NON-SINGULAR BLACK HOLE IN PLANCK ERA

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◇ The study of gravitational collapse in general relativity (GR) is fundamental for understanding the behaviour of the theory at high energies.

◇ The Oppenheimer-Snyder (OS) model still provides a paradigmatic example that serves as a good qualitative guide to the general collapse problem in GR [Oppenheimer & Snyder, 1923]; it can be solved analytically, as it simply assumes a collapsing homogeneous dust cloud of finite mass and radius, described by a FLRW metric and surrounded by a vacuum exterior which has to be **static** (Birkhoff, 1923).

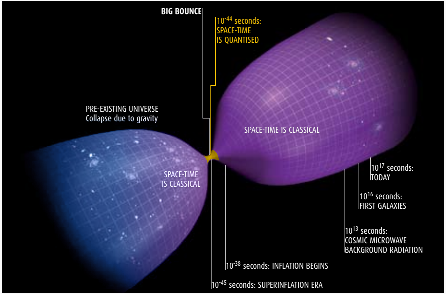
◇ Because of (generality of) the singular nature of the solutions to the Einstein's field equation (e.g., Gravitational collapse and Black hole solution), GR must be replaced by a quantum theory of gravity at Planck scale.

◇ Thus, it is important to consider quantum gravity modification to the gravitational system (or an alternative quantum theory of gravity) when studying them at high energy. For example: Late time stages of gravitational collapse & black hole interior; very early and late time (phantom dark energy) universe; and even when studying QFT in very early times (when the volume of the universe is some 75 orders of magnitude larger than the Planck volume $\ell_{\rm Pl}^3$).

◇ The key insight about the onset of quantum gravitational effects comes from quantum cosmology; according to loop quantum cosmology (LQC) the Friedmann equation that governs the dynamics of the scale factor $a(t)$ of the universe is modified by quantum gravitational effects:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \quad \xrightarrow{\text{LQC}} \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm crit}}\right)$$

where $\rho_{\rm crit} \sim 0.41\rho_{\rm Pl}$.



(Ashtekar-Pawlowski-Singh, 2006)

Effective repulsive force induced by LQG

$\Downarrow$

Quantum bounce

- ⋆ Semiclassical dynamics of horizon in scalar field collapse
  YT, J. Marto & A. Dapor IJMPD (2014);
- ⋆ Quantum gravitational collapse of homogeneous and inhomogeneous dust
  C. Bambi, D. Malafarina, L. Modesto, PRD (2013);
- ⋆ Planck stars
  C. Rovelli & F. Vidotto, IJMPD (2014);
- ⋆ Planck star phenomenology
  A. Barrau, C. Rovelli, PLB (2014);
- ⋆ Black hole fireworks: quantum-gravity effects outside the horizon spark black to white hole tunneling
  H. Haggard, C. Rovelli (2014);
- ⋆ Improved dynamics grav. collapse of tachyon field coupled to a barotropic fluid
  J. Marto, YT & P. V. Moniz, IJMPD (2015).

Let us consider an OS model; gravitational collapse with a homogeneous dust matter:

⋆ The Hamiltonian, $\mathcal{H}_{\mathrm{matt}}$, of the spherical collapsing cloud reads:

$$\mathcal{H}_{\mathrm{matt}} = \rho_{\mathrm{dust}} V_o \,, \qquad \text{where} \qquad \rho_{\mathrm{dust}} = \rho_0 \left(\frac{a_0}{a}\right)^3$$

is the constant total dust matter; $V_o = |p|^{3/2} := a^3$. The constants ρ_0 and a_0 are, respectively, initial energy density and scale of the matter ball.

⋆ The space-time inside the matter cloud is given by a flat FLRW geometry; the corresponding classical Hamiltonian constraint reads

$$\mathcal{H}_{\mathrm{class}} = -\frac{3}{\kappa\gamma^2} c^2 \sqrt{|p|} + \frac{\Lambda}{8\pi G}|p|^{3/2} + \mathcal{H}_{\mathrm{matt}}$$

where $|p| = a^2$, $c = \gamma\dot{a}$ and $\gamma = 0.24$ is the Barbero-Immirzi parameter.

By solving the Hamiltonian constraint, and matching the interior to a convenient exterior geometry (of Vaidya type metric), at the boundary of collapsing star Σ (with the radius r_b), it can be shown that, the exterior metric is written in a static form:

$$ds_+^2 \;=\; -H(R)d\tau^2 + H^{-1}(R)dR^2 + R^2 d\Omega^2$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ and the redshift function $H(R)$ is given by

$$H(R) \;=\; 1 - \frac{2Gm(R)}{R}$$

By introducing $M := \frac{4\pi}{3} R_0^3 \rho_0$ (the (constant) physical mass of dust), we get

$$m(R) \;=\; M + \frac{\Lambda}{6} R^3$$

where R belongs to the interval $0 < R < +\infty$, with $R = 0$ being the black hole singularity where the curvature of space-time diverges. The Ricci scalar for the exterior metric reads $\mathcal{R} = 4\Lambda$ which indicates to a static Schwartzchild de Sitter geometry.

An effective scenario is provided by the holonomy corrections in LQG; is given by replacing [Ashtekar, Pawlowski, Singh, PRD 2006]

$$\frac{3}{\kappa\gamma^2} c^2 \sqrt{|p|} \quad \xrightarrow{\text{LQG}} \quad \frac{3}{\kappa\gamma^2} \frac{\sin^2(\bar{\mu}c)}{\bar{\mu}^2} \sqrt{|p|}$$

So that, the resulting modified Hamiltonian constraint in this regime reads

$$\mathcal{H}_{\text{eff}} = -\frac{3}{\kappa\gamma^2\bar{\mu}^2} \sqrt{|p|} \sin^2(\bar{\mu}c) + \mathcal{H}_{\Lambda\text{M}}$$

where $\bar{\mu} = (3\sqrt{3}/2|\mu|)^{1/2}$, with μ being the eigenvalue of the triad operator $\hat{p}$ in quantum theory: $\hat{p}|\mu\rangle = (8\pi\gamma\ell_{\text{Pl}}^2/6)\mu|\mu\rangle$, where $\ell_{\text{Pl}} = \sqrt{\hbar G}$ is the Planck length (in units $c_{\text{Light}} = 1$ of speed of light). In addition,

$$\mathcal{H}_{\Lambda\text{M}} = \frac{\Lambda}{8\pi G} |p|^{3/2} + \mathcal{H}_{\text{matt}} =: (\rho_\Lambda + \rho_{\text{dust}}) V_o$$

The dynamics of the fundamental variable p is obtained as

$$\dot{p} = \{p, \mathcal{H}_{\text{eff}}\} = \frac{2a}{\gamma\bar{\mu}} \sin(\bar{\mu}c)\cos(\bar{\mu}c) \qquad \text{(Hamilton equation)}$$

where from $\mathcal{H}_{\text{eff}} = 0$ we have

$$\sin^2(\bar{\mu}c) = \frac{8\pi G\gamma^2\bar{\mu}^2}{3a} \mathcal{H}_{\Lambda\text{M}}$$

Using the last two equations together we obtain the Hubble rate, $H = \dot{p}^2/4p^2$:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_{\rm tot}\left(1 - \frac{\rho_{\rm tot}}{\rho_{\rm crit}}\right)$$

where $\rho_{\rm tot} = \rho_{\rm dust} + \rho_\Lambda$, and $\rho_{\rm crit} = \sqrt{3}/(16\pi^2\gamma^3)\rho_{\rm Pl} \sim 0.41\rho_{\rm Pl}$ with $\rho_{\rm Pl} = 1/(\hbar G^2)$ being the Planck density.

So, quantum gravity effect modifies the interior side of the boundary surface Σ, through the Friedmann equation (in terms of the area radius $R(r_b, t) = r_b a(t)$):

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho_{\rm tot}\left(1 - \frac{\rho_{\rm tot}}{\rho_{\rm crit}}\right)$$

This equation can be rewritten as

$$\dot{R}^2 = \frac{2GM}{R}\left(1 + 2\frac{\rho_\Lambda}{\rho_{\rm crit}}\right) - \frac{3G}{2\pi\rho_{\rm crit}}\frac{M^2}{R^4} + \frac{\Lambda}{3}R^2\left(1 - \frac{\rho_\Lambda}{\rho_{\rm crit}}\right)$$

where we have used $M := \frac{4\pi}{3}R_0^3\rho_0$, for $R_0 = r_b a(t_0)$ being the initial value for the area radius of the boundary shell.

We consider an exterior metric that could describe the resulting effective geometry: In Eddington-Finkelstein coordinates, the metric of the collapse can be written as

$$ds_+^2 \ = \ -\left(1-\frac{2Gm(r_v,v)}{r_v}\right)dv^2+2dvdr_v+r_v^2d\Omega^2$$

which is matched to the interior at the boundary surface Σ; in the coordinate (v,r_v,θ,φ), v is the retarded ingoing null coordinate, r_v and $m(v,r_v)$ are respectively, the Vaidiya radius and mass.

From matching the extrinsic curvature and area radius at Σ we have that

$$\begin{aligned} r_v|_\Sigma \ &= \ R(r_b,t) \\ 2Gm(r_v,v)/r_v \ &= \ \dot{R}^2(r_b,t) \end{aligned}$$

where $\dot{R}^2$ is already modified due to the quantum effects.

By using the resulting modified $\dot{R}$ in the second equation above we obtain

$$\begin{aligned} \dot{R}^2(r_b,t) \ &= \ \frac{2GM}{R}(1+2\delta)+\frac{\Lambda}{3}R^2(1-\delta)-\frac{3G}{2\pi\rho_{\rm crit}}\frac{M^2}{R^4} \\ &= \ \frac{2Gm(r_b,t)}{R} \end{aligned}$$

where we have defined $\delta := \frac{\rho_\Lambda}{\rho_{\rm crit}}$.

If we change the null coordinate by $dv = d\tau - dr_v/(1 - 2Gm/r_v)$ and $r_v|_\Sigma = R$, we can write the exterior metric in a rather general static form:

$$ds_+^2 \;=\; -\left(1 - \frac{2Gm(R)}{R}\right)d\tau^2 + \left(1 - \frac{2Gm(R)}{R}\right)^{-1} dR^2 + R^2 d\Omega^2$$

where the redshift function $H(R) = 1 - 2Gm(R)/R$ is modified due to modification of $m(R)$. By using the matching condition $2Gm(R)/R = \dot{R}^2$, we obtain

$$\boxed{m(R) \;=\; M\,(1 + 2\delta) + \frac{\Lambda}{6}R^3\,(1 - \delta) - \frac{3}{4\pi\rho_{\rm crit}}\frac{M^2}{R^3}}$$

NOW, R belongs to the interval $R_{\rm min} < R < +\infty$ with $R_{\rm min}$ is the area radius at the bounce (and $V_{\rm min} = R^3_{\rm min}$ is the volume of the bounce):

$$R_{\rm min} \;:=\; r_b a_{\rm min} = \left(\frac{3M}{4\pi\rho_{\rm crit}}\right)^{\frac{1}{3}} \quad\Rightarrow\quad V_{\rm min} \;=\; \left(\frac{M}{m_{\rm Pl}}\right)\ell^3_{\rm Pl}$$

Since R is bounded below as $R \geq R_{\rm min}$, thus, the (possible) final black hole is non-singular and the collapse will end with a bounce. Thus, the scalar curvature for the exterior (possible) static black hole remains finite during the collapse:

$$\mathcal{R} \;=\; 4\Lambda\left(1 - \frac{\rho_\Lambda}{\rho_{\rm crit}}\right) - \frac{9G}{\pi\rho_{\rm crit}}\frac{M^2}{R^6}$$

In the limit $\rho_{\rm crit} \to \infty$, this equation reduces to $\mathcal{R} = 4\Lambda$; this corresponds to the Ricci scalar in the classical Einstein equation for a vacuum exterior, namely, the Schwartzschild-de Sitter solution.

From an effective theory point of view, we can obtain a modified Raychadhuri equation, from which an effective pressure for the system is provided:

$$P_{\rm eff} \;=\; -\rho_\Lambda \left(1 - \frac{\rho_{\rm tot}}{\rho_{\rm crit}}\right) - \frac{\rho_{\rm tot}^2}{\rho_{\rm crit}}$$

This equation shows that, for $\Lambda = const.$ or $\Lambda = 0$ the modified matter pressure is non-zero, hence, the interior cannot be matched to a static Schwartzchild-de Sitter exterior geometry at Σ.

The new terms, other than those being present in a Schwarzschild de-Sitter space, represent a short-scale repulsive force due to quantum gravity effects.

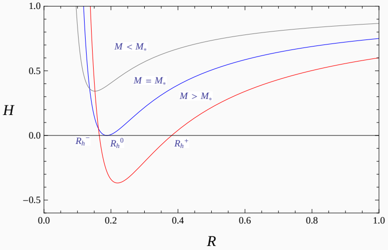
However, for a vacuum energy density evolving with time as

$$2\rho_\Lambda(a) \;=\; \rho_{\rm crit} - \sqrt{\rho_{\rm crit}^2 + 4\rho_{\rm dust}(a)}$$

the effective pressure vanishes. In other words, Schwarzschild solution implies the existence of a varying cosmological constant which must be negative and fine tuned with matter content.

HORIZON FORMATION

The numerical plot shows that the redshift function changes, depending on the initial mass of collapsing star M:



The extremum points of $H(R)$ are located at the roots of equation $dH/dR = 0$:

$$\tilde{\Lambda} R^6 - (3G\tilde{M})R^3 + (12G\hat{M}) = 0 \ ,$$

where $\tilde{\Lambda} := \Lambda\,(1-\delta)$; with $\delta := \frac{\rho_\Lambda}{\rho_{\mathrm{crit}}}$;

$$\tilde{M} := M\,(1+2\delta)\,, \quad \hat{M} := \frac{3M^2}{4\pi\rho_{\mathrm{crit}}}$$

Thus, solving this equation:

$\star$ For $\Lambda \neq 0$ we have two roots $R_- < R_+$ where $R_\pm = (3GM\xi_\pm/2\tilde{\Lambda})^{1/3}$ where

$$\xi_\pm := (1-2\delta) \pm \sqrt{(1-2\delta)^2 - 32\delta\,(1-\delta)}$$

We can show that at $R = R_-$ we have $d^2H(R_-)/dR^2 > 0$ so that the redshift function has a minimum at $H(R_-) = H_{\mathrm{min}}$.

If $H_{\rm min} > 0$, then $H(R)$ never cross the axis $R = 0$, that is $H(R) = 0$ has no root; no horizon forms during the collapse. The condition $H_{\rm min} = H(\mathrm{R}_-) > 0$ (the case with no horizon) can be translated to a constraint on the mass M; if the mass $M < M_\star$ where

$$M_\star = \Big[2(1-2\delta)(2\tilde{\Lambda}/3\xi_-)^{\frac{1}{3}} + (3/2\pi G\rho_{\rm crit})(2\tilde{\Lambda}/3\xi_-)^{\frac{4}{3}} \\ -(4\pi\rho_{\rm crit}/3)\delta(1-\delta)(2\tilde{\Lambda}/3\xi_-)^{-\frac{2}{3}}\Big]^{-\frac{3}{2}} G^{-1}$$

$\star$ For $\Lambda = 0$, equation above has only one extremum $\mathrm{R}_0 := (3M/\pi\rho_{\rm crit})^{\frac{1}{3}}$ for which $H(R)$ is minimum; positiveness of $H_{\rm min}$ in this case imposes a constraint on $M > M_\star$, which implies the threshold mass $M_\star$,

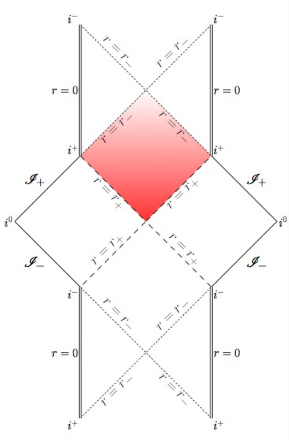
$$M_\star = \left(\frac{8}{9\pi G^3\rho_{\rm crit}}\right)^{\frac{1}{2}} \sim 0.83 m_{\rm Pl}$$

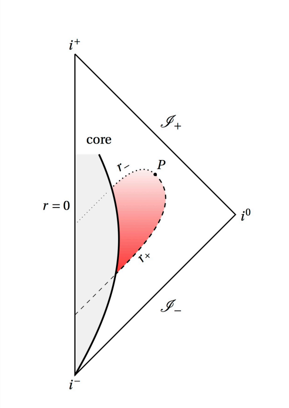
to be the minimum mass for the possible final black hole.

In summary, for the initial masses

(i) $M < M_\star \;\Rightarrow\; H_{\rm min} > 0$; no horizon formation;
(ii) $M = M_\star \;\Rightarrow\; H_{\rm min} = 0$; formation of one (double) horizon;
(iii) $M > M_\star \;\Rightarrow\; H_{\rm min} < 0$; formation of two horizons.

Possible scenarios for the case $M > M_\star$: (i) Evaporation of BH (Hayward, 2006) – (ii) Formation of non-singular Planck stars (Rovelli et. al., 2014).





Gravitational collapse of a non-singular and non-extremal BH.

THE CASE $\Lambda = 0$

The redshift function, $H(R) = 1 - 2Gm(R)/R$, reduces to:

$$H(R) \;=\; 1 - \frac{2GM}{R} + \frac{6G}{4\pi\rho_{\rm crit}}\frac{M^2}{R^4}$$

The extremal black hole mass in this case is

$$\boxed{M_\star \;=\; 0.83 m_{\rm Pl}}$$

The location of the horizon, R_h^0, for an (extreme) black hole with the mass $M_\star$ is $R_h^0 = \mathrm{R}_0 = \sqrt[3]{4}R_{\rm min}$. At the bounce, $4\pi\rho_{\rm crit}R_{\rm min}^3/3 = M_0$; with $m_{\rm Pl} = 4\pi\rho_{\rm Pl}\ell_{\rm Pl}^3/3$. So

$$\frac{M_0}{m_{\rm Pl}} \;\approx\; 0.41\left(\frac{R_{\rm min}}{\ell_{\rm Pl}}\right)^3$$

Therefore, for a star with initial mass $M_0 = M_\star = 0.83 m_{\rm Pl}$, the bounce occurs at the radius $R_{\rm min} \approx 2^{1/3}\ell_{\rm Pl}$. In this case, the location of the horizon is obtained as

$$\boxed{R_h^0 \;\approx\; 2\ell_{\rm Pl}}$$

Notice that $R_h^0 = 2^{\frac{2}{3}}R_{\rm min}$; the final bounce is covered by the horizon. Moreover,

$$R_S = (2G\bar{M}_\star/c^2) = 1.66\ell_{\rm Pl} < 2\ell_{\rm Pl}$$

The radius of such micro black hole is bigger than the (classical) Schwarzschild radius, R_S, of a black hole with the same mass $M_\star$.

$\star$ We studied an Oppenheimer-Snyder like model for gravitational collapse within an effective scenario provided by improved dynamics LQC: This quantum gravity correction gave rise to an exterior black hole geometry in which the classical singularity is resolved and replaced by a bounce.

$\star$ We found a threshold mass $M_\star = 0.83 m_{\rm Pl}$ to be the minimum mass for the black hole formation.

$\star$ For $M > M_\star$ there are different scenarios for collapse enestate such as: Evaporation of the resulting regular black hole; destruction of outer (classical) Schwarzschild horizon R_h^+ and inner (quantum) horizon R_h^- at some point $R_h^- < R < R_h^+$ and formation of a Planck star.

$\star$ For primordial (micro) black holes (or those may occur in LHC?!), for a mass $M = M_\star$ we have only one horizon $R_h^0 = 2\ell_{\rm Pl}$, for the extremal micro black hole, which is larger that the corresponding Schwarzschild radius: $R_h^0 > R_s = 1.6\ell_{\rm Pl}$.

IN PROGRESS...
(Thermodynamics of non-singular & and non-extremal (micro) black holes)

THANK YOU!