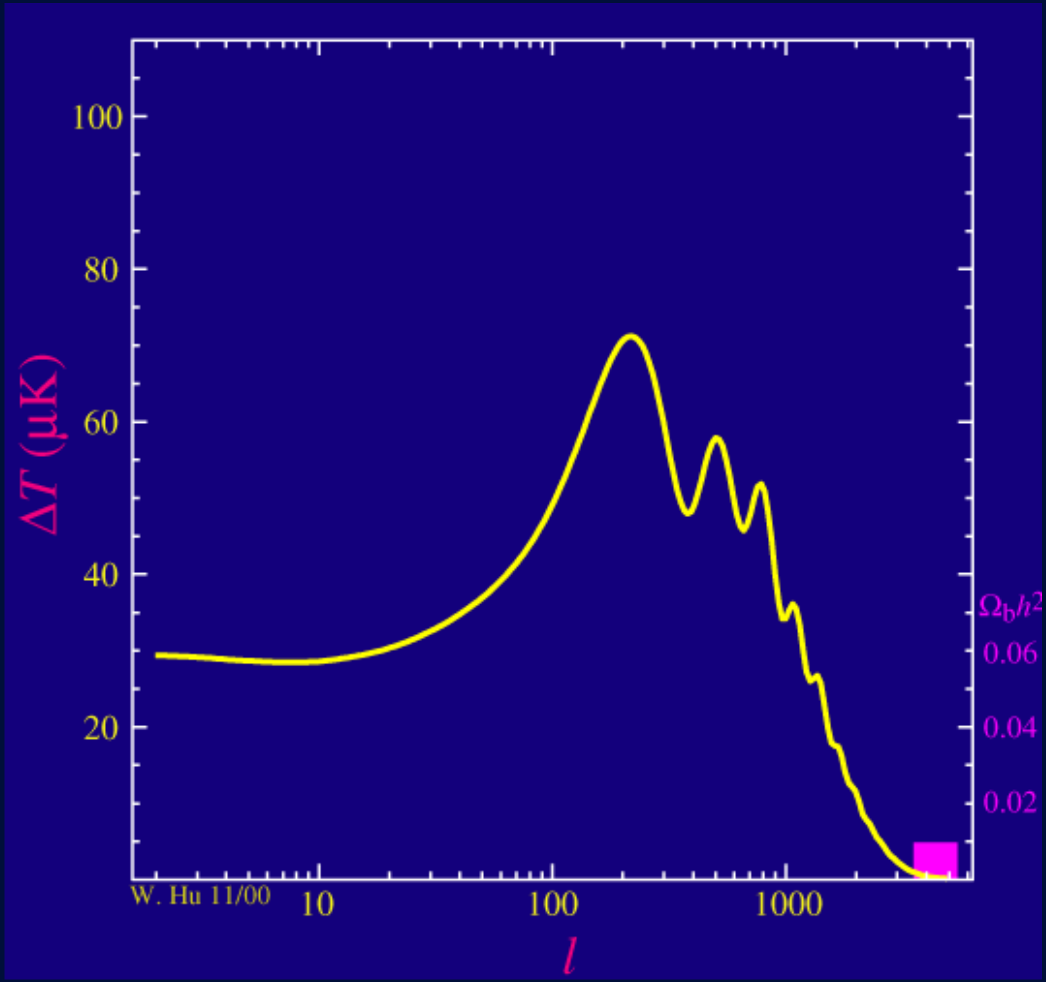
$$\left(\frac{1}{4}\delta_\gamma + \Psi\right)_* \approx [3R - (1+3R)\cos(ks_*)]\,\frac{1}{5}\zeta_\gamma \quad \text{for } k \gg k_{eq}$$

introduces asymmetry

– odd peaks ($ks_* = (2n+1)\pi$) higher than even peaks ($ks_* = 2n\pi$)



Effect of baryon loading



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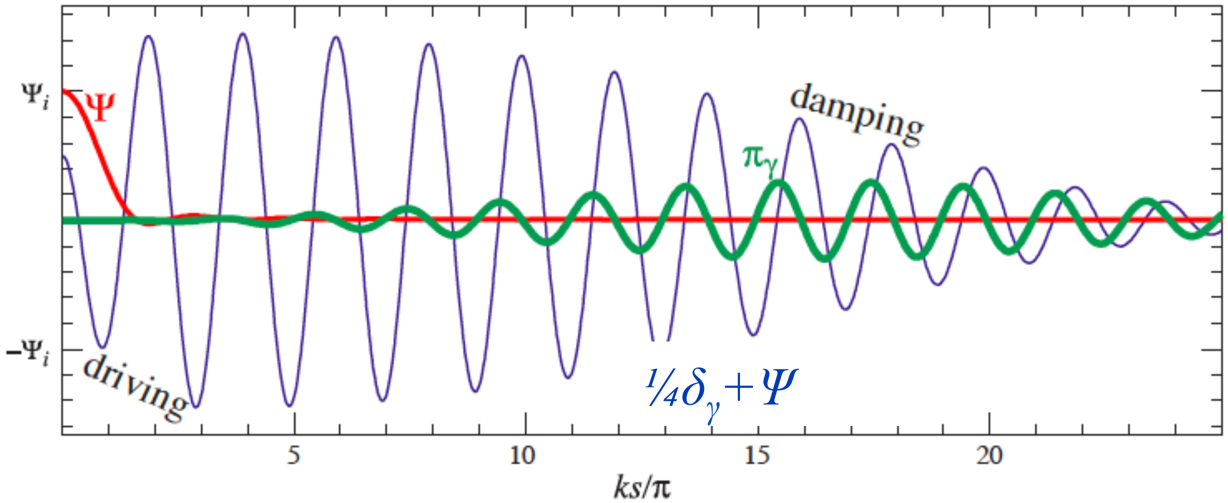
Acoustic oscillations (damping tail)



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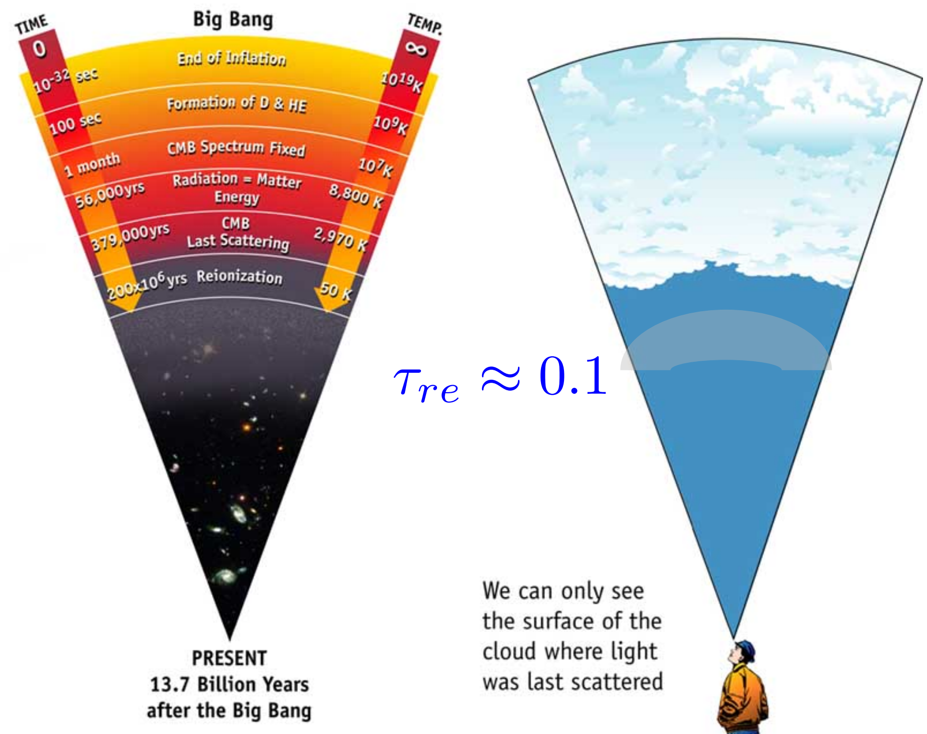
- Tight-coupling breaks down on small scales
  - Finite mean free path for photons
  - Anisotropic shear viscosity, π_γ
  - Photon diffusion due to random walk

$$\lambda_D \simeq 64.5 \text{ Mpc} \left(\frac{\Omega_m h^2}{0.14}\right)^{-0.278} \left(\frac{\Omega_b h^2}{0.024}\right)^{-0.18}$$

Reionisation: $H + \gamma \rightarrow p^{+} + e^{-}$

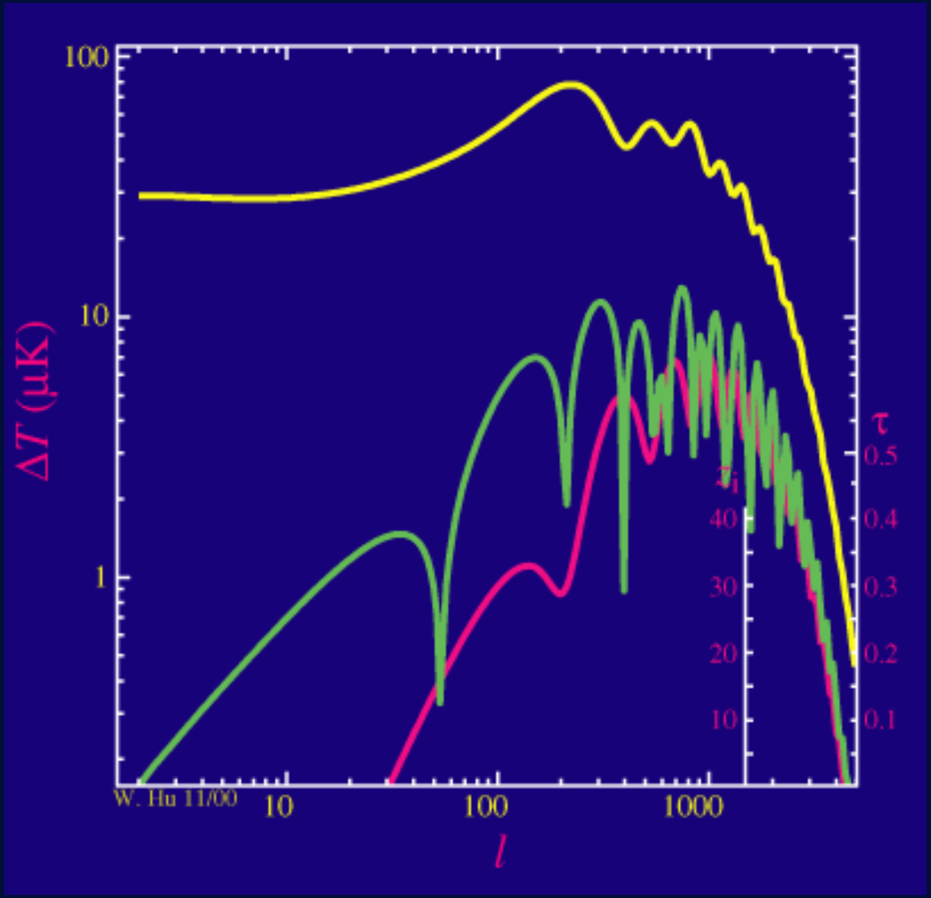
- Absence of Gunn-Peterson trough
  - No absorption by neutral H in quasar spectra
  - Neutral gas reionised by first stars at z>6
- Optically thin ($\tau \sim 0.1$) "smog" between us and recombination
  - re-scatters 10% of CMB photons
  - suppresses small-scale anisotropies



$$\tau_{re} \approx 0.1$$

The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

Effect of reionisation



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end of part three