

Introduction to Inflation

Lecture 3: Inflation phenomenology

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- ▶ Primordial perturbations from inflation:
 - ▶ initial conditions from sub-Hubble scale vacuum fluctuations
 - ▶ gauge-invariant field+metric fluctuations from inflaton field fluctuations
 - ▶ slow-roll induces weak scale-dependence
 - *confirmed by observations*
 - ▶ gravitational waves (tensors) also predicted
 - *but not yet seen*
- ▶ Observations can
 - ▶ constrain model parameters
 - ▶ discriminate between different kinds of inflation models

Outline

Models of inflation

- Large-field inflation
- Small-field inflation
- Natural inflation
- Higgs inflation
- Starobinsky inflation

Inflationary phenomenology

- Multi-field models and non-Gaussianity
- Stochastic inflation

Models of inflation

- Large-field inflation
- Small-field inflation
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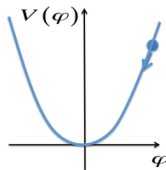
Inflationary phenomenology

- Multi-field models and
non-Gaussianity
- Stochastic inflation

Models of inflation: large-field inflation

- ▶ also known as chaotic or monomial inflation

$$V(\varphi) = \frac{\lambda_p}{p!} \frac{\varphi^p}{M_P^{p-4}}$$



- ▶ slow-roll parameters:

$$\epsilon_v = \frac{p^2}{16\pi} \left(\frac{M_P}{\varphi} \right)^2, \quad \eta_v = \frac{p(p-1)}{8\pi} \left(\frac{M_P}{\varphi} \right)^2$$

- ▶ slow-roll, $\epsilon, |\eta| \ll 1$, requires $\varphi \gg pM_P$

Models of inflation

Large-field inflation

Small-field inflation

Natural inflation

Higgs inflation

Starobinsky inflation

Inflationary phenomenology

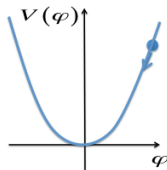
Multi-field models and
non-Gaussianity

Stochastic inflation

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- ▶ slow-roll, $\epsilon, |\eta| \ll 1$, requires $\varphi \gg pM_P$
- ▶ e-folds:

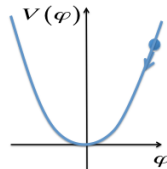
$$N(\varphi, \varphi_{\text{end}}) \simeq \int_{\varphi_{\text{end}}}^{\varphi} \sqrt{\frac{4\pi}{\epsilon_v}} \frac{d\varphi}{M_P} \simeq \int_{\varphi_{\text{end}}}^{\varphi} \frac{8\pi}{p} \frac{\varphi}{M_P} \frac{d\varphi}{M_P}$$

$$\Rightarrow \quad \varphi(N) \simeq \sqrt{\frac{pN}{4\pi}} M_P$$

Models of inflation: large-field inflation

- ▶ also known as chaotic inflation

$$V(\varphi) = \frac{\lambda_p}{p!} \frac{\varphi^p}{M_P^{p-4}}$$



- ▶ slow-roll solution $\varphi(N)$:

$$\epsilon \simeq \frac{p}{4N} \quad , \quad \eta \simeq \frac{p-1}{2N}$$

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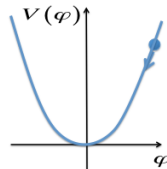
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Models of inflation: large-field inflation

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- ▶ slow-roll solution $\varphi(N)$:

$$\epsilon \simeq \frac{p}{4N} \quad , \quad \eta \simeq \frac{p-1}{2N}$$

- ▶ e.g., $p = 2$, quadratic inflaton, $V = m^2\varphi^2/2$

$$\epsilon \simeq \eta \simeq \frac{1}{2N} \simeq 0.01 \quad \text{for } N = 50$$

$$n_R - 1 \simeq -0.04 \quad , \quad r \simeq 0.16$$

Models of inflation

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Natural inflation

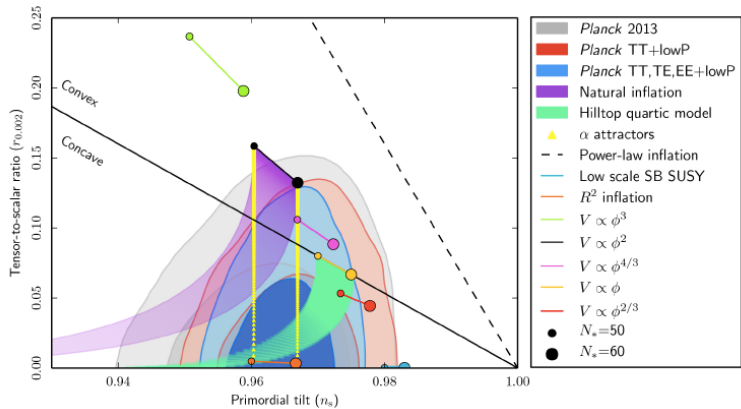
Higgs inflation

Starobinsky inflation

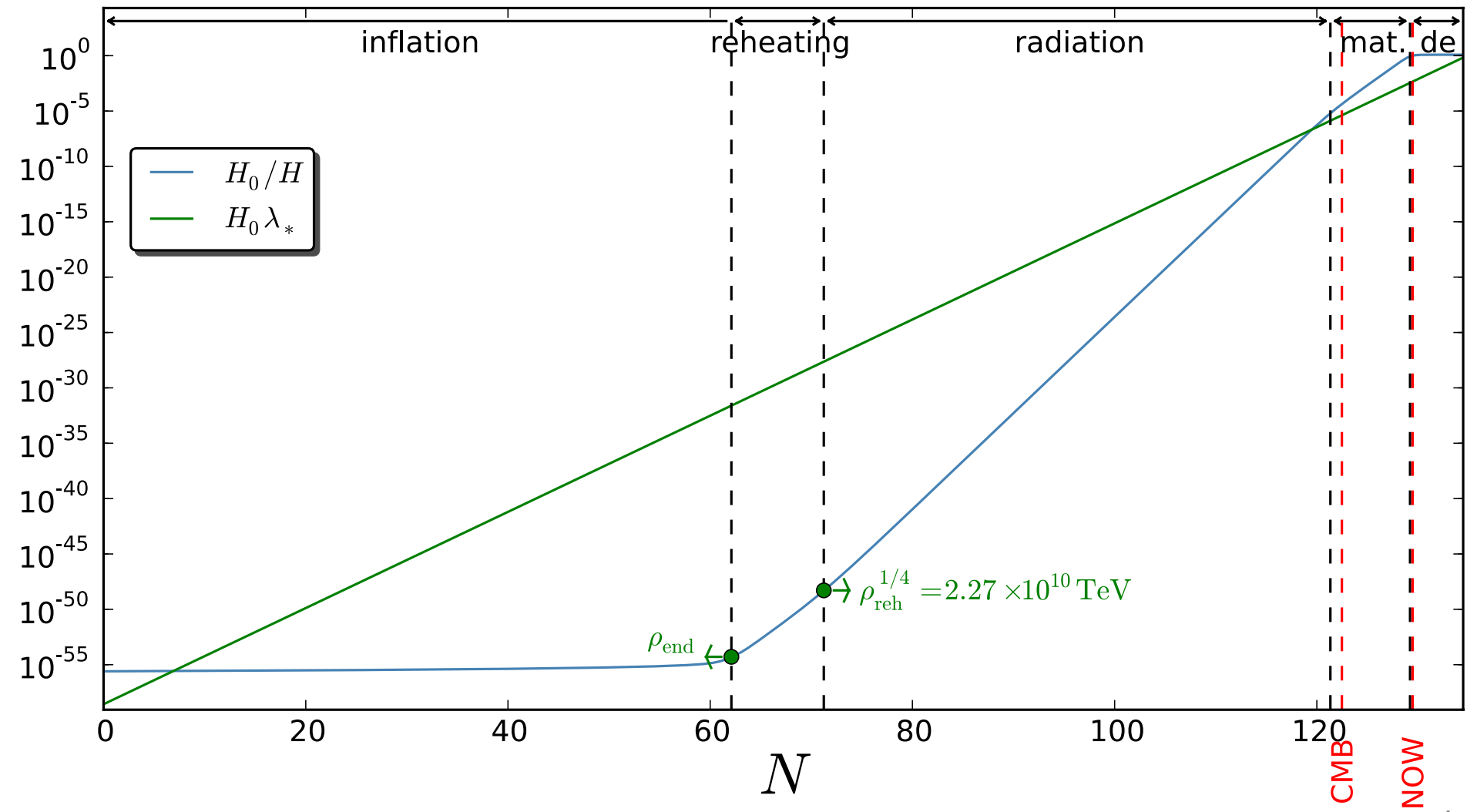
Inflationary phenomenology

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Role of Reheating



Reheating: simplest case

previously

Models of inflation

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- ▶ Coherent inflaton field oscillations are non-relativistic inflaton particles

$$\langle \rho_\varphi \rangle_t = mn_\varphi \propto a^{-3}$$

- ▶ Perturbative decay ($\Gamma < m$) to light (relativistic) particles

$$\mathcal{L}_{\text{decay}} = -\lambda_i \sigma \varphi \chi_i^2 - \lambda_j \varphi \bar{\psi}_j \psi_j$$

$$\Rightarrow \Gamma = \frac{\lambda_i^2 \sigma_i^2}{8\pi m} + \frac{\lambda_j^2 m}{8\pi}$$

$$\dot{\rho}_\varphi + 3H(\rho_\varphi + P_\varphi) = -\Gamma \varphi^2$$

(non-perturbative decay (preheating) can also be important)

- ▶ energy transferred to radiation when $H < \Gamma$

Reheating: temperature and e-folds

previously

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- decay followed by thermalisation to reheat temperature

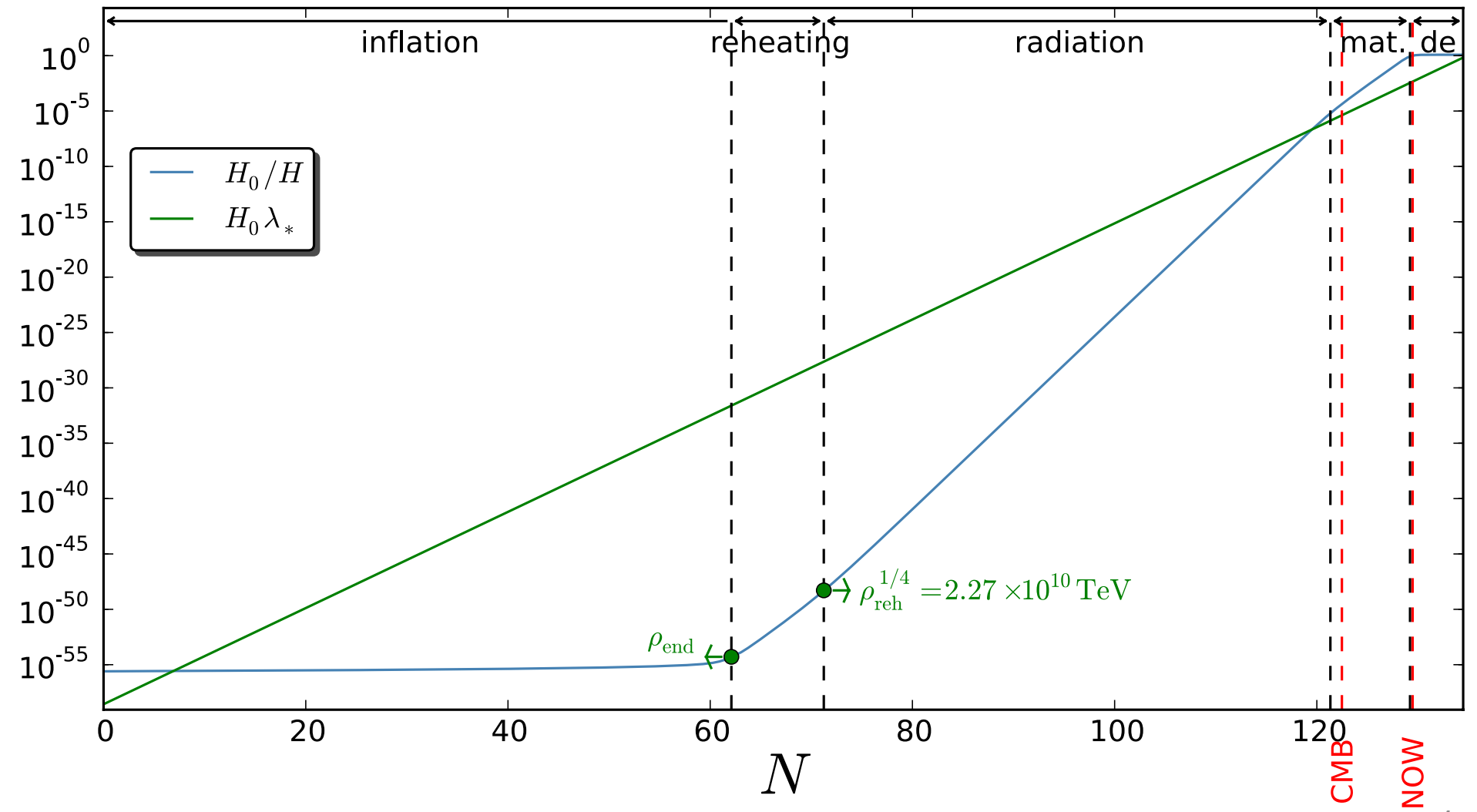
$$\rho_{\text{rh}} \leq \frac{3H_{\text{decay}}^2}{8\pi G} = \frac{3\Gamma^2}{8\pi G}$$

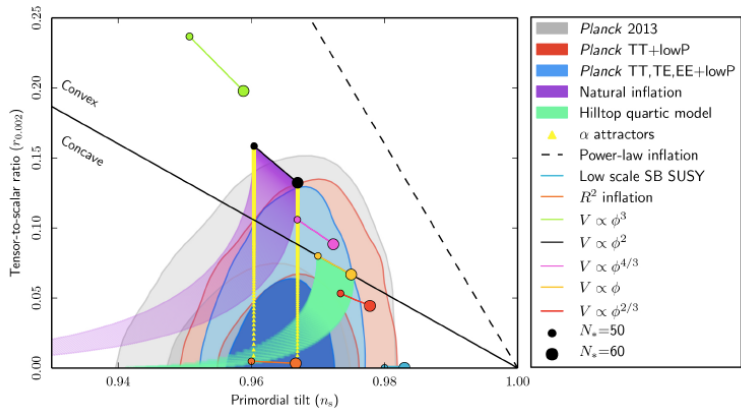
$$\Rightarrow T_{\text{rh}} \leq 0.2 \left(\frac{100}{g_{\text{rh}}} \right)^{1/4} (\Gamma M_P)^{1/2}$$

- Duration of reheating (and effective equation of state) affects expansion history after inflation and hence the number of e-folds when present horizon scale exits Hubble scale during inflation

$$N_* = 67 - \ln \left(\frac{k}{H_0} \right) + \frac{1}{4} \ln \left(\frac{V_*^2}{M_P^4 \rho_{\text{end}}} \right) + \frac{1}{12} \ln \left(\frac{\rho_{\text{rh}}}{\rho_{\text{end}}} \right) - \frac{1}{12} \ln(g_{\text{rh}})$$

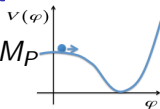
Role of Reheating





Models of inflation: small-field inflation

- ▶ also known as hill-top inflation: $\varphi < \mu < M_P$



$$V(\varphi) = V_0 \left(1 - \left(\frac{\varphi}{\mu} \right)^p + \dots \right) \quad \text{for } p \geq 2$$

- ▶ slow-roll parameters:

$$\epsilon \propto \left(\frac{M_P}{\mu} \right)^2 \left(\frac{\varphi}{\mu} \right)^{2(p-1)}, \quad \eta \propto \left(\frac{M_P}{\mu} \right)^2 \left(\frac{\varphi}{\mu} \right)^{p-2}$$

$$\Rightarrow \quad \frac{\epsilon}{|\eta|} = \frac{p}{2(p-1)} \left(\frac{\varphi}{\mu} \right)^p \ll 1 \quad \text{for } \varphi \ll \mu$$

Models of inflation

Large-field inflation

Small-field inflation

Natural inflation

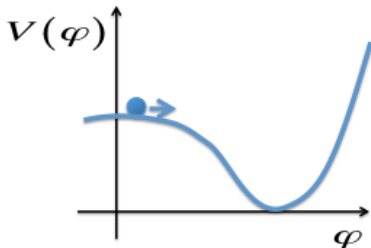
Higgs inflation

Starobinsky inflation

Inflationary phenomenology

Multi-field models and
non-Gaussianity

Stochastic inflation



- ▶ initial conditions problem for small field inflation?
 - ▶ require $\varphi_{\text{ini}} \ll \mu$ for $N \gg 1$
 - ▶ *how likely is this?*
 - ▶ need a theory of initial conditions for inflation
 - *stochastic inflation*

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Models of inflation: Starobinsky's R^2 inflation

Introduction to
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- ▶ $f(R)$ gravity:

$$f(R) = R + \frac{1}{6M^2}R^2$$

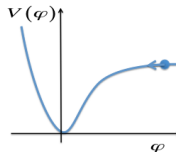
- ▶ conformal transformation to Einstein gravity

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = f'(R)g_{\mu\nu}$$

- ▶ potential for minimally-coupled field
 $\chi \propto M_P \ln f'(R)$

$$V(\chi) = M^2 M_P^2 \left(1 - e^{-\sqrt{3/2}\chi/M_P}\right)^2$$

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- Multi-field models and non-Gaussianity
Stochastic inflation

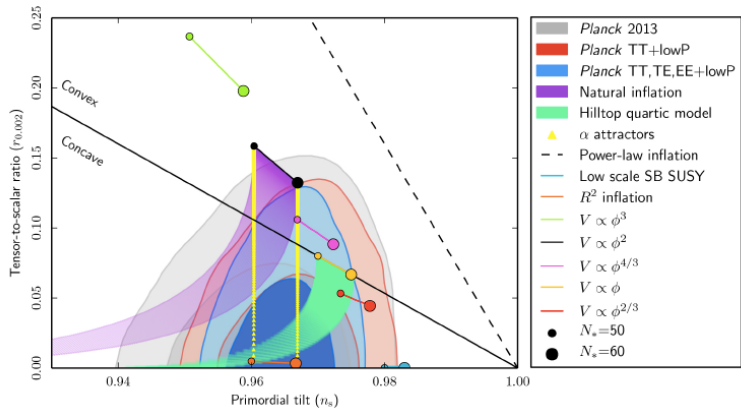
$$V(\chi) = M^2 M_P^2 \left(1 - e^{-\sqrt{3/2}\chi/M_P}\right)^2$$

- ▶ corresponds to “small-field” inflation in limit $p \rightarrow \infty$:

$$\eta \simeq -\frac{1}{N} \quad , \quad \epsilon \simeq \frac{3}{4N^2} \ll |\eta|$$

- ▶ observational parameters:

$$n_R - 1 \simeq -\frac{2}{N} \approx -0.04 \quad , \quad r \simeq \frac{12}{N^2} \approx 0.004$$



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- requires large non-minimal coupling $\xi \gg 1$ to curvature:

- conformal transformation to minimally-coupled field

$$V \rightarrow \tilde{V}(\psi) = \frac{\lambda}{4\xi^2} M_P^4 \left(1 - e^{-\psi/M_P} + \dots \right)$$

- $$\eta \simeq -\frac{1}{N} \quad , \quad \epsilon \sim \eta^2$$

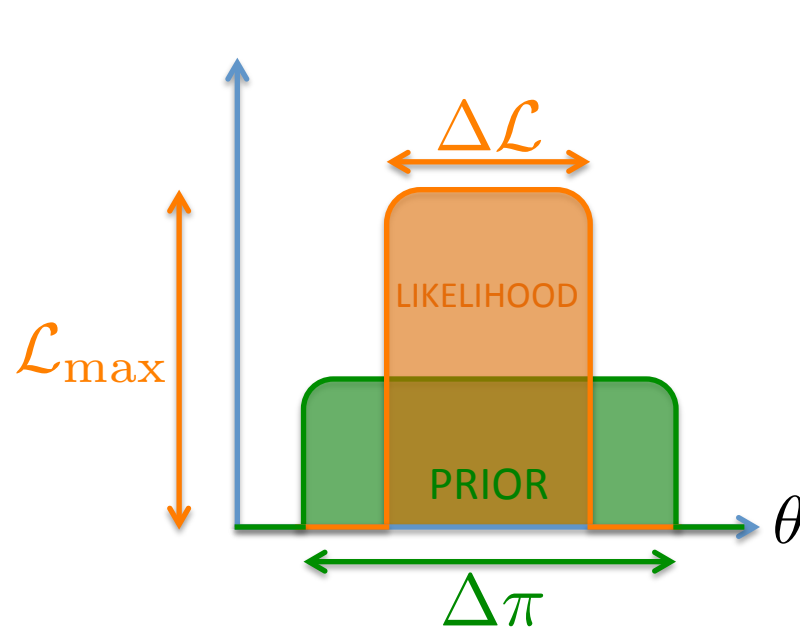
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Multi-field models and
non-Gaussianity
Stochastic inflation

Bayesian Approach

to model comparison

Bayesian evidence: Integral of the likelihood over parameter prior



$$\mathcal{E}(\mathcal{M}) = \mathcal{L}_{\max} \frac{\Delta \mathcal{L}}{\Delta \pi}$$

Compromise between **quality of fit** and **simplicity**

Bayes factor = ratio of evidence

$$B_{ij} = E(M_i) / E(M_j)$$

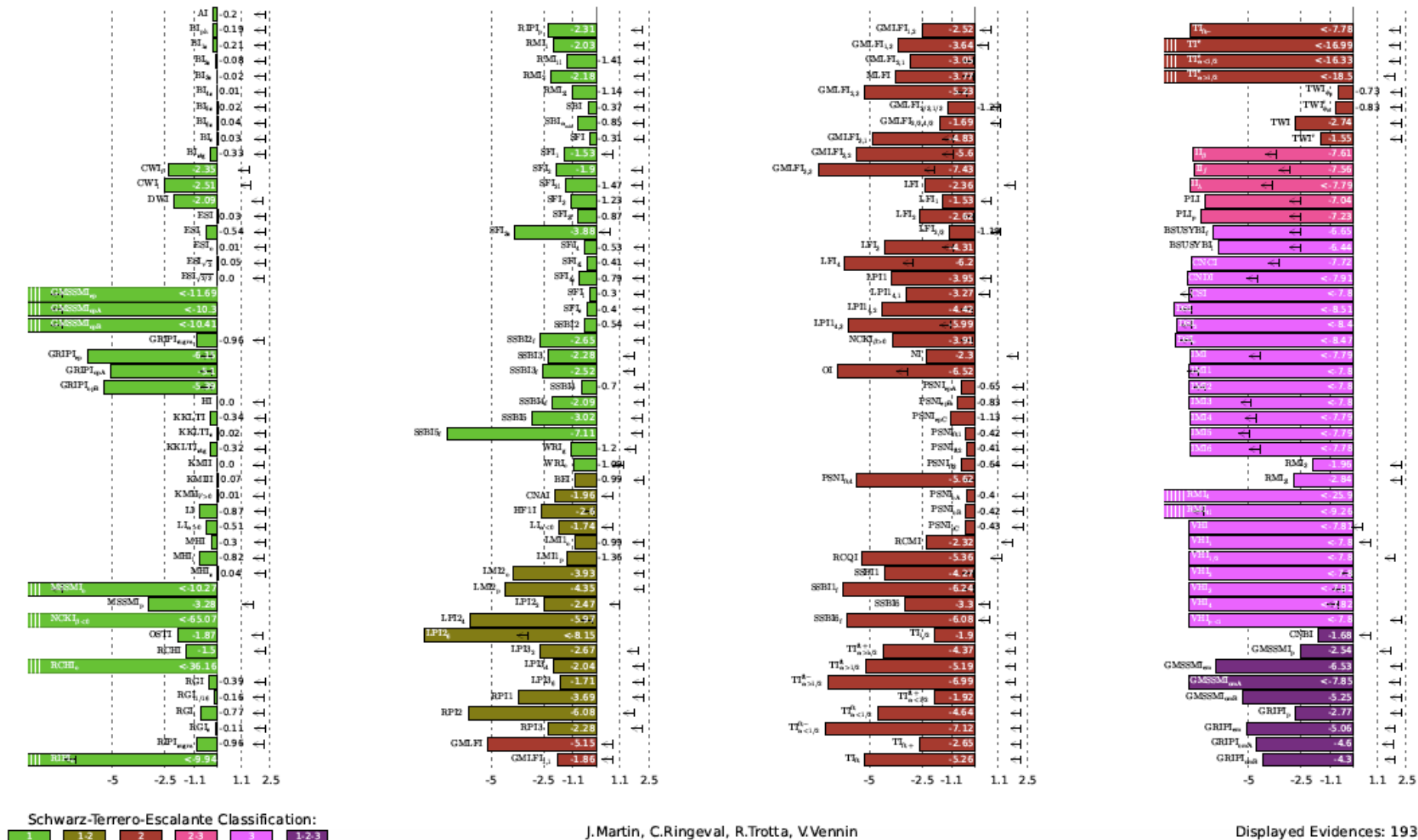
Jeffreys scale

- | | |
|---------------------|---------------------|
| • Strong evidence | $\ln(B_{ij}) > 5$ |
| • Moderate evidence | $\ln(B_{ij}) > 2.5$ |
| • Weak evidence | $\ln(B_{ij}) > 1$ |
| • Inconclusive | $\ln(B_{ij}) < 1$ |



Bayesian evidences computed with Planck

Martin, Ringeval, Trotta & Vennin (2014): 193 inflaton models

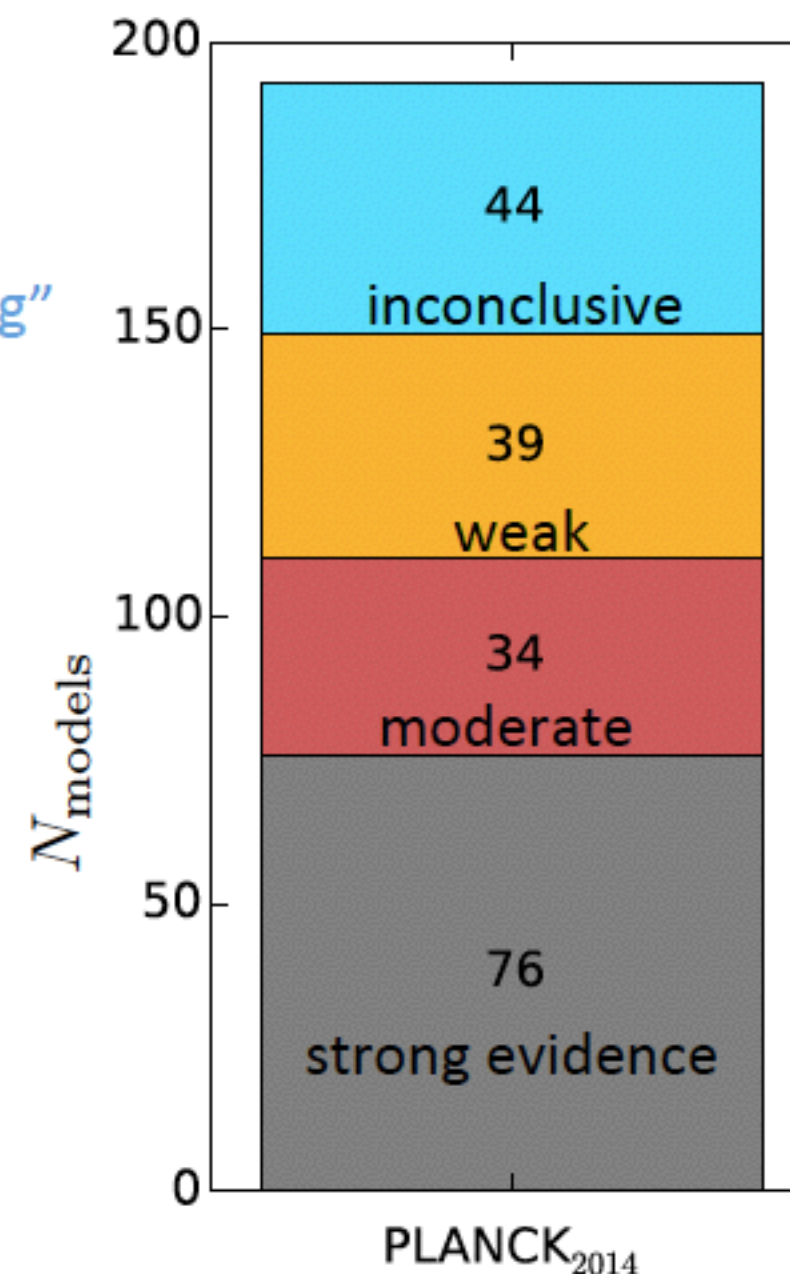


Bayesian evidence computed with Planck data

Summary of the results

- One third of the models are “ruled out”
 - $\ln (E / E_{\text{HI}}) < -5$
- Some of these models are killed by “fine-tuning”
- Planck favors “Plateau Inflation”,
e.g., Higgs or Starobinsky inflation

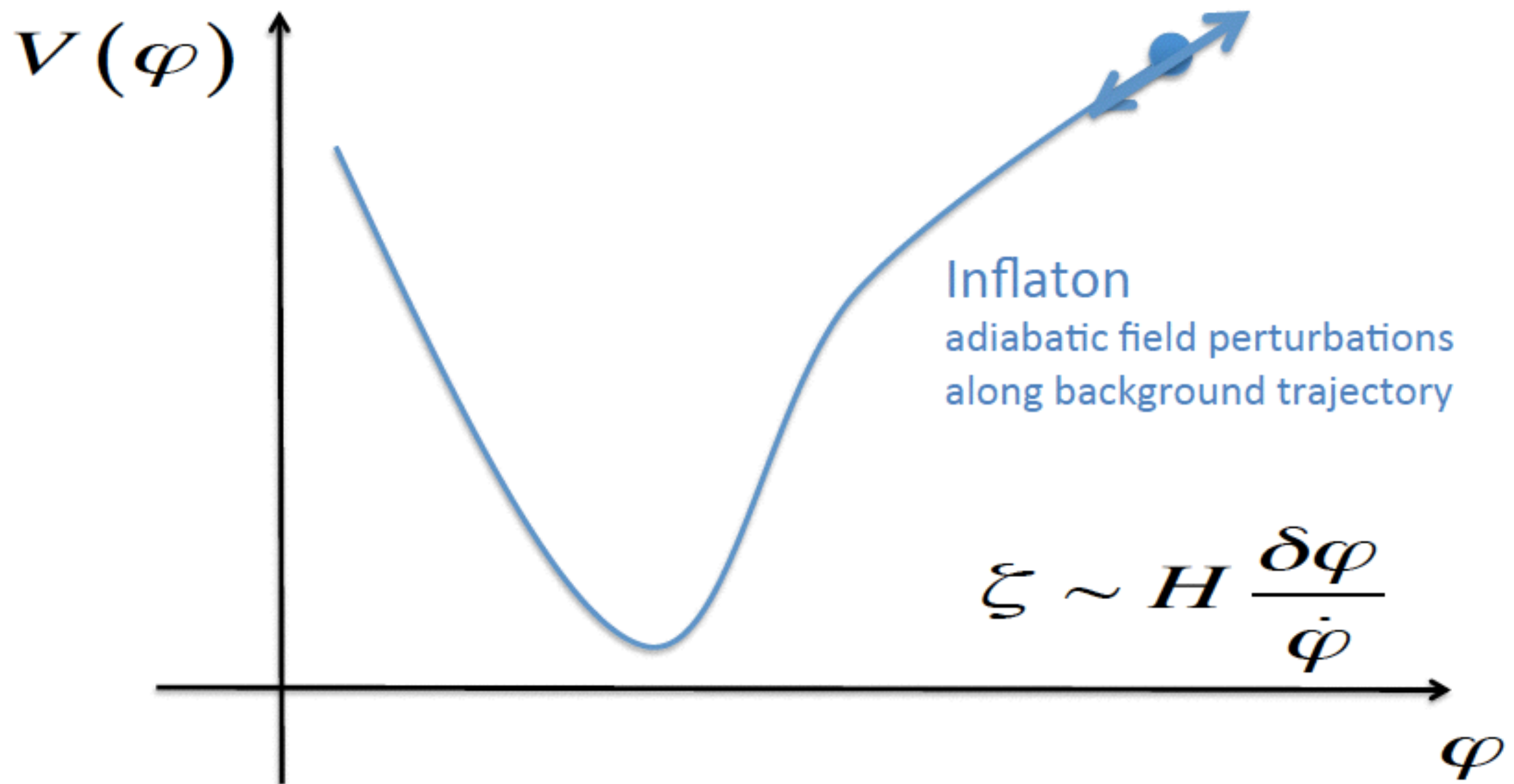
Model	$\Delta\chi^2$	$\ln B_{A,\text{ref}}$
R^2 inflation	+0.8	0
Power-law potential $\phi^{2/3}$	+6.5	-2.4
Power-law potential ϕ^2	+8.6	-4.7
Power-law potential ϕ^4	+43.3	-23.3
Natural inflation	+7.2	-2.4
SUSY α -attractor	+0.7	-1.8



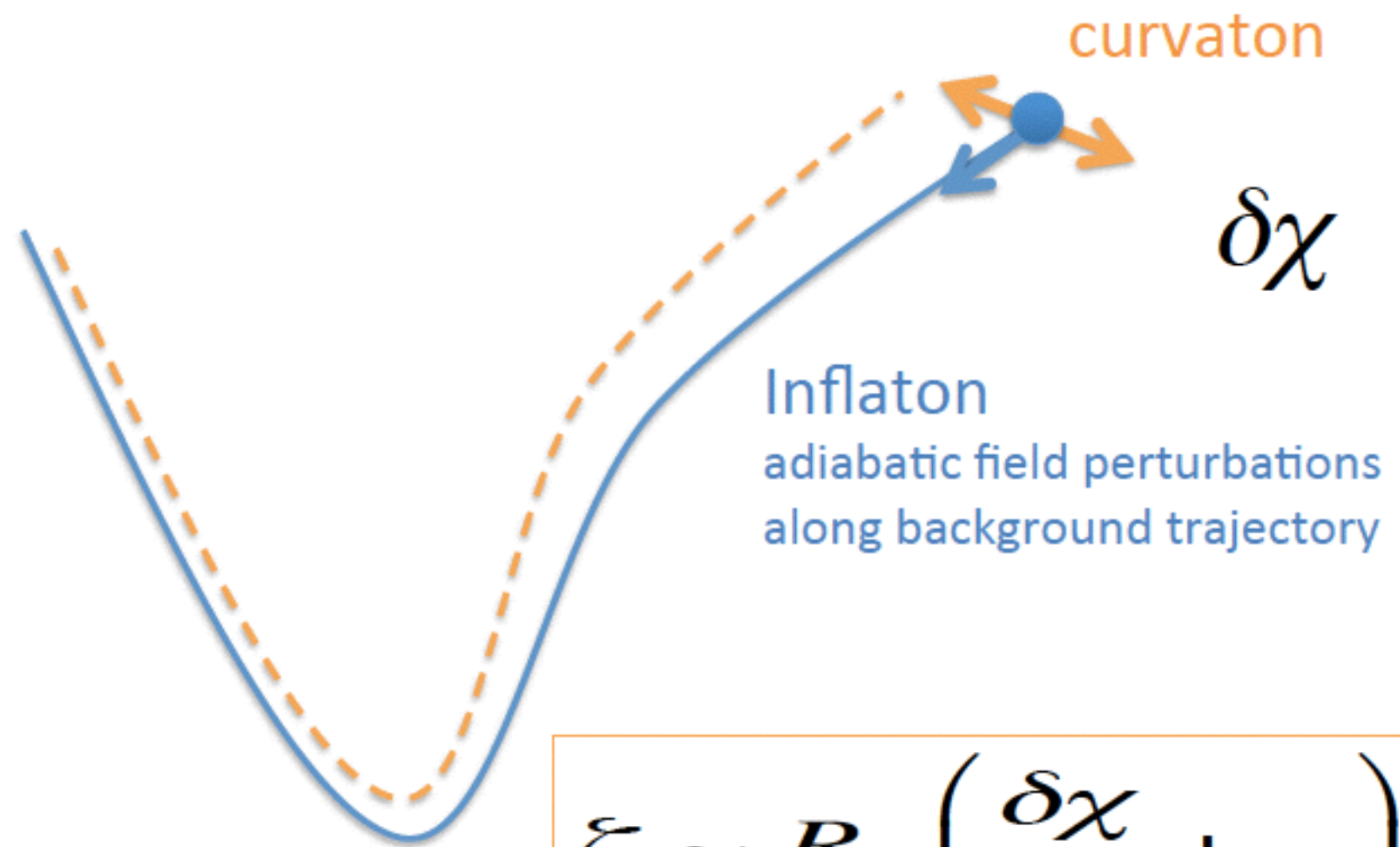
inflationary phenomenology

- Could inflation be very different from a simple single scalar field?
- and, if it was, how would we know?

single-field phenomenology



multi-field phenomenology



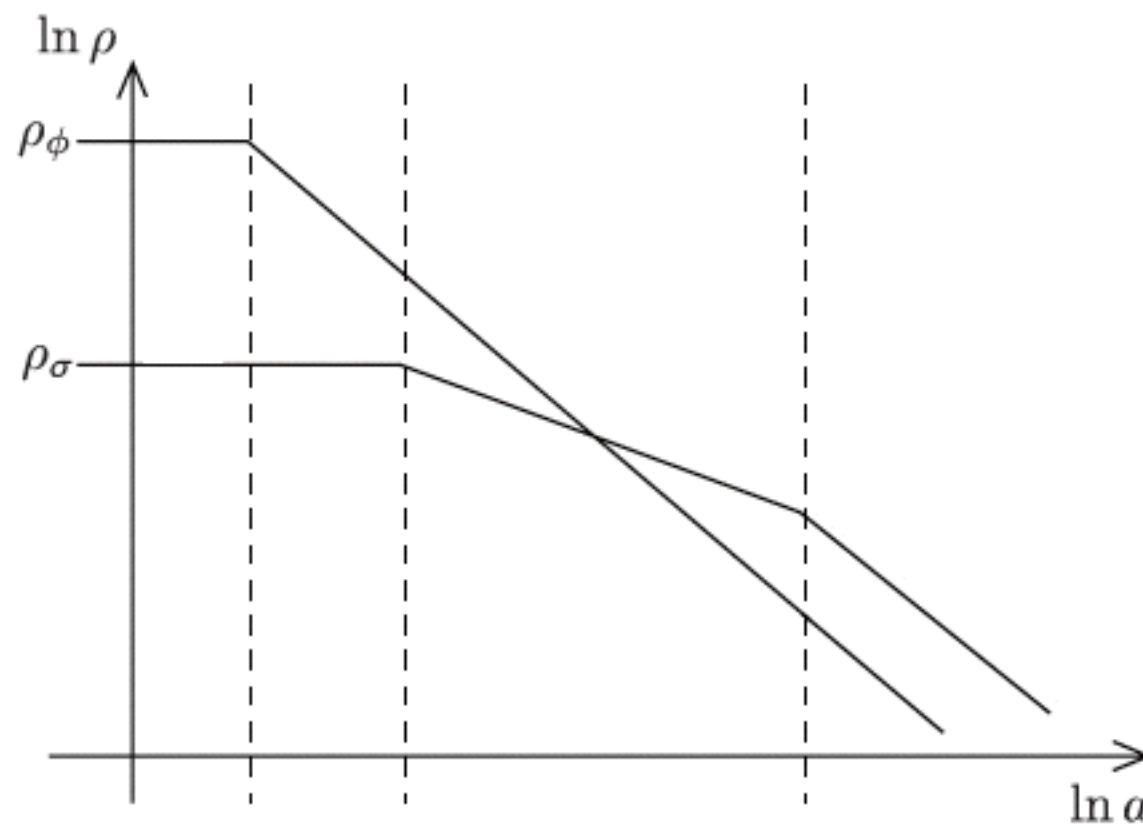
see also multi-field inflation,
modulated reheating,
inhomogeneous end of
inflation...

$$\xi \sim R_\chi \left(\frac{\delta\chi}{\chi} + \dots \right)$$

Inflaton models + curvaton field, χ

Curvaton scenarios with quadratic potential $V(\phi, \chi) = U(\phi) + m_\chi^2 \chi^2 / 2$

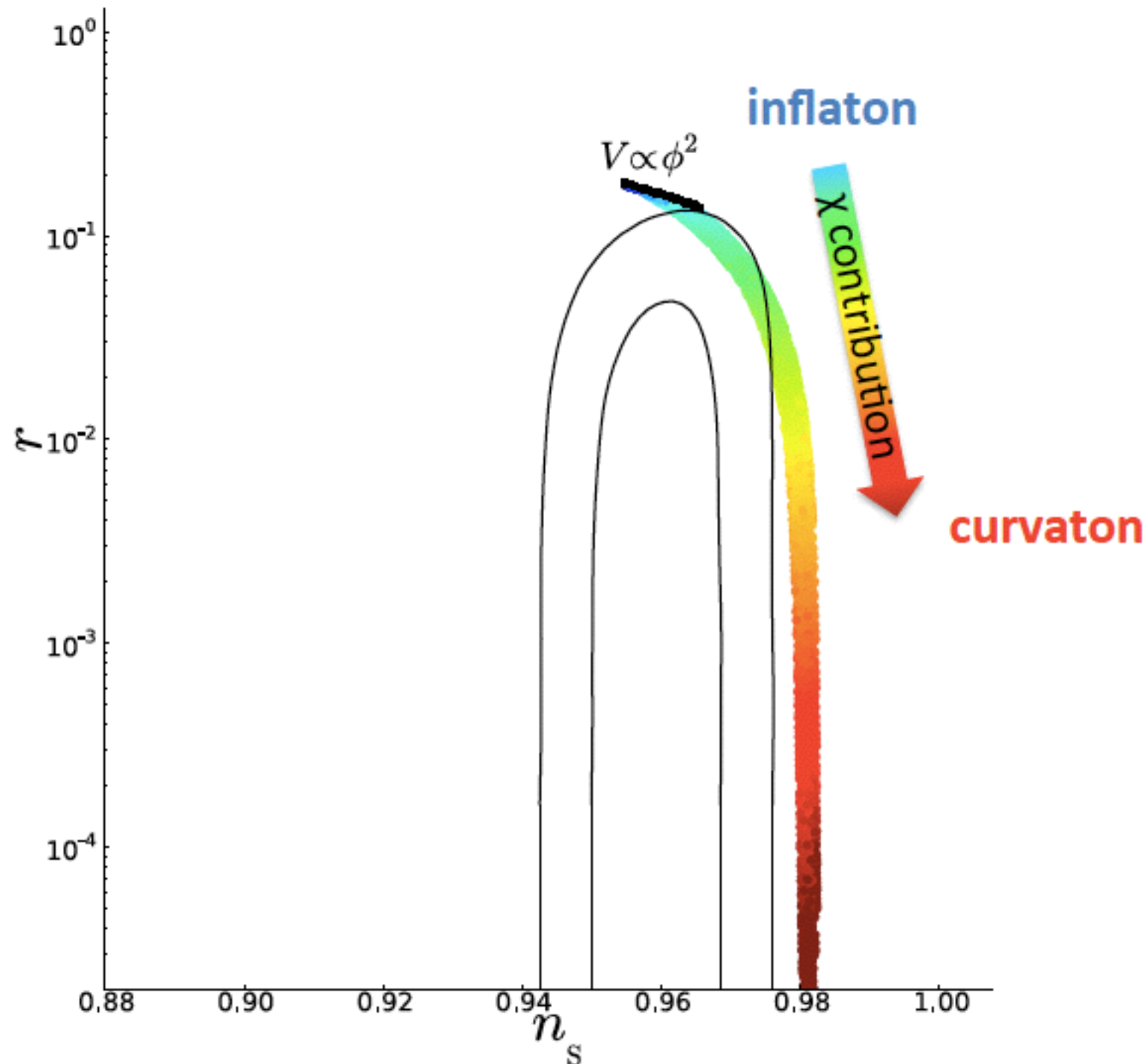
Linde and Mukhanov, 1997
Enqvist and Sloth, 2001
Lyth and Wands, 2001
Moroi and Takahashi, 2001
Langlois and Vernizzi, 2004



more reheating parameters: $\Gamma_\phi \rightarrow \Gamma_\phi, \Gamma_\chi, m_\chi, \chi_{\text{end}}$

primordial perturbations directly dependent on reheating
(not just through the expansion N_*)

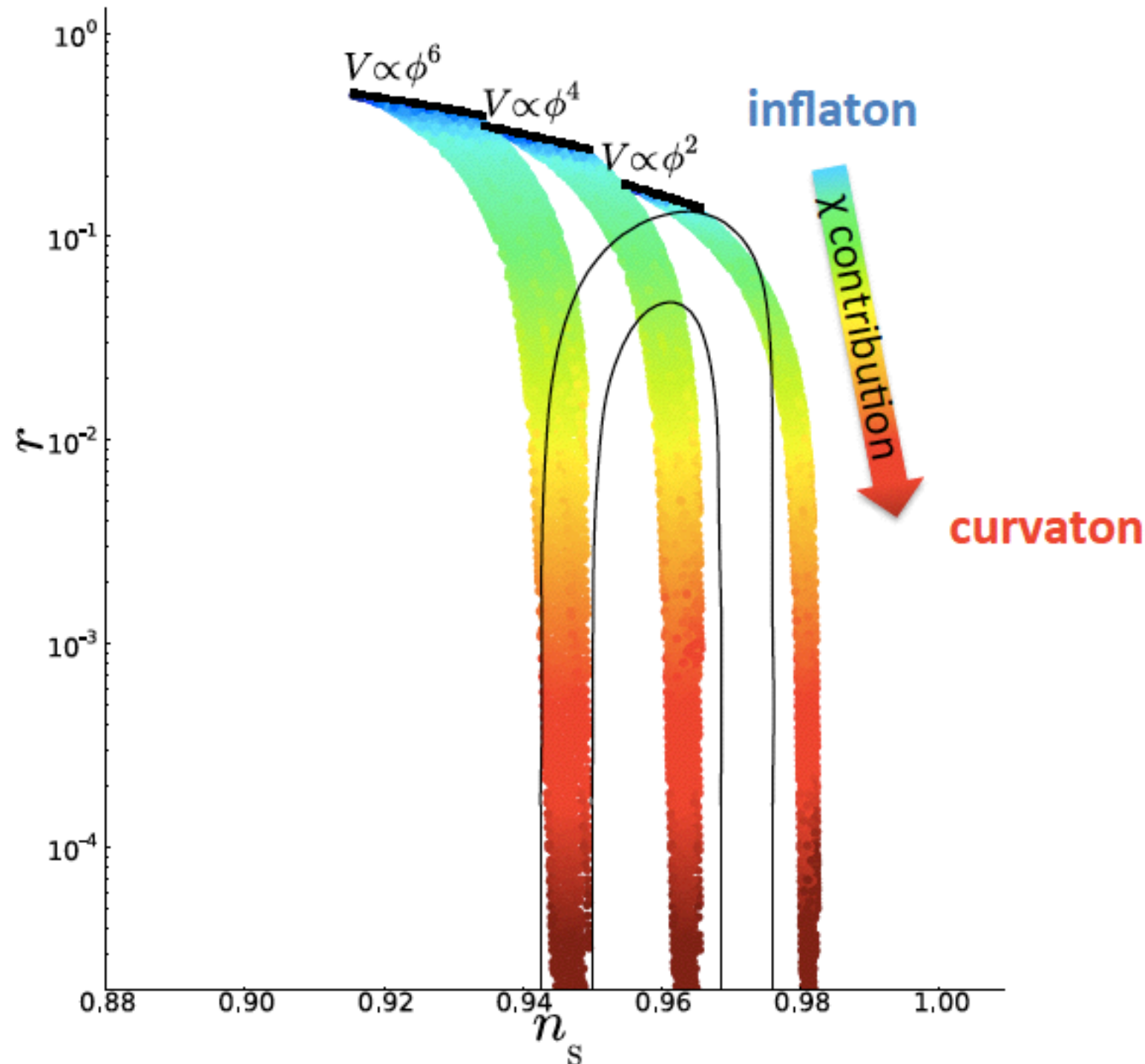
quadratic large-field (chaotic) inflaton plus quadratic curvaton, χ



Bartolo & Liddle (2002)
Ellis, Fairbairn & Sueiro (2014)
Byrnes, Cortes & Liddle (2014)
Smith & Grin (2015)
Vennin, Koyama & Wands (2015)

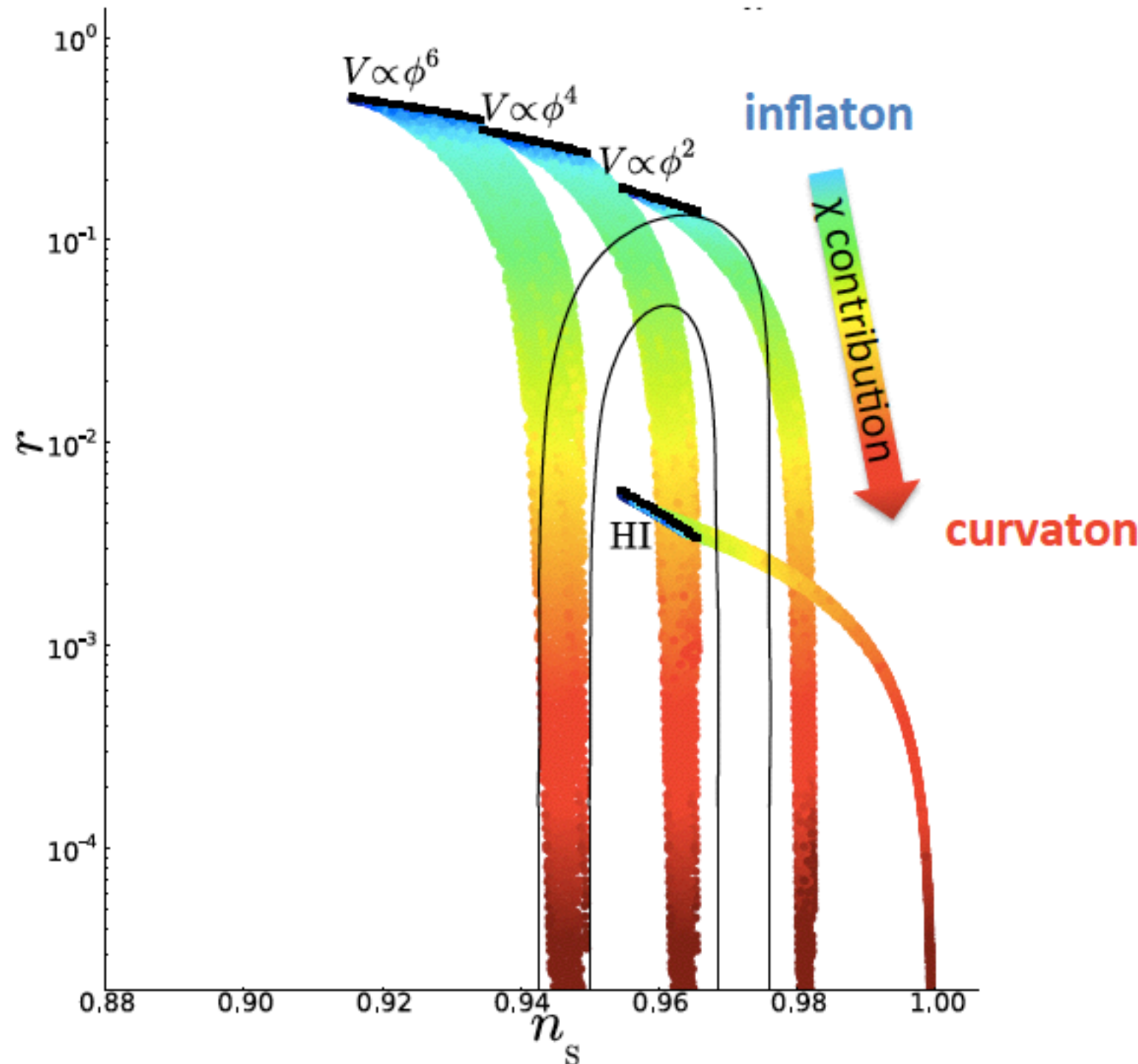
large-field inflaton (LFI) plus quadratic curvaton, χ

Vennin, Koyama and Wands (2015)



Higgs/Starobinsky inflation (HI) & LFI plus quadratic curvaton, χ

Vennin, Koyama and Wands (2015)

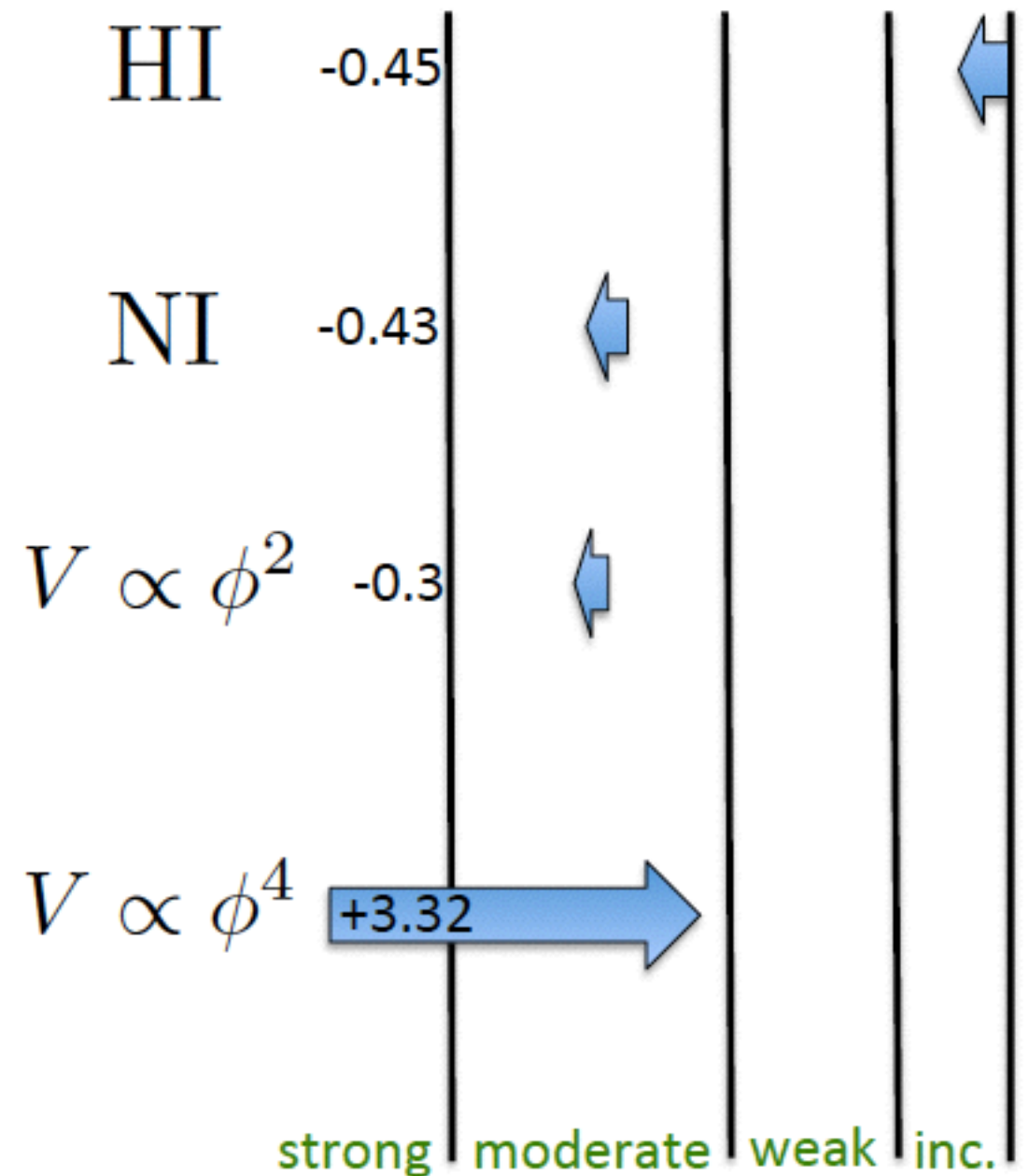
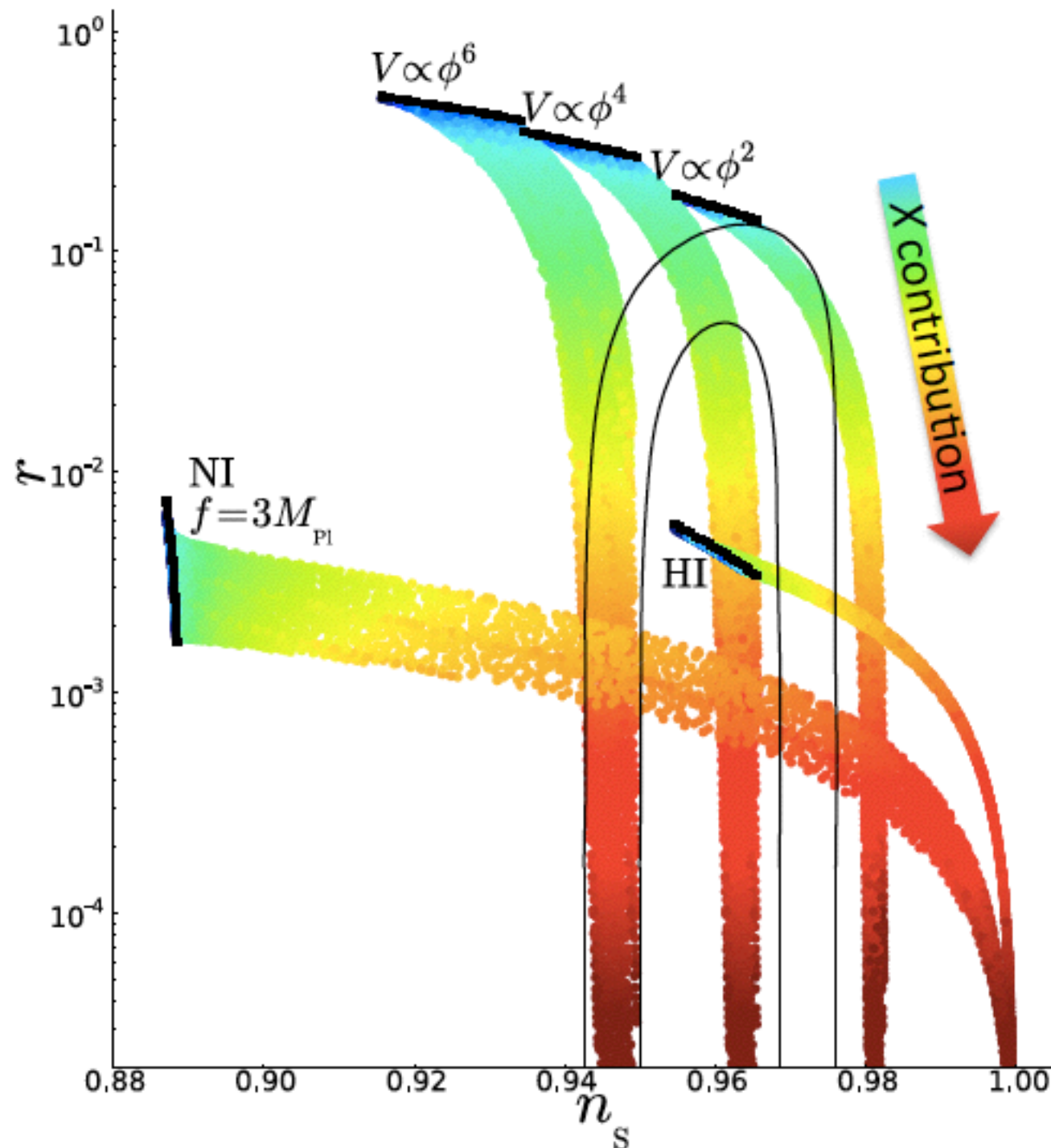


Add a weakly coupled scalar field, χ

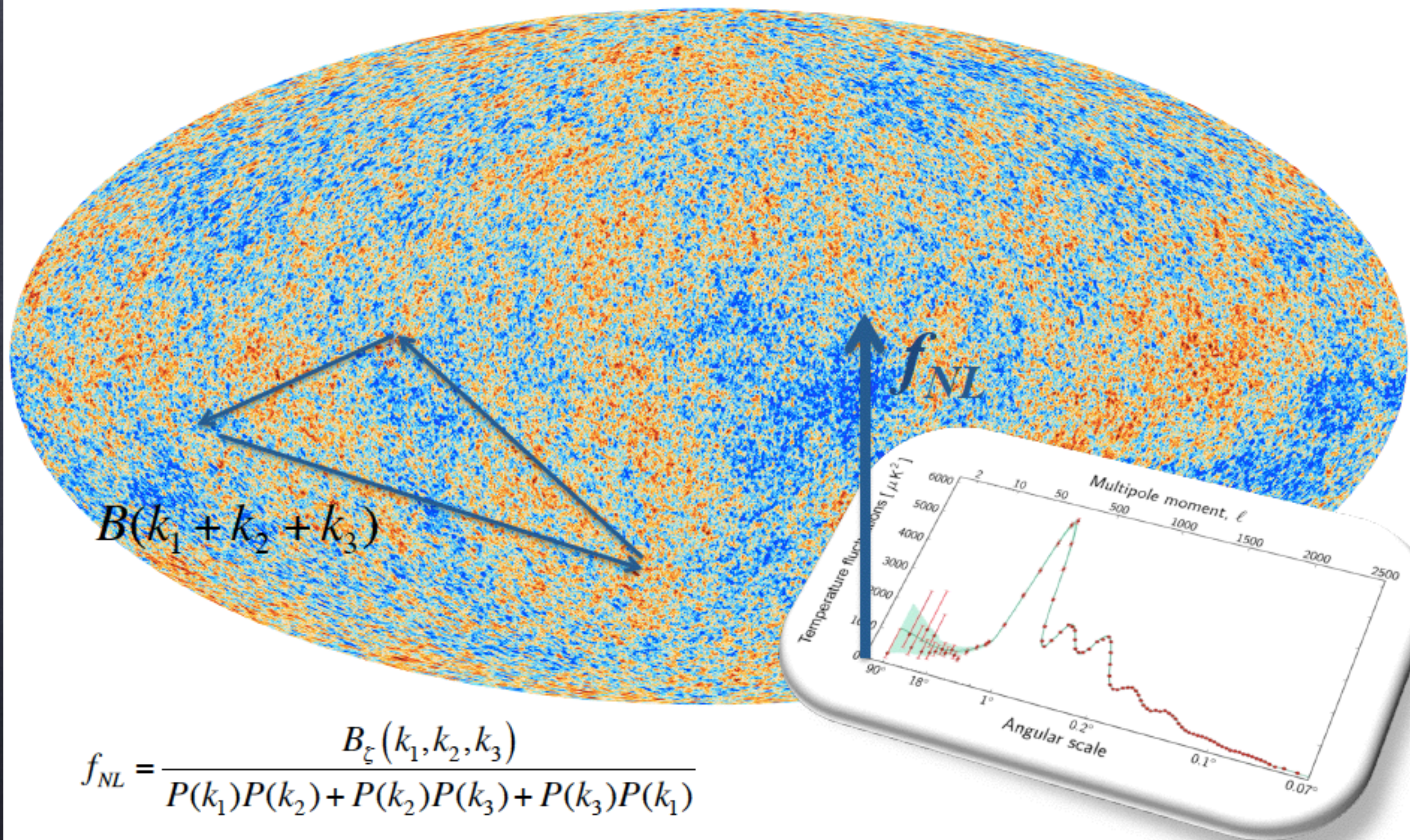
Vennin, Koyama and Wands (2015)

CASE # 5

Bayesian evidence, averaging over all cases:



more information in higher-order correlators...



$$f_{NL} = \frac{B_{\xi}(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$

curvaton scenario:

Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001



curvaton χ = weakly-coupled, late-decaying scalar field

- light field ($m < H$) during inflation acquires an almost scale-invariant, **Gaussian distribution of field fluctuations** on large scales
- **quadratic energy density** for free field, $\rho_\chi = m^2 \chi^2 / 2$
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3} \frac{\delta \rho_\chi}{\rho_\chi} \approx \frac{1}{3} \left(\frac{2\chi \delta\chi + \delta\chi^2}{\chi^2} \right)$$

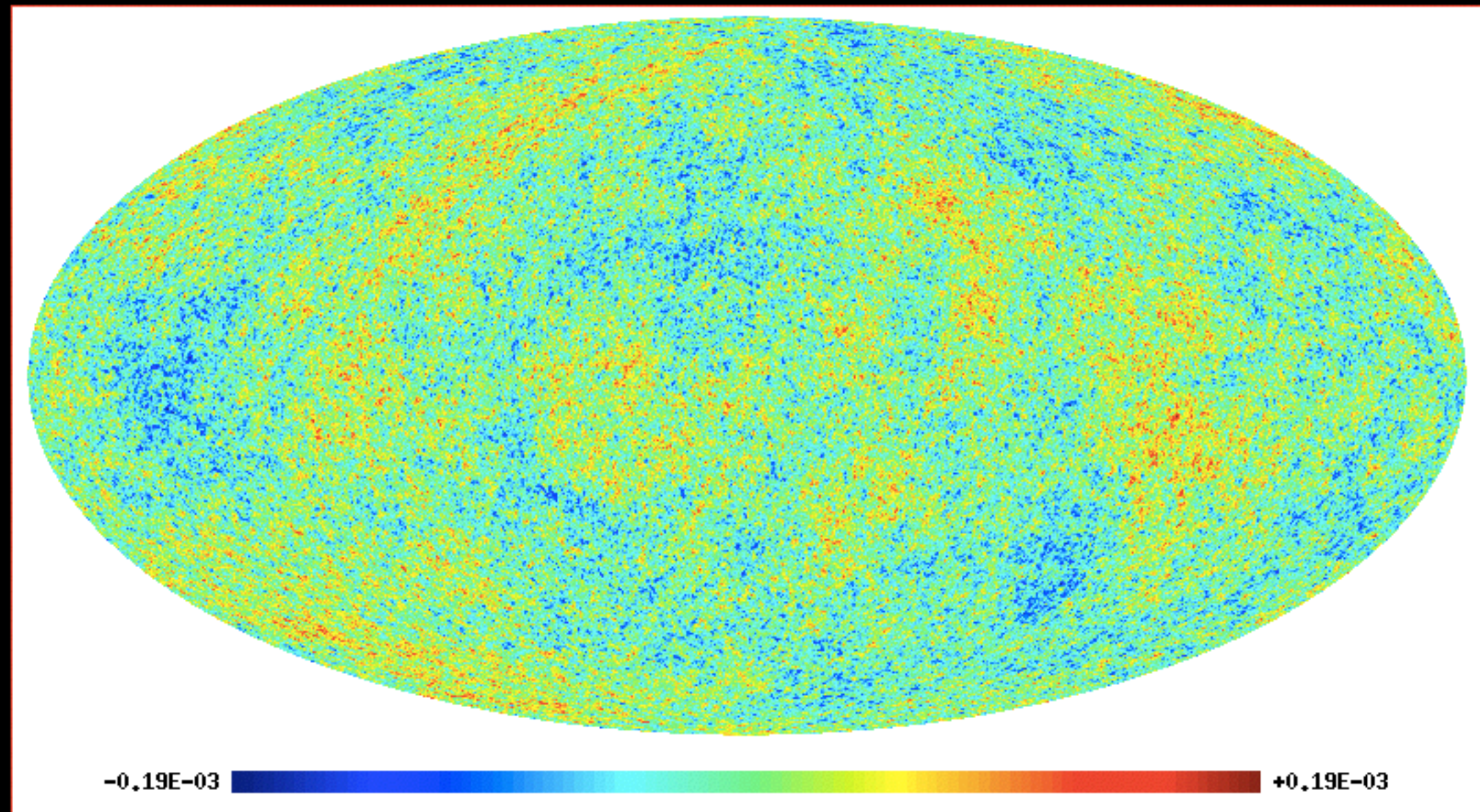
- **transferred to radiation when curvaton decays** after inflation with some **efficiency**, $0 < R_\chi < 1$, where $R_\chi \approx \Omega_{\chi, \text{decay}}$

$$\zeta = R_\chi \zeta_\chi \approx \frac{R_\chi}{3} \left(2 \frac{\delta\chi}{\chi} + \frac{\delta\chi^2}{\chi^2} \right)$$

$$= \zeta_G + \frac{3}{4R_\chi} \zeta_G^2 \Rightarrow f_{NL} = \frac{5}{4R_\chi}$$

Newtonian metric potential a *Gaussian random field*

$$\Phi(x) = \phi_G(x)$$



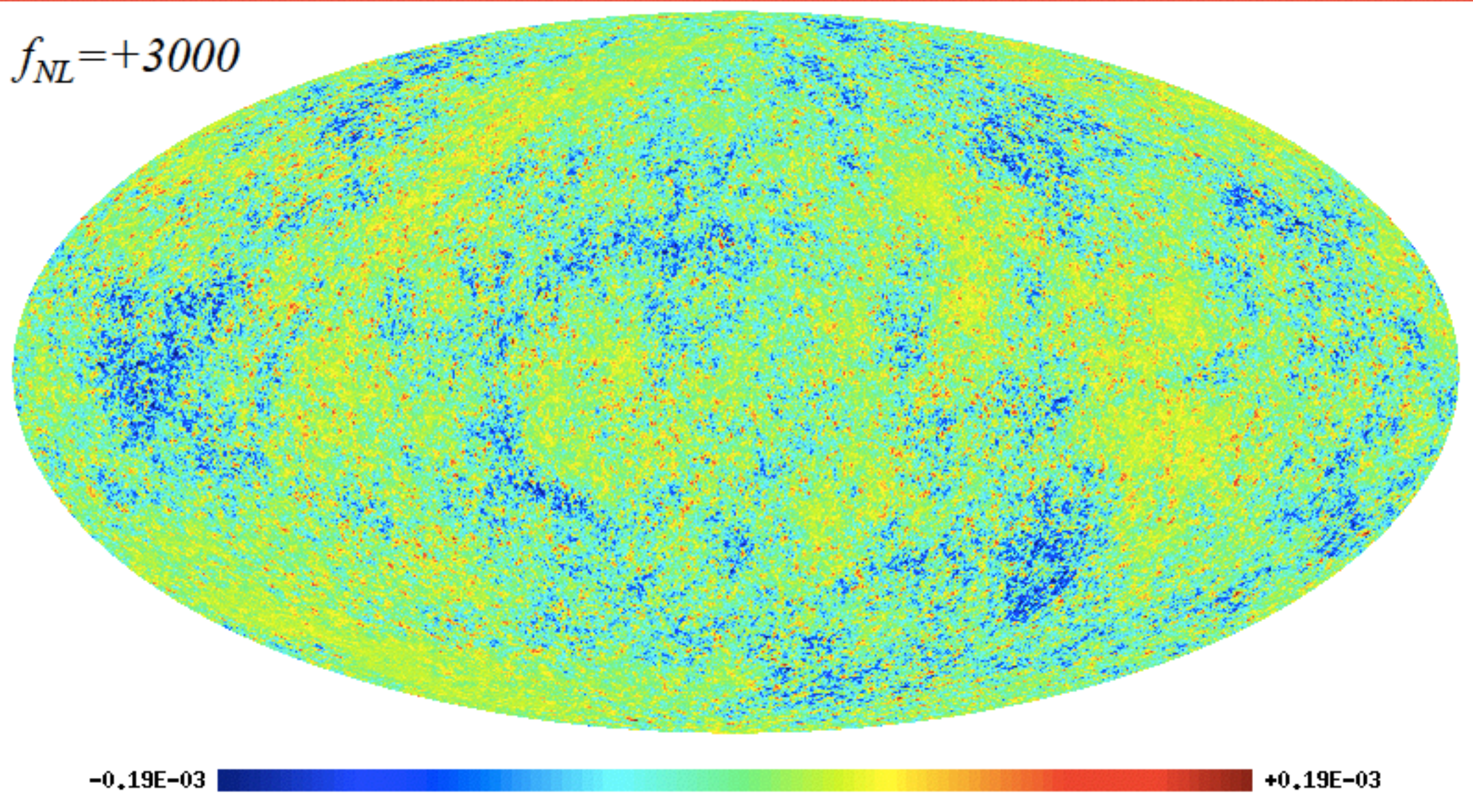
$$\Delta T/T \approx -\Phi/3 \approx -\zeta/5$$

Liguori, Matarrese and Moscardini (2003)

Newtonian metric *a local function of Gaussian random field*

$$\Phi(x) = \phi_G(x) + f_{NL} (\phi_G^2(x) - \langle \phi_G^2 \rangle)$$

$$f_{NL} = +3000$$



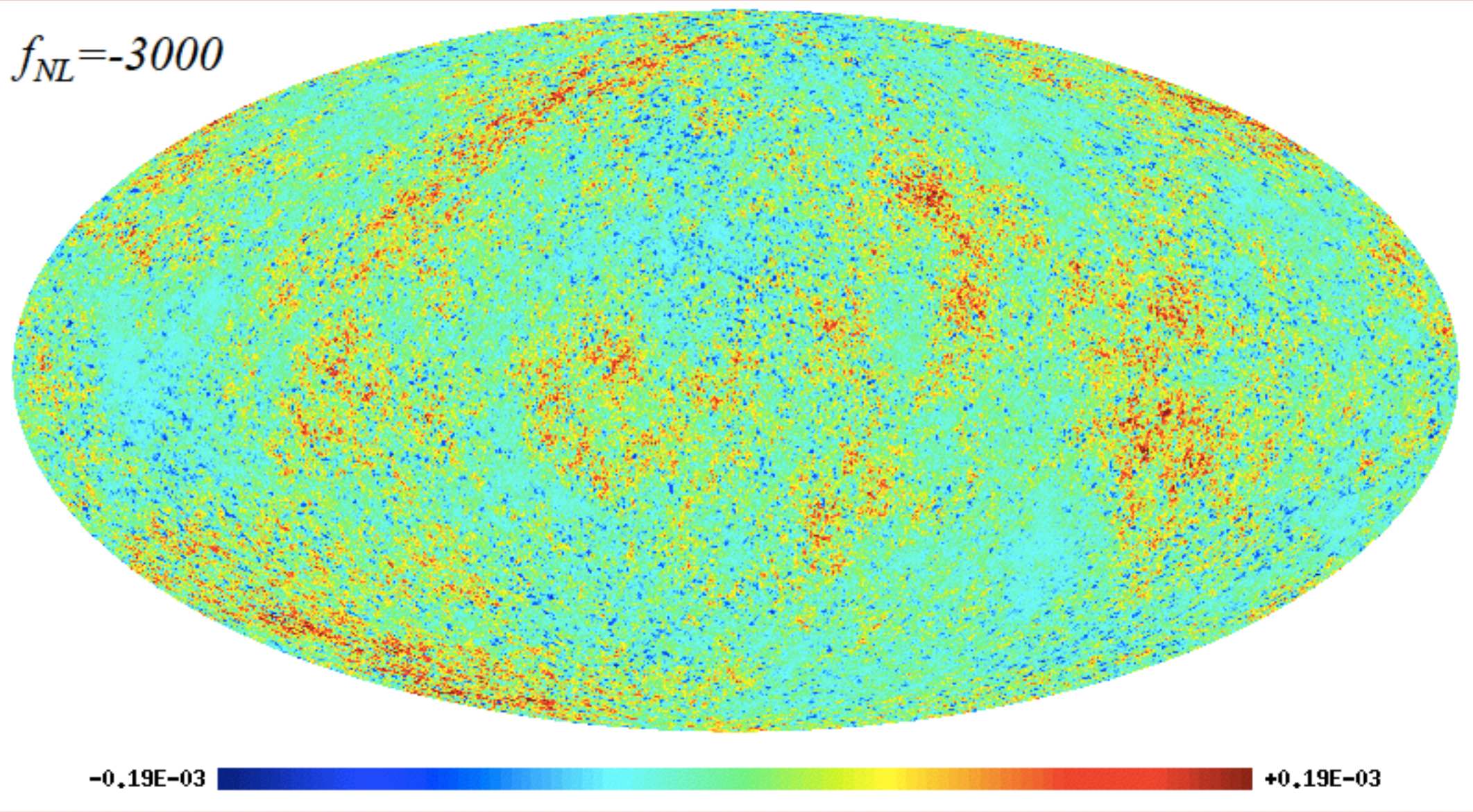
$\Delta T/T \approx -\Phi/3$, so positive $f_{NL} \Rightarrow$ more cold spots in CMB

Liguori, Matarrese and Moscardini (2003)

Newtonian potential *a local function of Gaussian random field*

$$\Phi(x) = \phi_G(x) + f_{NL} (\phi_G^2(x) - \langle \phi_G^2 \rangle)$$

$$f_{NL} = -3000$$



$\Delta T/T \approx -\Phi/3$, so negative $f_{NL} \Rightarrow$ more hot spots in CMB

Liguori, Matarrese and Moscardini (2003)

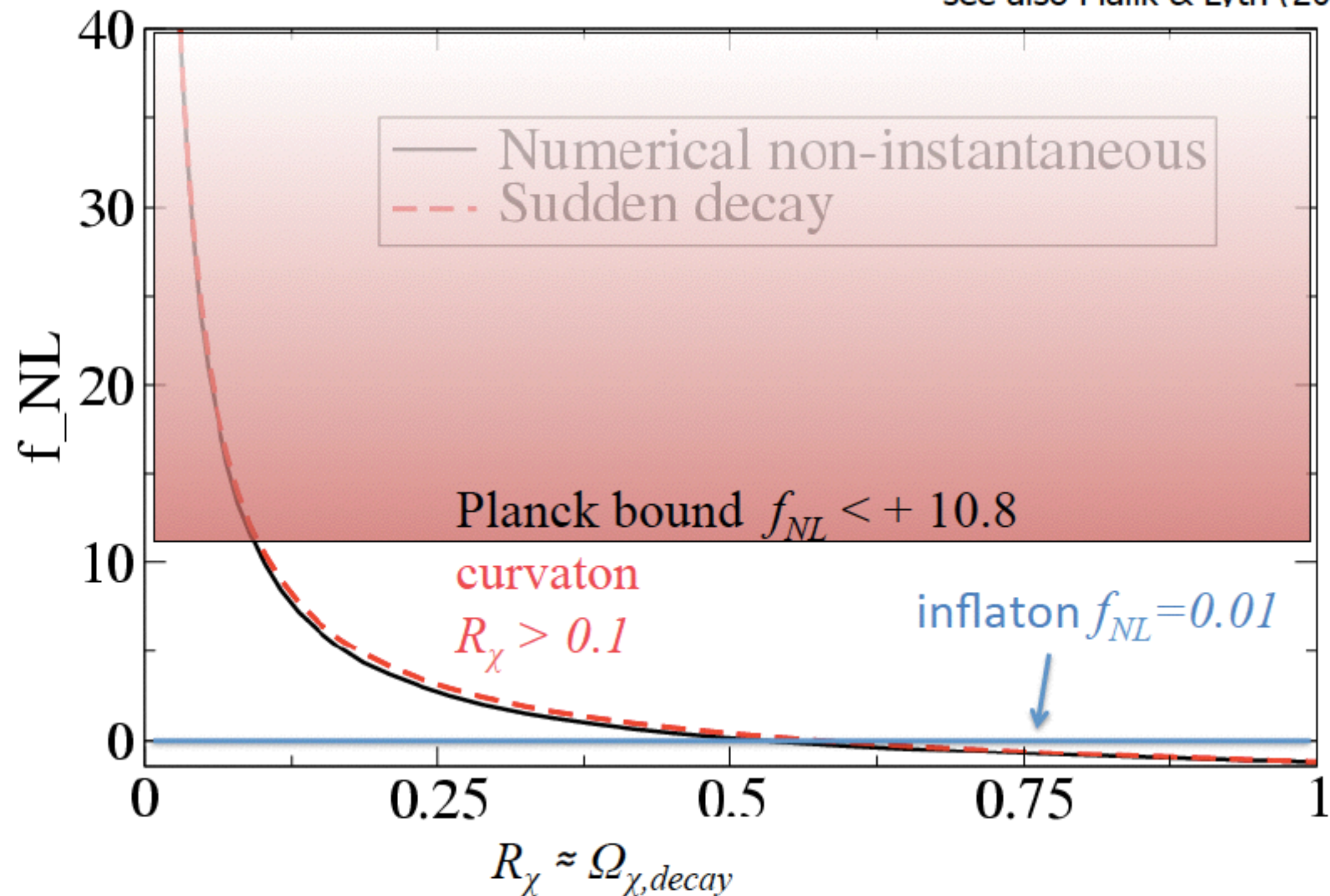
constraints on f_{NL}

- ◉ WMAP9 2-sigma constraints (Bennet et al 2012)
 - ◉ local: $-3 < f_{NL} < 77$
 - ◉ equilateral: $-121 < f_{NL} < 223$
 - ◉ orthogonal: $-445 < f_{NL} < -45$
- ◉ Planck2015 1-sigma constraints (Ade et al 2015)
 - ◉ local: $f_{NL} = 0.8 \pm 5.0$
 - ◉ equilateral: $f_{NL} = -4 \pm 43$
 - ◉ orthogonal: $f_{NL} = -26 \pm 21$
- ◉ SDSS BOSS galaxy power spectrum (Giannantonio et al 2015)
 - ◉ local: $f_{NL} = 5 \pm 21$

non-linearity parameter for quadratic curvaton

Sasaki, Valiviita & Wands (2006)

see also Malik & Lyth (2006)



future constraints on fNL?

- CORE-MS 1σ forecast bounds
 - local: $f_{NL} = 2.1$
 - equilateral: $f_{NL} = 21$
 - orthogonal: $f_{NL} = 9.6$
- SDSS BOSS galaxy bispectrum 1σ forecast (Tellarini et al 2016)
 - local: $f_{NL} = 1$
- BOSS + Euclid satellite 1σ forecast (Tellarini et al 2016)
 - local: $f_{NL} = 0.4$

an inflationary landscape:

- quantum fluctuations produce stochastic noise term for coarse-grained fields on super-Hubble scales

$$\frac{d\sigma}{dN} = \frac{dV/d\sigma}{3H^2} + \frac{H}{2\pi}\xi$$

- leads to probability distribution function for local field values

$$\frac{\partial}{\partial N}P(\sigma, N) = \frac{\partial}{\partial \sigma} \left[\frac{dV/d\sigma}{3H^2} P(\sigma, N) \right] + \frac{1}{2} \left(\frac{H}{2\pi} \right)^2 \frac{\partial^2}{\partial \sigma^2} P(\sigma, N)$$

stochastic inflation for the practical cosmologist

- stochastic probability distributions for initial field values in multiple field inflation:
 - the stochastic spectator (Hardwick, Vennin, Byrnes, Torrado & Wands, arXiv:1701.06473)
- stochastic delta-N (Vennin & Starobinsky, arXiv:1506.04732):
 - infinite inflation (divergent $\langle N \rangle$) can lead to infinite perturbations
 - regularise inflation at Planck scale to get finite primordial power spectrum (Vennin et al, arXiv: 1604.06017)

an inflationary cosmologist's agenda:

1. prove inflation really happened (Monday)
 - search for a smoking gun
2. show how inflation happened (Tuesday)
 - what is the correct inflation model?
3. explore inflationary phenomenology (Wednesday - Thursday)
 - primordial perturbations, gravitational waves, black holes, etc
 - particle production and reheating after inflation
4. understand the inflationary landscape (Friday)
 - multiverse, measure problem, anthropic arguments, alternatives