Matter growth in imperfect fluid cosmology

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Cosmological standard model

Two dynamically dominating components:

Dark Matter (DM), energy density ρ_m, pressure $p_m \ll \rho_m$

Dark Energy (DE), energy density ρ_x, pressure $p_x = w_x \rho_x$

Late-time acceleration of the scale factor in the standard model

One observational signature:
Growth rate of matter perturbations

Exotic matter or modified gravity?

Exotic matter:

- Scalar fields (quintessence, phantom)
- (Generalized) Chaplygin gas
- Viscous fluids

Modifying GR

- $f(R)$ theories
- Scalar-tensor theories
- Rastall's theory

Outline

- Effective Einsteinian description
  - General conservation equations for imperfect fluids
  - Effective two-fluid system
  - Scalar perturbations (anisotropic pressure and heat flux)
  - System of two second-order equations
- Anisotropic pressure and heat flux phenomenologically
- $e_{\Phi}\Lambda$CDM model
  - Jordan-Brans-Dicke inspired generalized Hubble rate
  - Influence on the growth rate of matter perturbations by
    - deviations from the standard background
    - anisotropic pressure
    - heat flux
  - Redshift-space-distortion (RSD) data

Effective Fluid description

- Rewrite field equations in an effective Einsteinian way
- Map additional (compared with GR) geometrical degrees of freedom onto an effective fluid component
- Effective two-fluid system within GR
- Imperfect-fluid degrees of freedom
- Phenomenological fluid dynamical equations

Example: JBD type scalar-tensor theory

Effective Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}$$

Total energy-momentum tensor

$$T_{\mu\nu} = T_{(m)\mu\nu} + T_{(x)\mu\nu}$$

Effective fluid

$$T_{(x)\mu\nu} \equiv \left(\frac{1}{\Phi} - 1\right) T_{(m)\mu\nu} + \frac{1}{\kappa^2 \Phi} \frac{\omega(\Phi)}{\Phi} \left[\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \left(\nabla\Phi\right)^2\right]$$

$$+ \frac{1}{\kappa^2 \Phi} \left[\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \Box \Phi - \frac{1}{2} g_{\mu\nu} U\right]$$

Two-component cosmic medium

- **Pressureless perfect-fluid type matter $\mathbf{T}_{(\mathbf{m})\mu\nu}$**
- **General imperfect fluid $\mathbf{T}_{(\mathbf{x})\mu\nu}$**

Total energy-momentum tensor (EMT)

$$\mathbf{T}_{\mu\nu} = \mathbf{T}_{(\mathbf{m})\mu\nu} + \mathbf{T}_{(\mathbf{x})\mu\nu}$$

Matter component

EMT

$$T_{(m)\mu\nu} = \rho_m u_{(m)\mu} u_{(m)\nu}$$

Matter four-velocity

$$u^{\mu}_{(m)} u_{(m)\mu} = -1$$

Matter energy density

$$\rho_m = T_{(m)\mu\nu} u^{\mu}_{(m)} u^{\nu}_{(m)}$$

Imperfect-fluid component

EMT

$$T_{(x)\mu\nu} = \rho_x u_{(x)\mu} u_{(x)\nu} + p_x h_{(x)\mu\nu} + \Pi_{(x)\mu\nu} + q_{(x)\mu} u_{(x)\nu} + q_{(x)\nu} u_{(x)\mu}$$

Four-velocity (e.g.,scalar-field gradient)

$$u^{\mu}_{(x)} u_{(x)\mu} = -1, \qquad h^{\mu\nu}_{(x)} = g^{\mu\nu} + u^{\mu}_{(x)} u^{\nu}_{(x)}$$

Energy density of component x

$$\rho_x = T_{(x)\mu\nu} u^{\mu}_{(x)} u^{\nu}_{(x)}$$

Isotropic pressure of component x

$$p_x = \frac{1}{3} h^{\mu\nu}_{(x)} T_{(x)\mu\nu}$$

Heat flux and anisotropic pressure

EMT

$$T_{(x)\mu\nu} = \rho_x u_{(x)\mu} u_{(x)\nu} + p_x h_{(x)\mu\nu} + \Pi_{(x)\mu\nu} + q_{(x)\mu} u_{(x)\nu} + q_{(x)\nu} u_{(x)\mu}$$

Heatflux of component x

$$q_{(x)\mu} = -h^{\nu}_{(x)\mu} T_{(x)\nu\sigma} u^{\sigma}_{(x)}$$

Anisotropic pressure of component x

$$\Pi_{(x)\mu\nu} = h^{\sigma}_{(x)(\mu} h^{\tau}_{(x)\nu)} T_{(x)\sigma\tau} - \frac{1}{3} h_{(x)\mu\nu} h^{\sigma\tau}_{(x)} T_{(x)\sigma\tau}$$

Orthogonality

$$h^{\mu\nu}_{(x)} u_{(x)\nu} = q_{(x)\mu} u^{\mu}_{(x)} = \Pi_{(x)\mu\nu} u^{\mu}_{(x)} = \Pi^{\mu}_{(x)\mu} = 0$$

Total cosmic medium

EMT

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} + \Pi_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu$$

Total energy density

$$\rho = T_{\mu\nu} u^\mu u^\nu$$

Total isotropic pressure

$$p = \frac{1}{3} h^{\mu\nu} T_{\mu\nu}$$

Four-velocity

$$u^\mu u_\mu = -1, \qquad h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Total heat flux and total anisotropic pressure

EMT

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu} + \Pi_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu$$

Total heat flux

$$q_\mu = -h^\nu_\mu T_{\nu\sigma} u^\sigma$$

Total anisotropic pressure

$$\Pi_{\mu\nu} = h^\sigma_{\langle\mu} h^\tau_{\nu\rangle} T_{\sigma\tau} \equiv h^\sigma_{(\mu} h^\tau_{\nu)} T_{\sigma\tau} - \frac{1}{3} h_{\mu\nu} h^{\sigma\tau} T_{\sigma\tau}$$

Orthogonality

$$h^{\mu\nu} u_\nu = q_\mu u^\mu = \Pi_{\mu\nu} u^\mu = \Pi^\mu_\mu = 0$$

Relation between u^μ, $u^\mu_{(m)}$ and $u^\mu_{(x)}$?

Conservation equations

Separate conservation

$$T^{\mu\nu}_{;\nu} = 0, \qquad T^{\mu\nu}_{(m)\ ;\nu} = 0, \qquad T^{\mu\nu}_{(x)\ ;\nu} = 0$$

Aim: Energy-density perturbations

$$\delta_x = \frac{\hat{\rho}_x}{\rho_x}, \qquad \delta = \frac{\hat{\rho}}{\rho}, \qquad \delta_m = \frac{\hat{\rho}_m}{\rho_m}$$

Matter growth rate determined by δ_m

Matter conservation equations

Projections

$$-u_{(m)\nu}T^{\nu\kappa}_{(m);\kappa} = \rho_{m,\alpha}u^{\alpha}_{(m)} + \Theta_m\rho_m = 0$$

and

$$h^{\alpha}_{(m)\nu}T^{\nu\kappa}_{(m);\kappa} = \rho_m\dot{u}^{\alpha}_{(m)} = 0$$

Expansion scalar and four-acceleration

$$\Theta_m \equiv u^{\mu}_{(m);\mu}, \qquad \dot{u}^{\alpha}_{(m)} \equiv u^{\alpha}_{(m);\beta}u^{\beta}_{(m)}$$

Homogeneous, isotropic background

$$\frac{d\rho_m}{dt} + 3H\rho_m = 0$$

Conservation equations x component

Projections

$$-u_{(x)\nu} T^{\nu\kappa}_{(x);\kappa} = \rho_{x,\alpha} u^{\alpha}_{(x)} + \Theta_x \left(\rho_x + p_x\right) + \nabla_\mu q^{\mu}_{(x)} = 0$$

and

$$h^{\alpha}_{(x)\nu} T^{\nu\kappa}_{(x);\kappa} = \left(\rho_x + p_x\right) \dot{u}^{a}_{(x)} + p_{x,\nu} h^{\alpha\nu}_{(x)}$$

$$+\nabla^{\kappa} \Pi_{(x)\mu\kappa} + h^{\nu}_{\mu(x)} \dot{q}_{(x)\nu} + \frac{4}{3} \Theta h_{\mu\kappa(x)} q^{\kappa}_{(x)} = 0$$

Expansion scalar and four-acceleration

$$\Theta_x \equiv u^{\mu}_{(x);\mu}, \qquad \dot{u}^{\alpha}_{(x)} \equiv u^{\alpha}_{(x);\beta} u^{\beta}_{(x)}$$

Background

$$\frac{d\rho_x}{dt} + 3H\left(1 + w_x\right)\rho_x = 0$$

Total conservation equations

Energy

$$\dot{\rho} + \Theta\left(\rho + p\right) + \nabla_{\mu} q^{\mu} = 0$$

Momentum

$$\left(\rho + p\right)\dot{u}_{\mu} + \nabla_{\mu} p + \nabla^{\kappa}\Pi_{\mu\kappa} + h^{\nu}_{\mu}\dot{q}_{\nu} + \frac{4}{3}\Theta h_{\mu\kappa} q^{\kappa} = 0$$

Background

$$\frac{d\rho}{dt} + 3H\left(1 + w\right)\rho = 0$$

EoS parameter

$$w \equiv \frac{p}{\rho} = \frac{p_x}{\rho} = w_x\Omega_x, \quad \Omega_x = \frac{\rho_x}{\rho} = 1 - \Omega_m, \quad \Omega_m = \frac{\rho_m}{\rho}$$

First-order perturbations

Metric

$$ds^2 = -\left(1 + 2\phi\right) dt^2 + a^2 \left(1 - 2\psi\right) \delta_{ab} dx^a dx^b$$

Perturbed time components of the four-velocities

$$\hat{u}_0 = \hat{u}^0 = \hat{u}^0_{(m)} = \hat{u}^0_{(x)} = \frac{1}{2}\hat{g}_{00} = -\phi$$

Velocity potentials

$$a^2 \hat{u}^a = \hat{u}_a \equiv v_{,a}$$

$$a^2 \hat{u}^a_{(m)} = \hat{u}_{(m)a} \equiv v_{m,a} \qquad a^2 \hat{u}^a_{(x)} = \hat{u}_{(x)a} \equiv v_{x,a}$$

Total and component quantities

Linear order

$$\hat{\rho} = \hat{\rho}_x + \hat{\rho}_m, \qquad \hat{p} = \hat{p}_x$$

$$(\rho + p)\,\hat{u}_a + q_a = \rho_m \hat{u}_{(m)a} + (\rho_x + p_x)\,\hat{u}_{(x)a} + q_{(x)a}$$

Scalar perturbations

$$q_{(x)a} = q_{x,a} \qquad q_a = q_{,a}$$

Relation beween velocities

$$v = \frac{\rho_m}{\rho + p} v_m + \frac{\rho_x + p_x}{\rho + p} v_x + \frac{q_x - q}{\rho + p}$$

Poisson-type equation

Combining 00 and 0a field equations

$$-\frac{2}{3}\frac{k^2}{H^2 a^2}\psi = \varepsilon$$

Generalized comoving density perturbations

$$\varepsilon = \delta - 3H\left[(1+w)\,v + \frac{q}{\rho}\right]$$

Relevant variables

Scalar part Π of the anisotropic stress tensor

$$\Pi_{ab} = \left(h^m_a h^n_b - \frac{1}{3} h_{ab} h^{mn} \right) \Pi_{,m,n}$$

Heat flux varable l

$$l \equiv 3H \frac{q}{\rho}$$

Generalized comoving pressure perturbations

$$\frac{\hat{p}^{\varepsilon}}{\rho} \equiv \frac{\hat{p}}{\rho} - 3H \frac{\dot{p}}{\dot{\rho}} \left[(1+w)\, v + \frac{q}{\rho} \right]$$

Inhomogeneous 2nd order equation for ε

Combination of total energy-momentum conservation and Raychaudhuri equation

$$\varepsilon'' + \left(\frac{3}{2} - \frac{15}{2}\frac{p}{\rho} + 3\frac{p'}{\rho'}\right)\frac{\varepsilon'}{a} - \left[\frac{3}{2} + 12\frac{p}{\rho} - \frac{9}{2}\frac{p^2}{\rho^2} - 9\frac{p'}{\rho'}\right]\frac{\varepsilon}{a^2}$$

$$-\frac{1}{a^2H^2}\frac{1}{a^2}\frac{\Delta\hat{p}^{\varepsilon}}{\rho} - \frac{3}{2}\left(1+\frac{p}{\rho}\right)\left[\frac{l'}{a} + \frac{3}{2a^2}\left(1-\frac{p}{\rho}\right)l\right]$$

$$-\frac{2}{a}\frac{\Delta\Pi'}{a^2\rho} - \left[3\left(1-\frac{p}{\rho}+2\frac{p'}{\rho'}\right)\frac{\Delta}{a^2H^2} + \frac{2}{3}\frac{\Delta^2}{a^4H^4}\right]\frac{H^2}{a^2}\frac{\Pi}{\rho} = 0.$$

Closed set of equations?

Relative perturbations

Difference between total and pure matter perturbations

$$S_m \equiv \frac{\delta}{1+w} - \delta_m$$

Non-adiabatic perturbations

$$\hat{p} = \frac{\dot{p}}{\dot{\rho}}\hat{\rho} + \hat{p}_{nad}$$

Coupling to S_m

$$\hat{p}_{nad} = \hat{p}_x^c - \frac{\dot{p}_x}{\dot{\rho}_x}\hat{\rho}_x^c + \rho_m \frac{\dot{p}_x}{\dot{\rho}_x} S_m$$

Inhomogeneous 2nd order equation for S_m

Combining total and separate matter conservation equations

$$S_m'' + \frac{3}{2}\left(1 - \frac{p}{\rho}\right)\frac{S_m'}{a} - \frac{1}{a^2 H^2}\frac{\Delta \hat{p}^{\varepsilon}}{a^2(\rho + p)}$$

$$+\frac{3}{a}\frac{\hat{p}'_{nad}}{\rho + p} + 9\left(\frac{7}{6} + \frac{p'}{\rho'} - \frac{1}{2}\frac{p}{\rho}\right)\frac{\hat{p}_{nad}}{a^2(\rho + p)}$$

$$-\frac{2}{3}\frac{1}{a^2 H^2}\frac{\Delta^2 \Pi}{a^4(\rho + p)} = 0$$

Isotropic pressure perturbations

Rest frame (of the x component)

$$\hat{p}_x^{\varepsilon} = c_{\varepsilon}^2 \hat{\rho}_x^{\varepsilon}$$

Comoving variables:

$$\hat{\rho}_x^{\varepsilon} \equiv \hat{\rho}_x - 3H\left[\left(1+w_x\right)\rho_x v_x + q_x\right]$$

$$\hat{p}_x^{\varepsilon} \equiv \hat{p}_x - 3H\frac{\dot{p}_x}{\dot{\rho}_x}\left[\left(1+w_x\right)\rho_x v_x + q_x\right]$$

Effective sound speed parameter c_{ε}^2

Generalized rest frame

Non-adiabatic and total pressure perturbations

Non-adiabatic part

$$\frac{\hat{p}_{nad}}{\rho} = \left(c_\varepsilon^2 - \frac{\dot{p}_x}{\dot{\rho}_x}\right)\left(1 - \frac{\Omega_m}{1+w}\right)\varepsilon + c_\varepsilon^2 \Omega_m S_m \qquad \left(\frac{a^2 H^2}{k^2} \ll 1\right)$$

Total (isotropic) pressure perturbation on subhorizon scales

$$\frac{\hat{p}^\varepsilon}{\rho} = c_\varepsilon^2 \left(1 - \frac{\Omega_m}{1+w}\right)\varepsilon + c_\varepsilon^2 \Omega_m S_m$$

Coupling to ε and S_m

Closes perfect-fluid dynamics

Anisotropic pressure perturbations

Coupling to ε and S_m ?

Ansatz

$$\frac{H^2 \Pi}{\rho} = \mu \varepsilon + \nu S_m$$

Similarly

$$I = \frac{3Hq}{\rho} = \alpha \varepsilon + \beta S_m$$

$\Rightarrow$ **Closed system of equations for ε and S_m**

Closed system of equation for ε and S_m

Coupling of total and relative density perturbations

$$\varepsilon'' + \left(\frac{3}{2} - \frac{15}{2}\frac{p}{\rho} + 3\frac{p'}{\rho'} + A_1(k)\right)\frac{\varepsilon'}{a}$$

$$- \left[\frac{3}{2} + 12\frac{p}{\rho} - \frac{9}{2}\frac{p^2}{\rho^2} - 9\frac{p'}{\rho'} - \frac{k^2}{a^2H^2}c_\varepsilon^2\Omega_x\frac{1+w_x}{1+w} - B_1(k)\right]\frac{\varepsilon}{a^2}$$

$$+ A_2(k)\frac{S_m{}'}{a} + \left(\frac{k^2}{a^2H^2}c_\varepsilon^2\Omega_m + B_2(k)\right)\frac{S_m}{a^2} = 0$$

2nd order equation for relative density perturbations

$$S_m'' + \left[\frac{3}{2}\left(1 - \frac{p}{\rho}\right) + \frac{3c_\varepsilon^2}{1+w}\Omega_m\right]\frac{S_m'}{a} + M_1(k)\frac{S_m}{a^2}$$

$$+E_1(k)\frac{\varepsilon'}{a} + E_2(k)\frac{\varepsilon}{a^2} = 0$$

Anisotropic pressure and gravitational potentials

Spatial part of Einstein's equations

$$\hat{G}^a_b - \frac{1}{3}\delta^a_b \hat{G}^m_m = 8\pi G \left(\hat{T}^a_b - \frac{1}{3}\delta_{ab} \hat{T}^m_m \right)$$

Left-hand side

$$\hat{G}^a_b - \frac{1}{3}\delta^a_b \hat{G}^m_m = \frac{1}{a^2} \left(\partial_a \partial_b - \frac{1}{3}\delta_{ab} \Delta \right) (\psi - \phi)$$

Right-hand side

$$\hat{T}^a_b - \frac{1}{3}\delta_{ab} \hat{T}^m_m = \pi^a_b = \frac{1}{a^2} \left(\partial_a \partial_b - \frac{1}{3}\delta_{ab} \Delta \right) \Pi$$

Gravitational slip

Difference of gravitational potentials

$$\psi - \phi = 8\pi G \Pi$$

With

$$\frac{H^2 \Pi}{\rho} = \mu \varepsilon + \nu S_m$$

Gravitational slip: $\frac{\phi}{\psi}$

$$\phi = \left[1 + 2\frac{k^2}{a^2 H^2}\left(\mu + \nu \frac{S_m}{\varepsilon}\right)\right]\psi$$

Solution of the system for ε and S_m required

Matter perturbations

Relative perturbations

$$S_m \equiv \frac{\delta}{1+w} - \delta_m$$

Comoving density perturbations

$$\varepsilon = \delta - 3H\left[(1+w)\,v + \frac{q}{\rho}\right]$$

Comoving matter perturbations

$$\delta_m = \frac{\varepsilon}{1+w_x\Omega_x} - S_m \qquad \left(\frac{a^2H^2}{k^2} \ll 1\right)$$

Matter perturbations determined by ε and S_m

Application

Extended ΛCDM model inspired by Jordan-Brans-Dicke theory

JBD type scalar-tensor theory

Jordan-frame action

$$\Sigma(g_{\mu\nu},\Phi) = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\Phi R - \frac{\omega(\Phi)}{\Phi}(\nabla\Phi)^2 - U(\Phi)\right] + \Sigma_m(g_{\mu\nu})$$

Matter part

$$\Sigma_m = \int d^4x\sqrt{-g}L_m(g_{\mu\nu})$$

Energy-momentum tensor

$$T_{(m)\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\Sigma_m}{\delta g^{\mu\nu}}$$

Field equations

Modified Einstein equations

$$\Phi\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=\kappa^2 T_{(m)\mu\nu}+\frac{\omega(\Phi)}{\Phi}\left(\partial_\mu\Phi\partial_\nu\Phi-\frac{1}{2}g_{\mu\nu}\left(\nabla\Phi\right)^2\right)$$

$$+\nabla_\mu\nabla_\nu\Phi-g_{\mu\nu}\Box\Phi-\frac{1}{2}g_{\mu\nu}U$$

Scalar-field equation

$$\Box\Phi=\frac{1}{2\omega(\Phi)+3}\left(\kappa^2 T-\frac{d\omega(\Phi)}{d\Phi}\left(\nabla\Phi\right)^2+\Phi\frac{dU}{d\Phi}-2U\right)$$

Matter conservation

$$T^{\mu\nu}_{(m);\nu}=0$$

Method

- Map additional (compared with GR) geometrical degrees of freedom onto an effective fluid component
- Effective two-fluid system within GR
- Gravitational dynamics does not depend on the details of the (underlying) scalar-field dynamics
- Phenomenological fluid dynamical equations

Effective Einstein equations

Rewrite scalar-tensor equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}$$

Total energy-momentum tensor

$$T_{\mu\nu} = T_{(m)\mu\nu} + T_{(x)\mu\nu}$$

Effective fluid

$$T_{(x)\mu\nu} \equiv \left(\frac{1}{\Phi} - 1\right) T_{(m)\mu\nu} + \frac{1}{\kappa^2 \Phi} \frac{\omega(\Phi)}{\Phi} \left[\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} \left(\nabla \Phi\right)^2\right]$$

$$+ \frac{1}{\kappa^2 \Phi} \left[\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \Box \Phi - \frac{1}{2} g_{\mu\nu} U\right]$$

Scalar-tensor theory

Background Hubble rate

$$H^2 = \frac{\kappa^2}{3}\frac{\rho_m}{\Phi} + \frac{1}{3\Phi}\left[\frac{1}{2}\frac{\omega(\Phi)}{\Phi}\left(\frac{\partial\Phi}{\partial t}\right)^2 - 3H\frac{\partial\Phi}{\partial t} + \frac{1}{2}U\right]$$

Solution of the dynamics?

- Choose a potential
- Solve the FLRW background dynamics (numerically)
- Use the known background dynamics to solve the (linear) perturbation equation (numerically)
- Calculate the growth rate of matter perturbations

Here: use only fluid dynamical equations

$e_\Phi\Lambda$CDM model: modified background dynamics

Hubble rate explicitly

$$\frac{H^2}{H_0^2} = \frac{A\Omega_{m0}a^{-3}}{\Phi} + [1 - A\Omega_{m0}]\,\Phi$$

$$\Phi = a^{\frac{-6m}{1+3m}} \qquad A \equiv \frac{(1+3m)^2}{1-m}$$

Consequence of the fluid dynamical equations

Scalar Φ modifies the cosmological dynamics

$\Phi = 1 \Leftrightarrow m = 0$: ΛCDM

W.C. Algoner, H.E.S. Velten, WZ, JCAP 11 (2016) 034

Effective EoS

Total energy density

$$\rho = A\frac{\rho_m}{\Phi} + \rho_0\left(1 - A\Omega_{m0}\right)\Phi$$

Define

$$\rho_x = \rho - \rho_m = \left(\frac{A}{\Phi} - 1\right)\rho_m + \rho_0\left[1 - A\Omega_{m0}\right]\Phi$$

$$\frac{d\rho_x}{dt} + 3H\left(1 + w_x\right)\rho_x = 0$$

Effective EoS

$$w_x = -1 + \frac{\frac{2m}{1+3m}\left[1 - A\Omega_{m0}\right]\Phi + \Omega_{m0}a^{-3}\left[\frac{1+m}{1+3m}A\Phi^{-1} - 1\right]}{\left[1 - A\Omega_{m0}\right]\Phi + \Omega_{m0}a^{-3}\left[A\Phi^{-1} - 1\right]}$$

Total EoS parameter

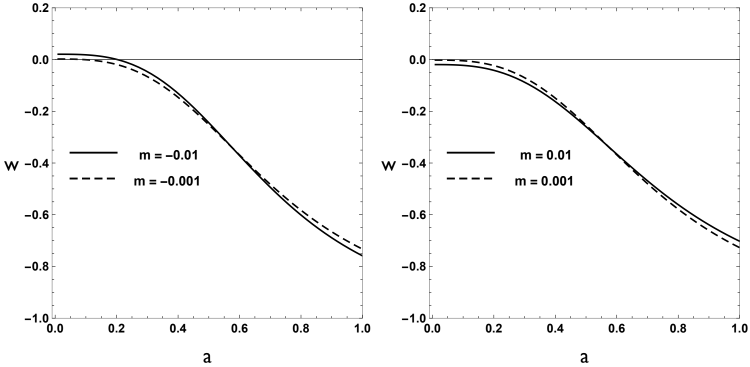

Figure: **Total EoS parameter for various negative (left panel) and positive (right panel) values of *m*.**

Growth for a typical scale

Growth function

$$\mathbf{f} = \frac{\mathrm{d}\ln\delta_{\mathrm{m}}}{\mathrm{d}\ln \mathrm{a}}$$

Observed quantity $\mathbf{f}\sigma_{\mathbf{8}}$

$\sigma_{\mathbf{8}}$:**rms mass fluctuation in spheres with radius** $\mathbf{8h}^{-\mathbf{1}}\mathrm{Mpc}$

Data sets of RSD measurements of $\mathbf{f}\sigma_{\mathbf{8}}$
(SDSS, BOSS, WiggleZ, Vipers, 6dFGS ...)

Typical scale: $\mathbf{k} = \mathbf{0.1h}\mathrm{Mpc}^{-\mathbf{1}}$: $\quad \frac{\mathbf{k^2}}{\mathbf{a^2H^2}} \approx \mathbf{1.8} \cdot \mathbf{10^5}$

Table: 23 data points.

Index	Data set	z	$f\sigma_8(z)$	Year	Notes
1	6dFGS + SnIa	0.02	0.428 $\pm$ 0.0465	2016	$(\Omega_{0m},\ h,\ \sigma_8)=(0.3,\ 0.683,\ 0.8)$
2	SnIa + IRAS	0.02	0.398 $\pm$ 0.065	2011	$(\Omega_{0m},\ \Omega_K)=(0.3,\ 0)$
3	2MASS	0.02	0.314 $\pm$ 0.048	2010	$(\Omega_{0m},\ \Omega_K)=(0.266{,}0)$
4	SDSS-veloc	0.10	0.370 $\pm$ 0.130	2015	$(\Omega_{0m},\ \Omega_K)=(0.3,\ 0)$
5	SDSS-MGS	0.15	0.490 $\pm$ 0.145	2014	$(\Omega_{0m},\ h,\ \sigma_8)=(0.31,\ 0.67,\ 0.83)$
6	2dFGRS	0.17	0.510 $\pm$ 0.060	2009	$(\Omega_{0m},\ \Omega_K)=(0.3,\ 0)$
7	GAMA	0.18	0.360 $\pm$ 0.090	2013	$(\Omega_{0m},\ \Omega_K)=(0.27,\ 0)$
8	GAMA	0.38	0.440 $\pm$ 0.060	2013	
9	SDSS-LRG-200	0.25	0.3512 $\pm$ 0.0583	2011	$(\Omega_{0m},\ \Omega_K)=(0.25,\ 0)$
10	SDSS-LRG-200	0.37	0.4602 $\pm$ 0.0378	2011	
11	BOSS-LOWZ	0.32	0.384 $\pm$ 0.095	2013	$(\Omega_{0m},\ \Omega_K)=(0.274,\ 0)$
12	SDSS-CMASS	0.59	0.488 $\pm$ 0.060	2013	$(\Omega_{0m},\ h,\ \sigma_8)=(0.307115,\ 0.6777,\ 0.8288)$
13	WiggleZ	0.44	0.413 $\pm$ 0.080	2012	$(\Omega_{0m},\ h)=(0.27,\ 0.71)$
14	WiggleZ	0.60	0.390 $\pm$ 0.063	2012	
15	WiggleZ	0.73	0.437 $\pm$ 0.072	2012	
16	Vipers PDR-2	0.60	0.550 $\pm$ 0.120	2016	$(\Omega_{0m},\ \Omega_b)=(0.3,\ 0.045)$
17	Vipers PDR-2	0.86	0.400 $\pm$ 0.110	2016	
18	FastSound	1.40	0.482 $\pm$ 0.116	2015	$(\Omega_{0m},\ \Omega_K)=(0.270,\ 0)$
19	SDSS-IV	1.52	0.420 $\pm$ 0.076	2018	$(\Omega_{0m},\ \Omega_b h^2,\ \sigma_8)=(0.26479,\ 0.02258,\ 0.8)$
20	SDSS-IV	1.52	0.396 $\pm$ 0.079	2018	$(\Omega_{0m},\Omega_b h^2,\ \sigma_8)=(0.31,\ 0.022,\ 0.8225)$
21	SDSS-IV	1.23	0.385 $\pm$ 0.099	2018	$(\Omega_{0m},\ \sigma_8)=(0.31,\ 0.8)$
22	SDSS-IV	1.526	0.342 $\pm$ 0.070	2018	
23	SDSS-IV	1.944	0.364 $\pm$ 0.106	2018	

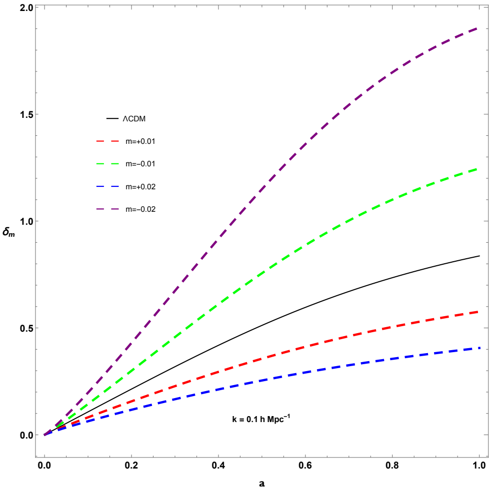

Figure: **Matter growth if only the background is changed.**

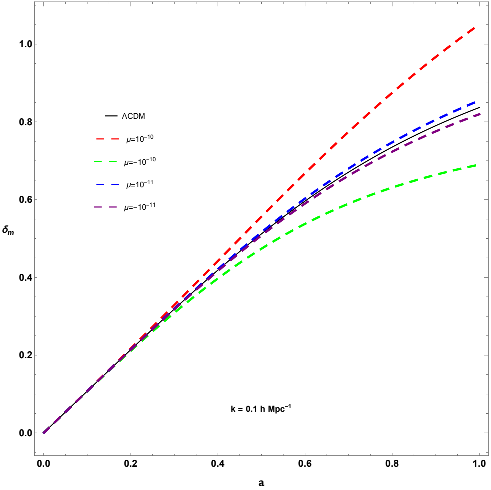

Figure: **Matter growth in the presence of anisotropic stresses.**

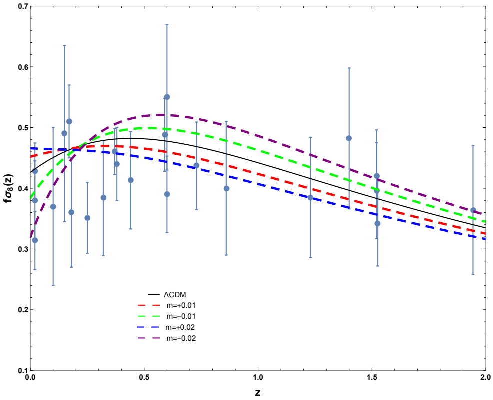

Figure: **Dependence of $f\sigma_8(z)$ on z if only the background dynamics is changed.**

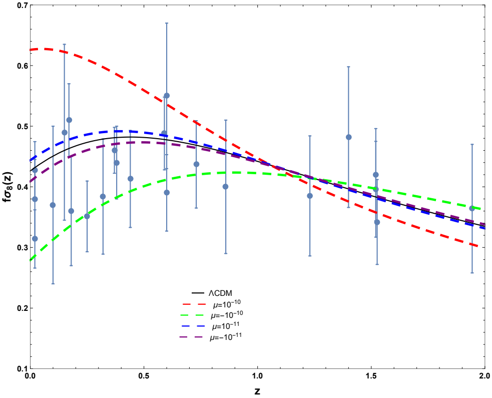

Figure: **Dependence of $f\sigma_8(z)$ on z in the presence of anisotropic stresses.**

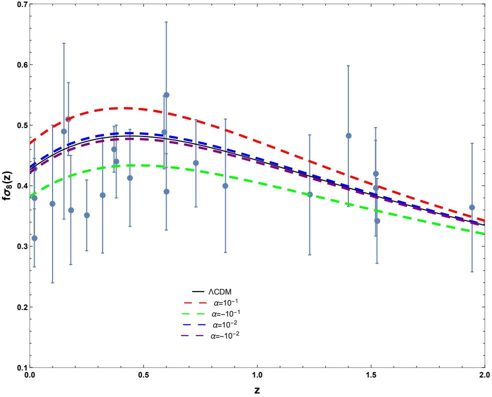

Figure: **Dependence of $f\sigma_8(z)$ on z in the presence of heat fluxes.**

Discussion: general dynamics

- General phenomenological scheme for implementing (effective) non-equilibrium effects in a fluid description of the cosmological dark sector
- Modified Poisson equation for the gravitational potential
- Phenomenological relations for anisotropic pressure and heat flux
- Manifestly gauge-invariant coupled system of two second-order differential equations for the total and the relative energy-density perturbations

Discussion: JBD inspired model

- Minimal extension of the standard model: $e_{\Phi}\Lambda$CDM
- Modified background dynamics analytically
- Phenomenological fluid dynamics: matter growth rate
- Observational constraints on anisotropic stresses and heat fluxes

Observational constraints

- **Deviations from the ΛCDM background** (parameter m)
- **Magnitude of anisotropic stresses** (parameters μ and ν)
- **Magnitude of heat fluxes** (parameters α and β)

HAPPY BIRTHDAY ILYA!