On the dark degeneracy and its observational consequences

Rodrigo von Marttens
and many collaborators...

Verão Quântico, Guarapari-ES
October 12, 2021

Observatório Nacional

The content of this presentation was obtained in collaboration with:

W. Zimdahl, J.E. Gonzalez, J. Alcaniz, L. Casarini, D.F. Mota, V. Marra, L. Lombriser, M. Kunz, U. Andrade, D. Anbajagane and D. Huterer.

1. RvM, L. Casarini, D.F. Mota and W. Zimdahl, “Cosmological constraints on parametrized interacting dark energy”, Phys. Dark Univ. 23 (2019) 100248.
2. RvM, V. Marra, L. Casarini, J.E. Gonzalez and J. Alcaniz, “Null test for interactions in the dark sector”, Phys.Rev. D99 (2019) no.4, 043521.
3. RvM, L. Lombriser, M. Kunz, V. Marra, L. Casarini and J. Alcaniz, “Dark degeneracy I: Dynamical or interacting dark energy?”, Phys. Dark Univ. 23 (2019) 100248.
4. RvM, J.E. Gonzalez, V. Marra, L. Casarini and J. Alcaniz, “A model-independent reconstruction of an interaction in the dark sector”, arXiv 2011.10846.
5. Uendert Andrade, Dhayaa Anbajagane, RvM, Dragan Huterer and J. Alcaniz, “A Test of the Standard Cosmological Model with Geometry and Growth”, arXiv 2107.07538.

TO DO LIST
1. Introduction
2. Dark degeneracy I (fluid)
3. Dynamical/Interacting mapping
4. Dark degeneracy II (MG)
5. Conclusions and Perspectives
Two announcements! :)

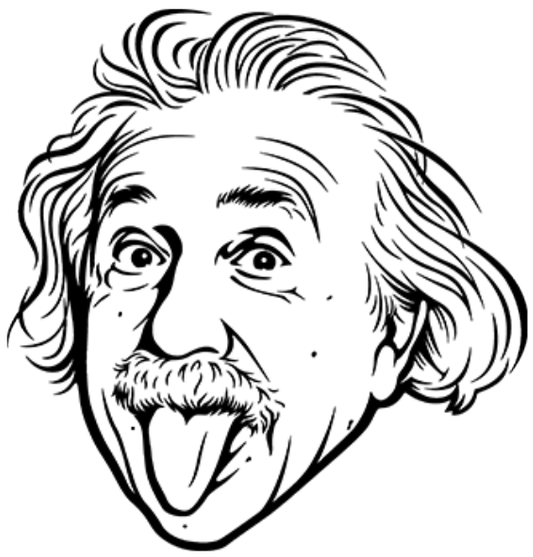
Introduction

Standard (late-time) cosmology:

Introduction

Standard (late-time) cosmology:

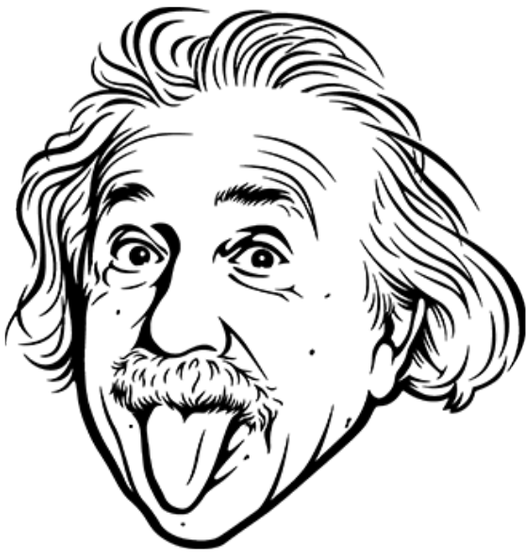
Theory of gravity



Introduction

Standard (late-time) cosmology:

Theory of gravity + Material content



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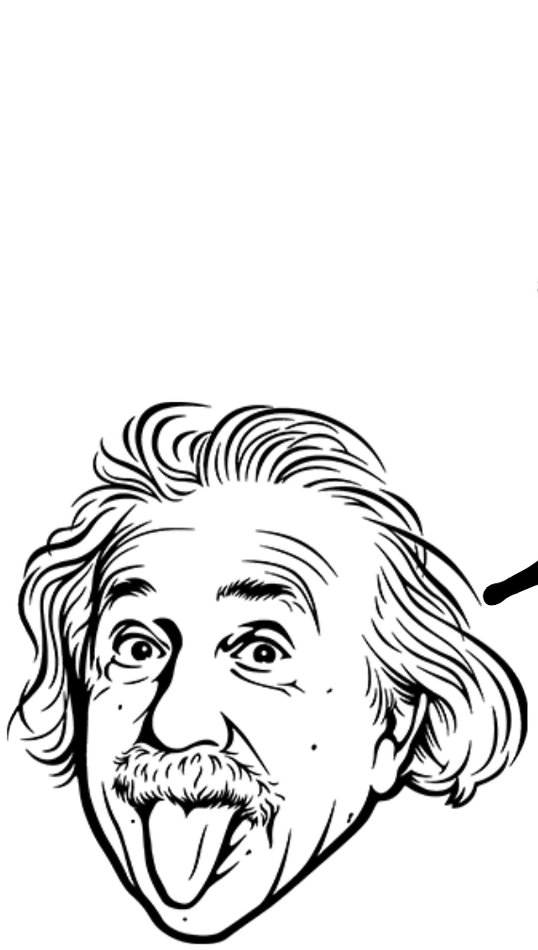
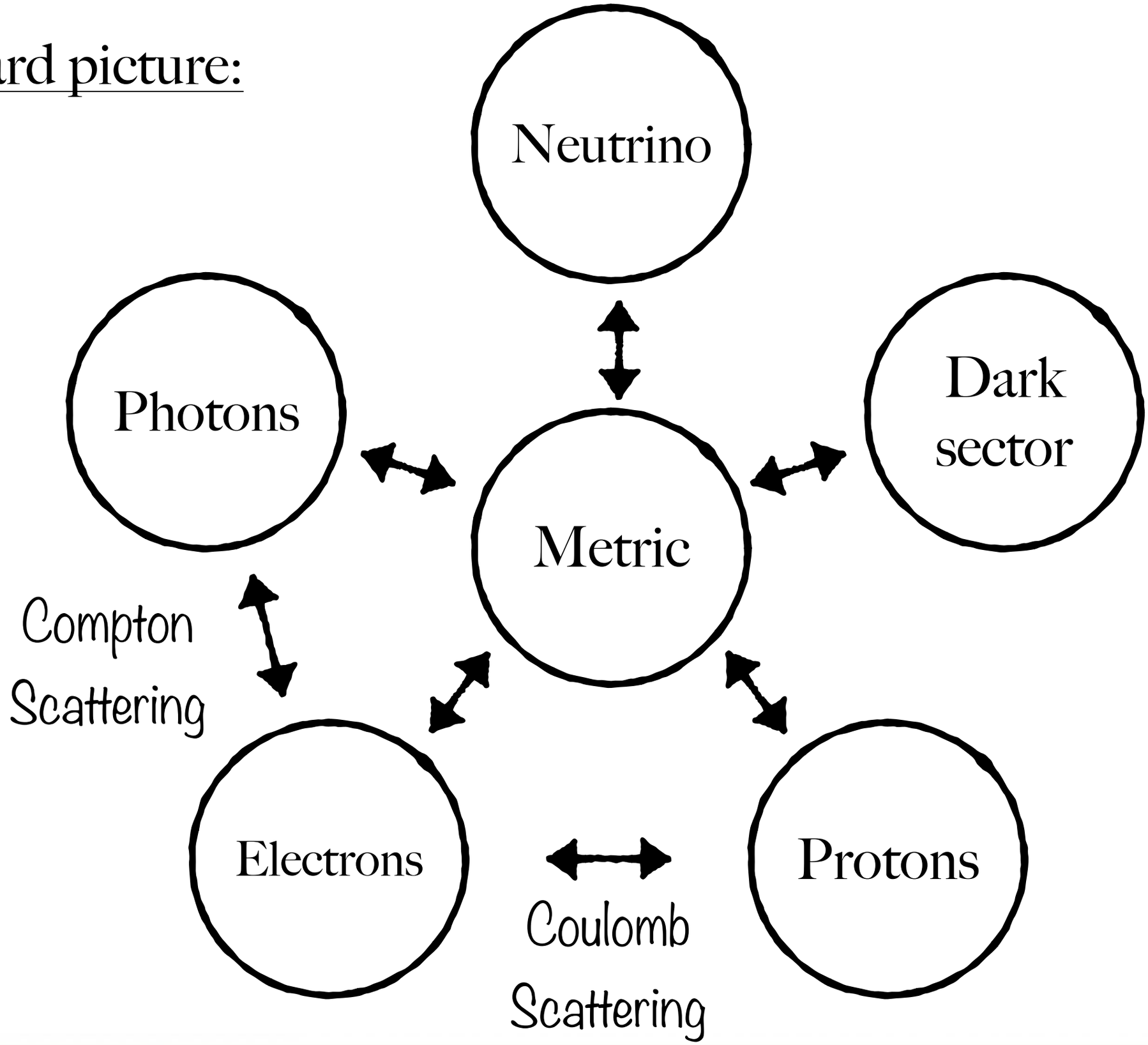


Einstein field equations relate geometry with TOTAL MATTER.

Introduction

Standard picture:



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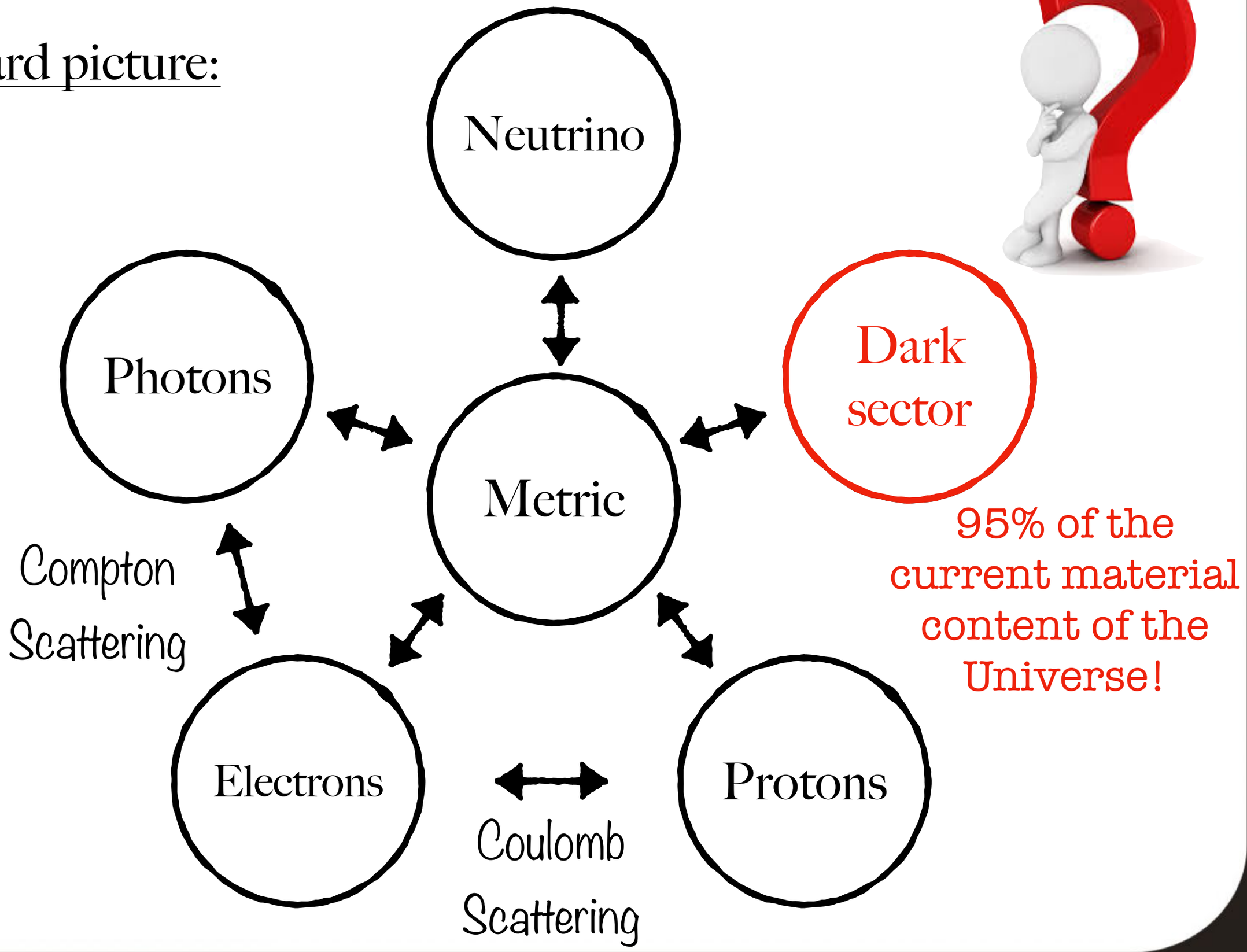
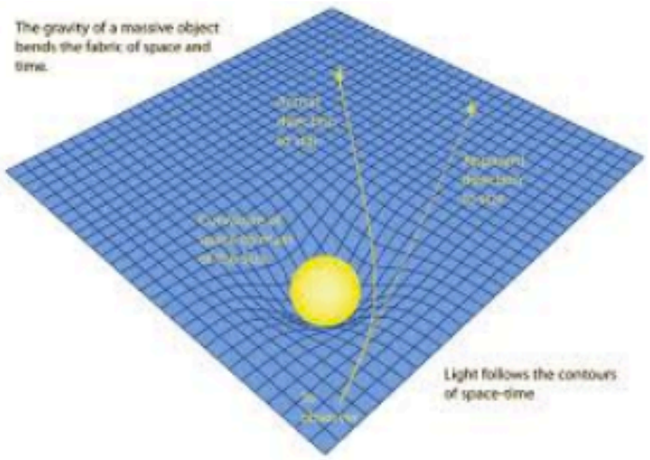


95% of the current material content of the Universe!

Introduction

(Late-time) Standard Cosmological Model:

GR and FLRW metric



\+

CDM

\+

Cosmological Constant

\+ Λ

ΛCDM model

Introduction

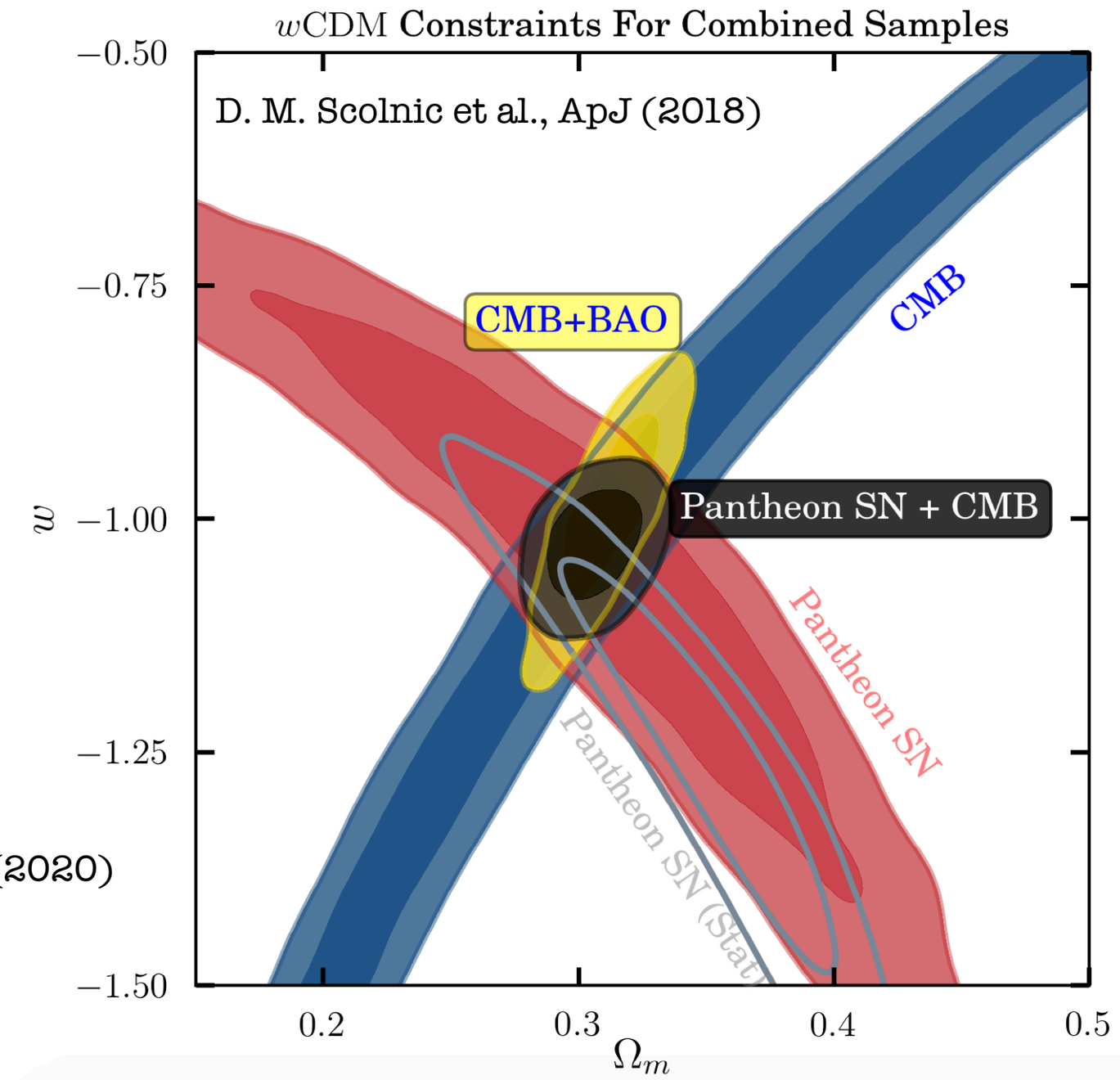
Current status of the ΛCDM model:

Observational Success

Planck 2018 results

Parameter	TT,TE,EE+lowE+lensing+BAO 68% limits
$\Omega_b h^2$	0.02242 ± 0.00014
$\Omega_c h^2$	0.11933 ± 0.00091
$100\theta_{MC}$	1.04101 ± 0.00029
τ	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.047 ± 0.014
n_s	0.9665 ± 0.0038

Planck Collaboration: N. Aghanim et al., A&A (2020)



Introduction

Current status of the ΛCDM model:

Theoretical challenges:

1. Cosmological Constant Problem;
   - S. Weinberg, "The Cosmological Constant Problem", Rev. Mod. Phys. 61 (1989).
   - J. Martin, "Everything You Always Wanted To Know About The Cosmological Constant Problem (But Were Afraid To Ask)", Comptes Rendus Physique 13 (2012) 566–665.
   - L. Lombriser, "On the cosmological constant problem", Phys. Lett. B797 (2019).
   - O. Piattella, "Does the cosmological constant stay hidden?", Phys.Rev.D 102 (2020) 10, 104039.

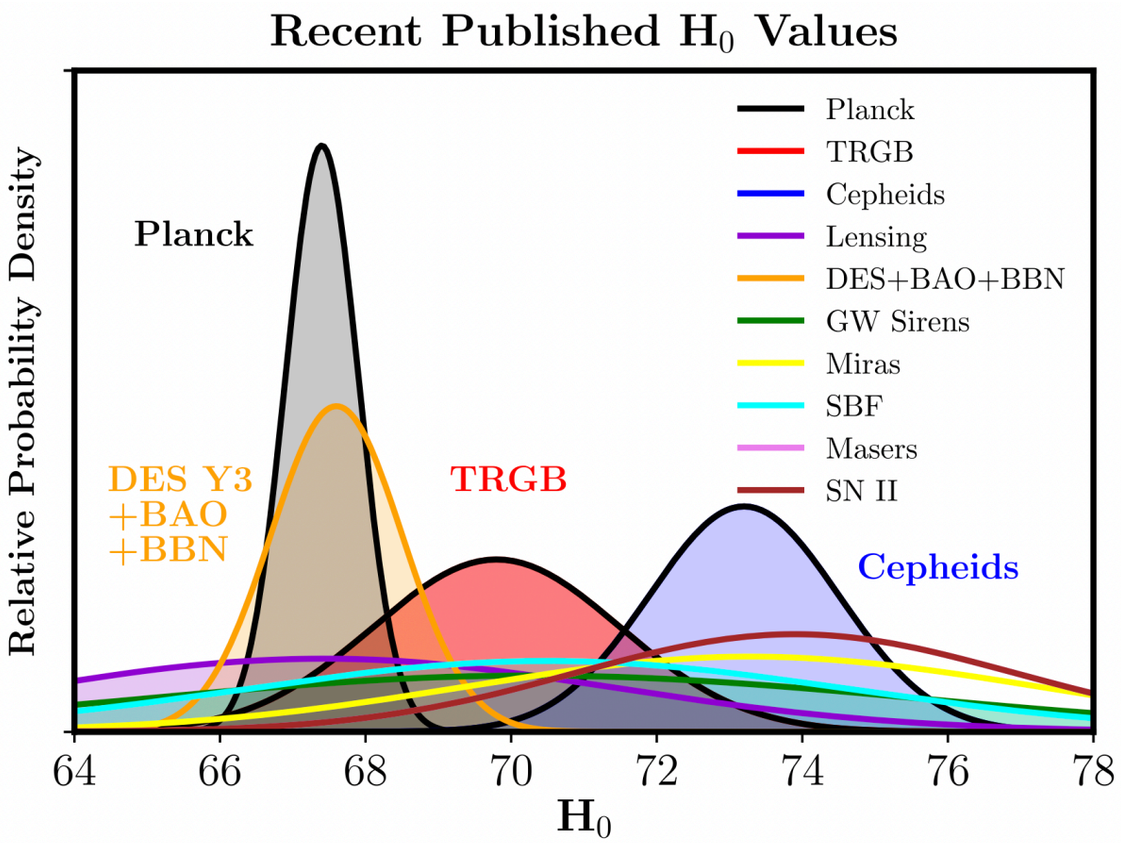
2. Cosmic Coincidence "Problem";
   - H. E. S. Velten, R. F. vom Marttens and W. Zimdahl. "Aspects of the cosmological 'coincidence problem'". Eur. Phys. J. C 74 no. 11, (2014)

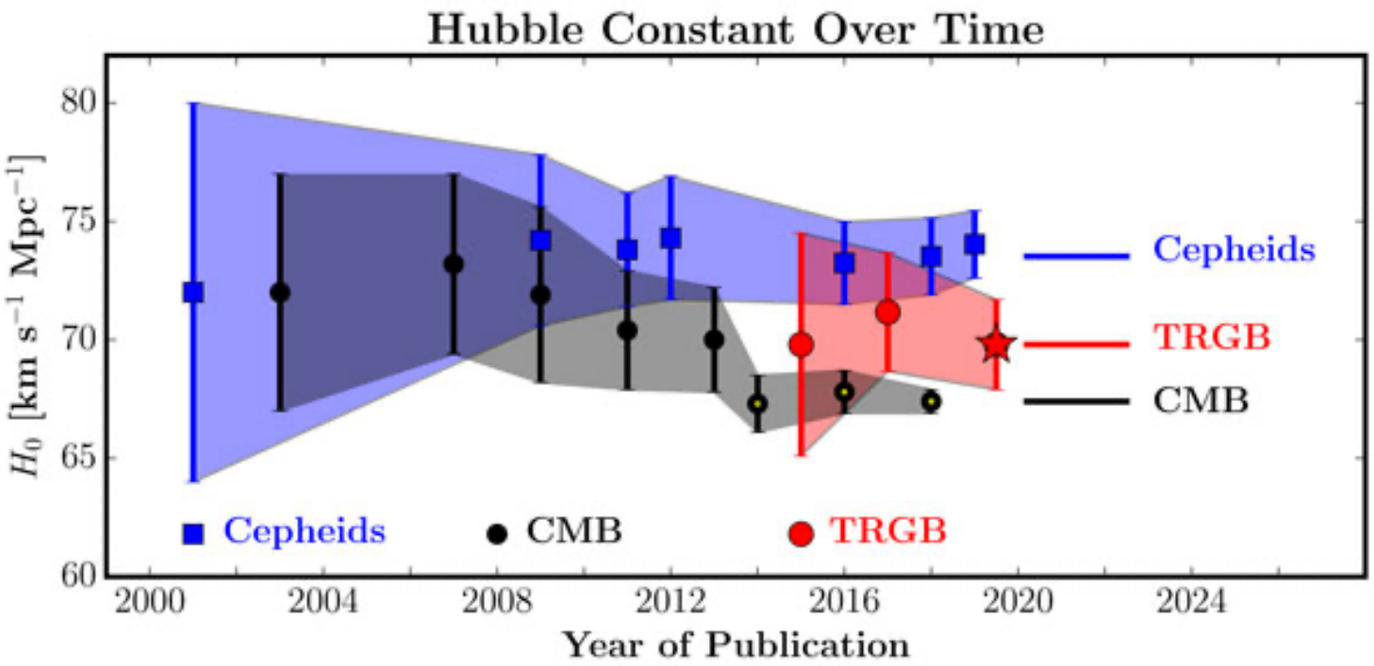
Introduction

Current status of the ΛCDM model:

Observational challenges:

1. H_0 tension;





Wendy L. Freedman ApJ 919 16 (2021).

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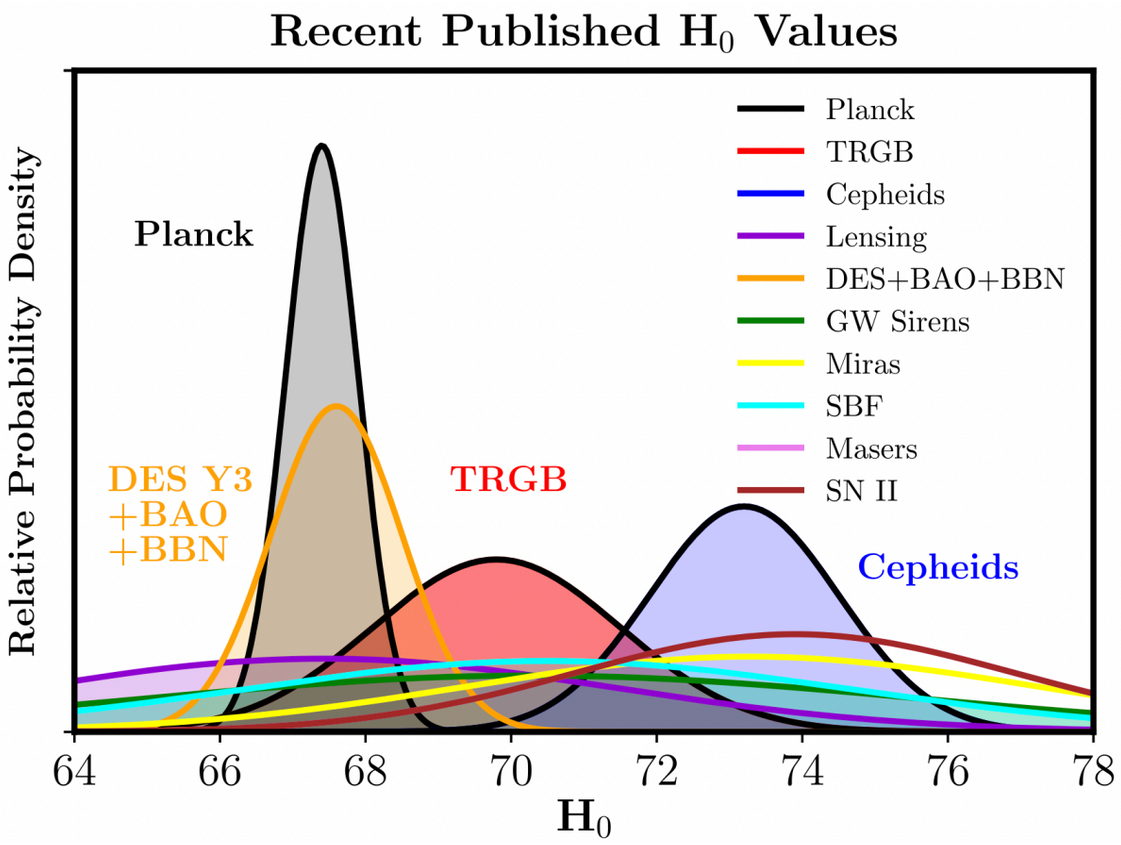

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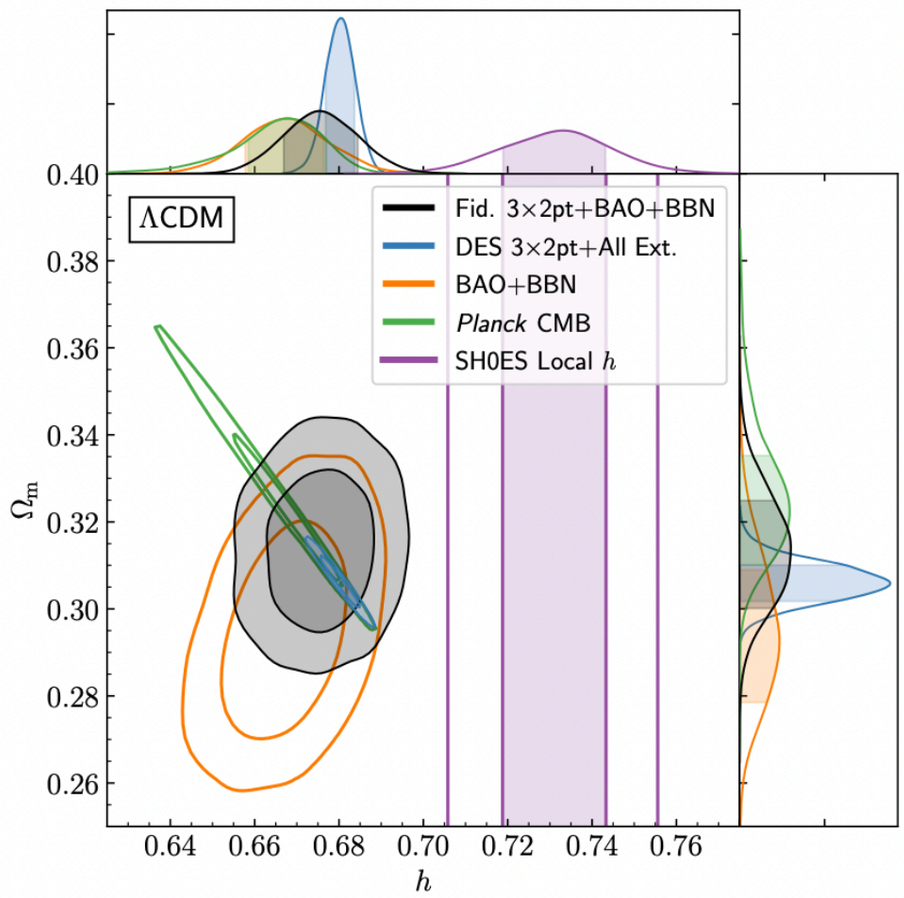
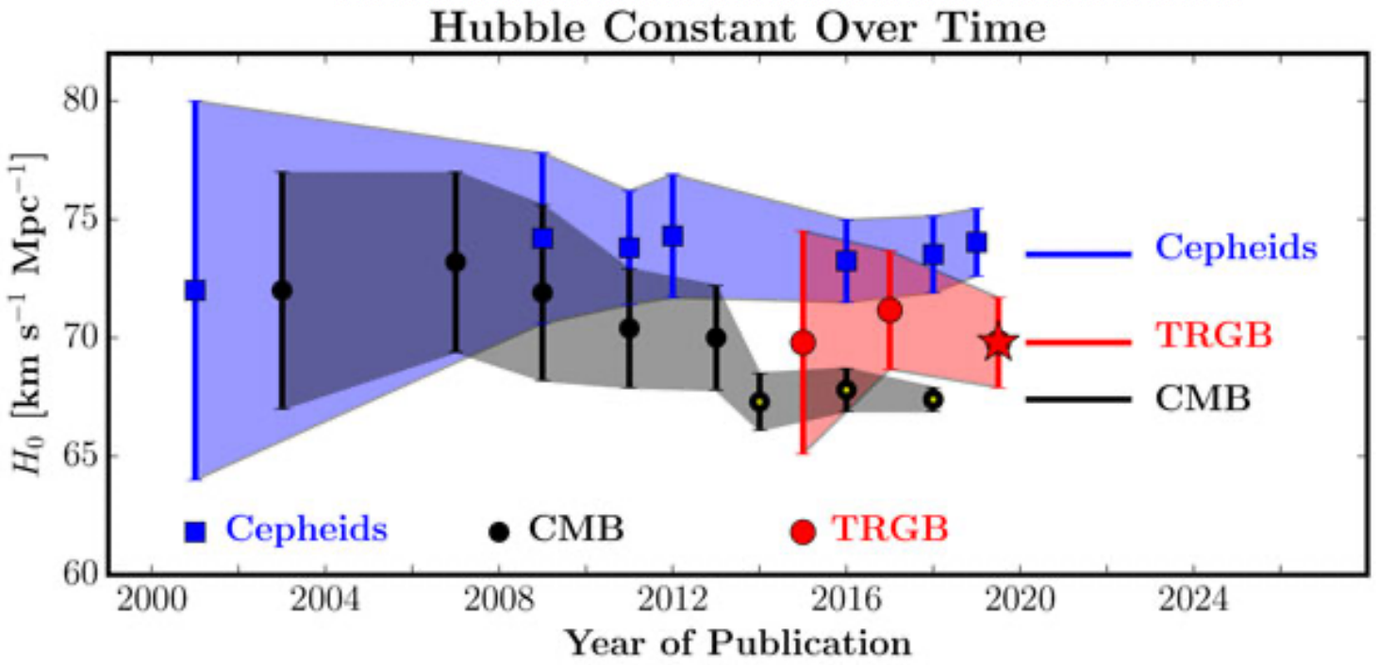
T. M. C. Abbott et al., arXiv:2105.13549





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Current status of the ΛCDM model:

A.G. Riess. Nature Reviews Physics (2020)

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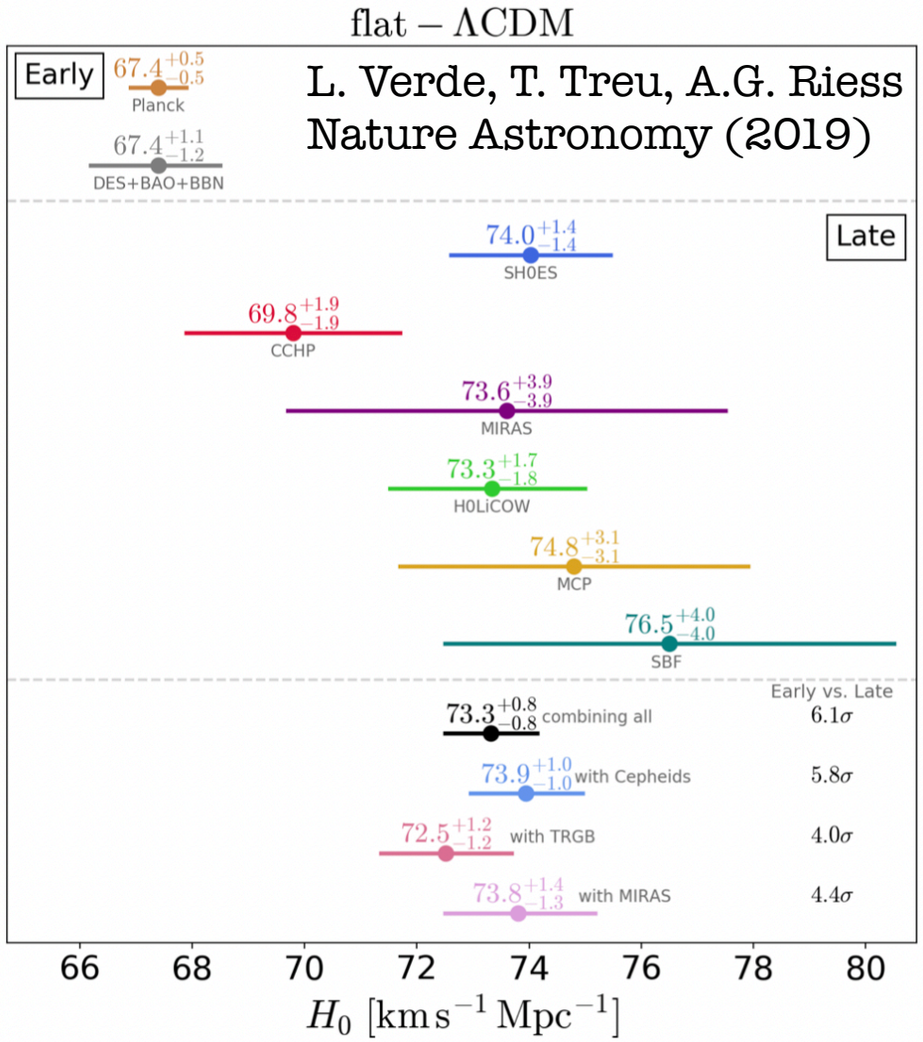

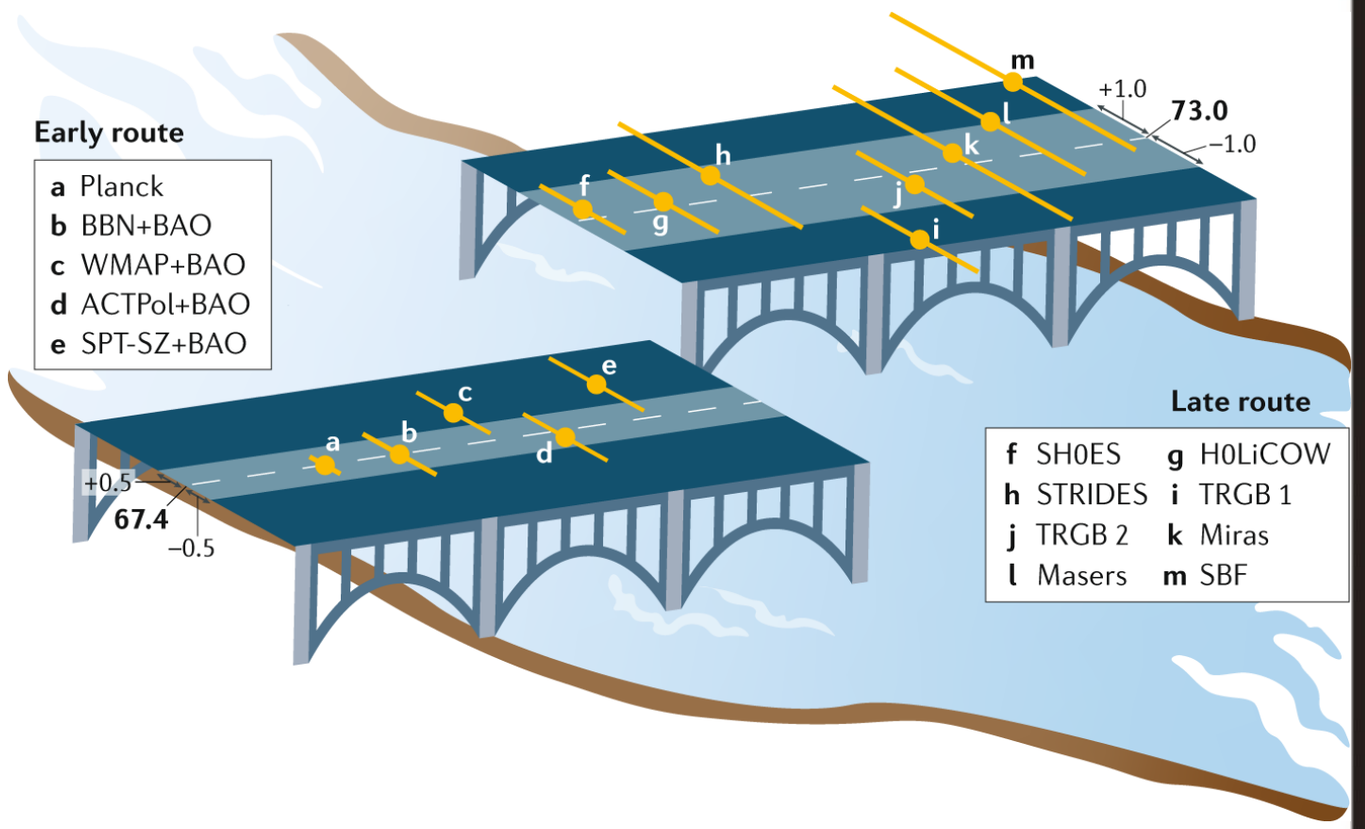

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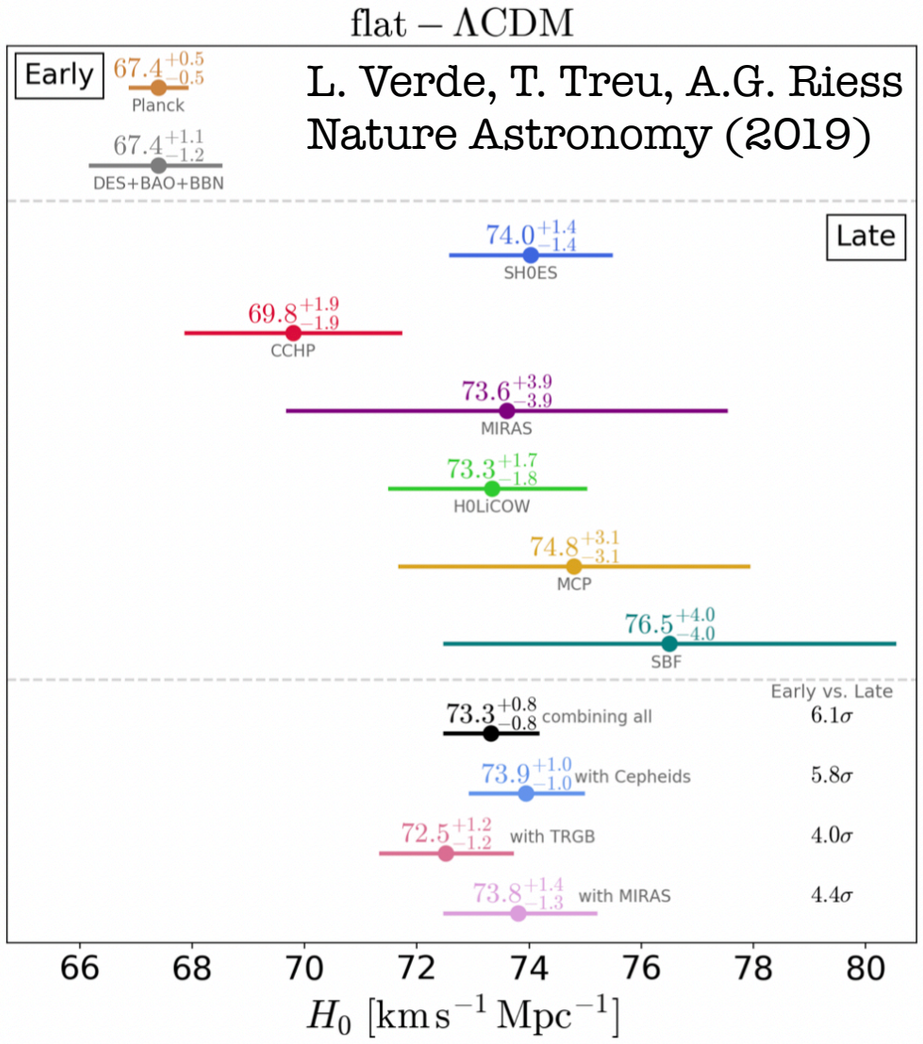
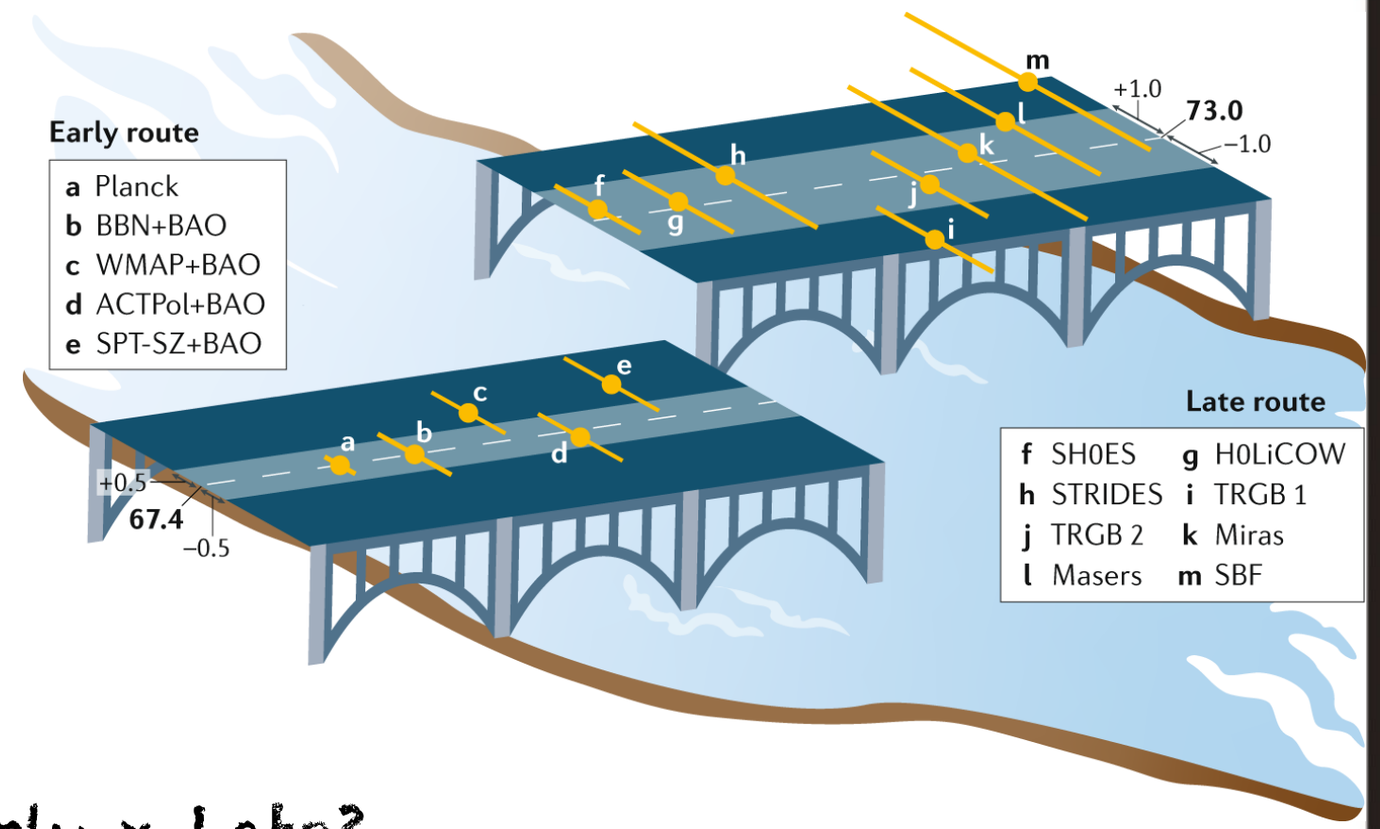




Early x Late?

The tension is emphatically not "early vs late" cosmology since baryon acoustic oscillations (BAO) distances (together with primordial element abundances [19–21] or marginalizing over or sidestepping the sound horizon at the baryon drag epoch [22, 23]), i.e. without use of the primordial CMB, gives the same answer as from the CMB.

Eric V. Linder, arXiv: 2105.02903

Introduction

Current status of the ΛCDM model:

Observational challenges:

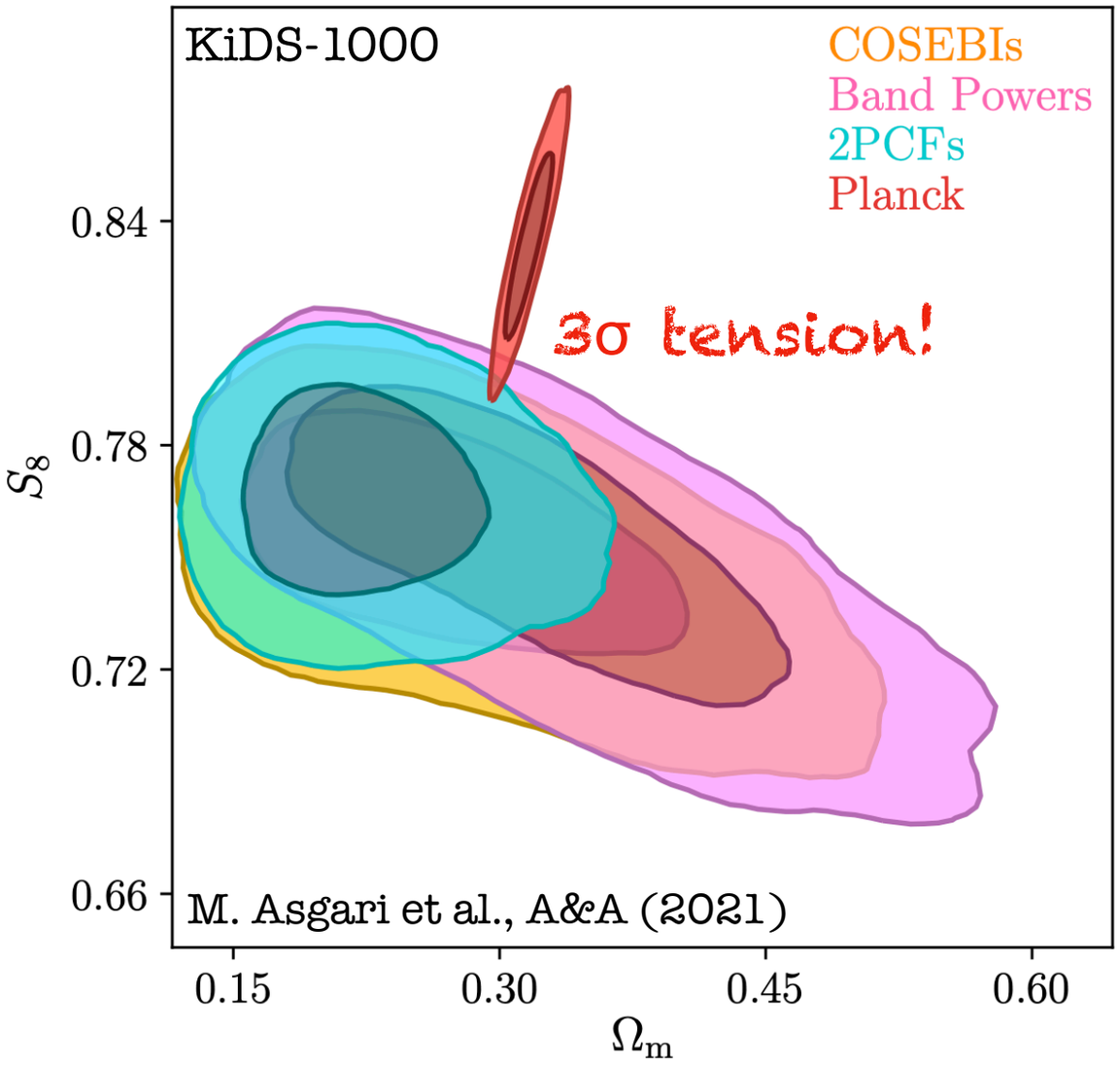
1. H_0 tension;
   - D. Camarena, V. Marra, "On the use of the local prior on the absolute magnitude of Type Ia supernovae in cosmological inference". Mon. Not. Roy. Astron. Soc. 504 (2021) 5164-5171
   - D. Camarena, V. Marra, "Local determination of the Hubble constant and the deceleration parameter". Phys.Rev.Res. 2 (2020) 1, 013028
   - G. Alestas, L. Kazantzidis, L. Perivolaropoulos, "H_0 tension, phantom dark energy, and cosmological parameter degeneracies". Phys. Rev. D 101 (2020) 12, 123516
   - George Efstathiou, "To H_0 or not to H_0?". Mon. Not. Roy. Astron. Soc. 505 (2021) 3, 3866-3872
   - George Efstathiou, "A Lockdown Perspective on the Hubble Tension (with comments from the SH0ES team)". arXiv: 2007.10716
   - Mikhail M. Ivanov, Yacine Ali-Haïmoud, Julien Lesgourgues, "H_0 tension or T_0 tension?". Phys.Rev.D 102 (2020) 6, 063515
   - B. Bose, L. Lombriser, "Easing cosmic tensions with an open and hotter universe". Phys. Rev. D 103, 081304 (2021)
   - C. A. P. Bengaly, J. E. Gonzalez, J. S. Alcaniz, "Is there evidence for a hotter Universe?". Eur. Phys. J. C 80, 936 (2020)
   - Nils Schöneberg et al., "The H_0 Olympics: A fair ranking of proposed models". arXiv 2107.10291

Introduction

Current status of the ΛCDM model:

Observational challenges:

2. S_8 tension;



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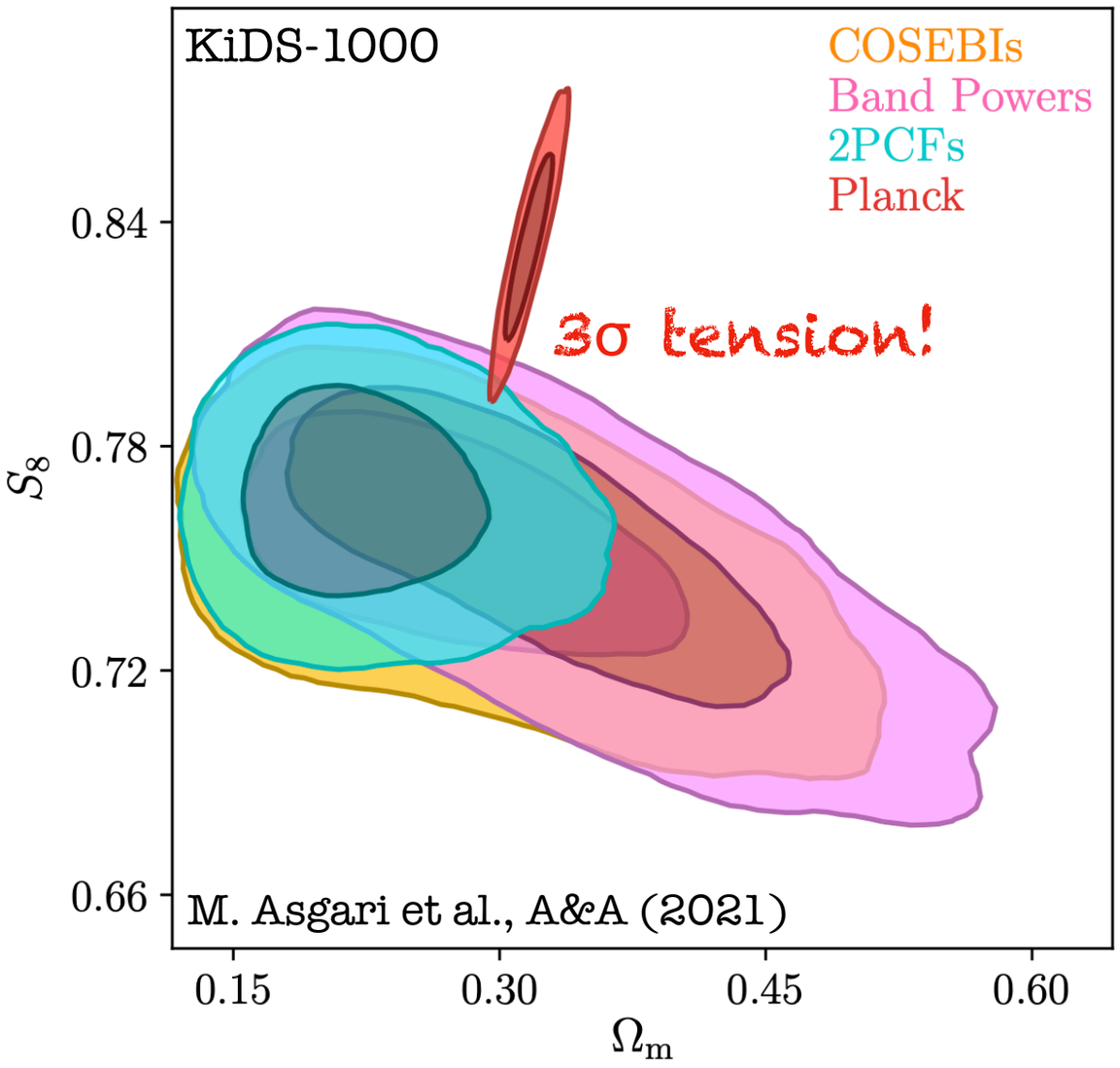
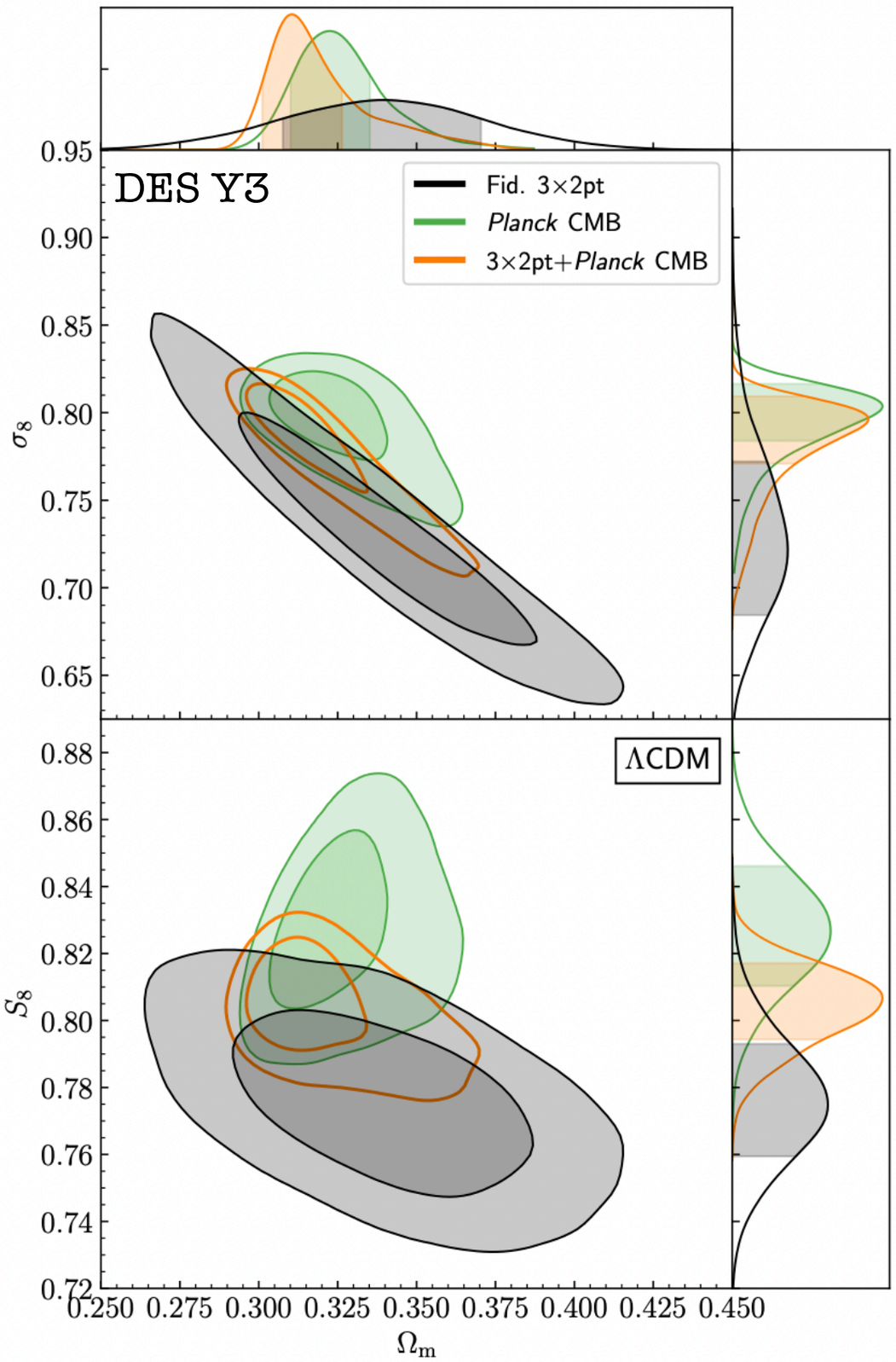


T. M. C. Abbott et al., arXiv:2105.13549



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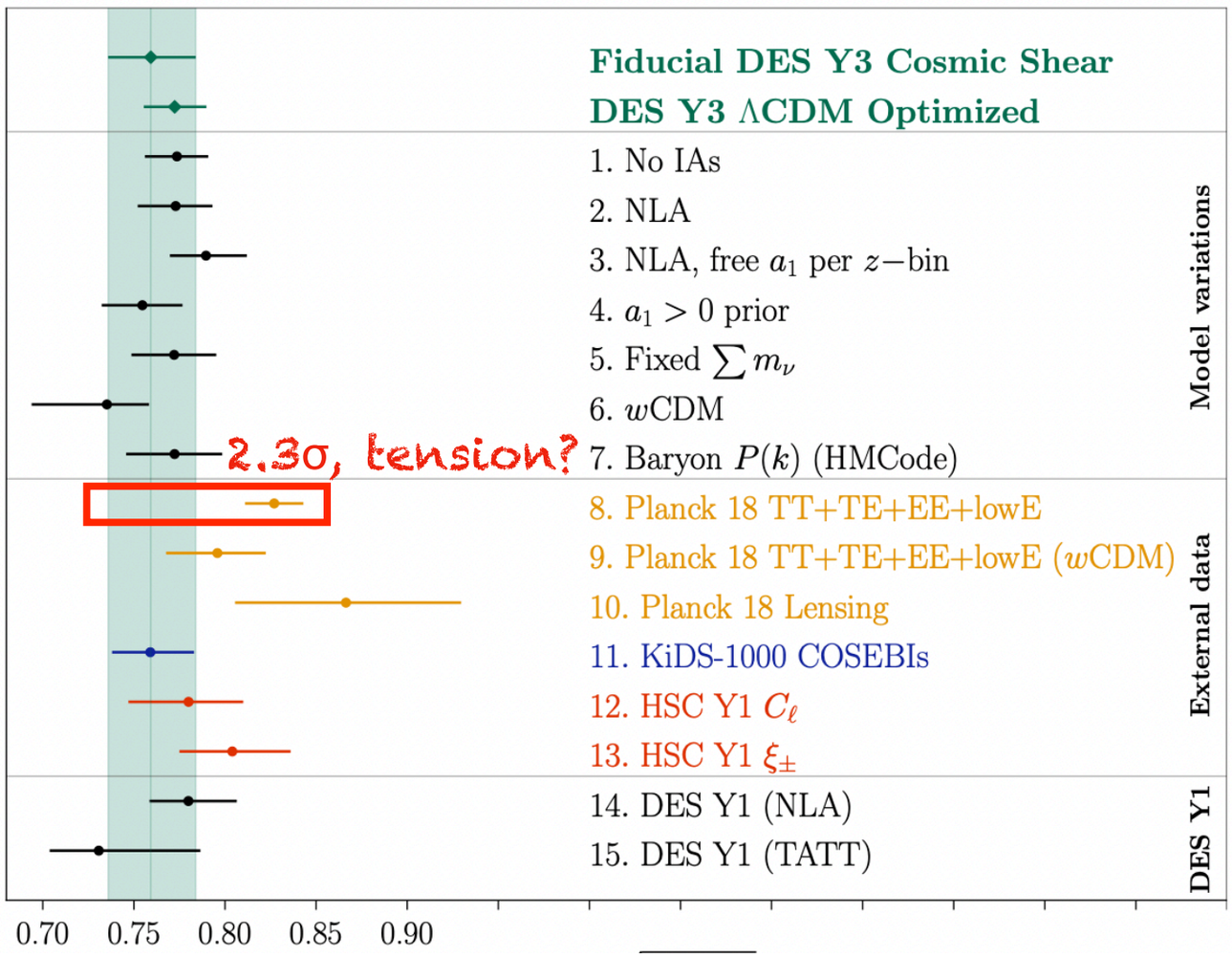

L. F. Secco et al., arXiv: 2105.13544

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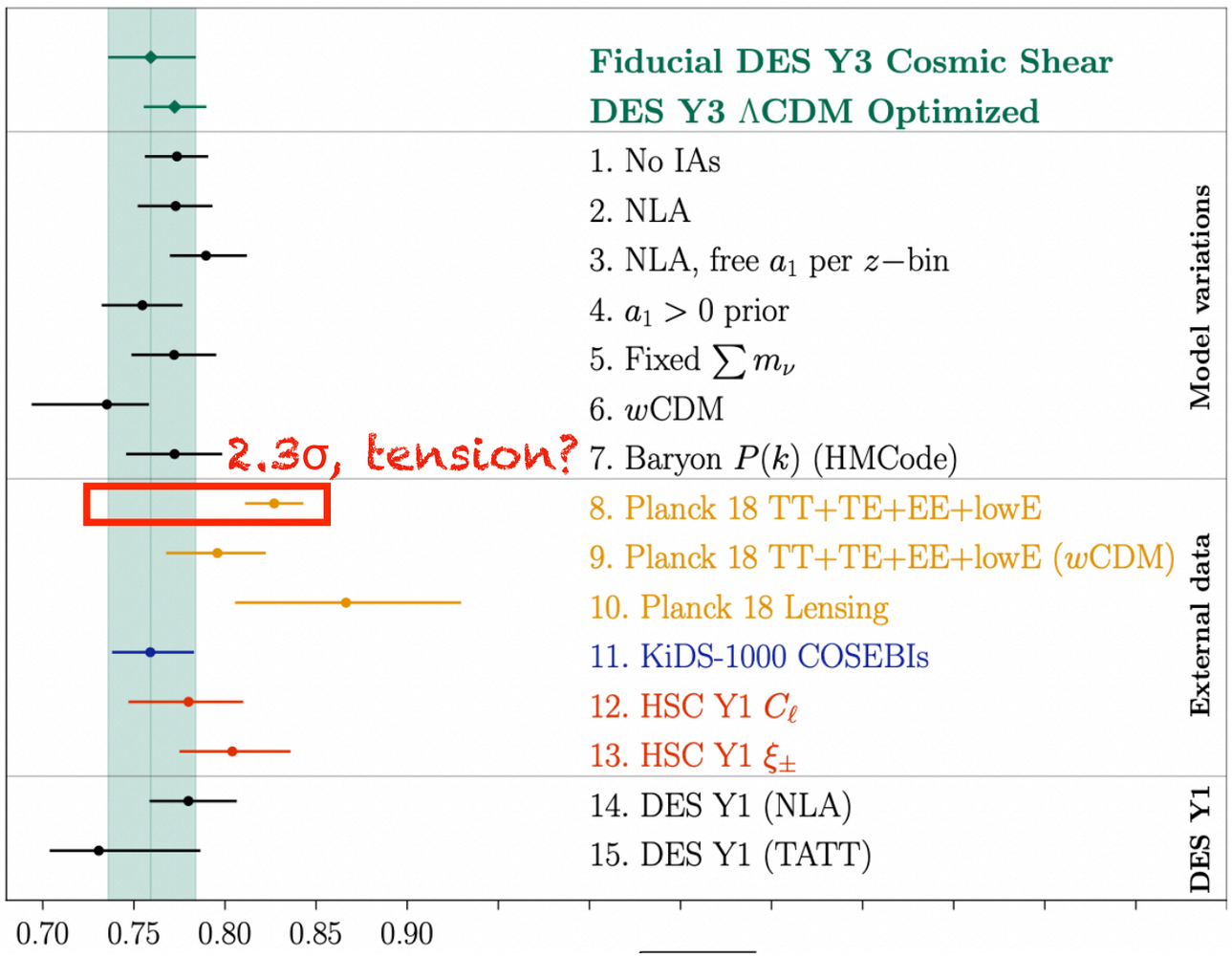

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twitter

[redacted]

Honest question (e.g. no sarcasm) : how can a 2.3sigma be no tension, but we're also so close to an epoch of holy "high precision cosmology", in which we will constrain parameters with ~1% accuracy...? I'm truly confused.

3

[redacted]

More generally: a 2-sigma "effect" (in this case, a discrepancy between 2 measurements) has no less than 28% probability of wrongly rejecting the null (ie, that there is no tension). The reason: p-vals vastly overstate the evidence against the null.

1 1

[redacted]

Yes but you have to put things in the context. A 2 sigma tension could bring a more than 3 sigma indication for Alens>1 when you combine Planck and DES. There is already a 2.8 sigma indication for Alens>1 from Planck data alone and it goes in the direction of lowering S8.

1

[redacted]

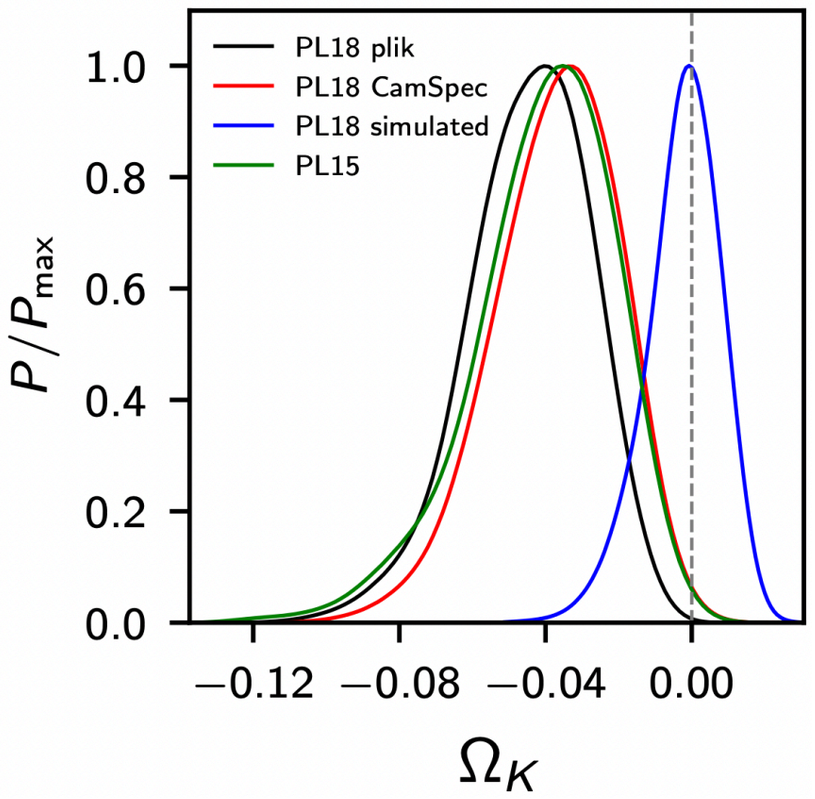
To be clear: planck is not in perfect agreement with lcdm but shows anomalies in the range 2.5-3.2 sigmas. A disagreement of about 2 sigmas could enhance the statistical significance of these anomalies to 3-4 sigmas. At that point or we have a systematic or we see something new.

Introduction

Current status of the ΛCDM model:

Observational challenges:

3. Curvature;



E. Di Valentino, A. Melchiorri and J. Silk, Nature Astronomy (2020)

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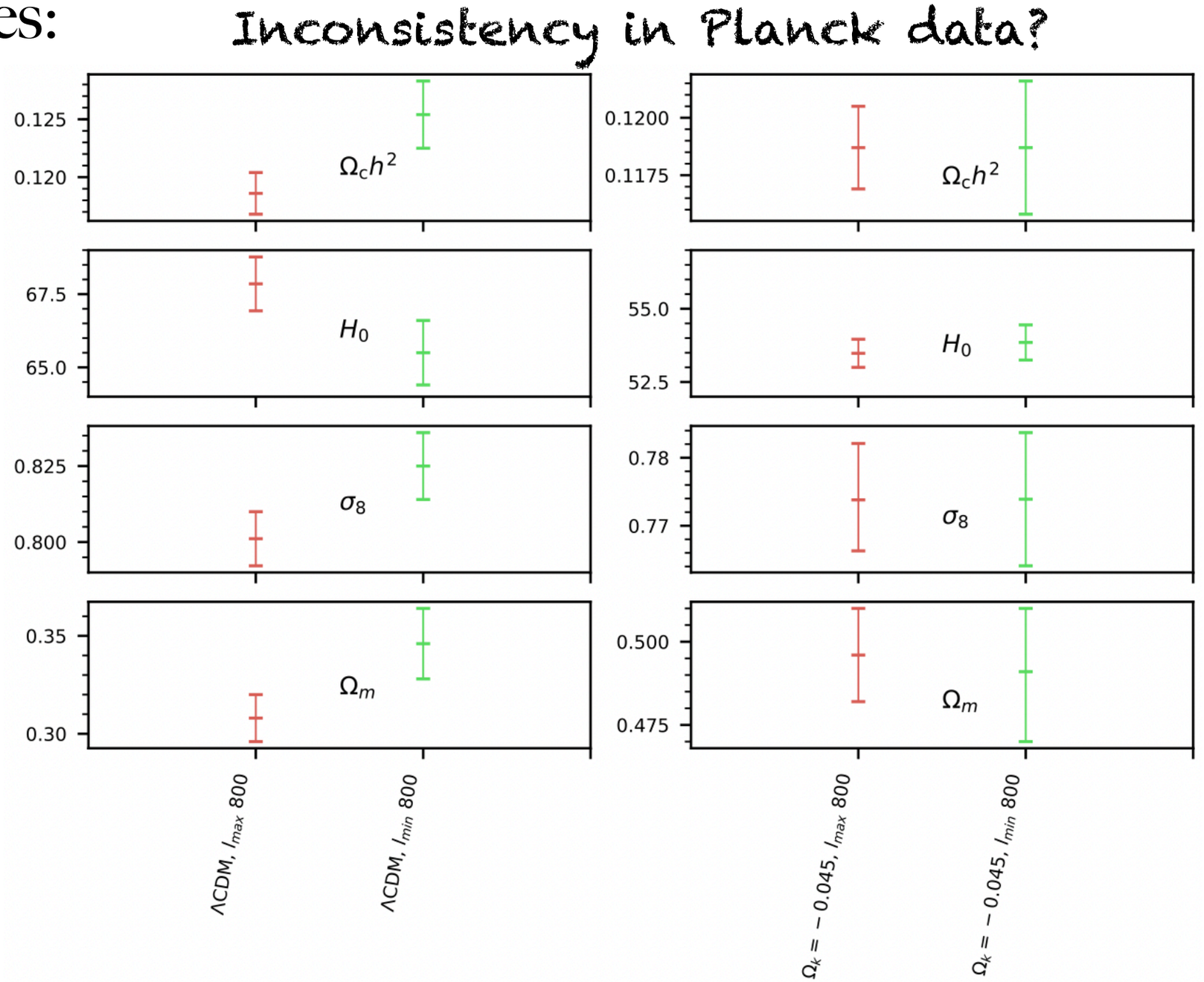

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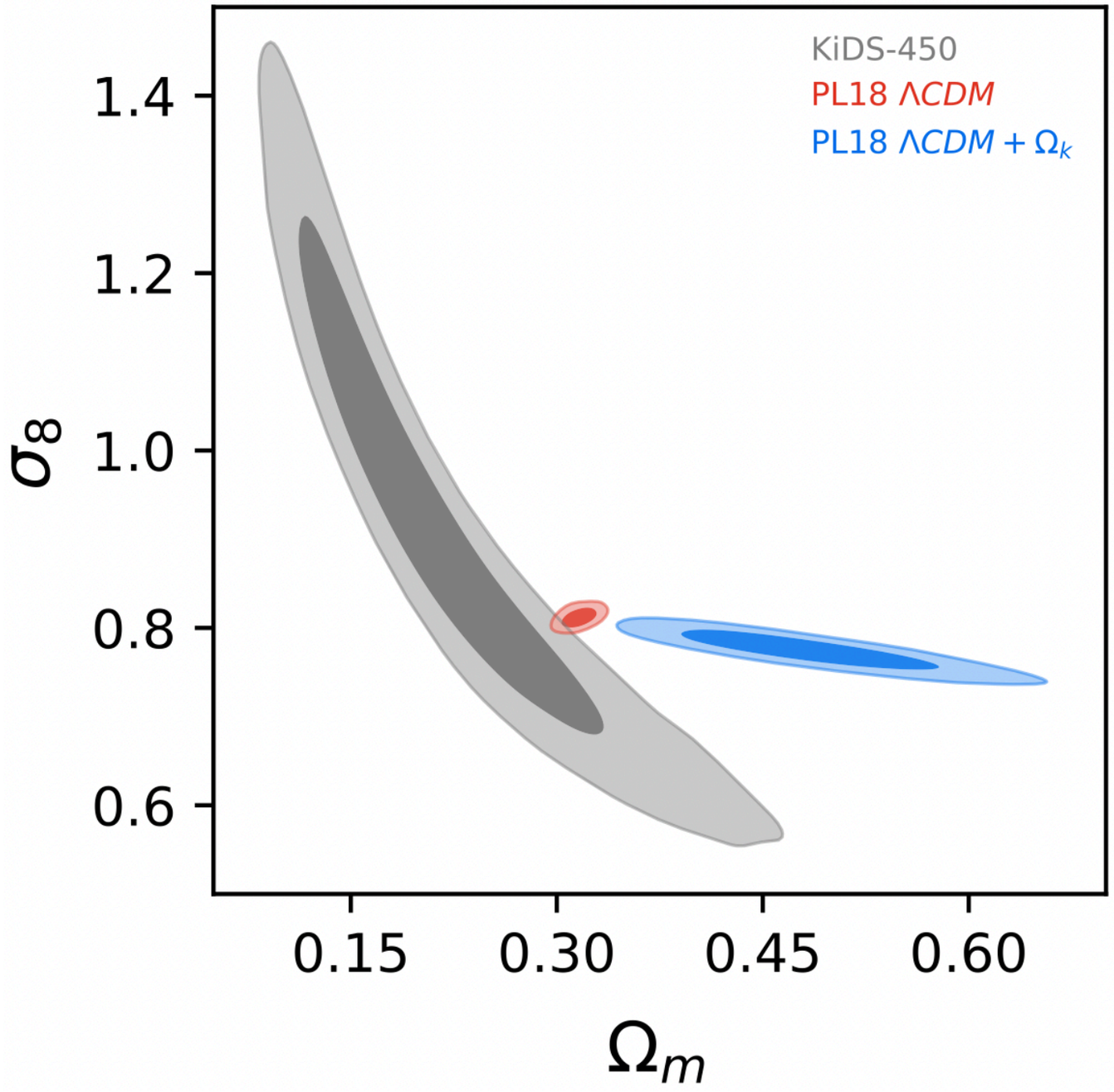

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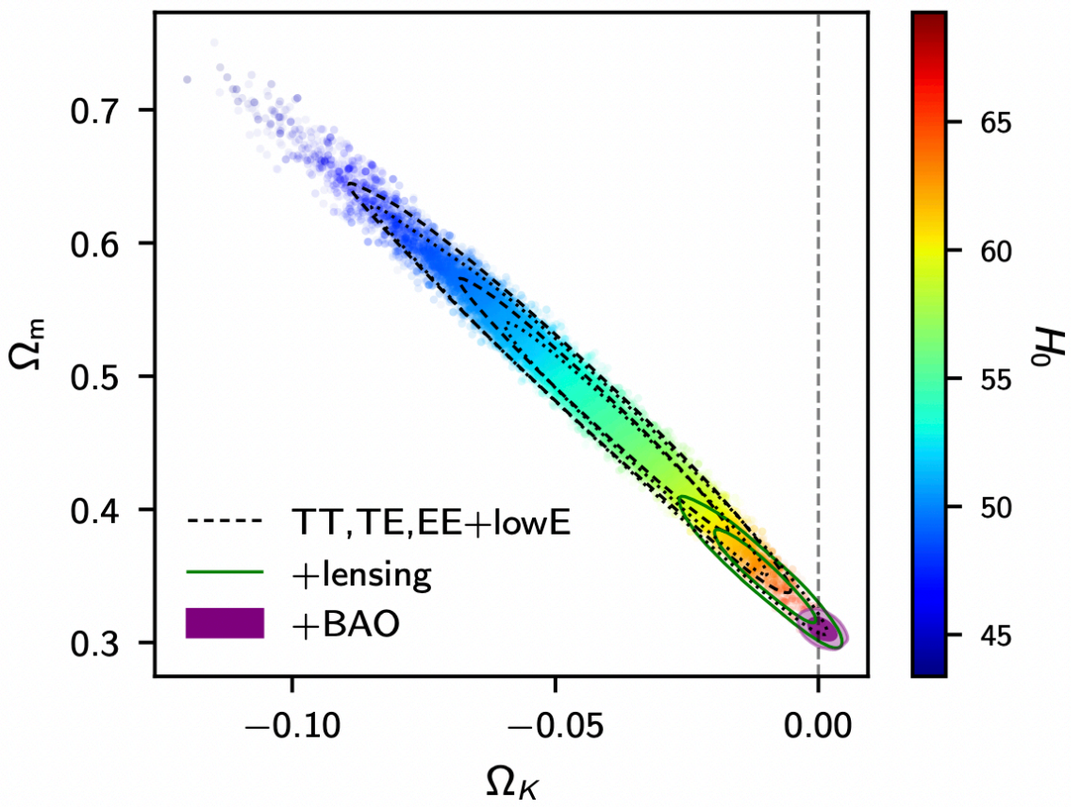

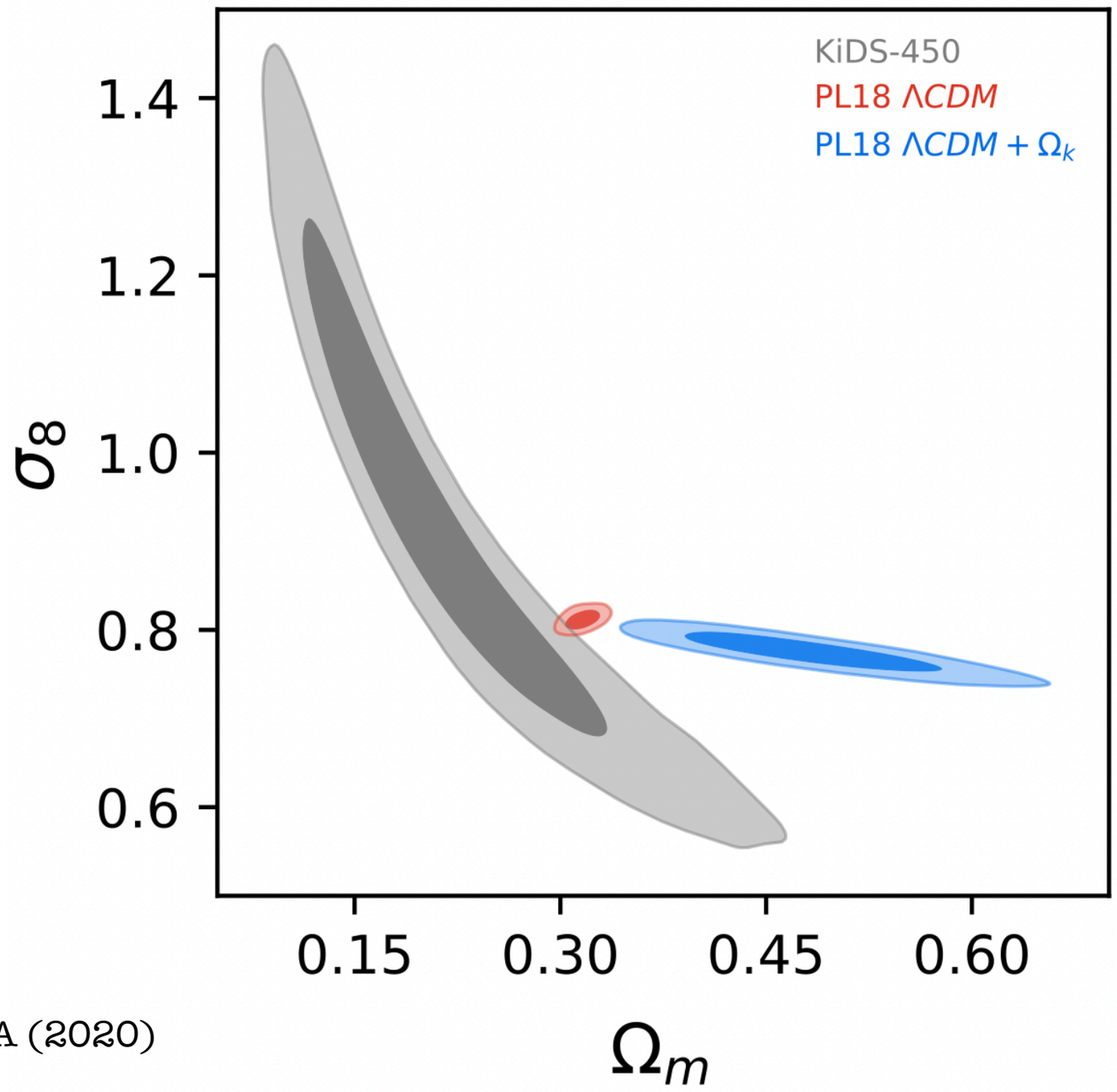

Planck Collaboration: N. Aghanim et al., A&A (2020)

Introduction

Is the ΛCDM model consistent with data?

Geometry/Growth split:

$$\Lambda\text{CDM split} : \Omega_M \longrightarrow \{\Omega_M^{\rm geom}, \Omega_M^{\rm grow}\}$$

$$w\text{CDM split} : \Omega_M, w \longrightarrow \{\Omega_M^{\rm geom}, \Omega_M^{\rm grow}, w^{\rm geom}, w^{\rm grow}\}$$

U. Andrade, D. Anbajagane, RvM, D. Huterer and J. Alcaniz, arXiv: 2107.07538.

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Ex 1: Luminosity distance

$$H(\tilde{z}) = H_0\sqrt{(1-\Omega_M^{\rm geom})(1+z)^{3(1+w^{\rm geom})} + \Omega_M^{\rm geom}(1+z)^3}$$

$$d_L = c(1+z)\int_0^z \frac{d\tilde{z}}{H(\tilde{z})}$$

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Ex 2: Sub-horizon matter fluctuations

$$\rho_m = \frac{8\pi G}{3H_0^2}\Omega_M^{\rm grow}(1+z)^3$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\rho_m\delta_m$$

$$H = H_0\sqrt{(1-\Omega_M^{\rm grow})(1+z)^{3(1+w^{\rm grow})} + \Omega_M^{\rm grow}(1+z)^3}$$

U. Andrade, D. Anbajagane, RvM, D. Huterer and J. Alcaniz, arXiv: 2107.07538.

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Geometry/Growth split

Cosmological Probe	Geometry	Growth
SN Ia	$H_0 D_L(z)$	——
BAO	$\{D_M(z); D_H(z)\}$	$r_d(z_d)$
CMB	$j_\ell[k\chi(z)]$	$S_T(k,z)$
Weak lensing	$\frac{d\chi(z)}{d(z)} g_i(z) g_j(z)$	$\Omega_m^2 P_\delta\left(\frac{\ell}{\chi}, z\right)$
RSD	——	$f(z)\sigma_8(z)$

U. Andrade, D. Anbajagane, RvM, D. Huterer and J. Alcaniz, arXiv: 2107.07538.

Introduction

Is the ΛCDM model consistent with data?

Geometry/Growth split

Paper	Data				
	CMB	Weak lensing	BAO/RSD	SN Ia	Galaxy clusters
This work	Planck 2018 (TTTEEE+lensing)	KiDS-1000 (COSEBIs)	eBOSS DR16	Pantheon	—
Wang et. al [20]	WMAP3, ACBAR, BOOMERanG and CBI (TTTEEE)	CTIO (Aperture mass statistic)	2dFGRS and SDSS LRG	SNLS	—
Ruiz & Huterer [21]	Planck 2013 (Shift parameter and early universe priors)	CFHTLens (Shear 2PCF $\xi_{\pm}^{ij}$)	6dFGS, SDSS LRG and BOSS CMASS	SNLS	MaxBCG (Cluster counts)
Bernal et. al [22]	Planck 2015 (TTTEEE+lensing)	—	6dFGS, SDSS-MGS, BOSS-LOWZ, BOSS-CMASS and BOSS-Lyα	JLA	Chandra X-ray and Planck tSZ $\left(\sigma_8 \left(\frac{\Omega_M}{\alpha}\right)^{\beta}\right)$
Muir et. al [23]	Planck 2018 (Shift parameter)	DES Y1 (Shear 2PCF $\xi_{\pm}^{ij}$)	DES Y1 and BOSS DR12	DES Y1	—
Ruiz-Zapatero et. al [24]	Planck 2018 (Shift parameter and primordial power spectrum)	KiDS-1000 (Band powers spectrum)	6dFGS, BOSS DR12 and BOSS DR14	—	—

U. Andrade, D. Anbajagane, RvM, D. Huterer and J. Alcaniz, arXiv: 2107.07538.

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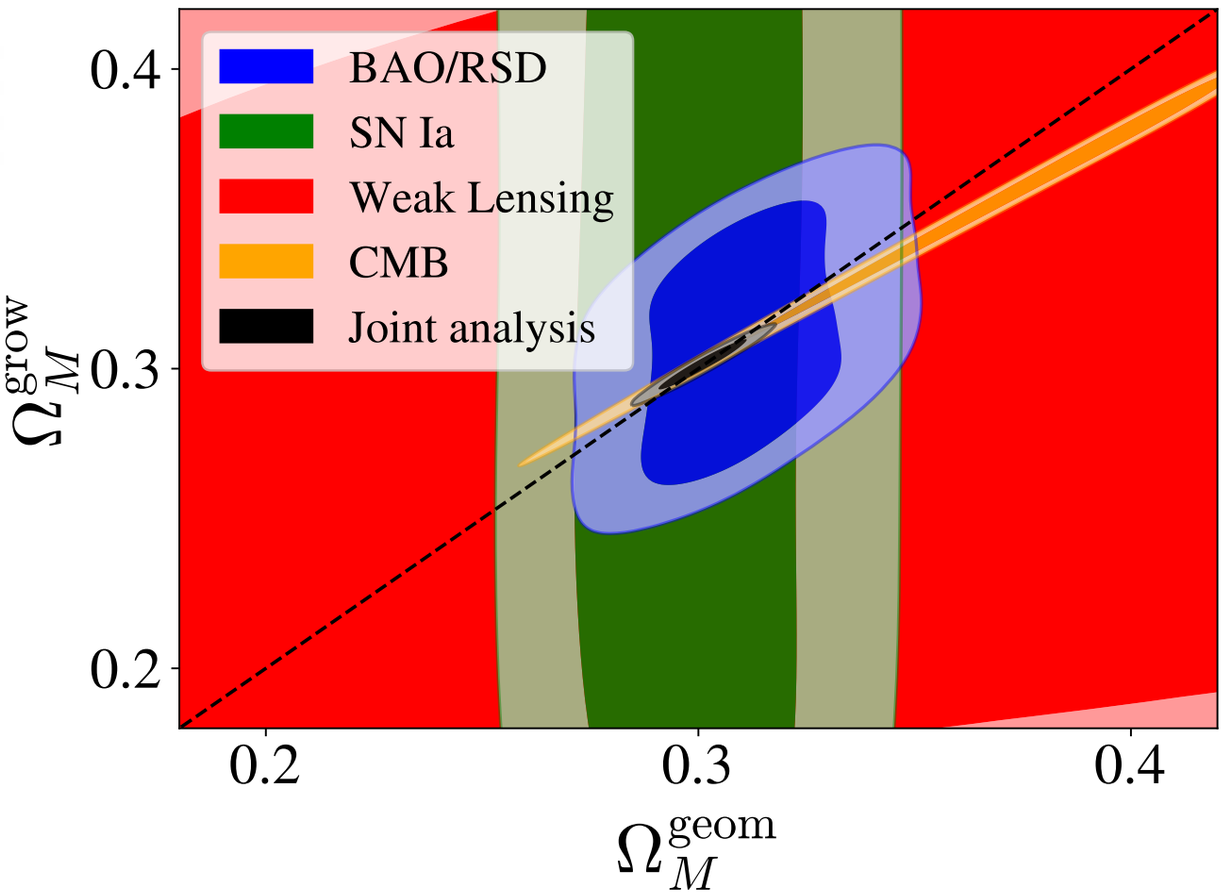
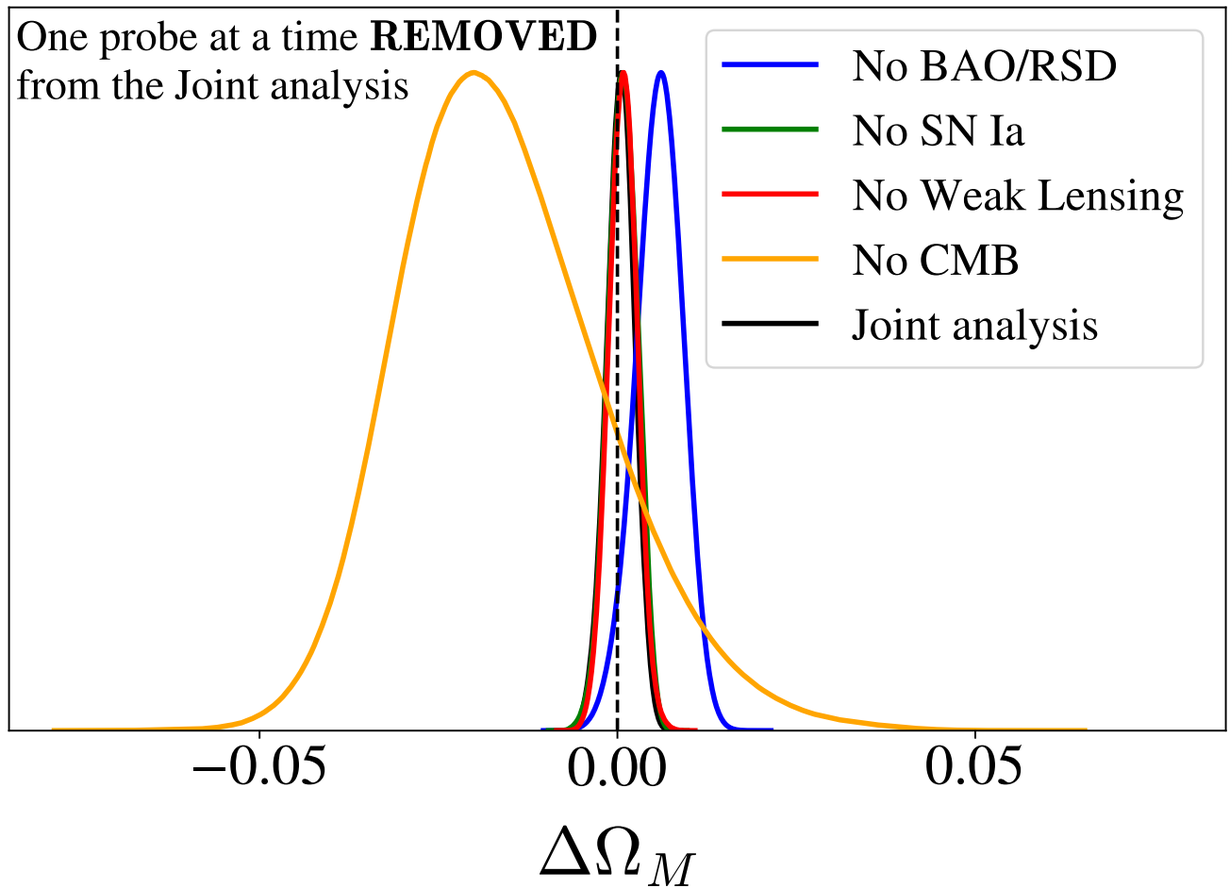
Geometry/Growth split





U. Andrade, D. Anbajagane, RvM, D. Huterer and J. Alcaniz, arXiv: 2107.07538.

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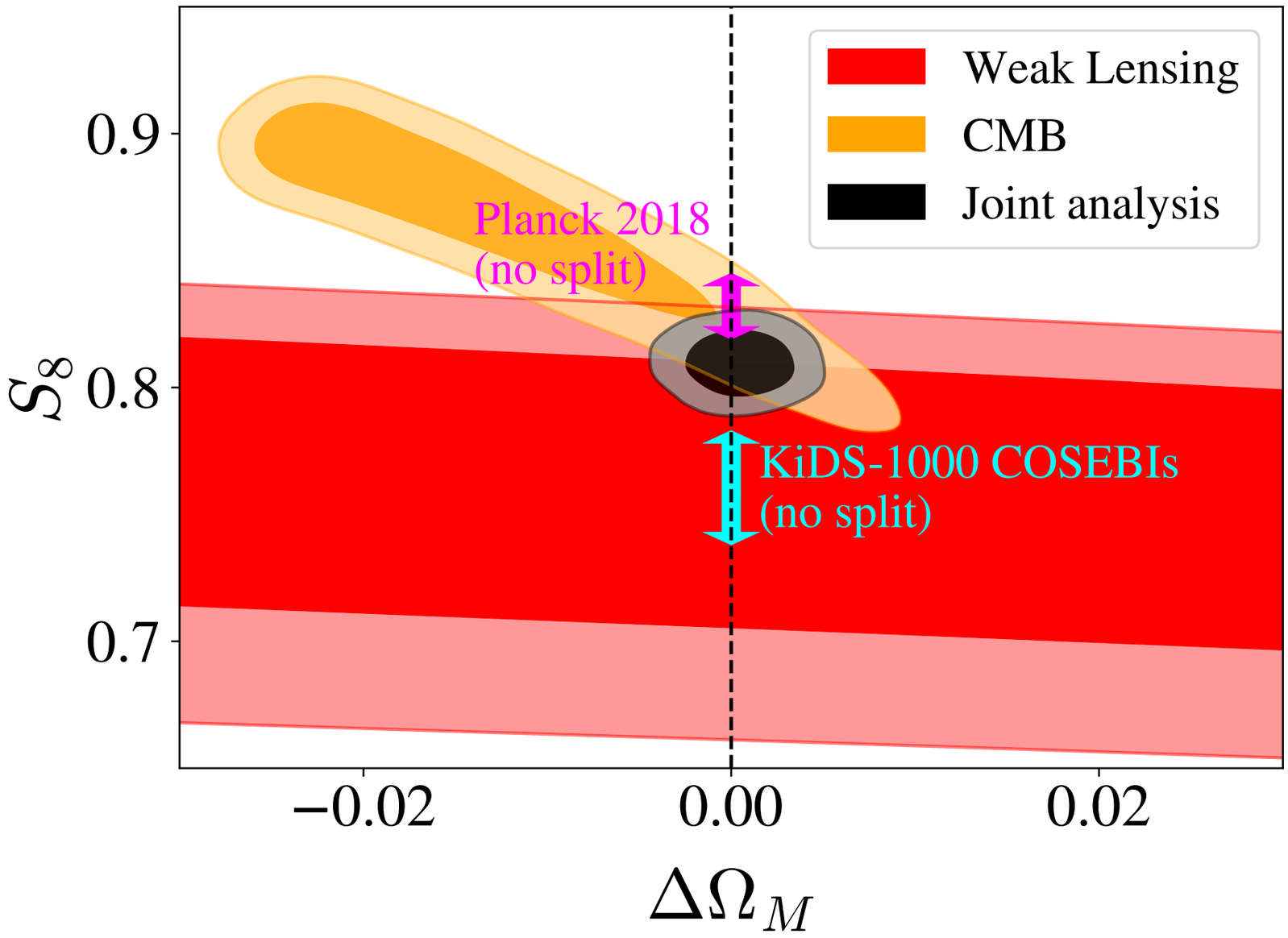

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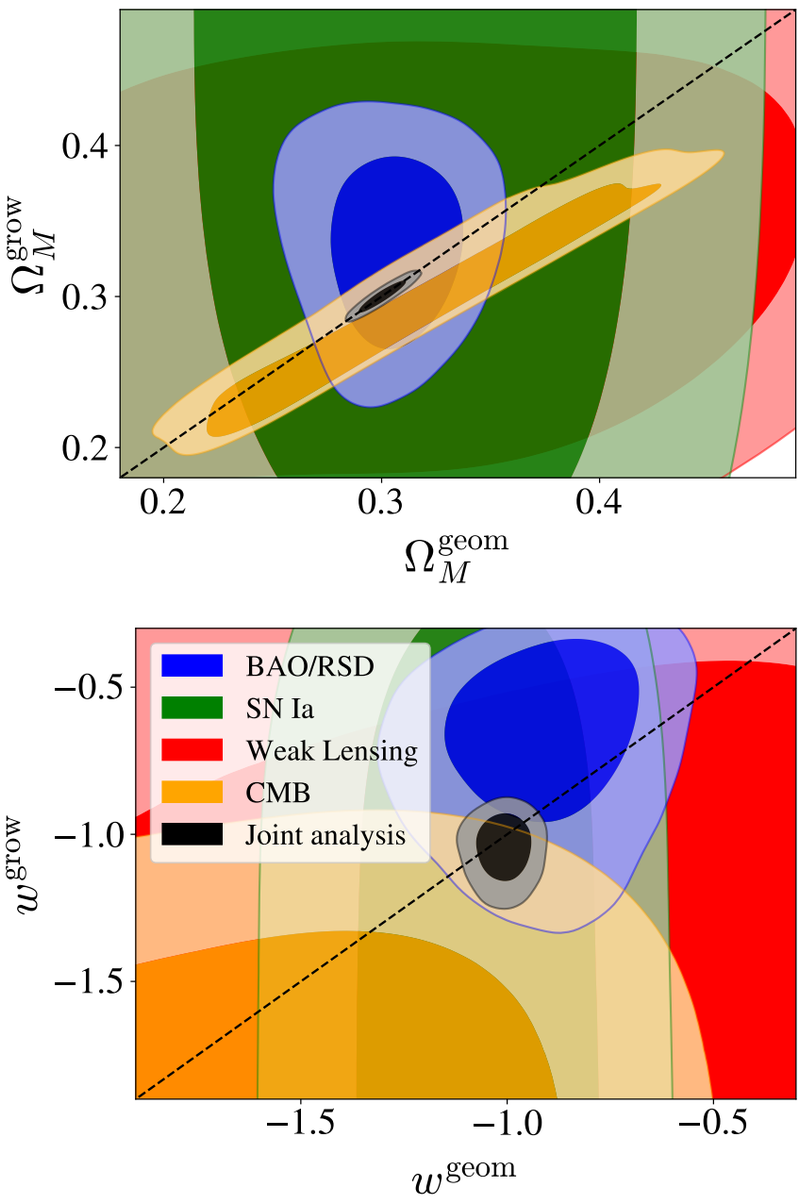
Introduction

Is the wCDM model consistent with data?

Geometry/Growth split



U. Andrade, D. Anbajagane, RvM, D. Huterer and J. Alcaniz, arXiv: 2107.07538.

Introduction

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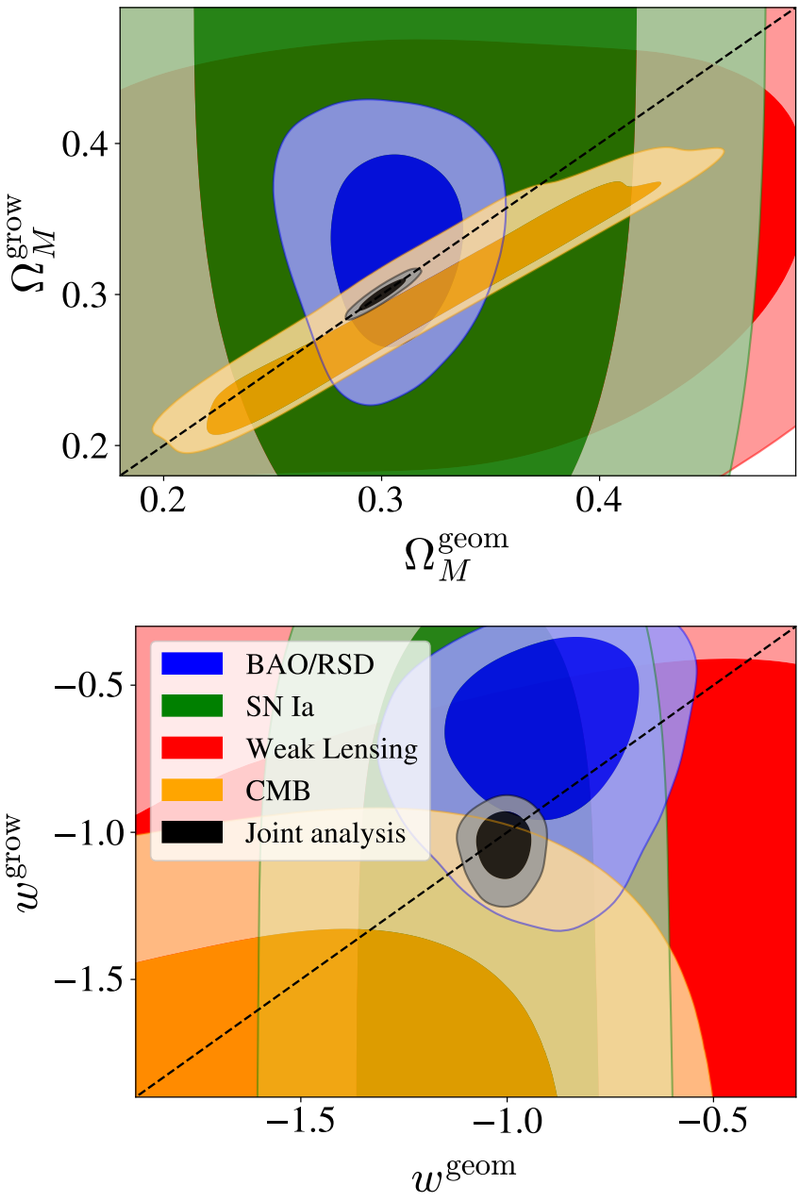

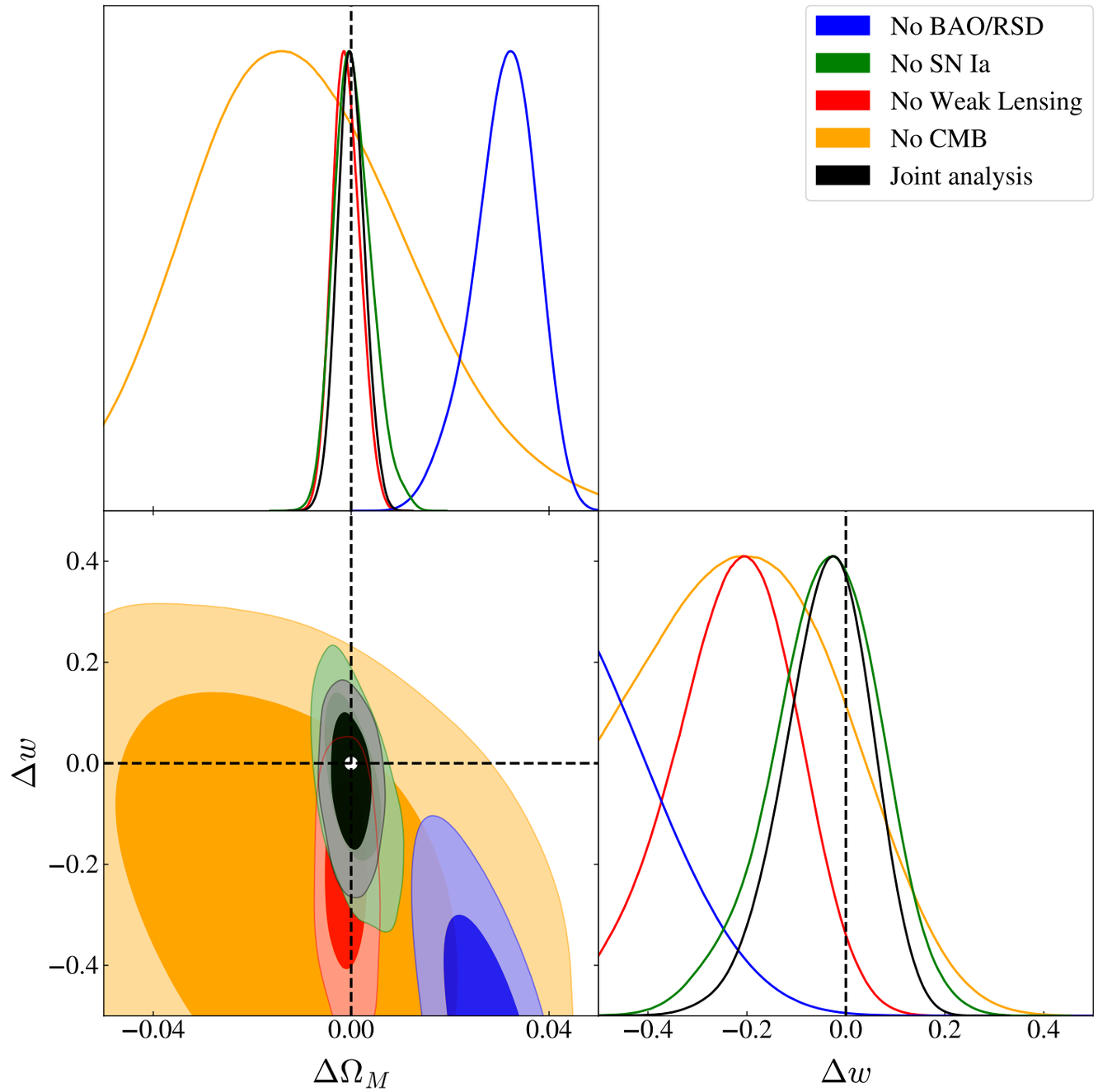

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Dark degeneracy I

Einstein's field equations:

$$T^{(d)}_{\mu\nu} = \frac{G_{\mu\nu}}{\kappa} - \sum_{i=1}^{n} T^{(i)}_{\mu\nu}$$

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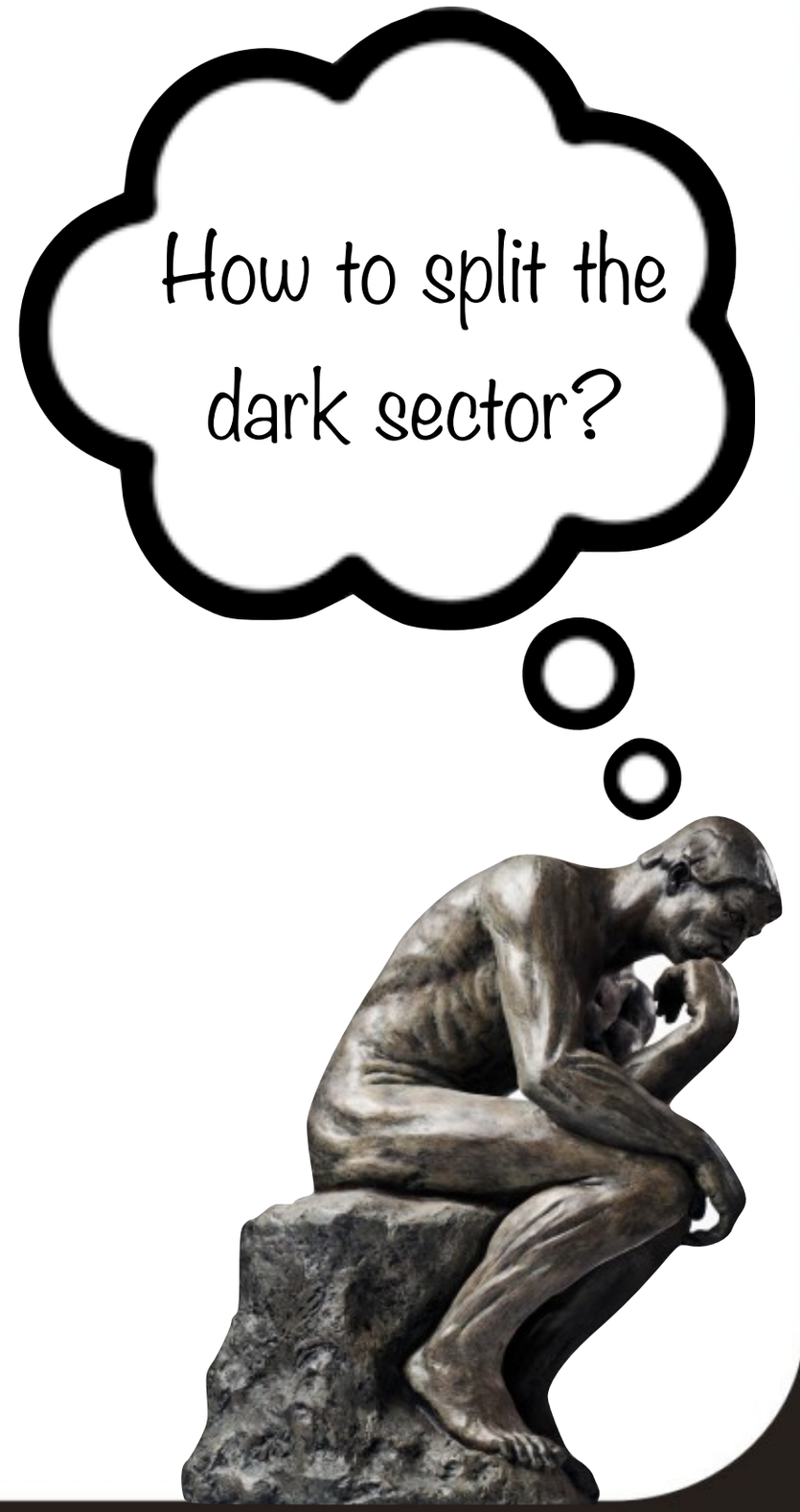

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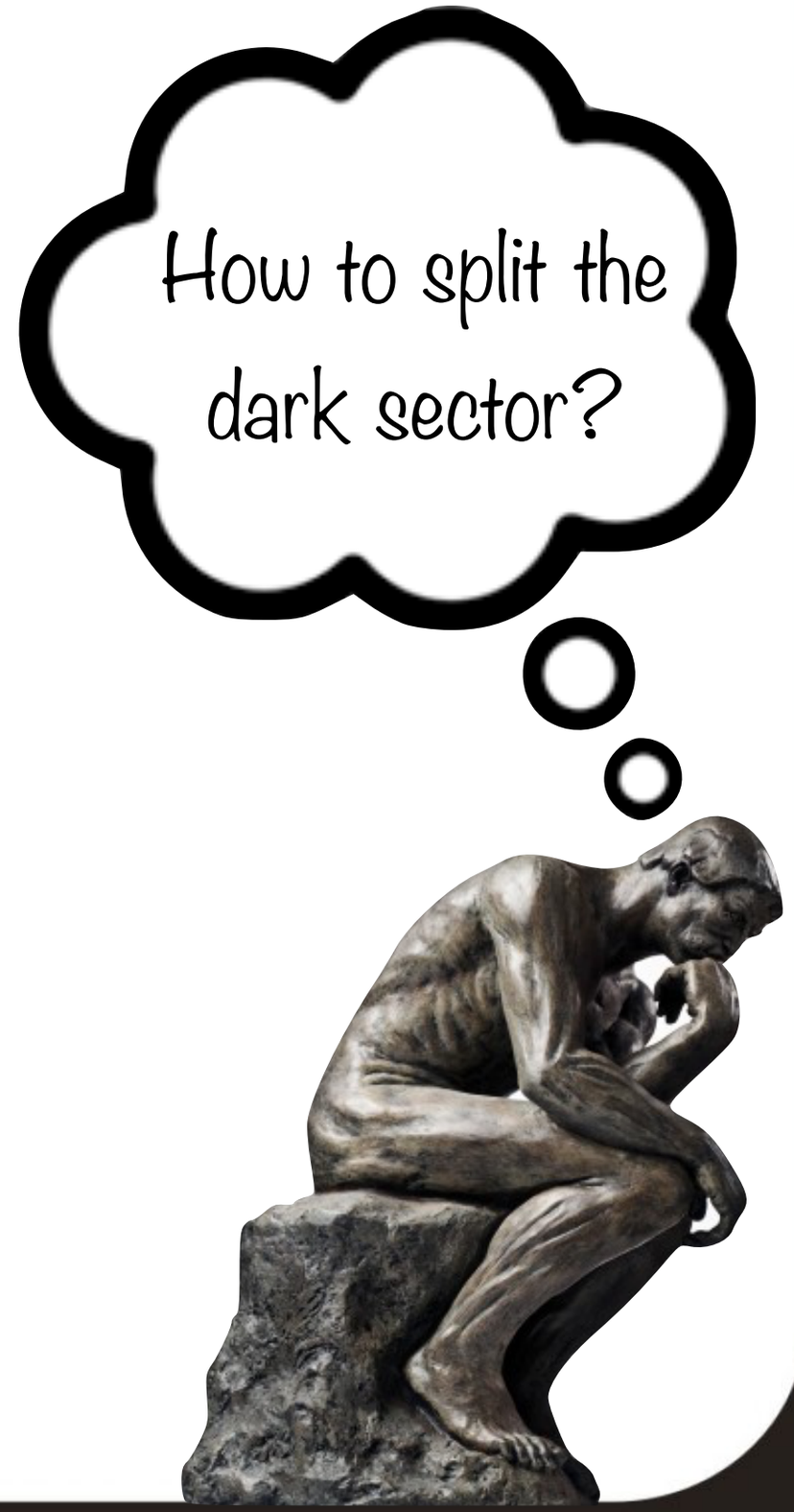

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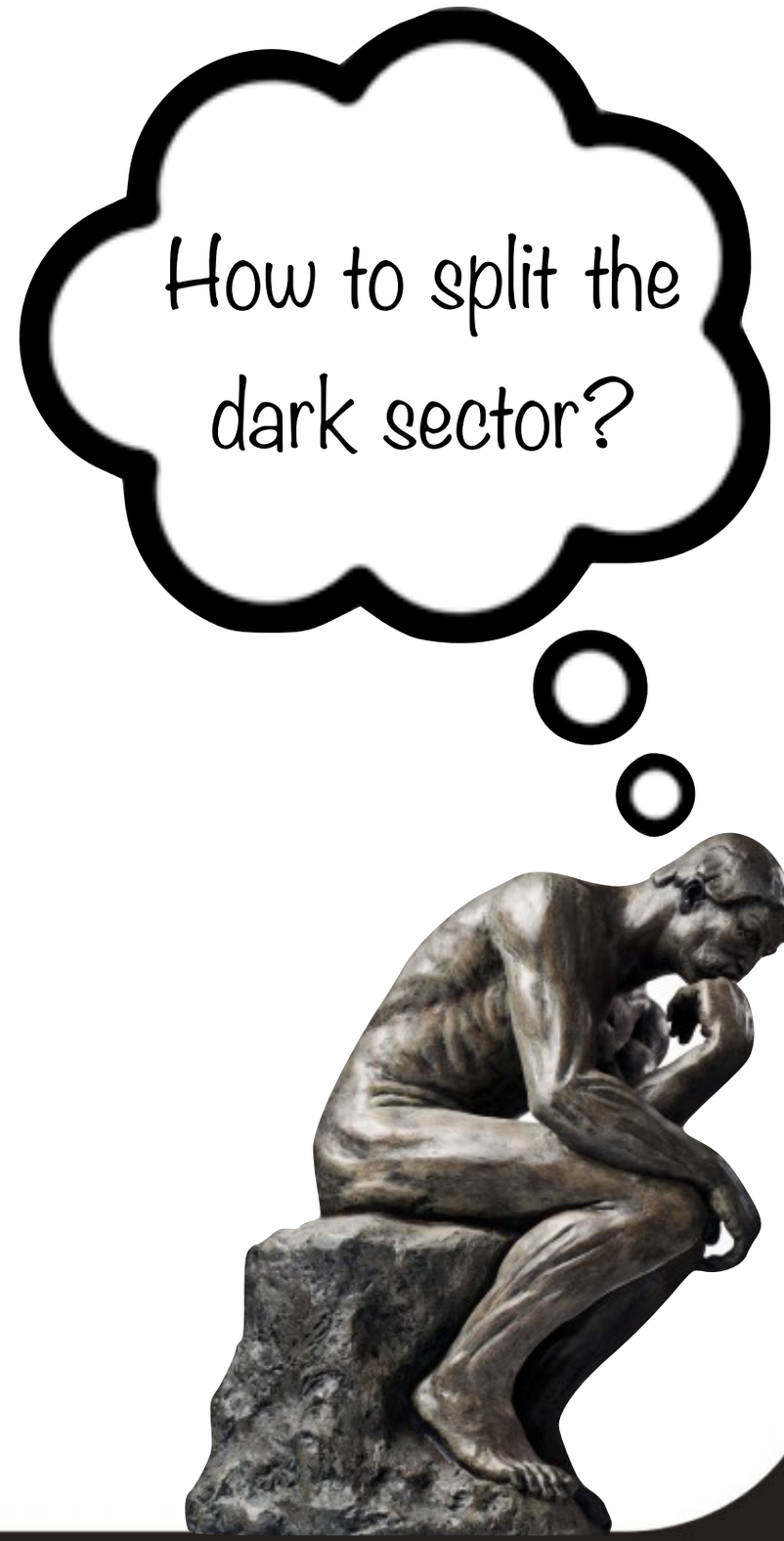

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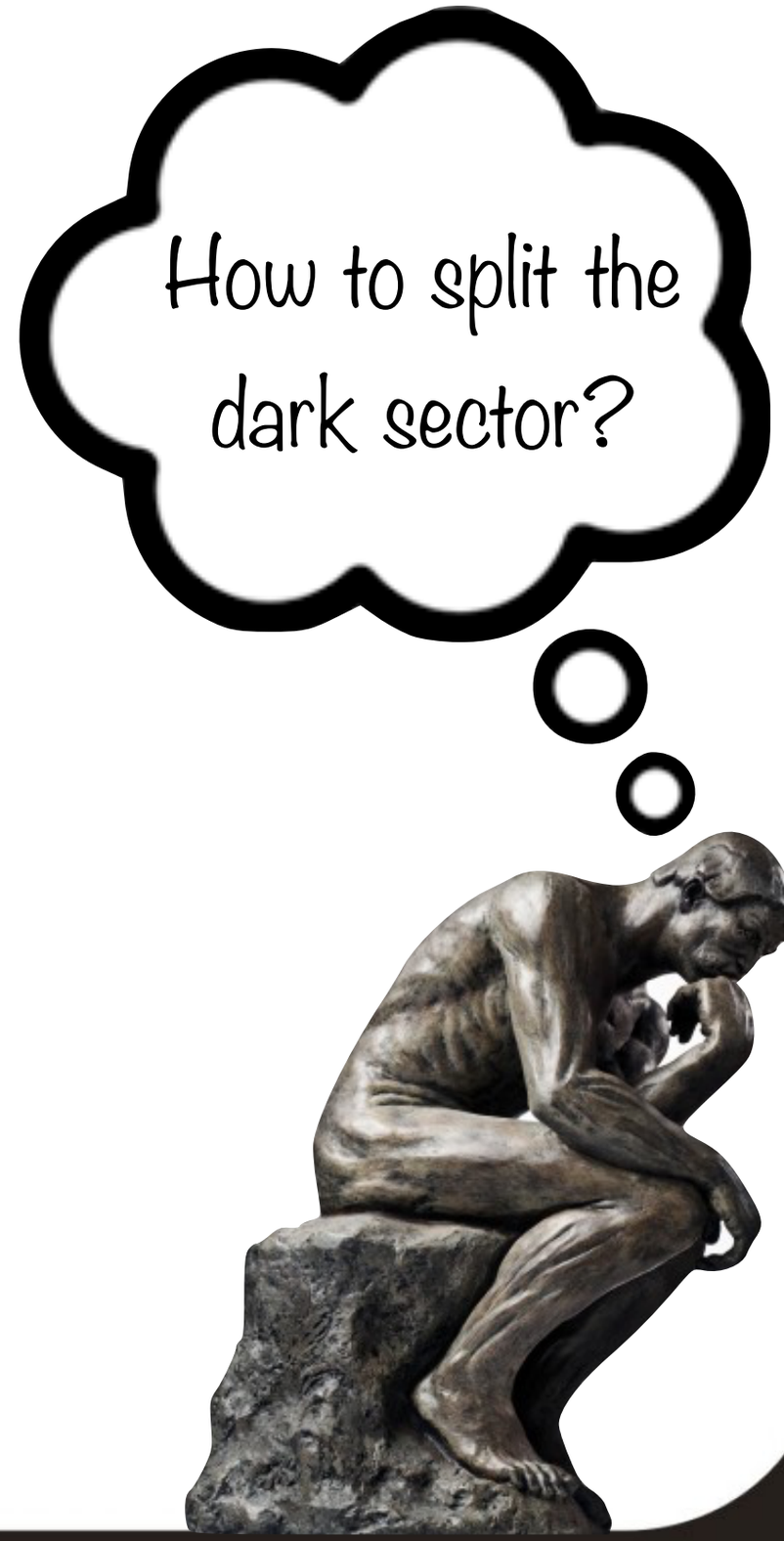

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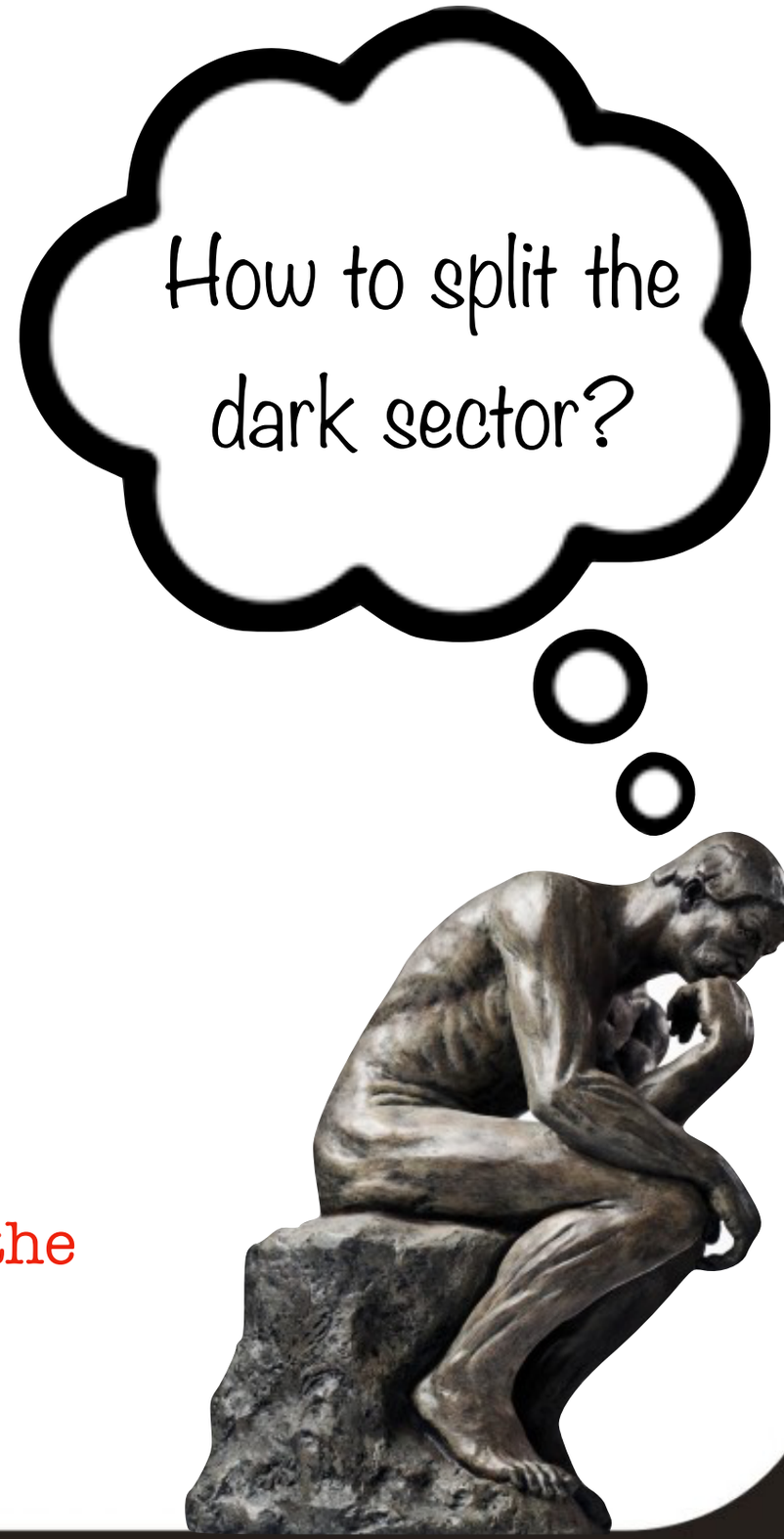

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In other words... In how many ways one can write the dark sector's energy-momentum tensor?

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Matter content: the Universe is composed by four components

1. Radiation ($w_r = 1/3$);
2. Baryons ($w_b = 0$);
3. Pressureless dark matter ($w_c = 0$);
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DE pressure

Sum of CDM and DE energy densities

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$$\dot{\rho}_d + 3H\rho_d\left[1 + w_d(a)\right] = 0 \quad \Rightarrow \quad \rho_d = \rho_{d0}\exp\left[-3\int\frac{1+w_d(\tilde{a})}{\tilde{a}}d\tilde{a}\right]$$

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At the background level, $w_d(a)$ is the most general (model-independent) quantity related to the dark sector!

$$w_d = \frac{w_x(a)}{1+r(a)}$$

Dark degeneracy I

Model-independent reconstruction of the dark fluid

Background solution for the unified dark sector:

$$\rho_d = \rho_{d0} \exp\left[-3\int \frac{1+w_d(a)}{a} da\right]$$

RvM, J.E. Gonzalez, V. Marra, L. Casarini, J. Alcaniz (arXiv:2011.10846 [astro-ph.CO])

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$$1 + w_d(z) = \frac{(1+z)}{3}\frac{d}{dz}\left\{\ln\left[\frac{1}{\Omega_{d0}}\left(\frac{H^2}{H_0^2} - \frac{\Omega_{b0}}{(1+z)^{-3}}\right)\right]\right\}$$

$$w_d(z) = \frac{E\left[3E - 2(1+z)E'\right]}{3\Omega_{b0}(1+z)^3 - 3E^2}$$

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Hubble rate can be reconstructed from the data!

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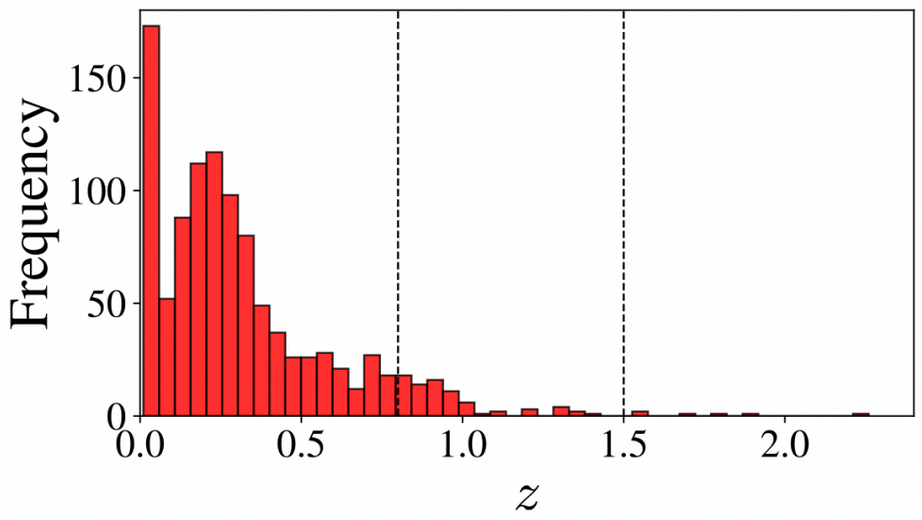

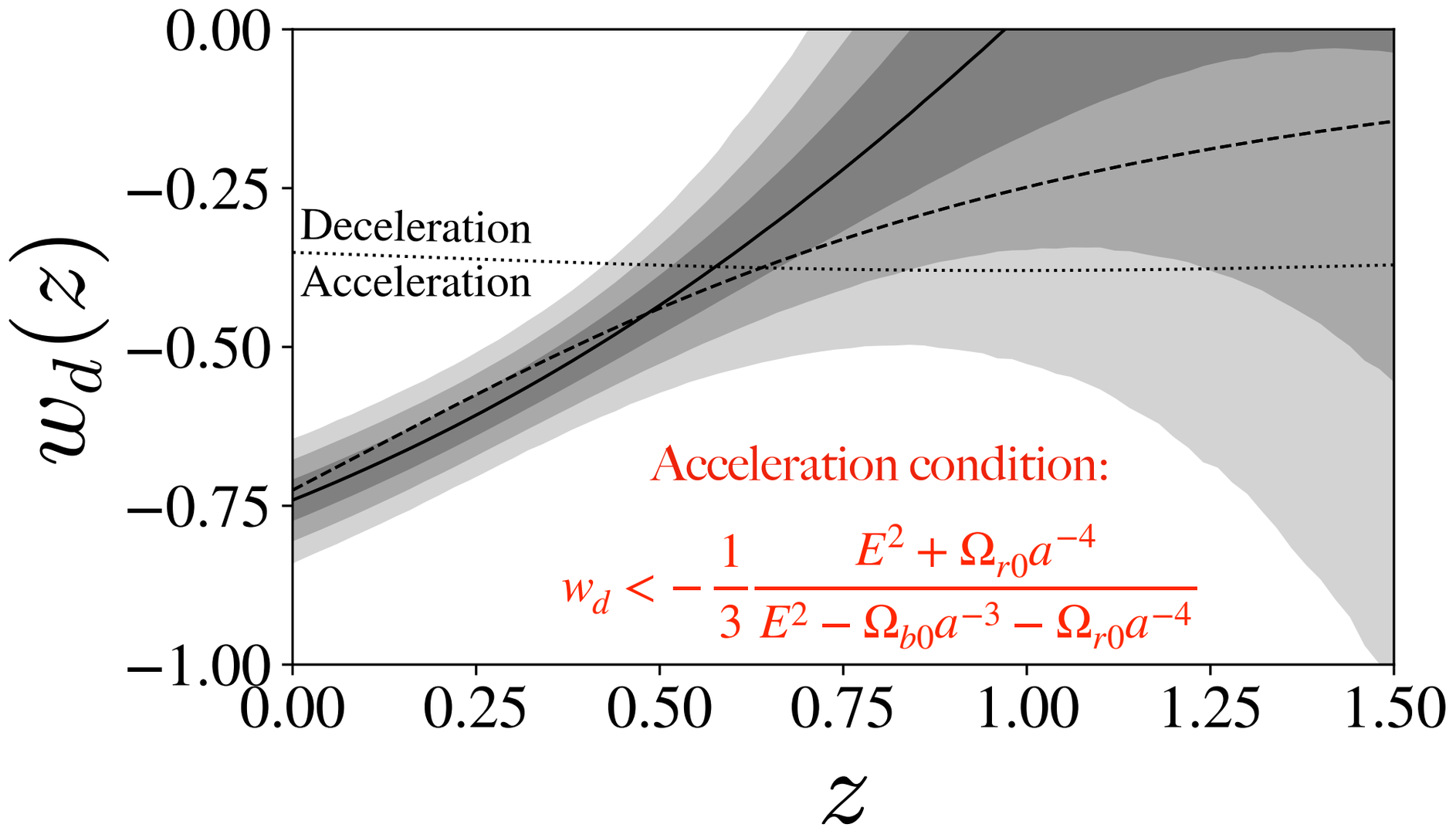

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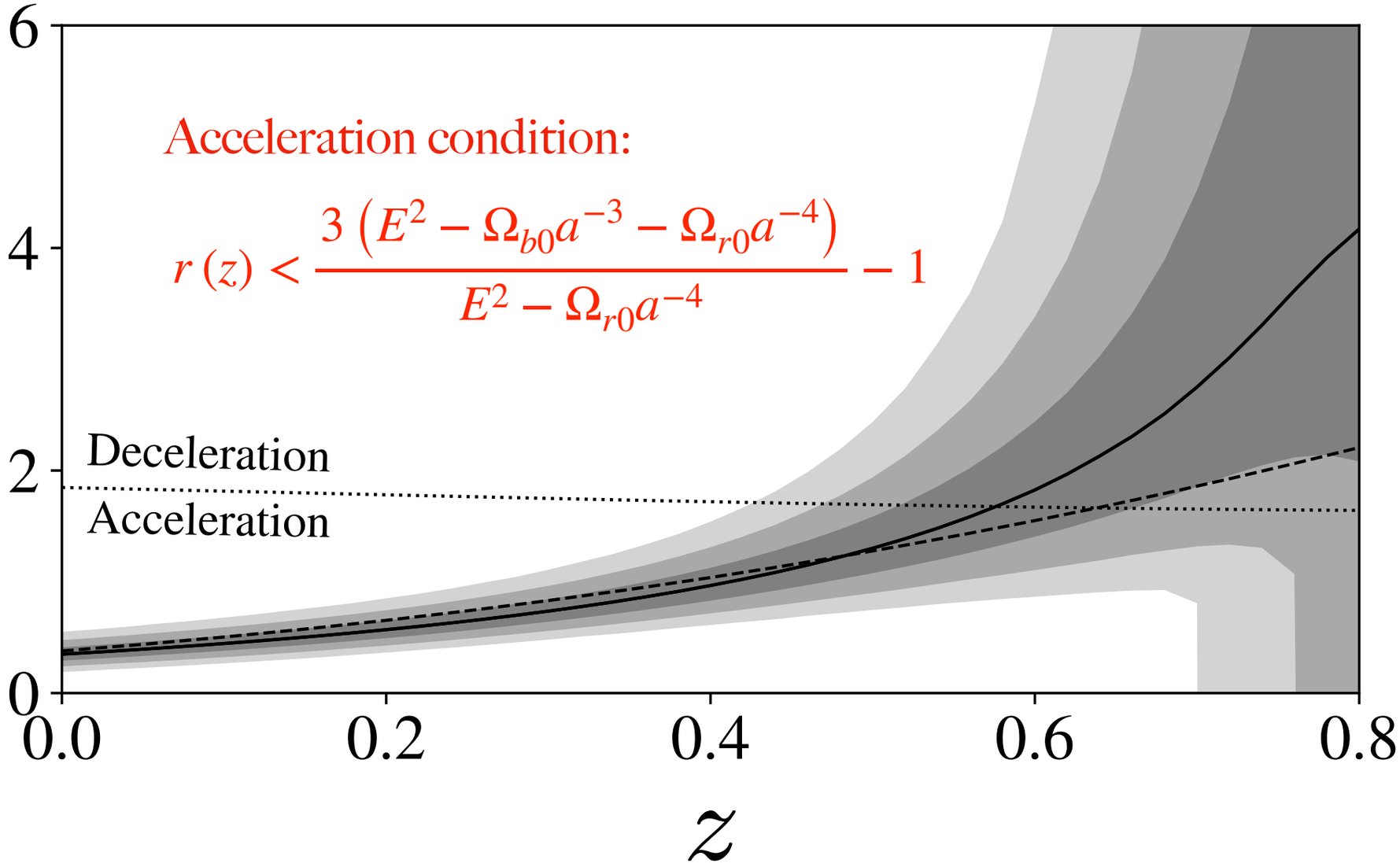
Dark degeneracy I

Ratio between CDM and DE energy densities

$$r(z) = \frac{-2(z+1)EE' + 6E^2 - 3\Omega_{b0}(z+1)^3}{E\left[3E - 2(z+1)E'\right]}$$



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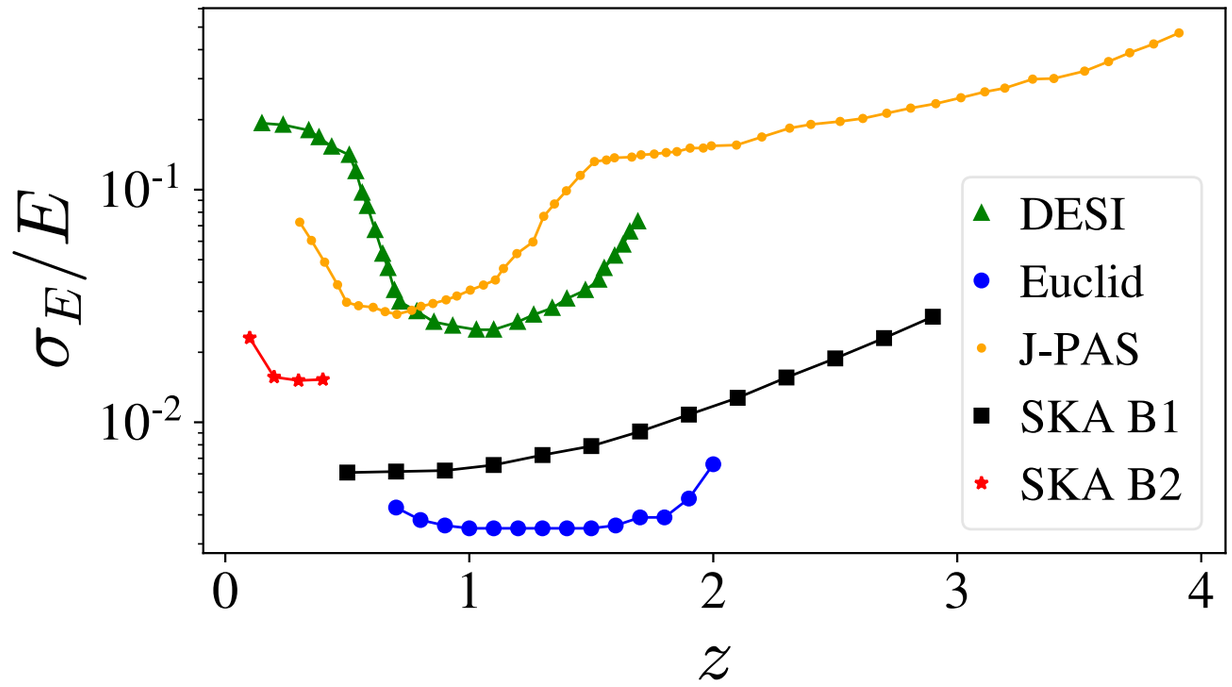
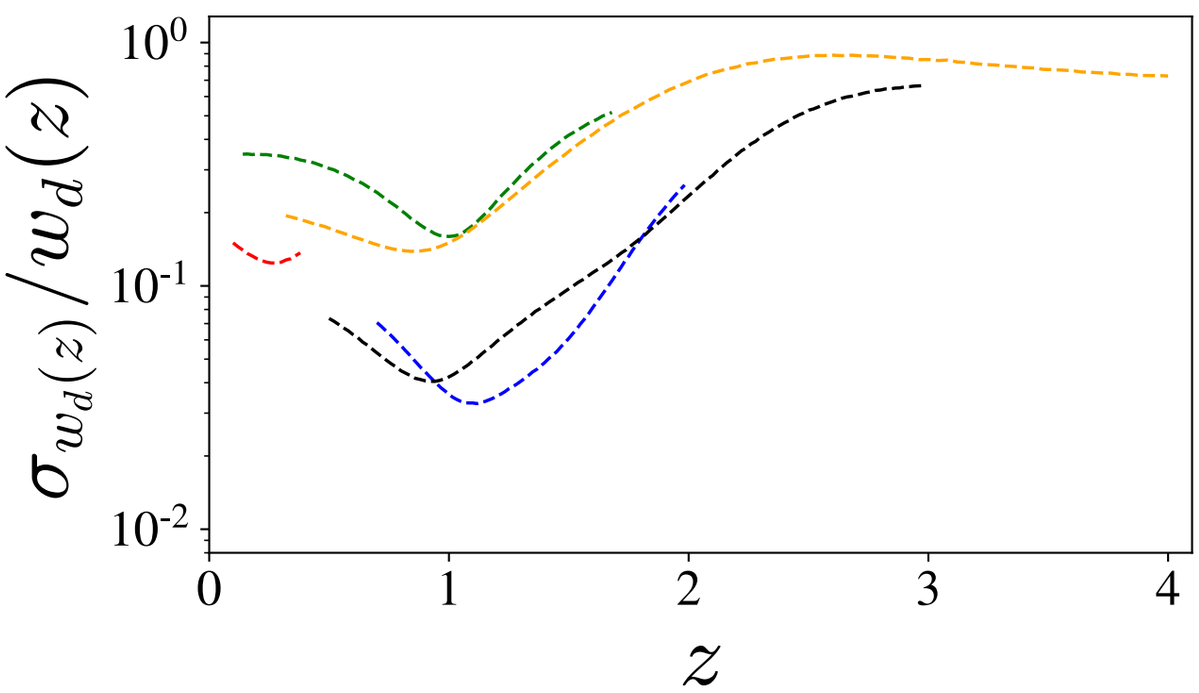
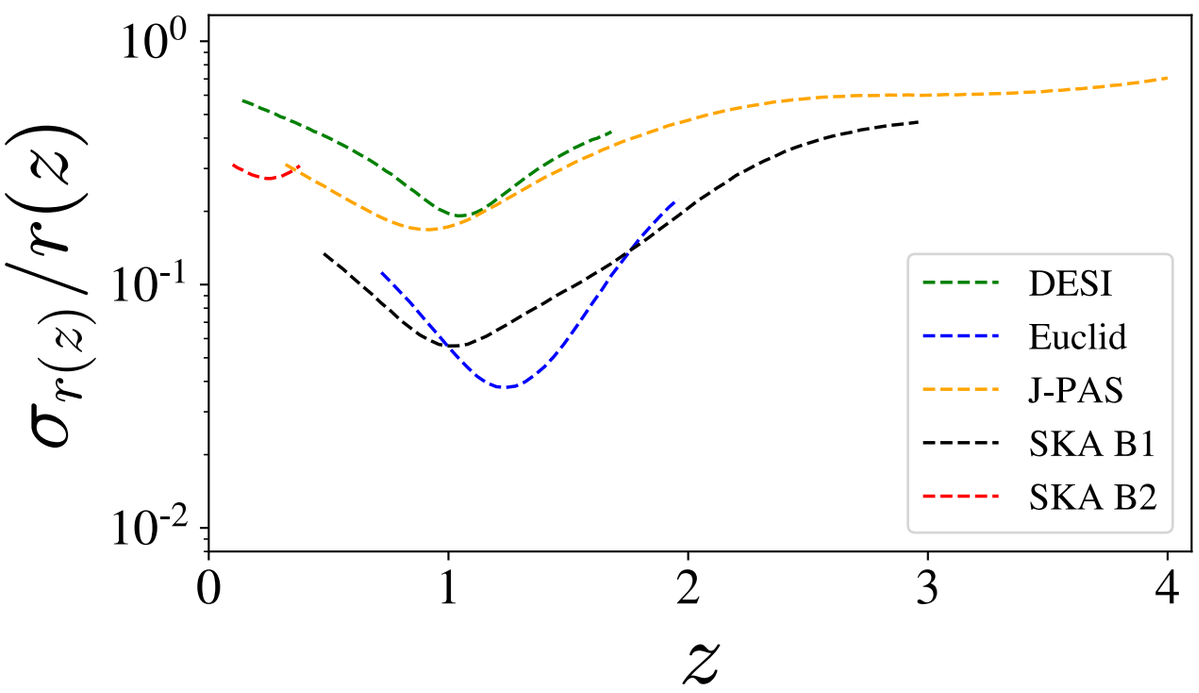
Dark degeneracy I

Future perspectives:



Forecast analysis





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Perturbative degeneracy: scalar perturbations

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Conditions on the anisotropic stress and sound speed

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Naturally satisfied (if others are!)

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Divergenceless and traceless

Einstein equation

$$h_{ij}'' + 2\mathscr{H} h_{ij}' + k^2 h_{ij} = 16\pi G a^2 \Pi_{ij}$$

Tensor part of the anisotropic stress

Dark degeneracy I

Perturbative degeneracy: tensor perturbations

Gauge-independent tensor metric perturbations

$$ds^2 = a^2(\tau)\left[-d\tau^2 + \left(\delta_{ij} + h_{ij}\right) dx^i dx^j\right]$$

Divergenceless and traceless

Einstein equation

$$h_{ij}'' + 2\mathscr{H} h_{ij}' + k^2 h_{ij} = 16\pi G a^2 \Pi_{ij}$$

Tensor part of the anisotropic stress

The only necessary condition for ensuring the dark degeneracy is that the tensor anisotropic stress is the same in both approaches. In the case where the tensor anisotropic stress is absent in both cases, the dark degeneracy for tensor perturbations is a trivial consequence of the background degeneracy.

Dynamical/Interacting mapping

Two approaches:

1. Dynamical DE: All components are independently conserved, and DE has a dynamical EoS parameter $w_X(a)$.

2. Interacting vacuum DE: Phenomenological source term Q responsible for energy transfer between CDM and DE ($w_X=-1$).

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From a theoretical point of view, the dark degeneracy consists in imposing that the left-hand side of Einstein's field equations must be equivalent in different approaches and therefore finding the conditions that ensure that the right-hand side is also identical.

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Ira Wasserman, Phys.Rev. D66 (2002) 123511

Martin Kunz, Phys.Rev.D80:123001,2009

S. Carneiro, H. A. Borges, JCAP 1406 (2014) 010

Dynamical/Interacting mapping

Snowmass2021 - Letter of Interest

E. di Valentino, L. A. Anchordoqui, Ö. Akarsu, Y. Ali-Haimoud, L. Amendola et al.

Cosmology Intertwined II: The Hubble Constant Tension

Possible solutions – Models addressing the H_0 tension are extremely difficult to concoct.

- Interacting dark energy (IDE) models, where dark matter (DM) and DE share interactions other than gravitational[52–64;210–213]. The IDE model solves the tension with R19 within one standard deviation, leading to a preference for a non-zero DE-DM coupling at more than 5 standard deviations[62;63], fixing the DE equation of state to a cosmological constant. However, this category can be further extended into two classes[63]: *(i)* models with $w < -1$ in which energy flows from DE to DM, *(ii)* models with $w > -1$ in which energy flows from DM to DE. Related models can be realized in string theory[162–164].

Cosmology Intertwined III: $f\sigma_8$ and S_8

Solutions – There are many papers investigating this tension[23;27;38–67], but the solutions proposed are not enough to put in agreement all the cosmological available data[68–70]. We can distinguish the following categories:

- Interacting dark energy models, where the energy flows from the dark matter to the dark energy[43;44].

Dynamical/Interacting mapping

Interaction in the dark sector:

$T^{\mu\nu}{}_{;\nu} = Q^{\mu}$ (Interaction term)

Dynamical/Interacting mapping

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perfect fluid $\Rightarrow$ $Q^{\mu} = \left(Q, \vec{0}\right)$

At the background level, the interaction is described through a scalar function!

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$$T^{\mu\nu}{}_{;\nu} = T^{\mu\nu}_{r\ ;\nu} + T^{\mu\nu}_{b\ ;\nu} + T^{\mu\nu}_{c\ ;\nu} + T^{\mu\nu}_{x\ ;\nu} = \left(0, \vec{0}\right)$$

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Background energy conservation: Coupled system!

$$\dot{\rho}_c + 3H\rho_c = -Q$$

$$\dot{\rho}_x + 3H\rho_x\left(1 + w_x\right) = Q$$

The interacting model is defined by the explicit expression of Q!

Dynamical/Interacting mapping

Physical ansatz: $Q = 3H\gamma R\left(\rho_c, \rho_x\right)$

Rewriting background energy conservation

Equations are still coupled!

$$\dot{\rho}_c + 3H\rho_c\left(\gamma\frac{R}{\rho_c} + 1\right) = 0 \quad \text{and} \quad \dot{\rho}_x + 3H\rho_x\left[1 + w_x(a) - \gamma\frac{R}{\rho_x}\right] = 0$$

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Introducing the ratio between CDM and DE energy densities...

$$\tilde{r} \equiv \frac{\tilde{\rho}_c}{\tilde{\rho}_x} \qquad \Rightarrow \qquad \dot{\tilde{r}} = \tilde{r}\left(\frac{\dot{\tilde{\rho}}_c}{\tilde{\rho}_c} - \frac{\dot{\tilde{\rho}}_x}{\tilde{\rho}_x}\right)$$

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... and using background energy conservation

$$\dot{r} + 3Hr\left(\gamma R\frac{\rho_c + \rho_x}{\rho_c\ \rho_x} - w_x\right) = 0$$

Dynamical/Interacting mapping

Alternative description of interacting models based on the ration between CDM and DE energy densities:

$$r \equiv \frac{\rho_c}{\rho_x}$$

Dynamical/Interacting mapping

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$$r \equiv \frac{\rho_c}{\rho_x}$$

$$f(r) \equiv \frac{Q}{3H\gamma}\left(\frac{\rho_c + \rho_x}{\rho_c\ \rho_x}\right)$$

$f(r)$	Q	$r(a)$
1	$3H\gamma\frac{\rho_c\rho_x}{\rho_c+\rho_x}$	$r_0 a^{-3(\gamma+1)}$
$1+r$	$3H\gamma\rho_c$	$\frac{(\gamma+1)\,r_0}{a^{3(\gamma+1)}\,(\gamma+\gamma r_0+1)-\gamma r_0}$
$1+\frac{1}{r}$	$3H\gamma\rho_x$	$\frac{a^{-3(\gamma+1)}\left(-\gamma a^{3(\gamma+1)}+\gamma+\gamma r_0+r_0\right)}{\gamma+1}$

Equivalence between the approaches

Dynamical/Interacting mapping

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Equivalence between the approaches

Differential equation for r:

$$\dot{r} + 3Hr\left[f(r) - w_x(a)\right] = 0$$

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Equivalence between the approaches

Differential equation for r:

$$\dot{r} + 3Hr\left[f(r) - w_x(a)\right] = 0$$

Separable conservation equations:

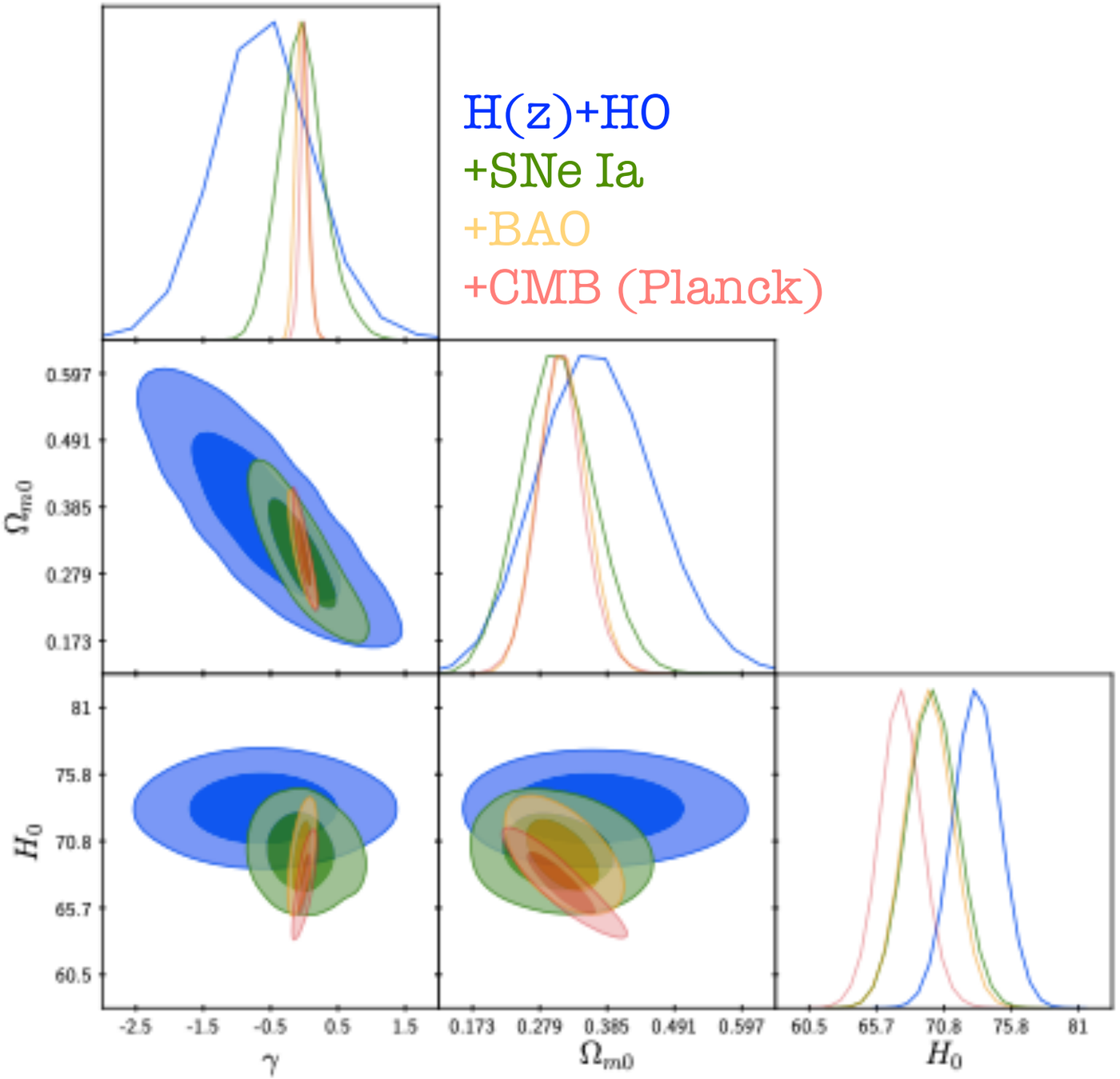
$$\dot{\rho}_c + 3H\rho_c\left[\gamma\frac{f(r)}{1+r} + 1\right] = 0$$

$$\dot{\rho}_x - 3H\gamma\rho_x\left(\gamma\frac{f(r)}{1+r^{-1}}\right) = 0$$

One can solve the equation for r, and one of the conservation equations!

Dynamical/Interacting mapping

Interacting Dark Energy Models: specific cases



Studied models

$f(r) = 1 \quad \Rightarrow \quad Q = 3H\gamma \dfrac{\rho_c \rho_x}{\rho_c + \rho_x}$
$f(r) = r \quad \Rightarrow \quad Q = 3H\gamma \dfrac{\rho_c^2}{\rho_c + \rho_x}$
$f(r) = \dfrac{1}{r} \quad \Rightarrow \quad Q = 3H\gamma \dfrac{\rho_x^2}{\rho_c + \rho_x}$
$f(r) = 1 + r \quad \Rightarrow \quad Q = 3H\gamma\rho_c$
$f(r) = 1 + \dfrac{1}{r} \quad \Rightarrow \quad Q = 3H\gamma\rho_x$

RvM, L. Casarini, D.F. Mota, W. Zimdahl, Phys. Dark Univ. 23 (2019)

Dynamical/Interacting mapping

Background degeneracy:

At background level, one must impose that the Hubble rate (or equivalently the scale factor as a function of time) is identical in both approaches...

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... all the background information in Hubble rate about dark sector comes from the unified dark EoS parameter

$$\frac{H^2}{H_0^2} = \Omega_{r0}a^{-4} + \Omega_{b0}a^{-3} + \Omega_{d0}\exp\left[-3\int\frac{1+w_d(a')}{a'}da'\right]$$

Background degeneracy condition:

$$\boxed{\bar{w}_d(a) = \tilde{w}_d(a)}$$

Dynamical/Interacting mapping

Background degeneracy: degeneracy condition

Mapping between dynamical and interacting approaches

$$\tilde{r}(a) = -\frac{1 + \bar{r}_0 a^{-3} \exp\left[3\int \frac{1+\bar{w}_x(\hat{a})}{\hat{a}} d\hat{a}\right]}{\bar{w}_x(a)} - 1$$

This equation shows how any dynamical DE parameterisation, characterized by $w_x(a)$, can be mapped into an interacting dark sector description, characterized by r(a), which yields exactly the same background expansion dynamics!

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Interacting source function

$$\tilde{f}(\tilde{r}) = -\frac{\tilde{r}'a}{3\tilde{r}} - 1$$

Dynamical/Interacting mapping

Background degeneracy: energy density solutions

It is not necessary to solve the background energy conservation equations to compute energy densities in the interacting approach.

$$\bar{w}_x \frac{\bar{\rho}_x}{\bar{\rho}_c + \bar{\rho}_x} = -\frac{\tilde{\rho}_x}{\tilde{\rho}_c + \tilde{\rho}_x}$$

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Identical!

Dynamical/Interacting mapping

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$$\bar{w}_x \frac{\bar{\rho}_x}{\bar{\rho}_c + \bar{\rho}_x} = -\frac{\tilde{\rho}_x}{\tilde{\rho}_c + \tilde{\rho}_x} \quad \Rightarrow \quad \begin{cases} \tilde{\rho}_c = \bar{\rho}_c + \bar{\rho}_x \left(1 + \bar{w}_x\right) \\ \tilde{\rho}_x = -\bar{w}_x \bar{\rho}_x \end{cases}$$

Identical!

Even if the Hubble rate is the same in both approaches, the current values of the CDM and DE density parameters are different!

The following conditions are satisfied:

$$\bar{w}_x(a=1) = w_0 \qquad \Rightarrow \qquad \tilde{\Omega}_{c0} = \bar{\Omega}_{c0} + \bar{\Omega}_{x0}\left(1 + w_0\right)$$

Dynamical/Interacting mapping

Specific case: CPL

1. DE EoS:

$$\bar{w}(z) = \bar{w}_0 + (1 - a)\,\bar{w}_a$$

In our paper also wCDM and BA parameterisation are also considered

Dynamical/Interacting mapping

Specific case: CPL

1. DE EoS:

$$\bar{w}(z) = \bar{w}_0 + (1-a)\,\bar{w}_a$$

In our paper also wCDM and BA parameterisation are also considered

2. The degeneracy is broken at linear level, and one can assess the observational consequences in the CMB power spectrum and in linear matter power spectrum:

$$\sigma_x = \tilde{\sigma}_x = 0 \qquad\qquad c_s^2 = \tilde{c}_s^2 = 1$$

Only the second assumption breaks the dark degeneracy and it is a common choice for dynamical and interacting models.

Dynamical/Interacting mapping

Specific case: dynamical CPL

CDM energy density

$$\bar{\rho}_c = \frac{3H_0^2}{8\pi G}\bar{\Omega}_{c0}a^{-3}$$

DE energy density

$$\bar{\rho}_x = \frac{3H_0^2}{8\pi G}\bar{\Omega}_{x0}a^{-3\left(1+w_0+w_a\right)}\exp\left[3w_a\left(a-1\right)\right]$$

Ratio between CDM and DE energy densities

$$\bar{r}\left(a\right) = \bar{r}_0\exp\left[3w_a\left(1-a\right)\right]a^{3\left(w_0+w_a\right)}$$

Dynamical/Interacting mapping

Specific case: "interacting" CPL

CDM energy density

$$\tilde{\rho}_c = \frac{3H_0^2}{8\pi G}\left\{\bar{\Omega}_{c0} + \bar{\Omega}_{x0}\left[1 + w_0 + w_a(1-a)\right]\exp\left[-3w_a(1-a)\right]a^{-3(w_0+w_a)}\right\}a^{-3}$$

DE energy density

$$\tilde{\rho}_x = -\frac{3H_0^2}{8\pi G}\bar{\Omega}_{x0}\left[w_0 + w_a(1-a)\right]\exp\left[-3w_a(1-a)\right]a^{-3(1+w_0+w_a)}$$

Ratio between CDM and DE energy densities

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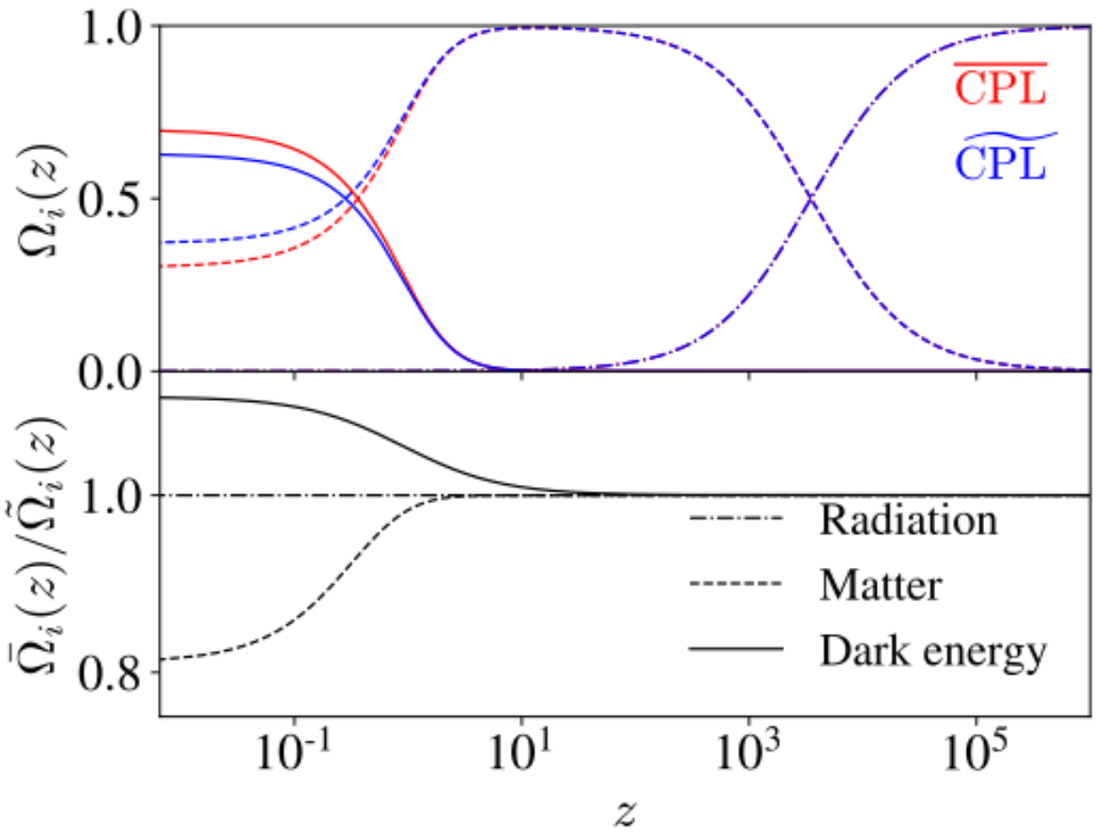

RvM, L. Lombriser, M. Kunz, V. Marra, L. Casarini, J. Alcaniz, Phys. Dark Univ. 28 (2020)

Dynamical/Interacting mapping

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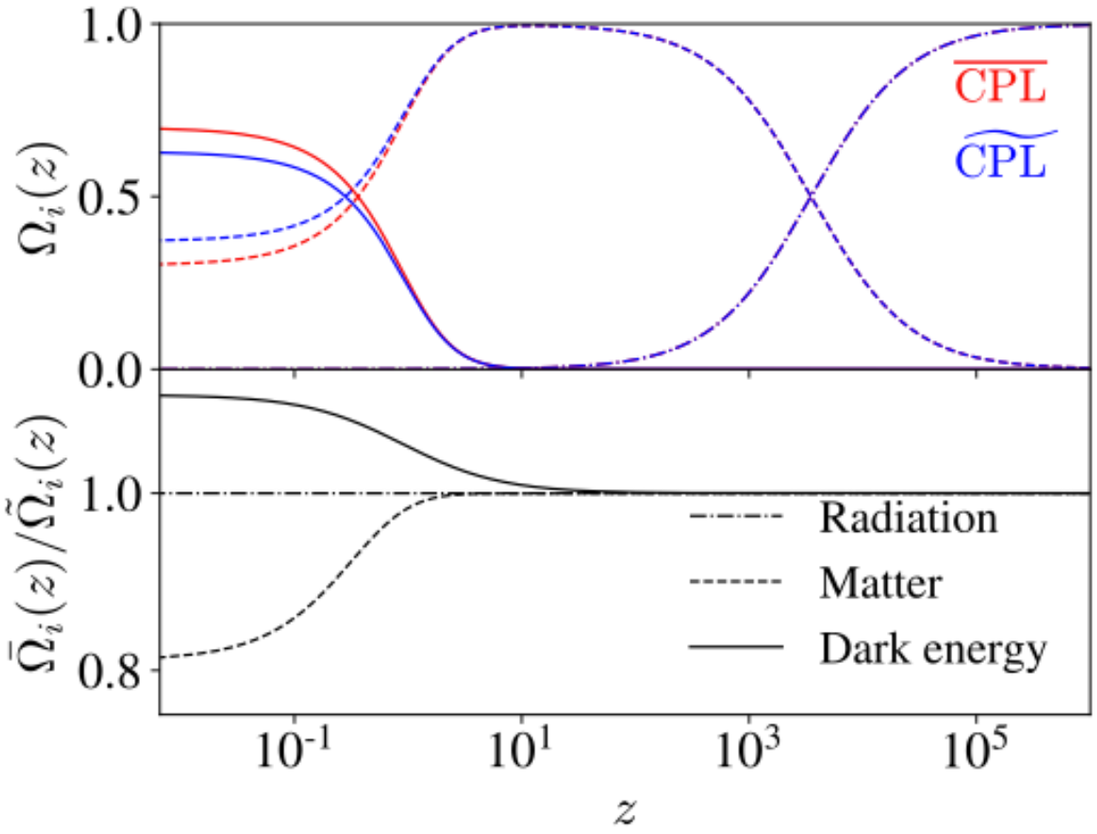

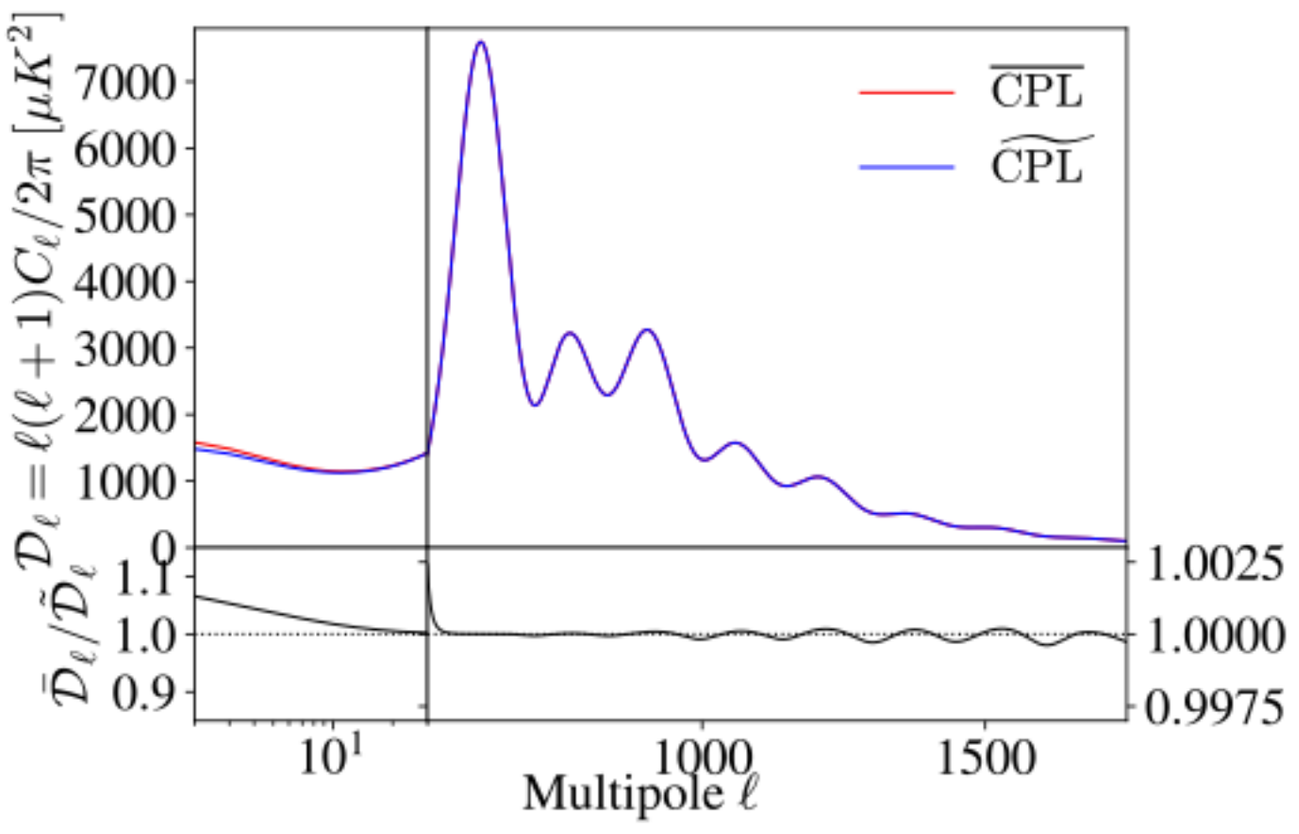

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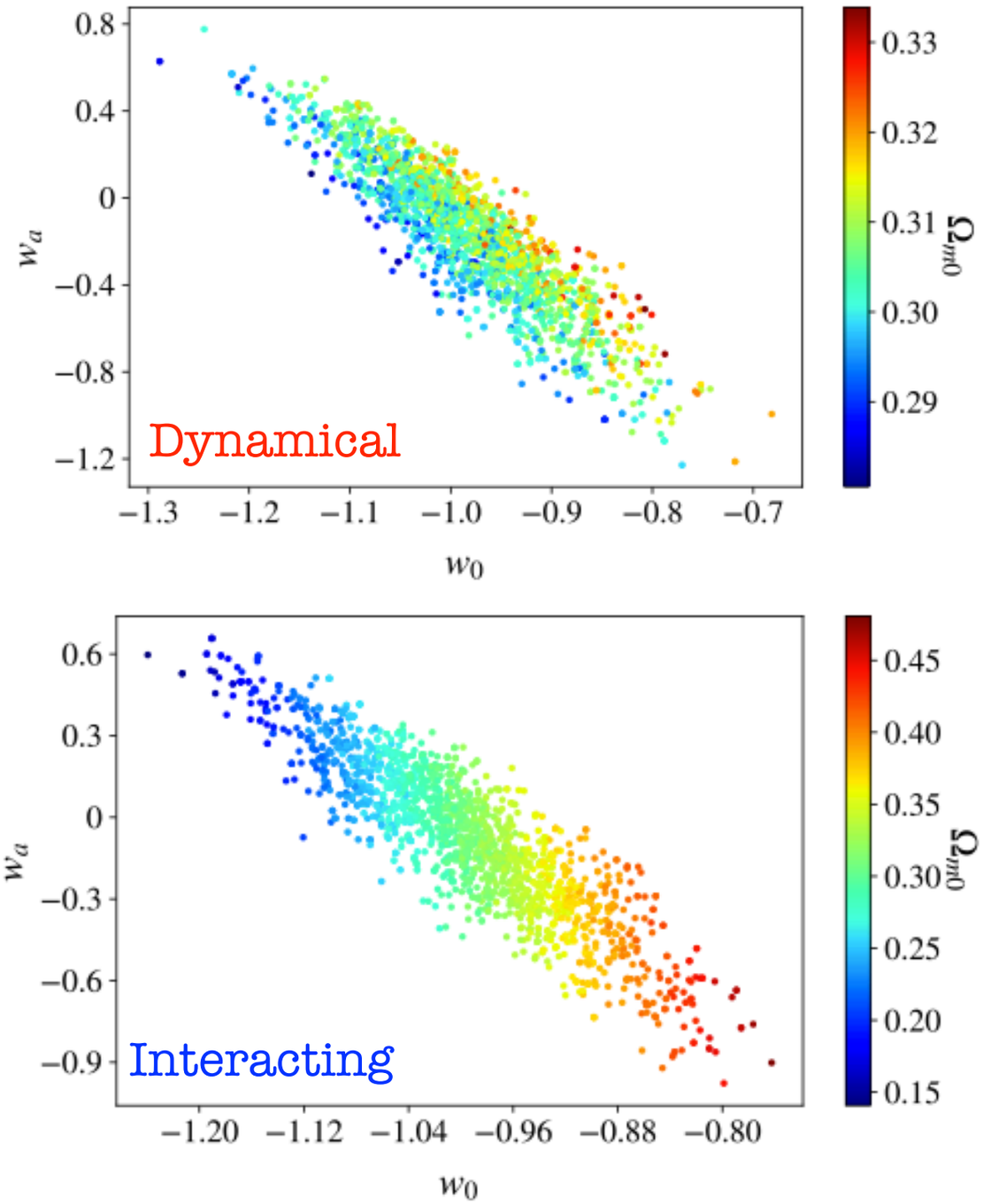
Dynamical/Interacting mapping

Two important features:



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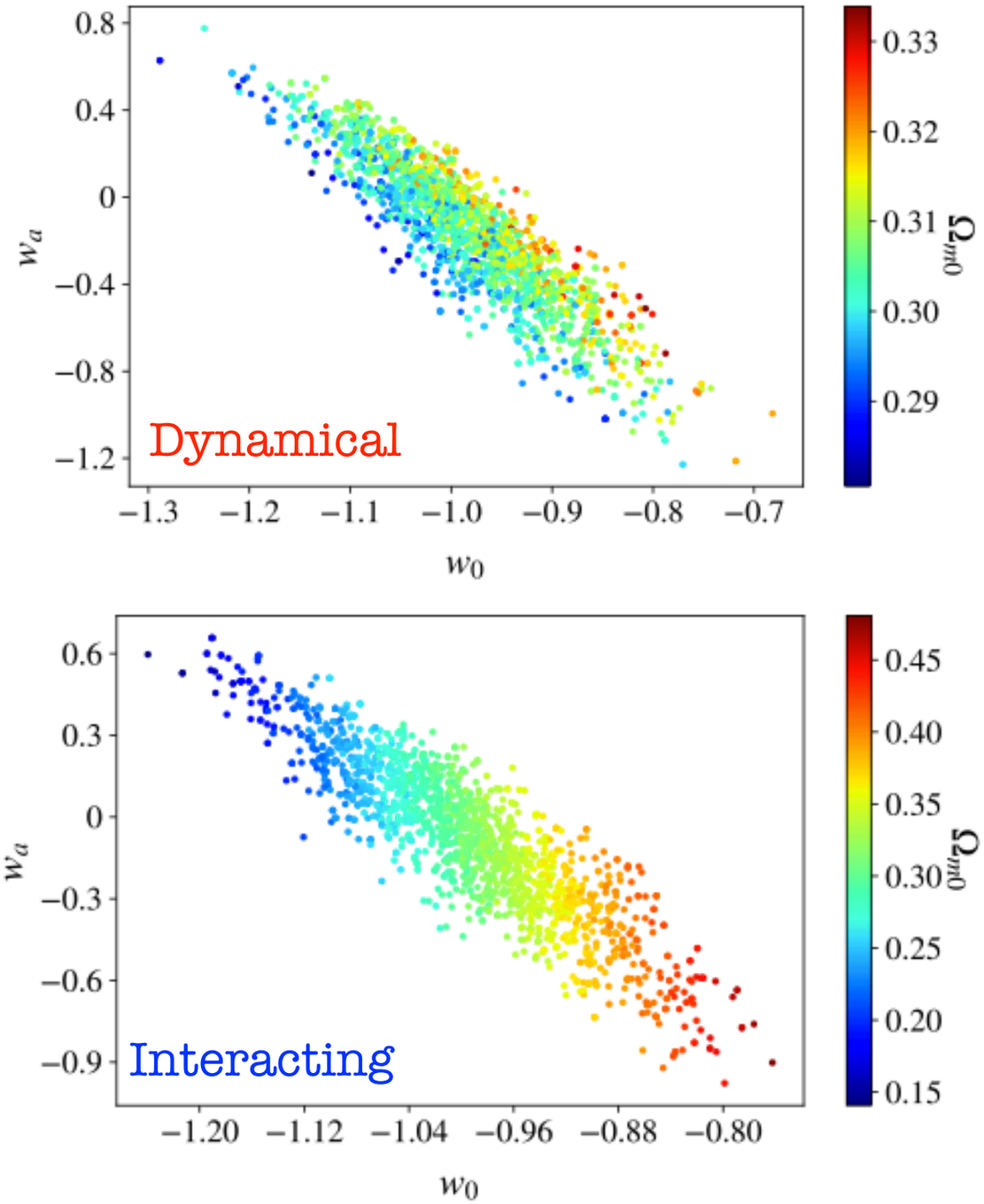
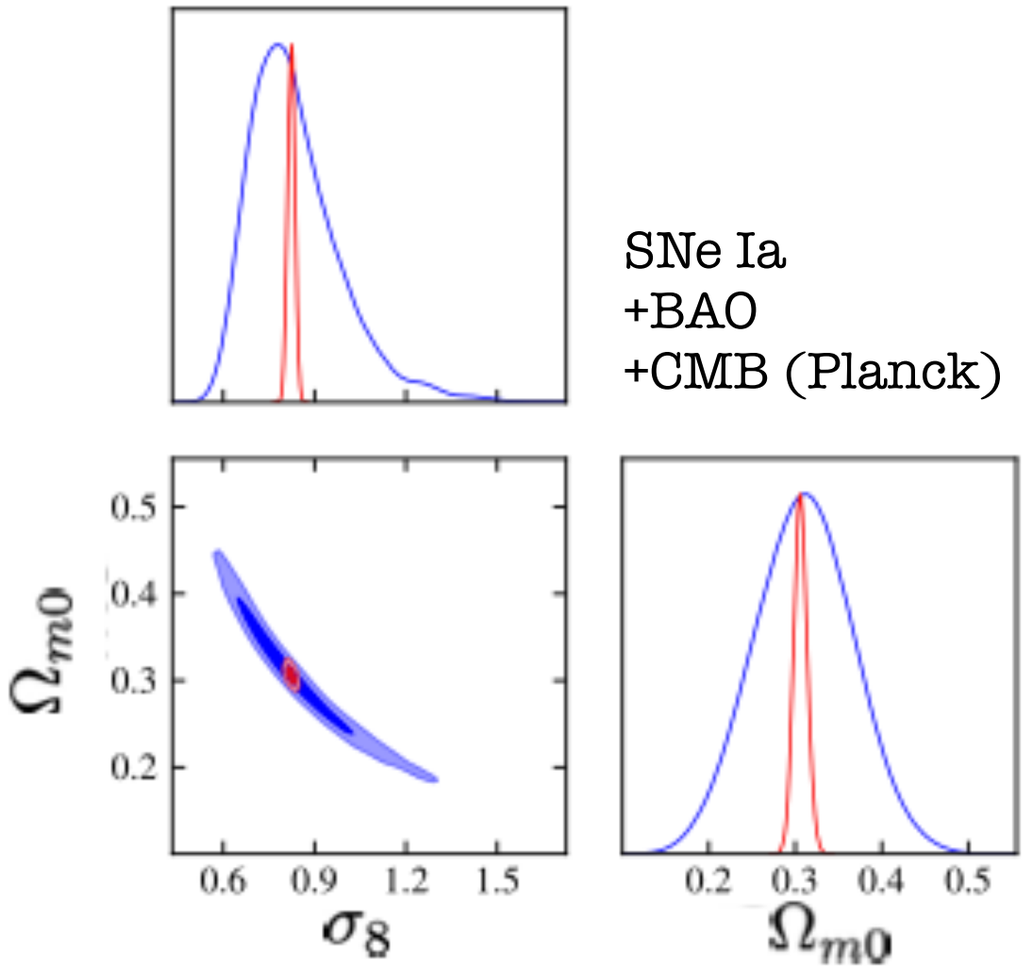


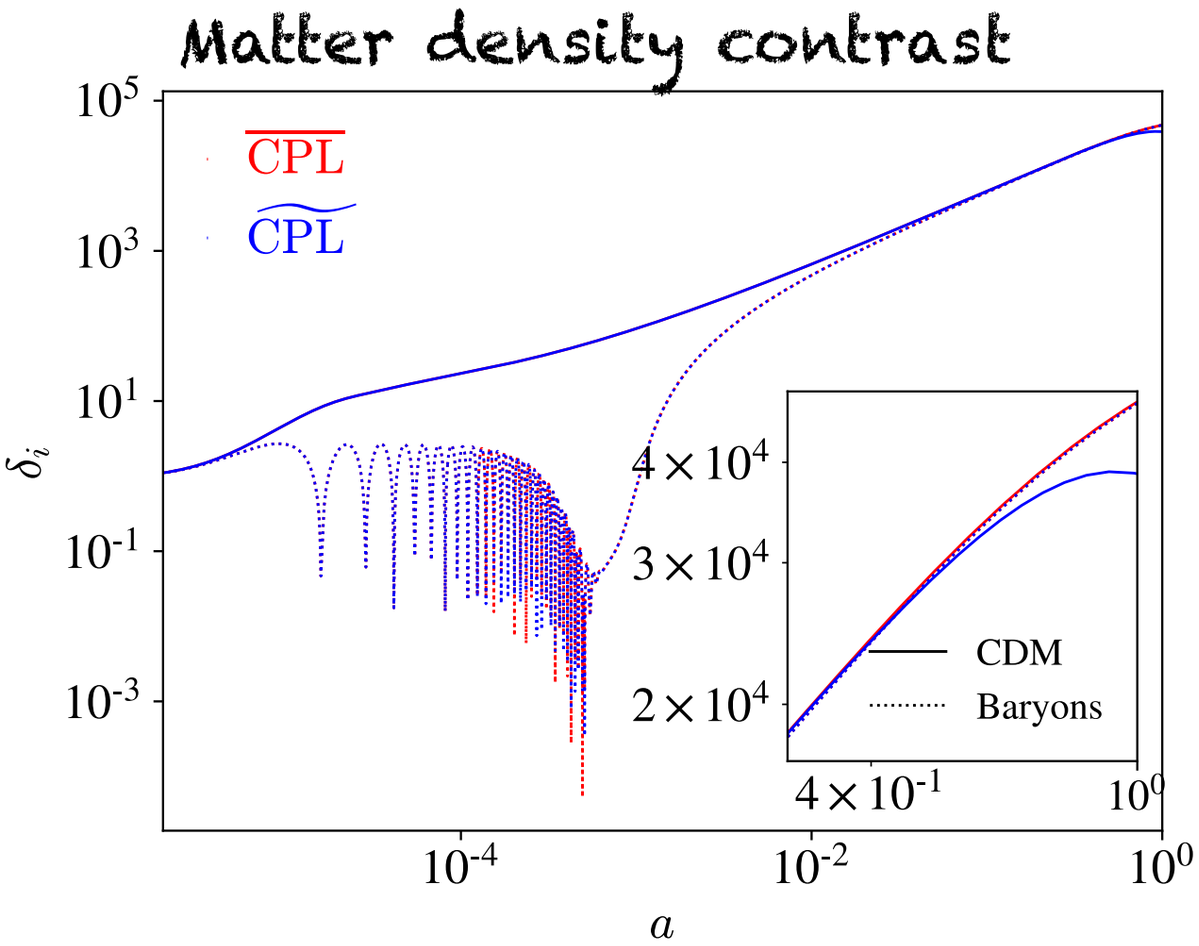
Interacting approach can be used to ease σ_8 tension?

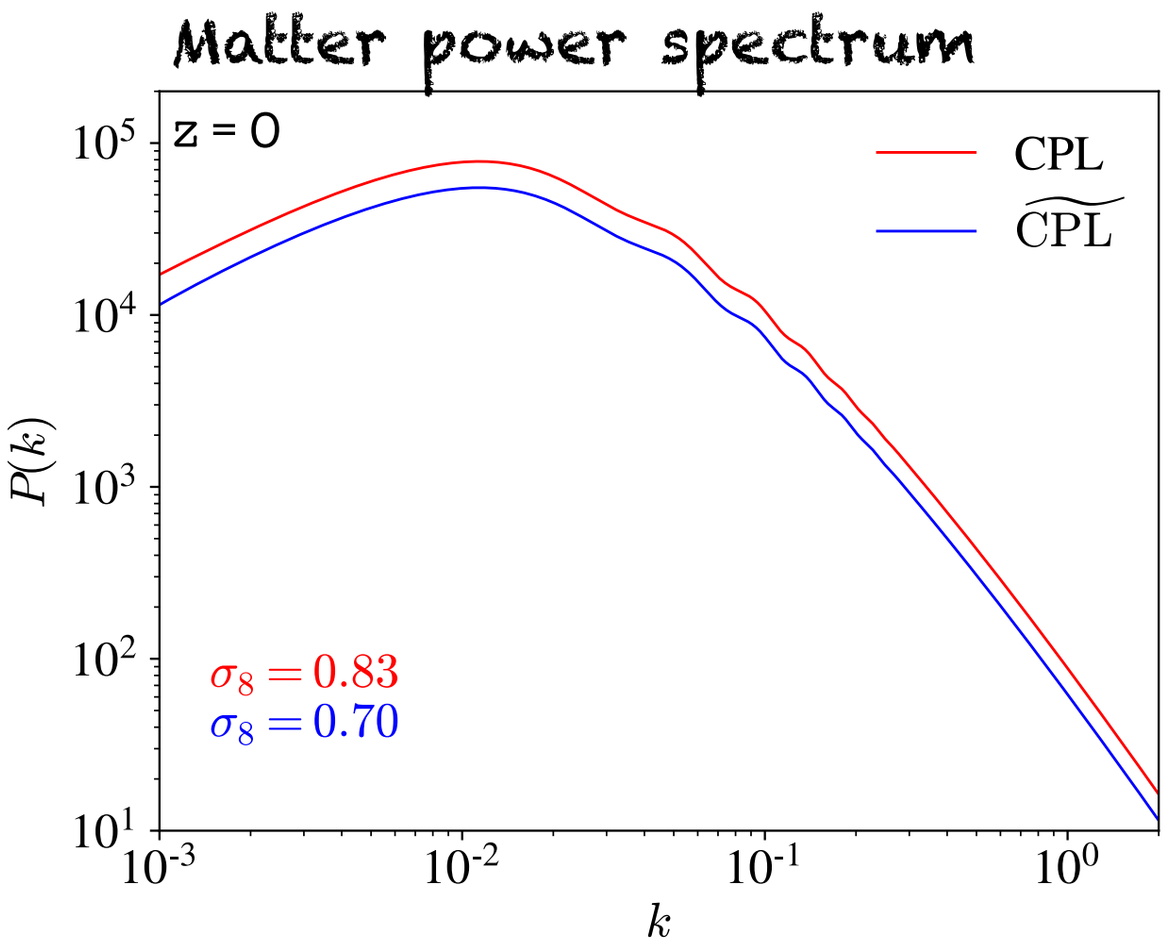


RvM, L. Lombriser, M. Kunz, V. Marra, L. Casarini, J. Alcaniz, Phys. Dark Univ. 28 (2020)

Dynamical/Interacting mapping

Structure formation: matter perturbation





RvM, L. Lombriser, M. Kunz, V. Marra, L. Casarini, J. Alcaniz, Phys. Dark Univ. 28 (2020)

Dark degeneracy II

General action: Modified gravity + Non-minimally coupled matter

$$\mathcal{S} = \int d^4x \left\{ \sqrt{-g}\mathcal{L}_g \left[g_{\mu\nu}; \phi \right] + \sqrt{-\tilde{g}}\mathcal{L}_m \left[\tilde{g}_{\mu\nu}; \psi \right] \right\}$$

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Gravity part: Horndeski picture

1. Kineticity (α_K): Generalization of the kinetic term;

2. Braiding ($\alpha_{\rm B}$): Mixing of the kinetic terms and the scalar field terms;

3. Varying Planck mass ($\alpha_{\rm M}$): $\alpha_M \equiv \dfrac{d\ \ln(M^2)}{d\ \ln(a)}$

4. Deviations of the speed of GWs ($\alpha_{\rm T}$): $c_T = 1 + \alpha_T$

E.Bellini and I.Sawicki, JCAP (2014).

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} Anisotropic stress

E.Bellini and I.Sawicki, JCAP (2014).

Dark degeneracy II

Model Class		$\alpha_{\rm K}$	$\alpha_{\rm B}$	$\alpha_{\rm M}$	$\alpha_{\rm T}$
ΛCDM		***0***	***0***	***0***	***0***
cuscuton ($w_X \neq -1$)	[71]	0	0	0	0
quintessence	[1, 2]	$(1-\Omega_{\rm m})(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_{\rm m})(1+w_X)}{c_{\rm s}^2}$	0	0	0
kinetic gravity braiding	[47–49]	$m^2(n_m+\kappa_\phi)/H^2M_{\rm Pl}^2$	$m\kappa/HM_{\rm Pl}^2$	0	0
galileon cosmology	[57]	$-3/2\alpha_{\rm M}^3H^2r_{\rm c}^2e^{2\phi/M}$	$\alpha_{\rm K}/6-\alpha_{\rm M}$	$-2\dot{\phi}/HM$	0
BDK	[26]	$\dot{\phi}^2K_{,\dot{\phi}\dot{\phi}}e^{-\kappa}/H^2M^2$	$-\alpha_{\rm M}$	$\dot{\kappa}/H$	0
metric $f(R)$	[3, 72]	0	$-\alpha_{\rm M}$	$B\dot{H}/H^2$	0
MSG/Palatini $f(R)$	[73, 74]	$-3/2\alpha_{\rm M}^2$	$-\alpha_{\rm M}$	$2\dot{\phi}/H$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot{\xi}}{M^2+H\dot{\xi}}$	$\frac{\dot{H}\dot{\xi}+H\ddot{\xi}}{H\left(M^2+H\dot{\xi}\right)}$	$\frac{\ddot{\xi}-H\dot{\xi}}{M^2+H\dot{\xi}}$

E.Bellini and I.Sawicki, JCAP (2014).

Dark degeneracy II

General action: Modified gravity + Non-minimally coupled matter

$$\mathcal{S} = \int d^4x \left\{ \sqrt{-g}\mathcal{L}_g \left[g_{\mu\nu}; \phi \right] + \sqrt{-\tilde{g}}\mathcal{L}_m \left[\tilde{g}_{\mu\nu}; \psi \right] \right\}$$

Matter part: Conformal/Disformal coupling

Dark degeneracy II

General action: Modified gravity + Non-minimally coupled matter

$$\mathcal{S} = \int d^4x \left\{ \sqrt{-g}\mathcal{L}_g \left[g_{\mu\nu}; \phi \right] + \sqrt{-\tilde{g}}\mathcal{L}_m \left[\tilde{g}_{\mu\nu}; \psi \right] \right\}$$

Matter part: Conformal/Disformal coupling

$$\tilde{g}_{\mu\nu} = C\left(\phi\right) g_{\mu\nu} + D\left(\phi\right) \phi_{,\mu}\phi_{,\nu}$$

1. Conformal transformation ($\alpha_{\rm C}$): $\alpha_C \equiv \dfrac{\phi'}{2}\dfrac{d\ln(C)}{d\phi}$

2. Disformal transformation ($\alpha_{\rm D}$): $\alpha_D \equiv \dfrac{(H\phi')D}{C-(H\phi')D}$

Dark degeneracy II

General dynamics

Einstein equations:

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0$$

Conservation equation: invariance of the matter action under an diffeomorphism $x^\mu \to x^\mu + \xi^\mu$

$$\nabla_\mu T^\mu_\nu + Q\partial_\nu\phi = 0$$

$$Q \equiv -\frac{1}{\sqrt{-g}}\frac{\delta \mathcal{S}}{\delta\phi} = -\frac{C_{,\phi}}{2C}T - \frac{D_{,\phi}}{2C}T^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \nabla_\mu\left(T^{\mu\nu}\partial_\nu\phi\frac{D}{C}\right)$$

Scalar field equation:

$$\frac{1}{\sqrt{g}}\frac{\delta S_g}{\delta\phi} - Q = 0$$

Dark degeneracy II

Background equations

Einstein equations:

$$H^2 = \frac{1}{3M^2}\left(\rho_m + \rho_x\right) \quad \text{and} \quad H' = -\frac{3}{2}H - \frac{1}{2HM^2}\left(p_m + p_x\right)$$

Conservation equation:

$$Q = \frac{H\rho_m}{1+\alpha_D}\left\{\alpha_C\left[1 - 3w_m\left(1+\alpha_D\right)\right] + \alpha_D\left(3 + \frac{\rho'_m}{\rho_m}\right) + \frac{\alpha'_D}{2\left(1+\alpha_D\right)}\right\}$$

$$\rho'_m + 3\left(1 + w_m - \gamma\right)\rho_m = 0$$

Scalar field equation:

$$\gamma \equiv \frac{1}{3}\alpha_C\left[1 - 3w_m\left(1+\alpha_D\right)\right] - w_m\alpha_D + \frac{\alpha'_D}{6\left(1+\alpha_D\right)}$$

$$\rho'_x + \left[3\left(1+w_x\right) - \alpha_M\right]\rho_x = \left(\alpha_M - 3\gamma\right)\rho_m$$

Dark degeneracy II

Linear equations

Einstein equations:

$$6\left(\alpha_B+1\right)\Phi'+2k_H^2\Phi-\left(12\alpha_B-\alpha_K+6\right)\Psi+\left(6\alpha_B-\alpha_K\right)H\pi'$$
$$-2\left[-k_H^2\alpha_B+3\left(\alpha_B+1\right)\frac{H'}{H}+\frac{3\rho_m}{2M^2H^2}\left(1+w_m-\gamma_m\right)\right]H\pi=-\frac{\delta\rho_m}{M^2H^2}$$

$$\Phi'-\left(\alpha_B+1\right)\Psi+\alpha_BH\pi'-\left[\frac{H'}{H}+\frac{\rho_m}{2M^2H^2}\left(1+w_m\right)\right]H\pi=-\frac{\rho_m}{2M^2H^2}\left(1+w_m\right)\frac{v_m}{k_H}$$

$$\Psi+\left(\alpha_T+1\right)\Phi+\left(\alpha_M-\alpha_T\right)H\pi=\frac{\sigma_m}{M^2}$$

$$\Phi''+\left(\alpha_M+\frac{H'}{H}+3\right)\Phi'-\left(\alpha_B+1\right)\Psi'-\left[\alpha_B'+\left(\alpha_B+1\right)\left(\alpha_M+\frac{H'}{H}+3\right)\right.$$
$$\left.-\frac{\rho_m}{2M^2H^2}\left(1+w_m\right)\right]\Psi+\alpha_BH\pi''+\left[\alpha_B'+\alpha_B\left(\alpha_M+2\frac{H'}{H}+3\right)-\frac{H'}{H}\right.$$
$$\left.-\frac{\rho_m}{2M^2H^2}\left(1+w_m\right)\right]H\pi'-\left\{\left(\alpha_M+\frac{H'}{H}+3\right)\frac{H'}{H}+\frac{H''}{H}\right.$$
$$\left.+\frac{\rho_m}{2M^2H^2}\left[w_m'-3\left(1+w_m-\gamma_m\right)\right]\right\}H\pi=\frac{\delta p_m}{2M^2H^2}-\frac{1}{3}k_H^2\frac{\sigma_m}{M^2}$$

Dark degeneracy II

Linear equations

Conservation equations:

$$\delta'_m = -3\left(\alpha_C+1\right)\left(\alpha_D+1\right)\left(\frac{\delta p_m}{\rho_m}-w_m\delta_m\right)+\left(1+w_m\right)k_H v_m+3\left[1+\left(\alpha_D+1\right)w_m\right]\Phi'$$
$$-2\left(\alpha_D+1\right)\left[\alpha_C\left(1-3w_m\right)-3\gamma\right]\Psi+\alpha_D\left(\Psi'-H\pi''+w_m k_H^2 H\pi\right)+\Bigg[2\alpha_C\left(\alpha_D+1\right)$$
$$\left(1-3w_m\right)-\frac{H'}{H}-3w_m\alpha_D-3\gamma\left(2\alpha_D+3\right)\Bigg]H\pi'-3\Bigg[\frac{H'}{H}\gamma+w_m\alpha_D\frac{H'}{H}\Big(\alpha_C+\alpha_D$$
$$\left(\alpha_C+1\right)\Big)w'_m+\gamma'\Bigg]H\pi$$

$$\frac{v'_m}{k_H}=-\frac{v_m}{k_H}\left(1-3w_m+3\gamma+\frac{w'_m}{1+w_m}\right)-\frac{\delta p_m}{\left(1+w_m\right)\rho_m}+\Psi+\frac{2H^2}{3\left(1+w_m\right)\rho_I}k_H^2\sigma_m$$
$$+\frac{3\gamma_m}{1+w_m}H\pi$$

Dark degeneracy II

Linear equations

Scalar field equation:

$$\left(\alpha_K + \frac{\rho_m}{M^2H^2}\alpha_D\right) H\pi'' + \left\{\alpha_K' + \alpha_K\left[\alpha_M + 3\left(\frac{H'}{H}+1\right)\right] + \frac{\rho_m}{M^2H^2}\left[\frac{H'}{H}\alpha_D\right.\right.$$

$$\left.\left. -2\alpha_C\left(\alpha_D+1\right)\left(1-3w_m\right) - 3w_m\alpha_D - 6\gamma\left(\alpha_D+1\right)\right]\right\} H\pi' + 6\left\{\frac{H'}{H}\left[\alpha_B' + \alpha_B\right.\right.$$

$$\left.\left(\alpha_M + \frac{H''}{H} + 2\frac{H'}{H} + 3\right) + \frac{H'}{H} + \frac{\rho_m}{2M^2H^2}\left(1+w_m\right) + \frac{\rho_m}{2M^2H^2}w_m\alpha_D\right]$$

$$\left. + \frac{\rho_m}{2M^2H^2}\left[w_m'\left(\alpha_D + \alpha_C\left(\alpha_D+1\right)\right) + 3\gamma\left(1+w_m-\gamma\right)\right]\right\} H\pi$$

$$-2k_H^2\left[\alpha_B' - \alpha_M + \alpha_T + \alpha_B\left(\alpha_M + \frac{H'}{H} + 1\right) + \frac{H'}{H} + \frac{\rho_m}{2M^2H^2}\left(1+w_m\right) + \frac{\rho_m}{2M^2H^2}\right.$$

$$\left. w_m\alpha_D\right] H\pi - 6\alpha_B\Phi'' - 6\left[\alpha_B' + \alpha_B\left(\alpha_M + 2\frac{H'}{H} + 3\right) + \frac{H'}{H} + \frac{\rho_m}{2M^2H^2}\left(1+w_m\right)\ldots = 0\right.$$

Dark degeneracy II

Semi-dynamical approach

Modified Einstein equations:

$$\Phi'' + A_1\Phi' + A_2\Phi = A_3\delta\rho_m + A_4\rho_m\frac{v_m}{k_H}$$

$$\Phi' + B_1\Phi + B_2\Psi = B_3\delta\rho_m + B_4\rho_m\frac{v_m}{k_H}$$

Dark degeneracy II

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$$\Phi' + B_1\Phi + B_2\Psi = B_3\delta\rho_m + B_4\rho_m\frac{v_m}{k_H}$$

Extension of the quasi-static approximation:

$$\Phi' \equiv f_\Phi\Phi$$

$$\zeta' \equiv f_\zeta\Phi \qquad \text{Comoving curvature perturbation} \quad \zeta \equiv \Phi - \frac{v_m}{k_H}$$

L. Lombriser and A.Taylor JCAP (2015).

Dark degeneracy II

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L. Lombriser and A.Taylor JCAP (2015).

Total matter velocity equation:

$$\frac{v_m}{k_H} = \frac{(f_\Phi - f_\zeta)\,\Phi - \Psi + 3\gamma_m\dfrac{[\Psi + (1+\alpha_T)\,\Phi]}{\alpha_M - \alpha_T}}{\dfrac{H'}{H} - 3\gamma_m}$$

Dark degeneracy II

Semi-dynamical approach

Modified Einstein equations:

$$\left[f'_\Phi + f_\Phi^2 + A_1 f_\Phi + A_2 + A_4 \rho_m \frac{(\alpha_M - \alpha_T)(f_\zeta - f_\Phi) - 3(\alpha_T + 1)\gamma_m}{(\alpha_M - \alpha_T)\left(3\gamma - \frac{H'}{H}\right)}\right]\Phi$$

$$+ \left[A_4 \rho_m \frac{\alpha_M - \alpha_T - 3\gamma}{(\alpha_M - \alpha_T)\left(3\gamma - \frac{H'}{H}\right)}\right]\Psi = A_3 \delta\rho_m$$

$$\left[f_\Phi + B_1 + B_4 \rho_m \frac{(\alpha_M - \alpha_T)(f_\zeta - f_\Phi) - 3(\alpha_T + 1)\gamma}{(\alpha_M - \alpha_T)\left(3\gamma - \frac{H'}{H}\right)}\right]\Phi$$

$$+ \left[B_2 + B_4 \rho_m \frac{\alpha_M - \alpha_T - 3\gamma}{(\alpha_M - \alpha_T)\left(3\gamma - \frac{H'}{H}\right)}\right]\Psi = B_3 \delta\rho_m$$

Dark degeneracy II

Parameterizing deviations from standard cosmology

Effective modification of the Poisson:

$$\mu\left(a,k\right)\equiv-\frac{2H^{2}k_{H}^{2}\Psi}{\kappa^{2}\delta\rho_{m}}=\frac{1}{\kappa^{2}M^{2}}\left(\frac{\mu_{+2}k_{H}^{2}+\mu_{+4}k_{H}^{4}+\mu_{+6}k_{H}^{6}}{\mu_{-0}+\mu_{-2}k_{H}^{2}+\mu_{-4}k_{H}^{4}+\mu_{-6}k_{H}^{6}}\right)$$

Slip parameter:

$$\gamma\left(a,k\right)\equiv-\frac{\Phi}{\Psi}=\frac{\gamma_{+0}+\gamma_{+2}k_{H}^{2}+\gamma_{+4}k_{H}^{4}+\gamma_{+6}k_{H}^{6}+\gamma_{+8}k_{H}^{8}+\gamma_{+10}k_{H}^{10}}{\gamma_{-0}+\gamma_{-2}k_{H}^{2}+\gamma_{-4}k_{H}^{4}+\gamma_{-6}k_{H}^{6}+\gamma_{-8}k_{H}^{8}+\gamma_{-10}k_{H}^{10}}$$

Dark degeneracy II

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Question: At small scales, can we combine an interacting dark sector and modified gravity in order to recover μ = γ = 1?

Dark degeneracy II

Parameterizing deviations from standard cosmology

Effective modification of the Poisson:

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Question: At small scales, can we combine an interacting dark sector and modified gravity in order to recover μ = γ = 1?

Dark degeneracy II

Parameterizing deviations from standard cosmology

Small scales limit:

$$\lim_{k\to\infty} \mu(a,k) = 1 \quad \text{and} \quad \lim_{k\to\infty} \gamma(a,k) = 1$$

$$\alpha_C = 2(\alpha_B+1)(\alpha_D+1)\frac{H'}{H}((\alpha_B+2)\alpha_T - 2\alpha_M) + (\alpha_B\,(2(\alpha_D+1)\,(\alpha_T\,(\alpha_B' + 4\alpha_T + \rho_m + 2)$$
$$-\alpha_M(5\alpha_T+2) + \alpha_M^2\big) + \alpha_D\prime(\alpha_M - \alpha_T)\big) + 4(\alpha_D+1)(\alpha_M-\alpha_T)\,(-\alpha_B' + \alpha_M - \alpha_T - \rho_m)$$
$$+\alpha_B^2\,(-\alpha_T\alpha_D\prime - 2(\alpha_D+1)(\alpha_T+1)(\alpha_M-2\alpha_T))\big)\,(2\alpha_B(\alpha_D+1)((\alpha_B+1)\alpha_T - \alpha_M))^{-1}$$

$$\alpha_D' = 2(\alpha_D+1)\,\Big(\big(-\alpha_B\,\big(\alpha_T\,(\alpha_B\prime - \alpha_C + 4\alpha_T + \rho_m + 2) + \alpha_M(\alpha_C - 5\alpha_T - 2) + \alpha_M^2\big) + 2(\alpha_M - \alpha_T)$$
$$(\alpha_B\prime - \alpha_M + \alpha_T + \rho_m) + \alpha_B^2\,\big(\alpha_T(\alpha_C + \alpha_M - 2) + \alpha_M - 2\alpha_T^2\big)\big) - (\alpha_B+1)\frac{H'}{H}((\alpha_B+2)\alpha_T - 2\alpha_M)\Big)$$
$$[\alpha_B((\alpha_B+1)\alpha_T - \alpha_M)]^{-1}$$

Dark degeneracy II

Can we go beyond small scales?

$$\mathcal{S} = \int d^4x \left\{ \sqrt{-g}\mathcal{L}_g \left[g_{\mu\nu}; \phi \right] + \sum_{I=1}^{N} \sqrt{-\tilde{g}^{(I)}} \mathcal{L}_m^{(I)} \left[\tilde{g}_{\mu\nu}^{(I)}; \psi^{(I)} \right] \right\}$$

Dark degeneracy II

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In how many interacting matter species do you believe?

Unified background description:

$$\rho_m' + 3\left(1 + w_m - \gamma_m\right)\rho_m = 0 \quad \text{with} \quad \gamma_m = \frac{\sum \rho_I \gamma_I}{\rho_m}$$

Modified Einstein equations:

$$\Phi'' + A_1\Phi' + A_2\Phi = A_3 \sum \delta\rho_I + \bar{A}_3 \sum \gamma_I \delta\rho_I + A_4 \sum \rho_I \frac{v_I}{k_H}$$

$$\Phi' + B_1\Phi + B_2\Psi = B_3 \sum \delta\rho_I + B_4 \sum \rho_I \frac{v_I}{k_H}$$

Conclusions and perspectives

1. ΛCDM model is still the one to explain late-time accelerated expansion! However, we must keep trying to understand its theoretical and observational issues, they can be the key for a more fundamental description;

2. Interacting models are interesting and viable options for addressing the current tensions on Cosmology. They can be "observationally equivalent" for the background and CMB data. However, due to the fact that interaction parameters affect CDM evolution, interesting features are introduced;

3. In order to avoid the dark degeneracy, we should try to connect the specific signatures of each kind of descriptions for the dark components with data in a multidisciplinary way (particle physics?);

4. Non-linear level;

Astrocientistas

I Encontro Brasileiro de Meninas e Mulheres da Astrofísica, Gravitação e Cosmologia

CICLO DE SEMINÁRIOS

da Coordenação de Astronomia e Astrofísica

14.10

TRANSMISSÃO AO VIVO

YOUTUBE.COM/OBSERVATORIONACIONAL

YouTube | 10H

Searching for dark energy off the beaten track

Palestrante

Sunny Vagnozzi

Kavli Institute for Cosmology and Institute of Astronomy, University of Cambridge **(UK)**

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