Relativistic Horndeski gravity N-body simulations

Guilherme Brando

Based on 2105.04491 and 21??.?????

In collaboration with Kazuya Koyama, David Wands, Iggy Sawicki, Emilio Bellini, +++

Universidade Federal do Espírito Santo

PPGCosmo

Introduction

- Euclid, DESI, LSST → bulk of information will be on non-linear scales → Simulations

- In Mod. Grav. Newtonian simulations only account for dark energy effects dominated by matter perturbations (small scales): Fifth force

$$\ddot{\delta}_m^{\mathrm{N}} + \mathcal{H}\dot{\delta}_m^{\mathrm{N}} - 4\pi G_{\mathrm{eff}} a^2 \rho_m \delta_m^{\mathrm{N}} = 0 \qquad \nabla^2\Phi \propto G_{\mathrm{eff}}(\tau, k)\rho_m\delta_m$$

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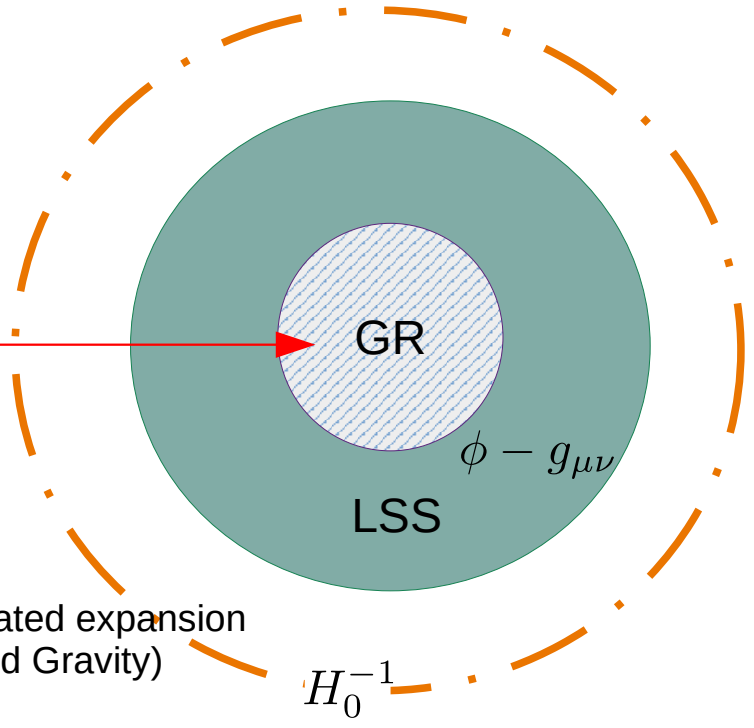
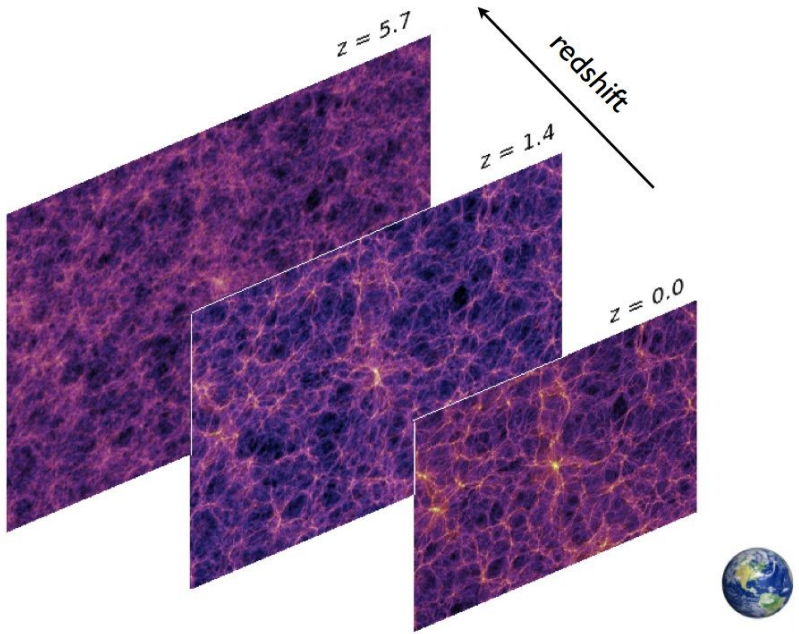


Introduction

- But where do Newtonian simulations live in a relativistic scenario?



$$\Longleftrightarrow \qquad ds^2 = ??$$

N-Body gauge

- CDM dynamics:

$$k^2\Phi^{\rm N} = 4\pi G a^2 \bar{\rho}^{\rm N}_{\rm cdm}\delta^{\rm N}_{\rm cdm}\,,$$
$$\dot{\delta}^{\rm N}_{\rm cdm} + k v^{\rm N}_{\rm cdm} = 0\,,$$
$$\left[\partial_\tau + \mathcal{H}\right] v^{\rm N}_{\rm cdm} = -k\Phi^{\rm N}$$

Newtonian

$$k^2\Phi = 4\pi G a^2\left[\bar{\rho}\delta + 3\mathcal{H}\left(\bar{\rho}+\bar{p}\right)(v-B)/k\right],$$
$$\dot{\delta}_{\rm cdm} + k v_{\rm cdm} = -3\dot{H}_{\rm L}\,,$$
$$\left[\partial_\tau + \mathcal{H}\right] v_{\rm cdm} = -k\left[\Phi + (\partial_\tau + \mathcal{H})\dot{H}_{\rm T} + 12\pi G a^2\left(\rho + p\right)\sigma\right]$$

GR

$$g_{00} = -a^2\left(1+2A\right)\,,$$
$$g_{0i} = a^2\,{\rm i}\hat{k}_i B\,,$$
$$g_{ij} = a^2\left[\delta_{ij}\left(1+2H_{\rm L}\right) + 2\left(\delta_{ij}/3 - \hat{k}_i\hat{k}_j\right)H_{\rm T}\right]$$

N-Body gauge

- Setting: $H_{\rm L} = 0, \quad B = v_m$

$$
\begin{aligned}
k^2 \Phi^{\rm Nb} &= 4\pi G a^2 \bar{\rho}_{\rm cdm} \delta^{\rm Nb}_{\rm cdm} \,, \\
\dot{\delta}^{\rm Nb}_{\rm cdm} + k v^{\rm Nb}_{\rm cdm} &= 0 \,, \\
\left[\partial_\tau + \mathcal{H}\right] v^{\rm Nb}_{\rm cdm} &= -k \left(\Phi^{\rm Nb} + \gamma^{\rm Nb}\right) \,, \\
k^2 \gamma^{\rm Nb} &= -(\partial_\tau + \mathcal{H}) \dot{H}^{\rm Nb}_{\rm T} + 12\pi G a^2 \left(\rho + p\right) \sigma,
\end{aligned}
$$

- Comoving gauge

$$
\dot{H}^{\rm Nb}_{\rm T} = 3 \frac{\mathcal{H}}{\rho + p} \left[\left(\rho + p\right) \sigma - \delta p^{\rm S/P} + \dot{p} \frac{\theta^{\rm S/P}_{\rm tot}}{k^2} \right] ,
$$

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$$

N-Body gauge

- In summary, in the N-Body gauge:

  i. We can introduce effects of photons, neutrinos and dark energy at large sacales:

$$\ddot{\delta}_m^{\rm Nb} + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb} = 4\pi G a^2 \delta\rho_{\rm GR}$$

Same as Newtonian at small scales

Relevant at large scales

Modified Gravity

- Horndeski theory: most general scalar tensor theory with 2nd order equations in both fields.
- Big and messy:

$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\left[\sum_{i=2}^{5}\frac{1}{8\pi G}\mathcal{L}_i[g_{\mu\nu},\phi] + \mathcal{L}_{\mathrm{m}}[g_{\mu\nu},\psi_M]\right]$$

- But can be made simpler:

E. Bellini and I. Sawicki 1404.3713

$H(z)$	Fixes background	
$\alpha_{\mathrm{K}}(z)$	Kineticity	Linear Pert.
$\alpha_{\mathrm{B}}(z)$	Braiding	
$\alpha_{\mathrm{M}}(z)$	Running Planck Mass	
$\alpha_{\mathrm{T}}(z)$	Tensor excess	

Modified Gravity as a fluid

- Need to compute anisotropic stress and pressure perturbation of DE!
- Effective Fluid Description:
  - Bianchi Identities + Conservation of Energy Momentum Tensor
  - Move MG to the RHS of Einstein Equation:

$$G_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{MG}_{\mu\nu}$$

GR

Horndeski

- In this way we can compute $\delta\rho_{\rm DE}, \theta_{\rm DE}, \sigma_{\rm DE}, \delta p_{\rm DE}$ and feed it to $\dot{H}^{\rm Nb}_{\rm T}$

Modified Gravity

- Status:

  1) Modified gravity Newtonian simulations: 5th force

$$\nabla^2\Phi \propto G_{\rm eff}(\tau,k)\rho_m\delta_m \qquad\qquad \ddot{\delta}_m^{\rm N} + \mathcal{H}\dot{\delta}_m^{\rm N} - 4\pi G_{\rm eff}a^2\rho_m\delta_m^{\rm N} = 0.$$

  2) N-Body gauge:

$$\delta\rho_{\rm GR} = \delta\rho_\gamma^{\rm Nb} + \delta\rho_\nu^{\rm Nb} + \delta\rho_{\rm DE}^{\rm Nb} + \delta\rho_{\rm metric}^{\rm Nb} \qquad\qquad \ddot{\delta}_m^{\rm Nb} + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2\rho_m\delta_m^{\rm Nb} = 4\pi G a^2\delta\rho_{\rm GR}$$

  3) How can we connect the two?

Modified Gravity

- Separation small from large → Quasi-Static Aprroximation (QSA) → Mod. Grav. Simulations

- QSA:

  1) Write the small scale contributions of the dark energy density perturbations and anisotropic stress (diff. between Newtonian Potentials)

$$\delta\rho_{\rm DE} = \delta\rho_{\rm DE}^{\rm QSA} + \delta\rho_{\rm DE,\ rel.}$$
$$\sigma_{\rm DE} = \sigma_{\rm DE}^{\rm QSA} + \sigma_{\rm DE,\ rel.}$$

$$\left.\begin{aligned}\delta\rho_{\rm DE}^{\rm QSA} &\propto \delta\rho_m \\ \sigma_{\rm DE}^{\rm QSA} &\propto \delta\rho_m\end{aligned}\right\} \quad G_{\rm eff} = 1 + \frac{c_{\rm sN}^2\left(2 - 2M_*^2 + 2\alpha_{\rm T}\right) + \left(\alpha_{\rm B} + 2\alpha_{\rm M} - 2\alpha_{\rm T} + \alpha_{\rm B}\alpha_{\rm T}\right)^2}{2c_{\rm sN}^2 M_*^2}$$

Results

- Status:

  1) Modified gravity Newtonian simulations: 5th force

$$\nabla^2\Phi \propto G_{\rm eff}(\tau,k)\rho_m\delta_m \qquad\qquad \ddot{\delta}_m^{\rm N} + \mathcal{H}\dot{\delta}_m^{\rm N} - 4\pi G_{\rm eff}a^2\rho_m\delta_m^{\rm N} = 0.$$

  2) N-Body gauge:

$$\delta\rho_{\rm GR} = \delta\rho_\gamma^{\rm Nb} + \delta\rho_\nu^{\rm Nb} + \delta\rho_{\rm DE}^{\rm Nb} + \delta\rho_{\rm metric}^{\rm Nb} \qquad\qquad \ddot{\delta}_m^{\rm Nb} + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2\rho_m\delta_m^{\rm Nb} = 4\pi G a^2\delta\rho_{\rm GR}$$

  3) QSA:

Results

- Status:

  1) Modified gravity Newtonian simulations: 5th force

$$\nabla^2\Phi \propto G_{\rm eff}(\tau,k)\rho_m\delta_m \qquad \ddot{\delta}^{\rm N}_m + \mathcal{H}\dot{\delta}^{\rm N}_m - 4\pi G_{\rm eff}a^2\rho_m\delta^{\rm N}_m = 0.$$

$P^{\rm Geff}_{\rm N}$

  2) N-Body gauge:

$$\delta\rho_{\rm GR} = \delta\rho^{\rm Nb}_\gamma + \delta\rho^{\rm Nb}_\nu + \delta\rho^{\rm Nb}_{\rm DE} + \delta\rho^{\rm Nb}_{\rm metric} \qquad \ddot{\delta}^{\rm Nb}_m + \mathcal{H}\dot{\delta}^{\rm Nb}_m - 4\pi G a^2\rho_m\delta^{\rm Nb}_m = 4\pi G a^2\delta\rho_{\rm GR}$$

$P_{\rm GR}$

  3) QSA:

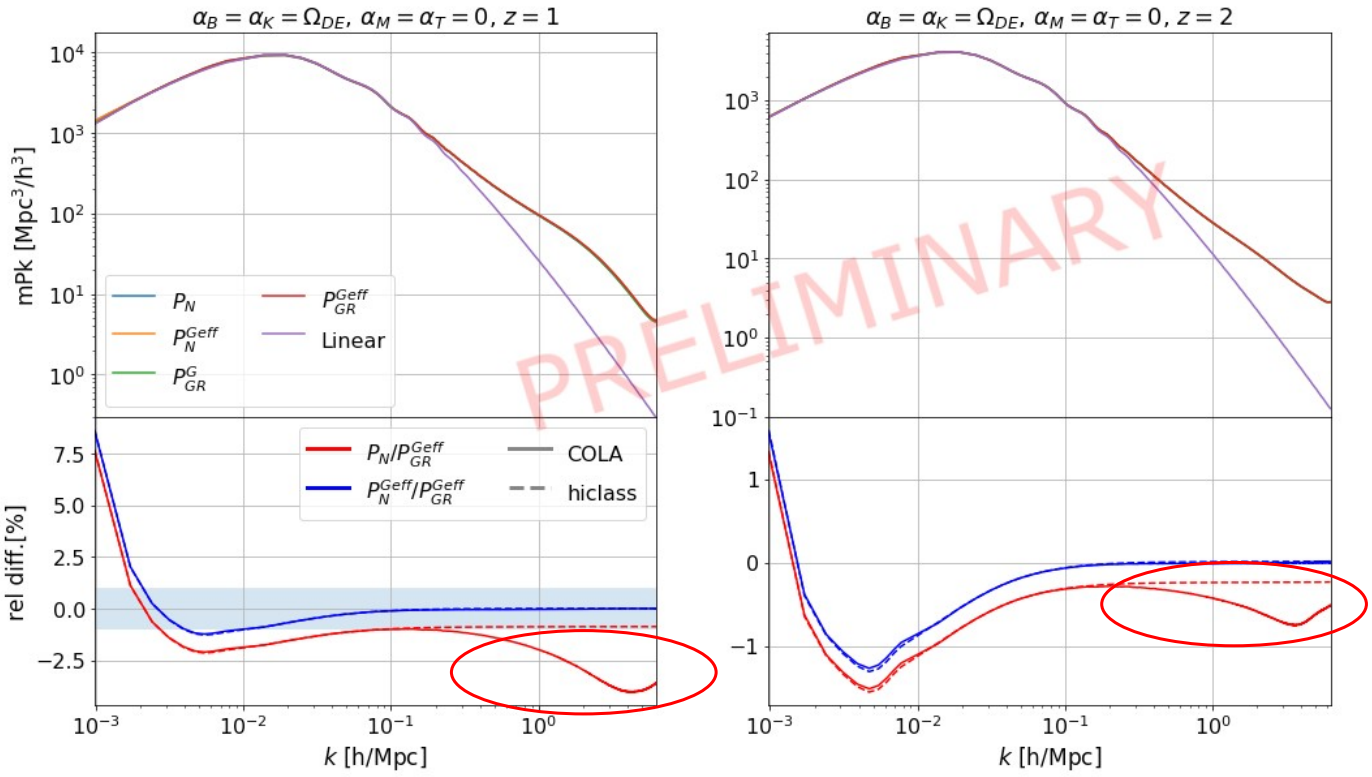
$$\delta\rho_{\rm GR,\ rel.} = \delta\rho^{\rm Nb}_\gamma + \delta\rho^{\rm Nb}_\nu + \delta\rho^{\rm Nb}_{\rm DE,\ rel.} + \delta\rho_{\rm metric,\ rel.},$$
$$\delta\rho^{\rm Nb}_{\rm DE} = \delta\rho^{\rm Nb}_{\rm DE,\ rel.} + \delta\rho^{\rm QSA}_{\rm DE},$$
$$\delta\rho_{\rm metric,rel.} = \delta\rho_{\rm metric} - (\rho_{\rm DE} + p_{\rm DE})\,\sigma^{\rm QSA}_{\rm DE}$$

$$\ddot{\delta}^{\rm Nb}_m + \mathcal{H}\dot{\delta}^{\rm Nb}_m - 4\pi G_{\rm eff}a^2\rho_m\delta^{\rm Nb}_m = 4\pi G a^2\delta\rho_{\rm GR,\ rel.}$$

$P^{\rm Geff}_{\rm GR}$

Results

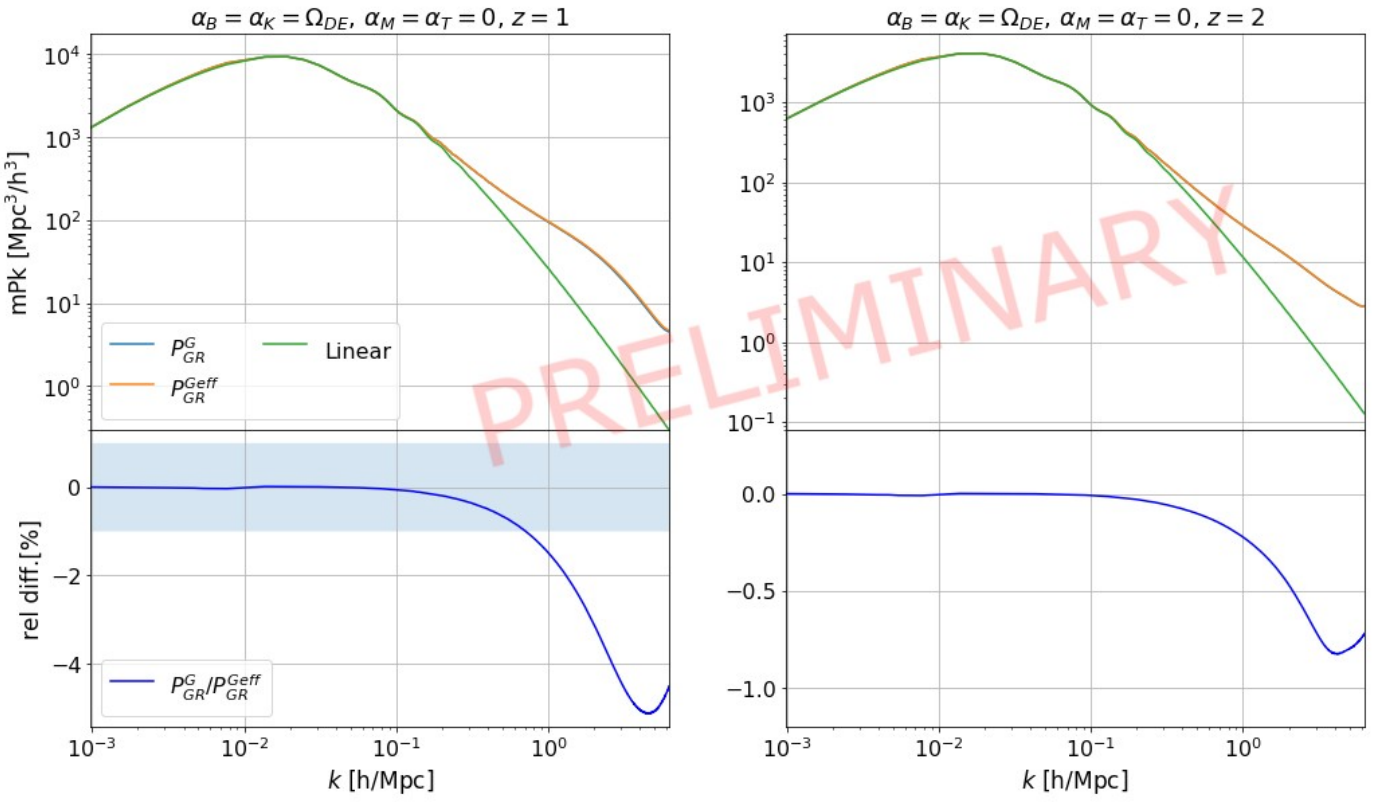
- Non-linear corrections from Geff
- Blue curves agree as they both share the same small scale physics
- Relativistic corrections affect primarily large scale physics



Non-linearities in Geff

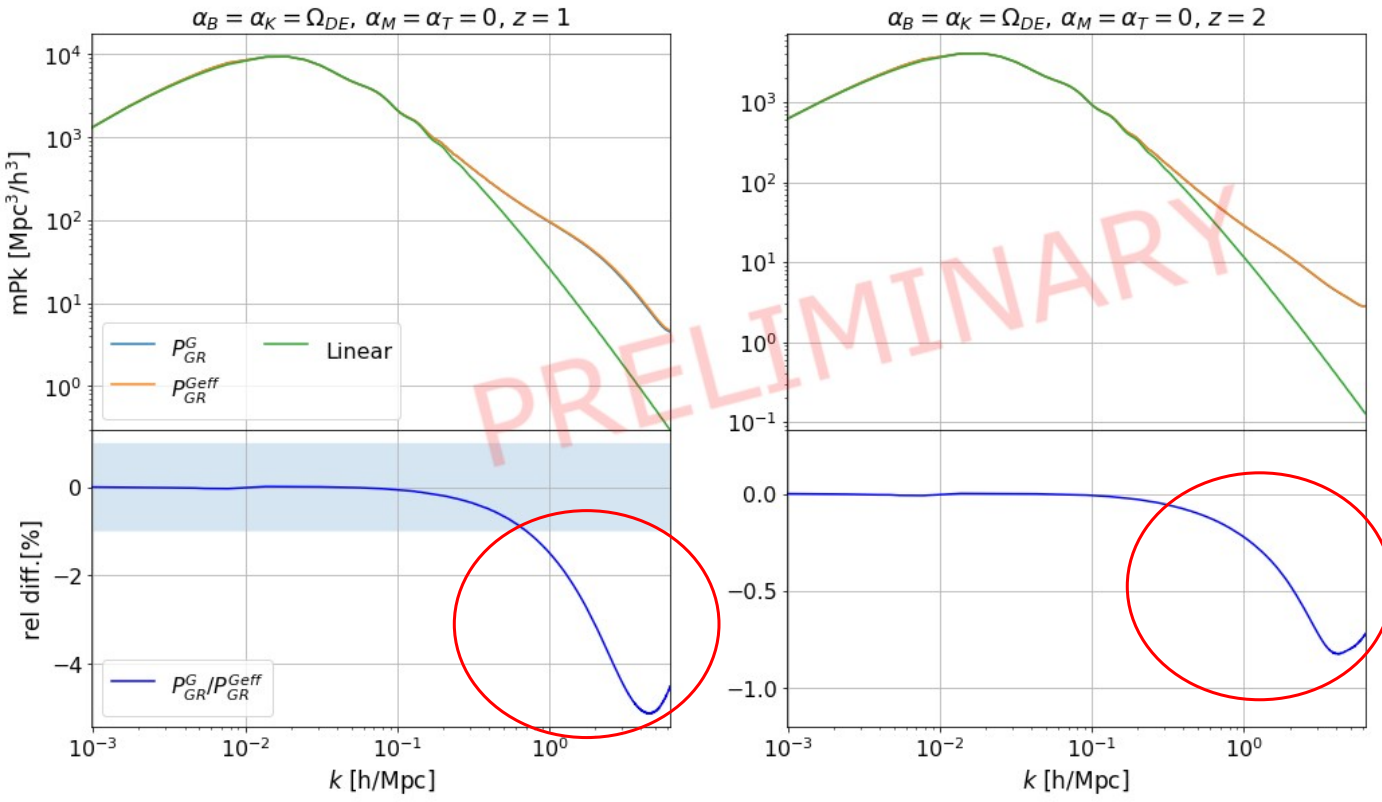
Results

- Non-linear corrections from Geff



Results

- Still difference at small scales
- Why?



Results

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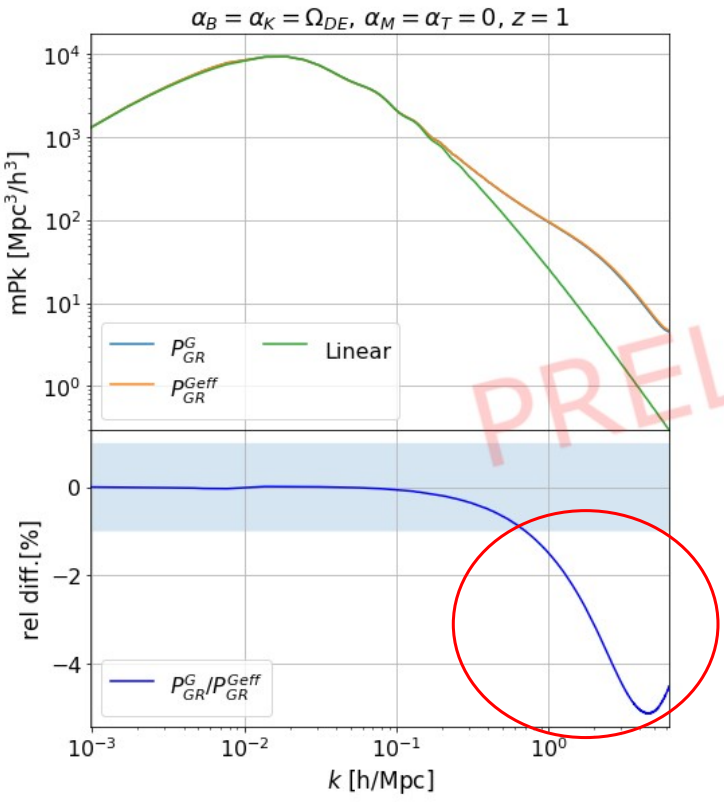
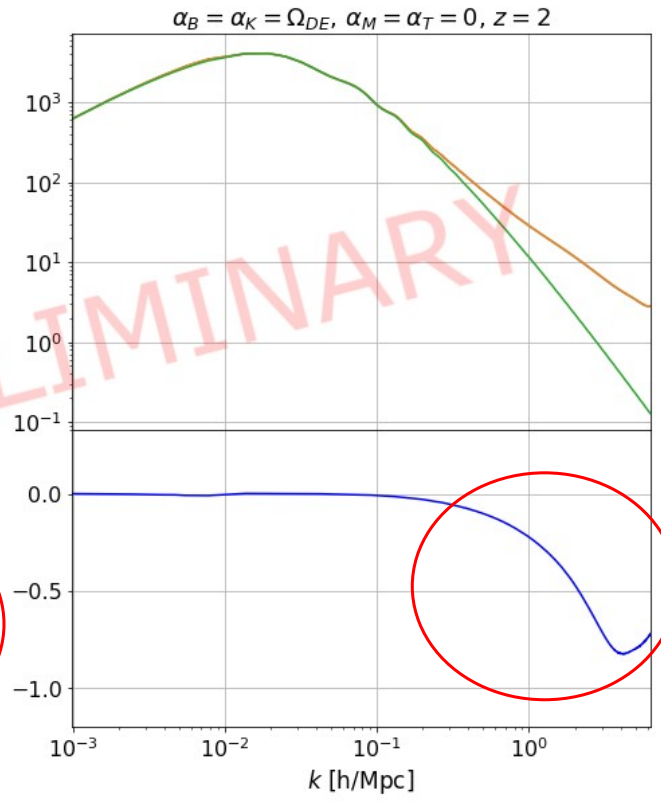
$$\left.\begin{aligned} \delta\rho_{\rm DE}^{\rm QSA} &\propto \delta\rho_m \\ \sigma_{\rm DE}^{\rm QSA} &\propto \delta\rho_m \end{aligned}\right\} \; G_{\rm eff}$$





PRELIMINARY

Results

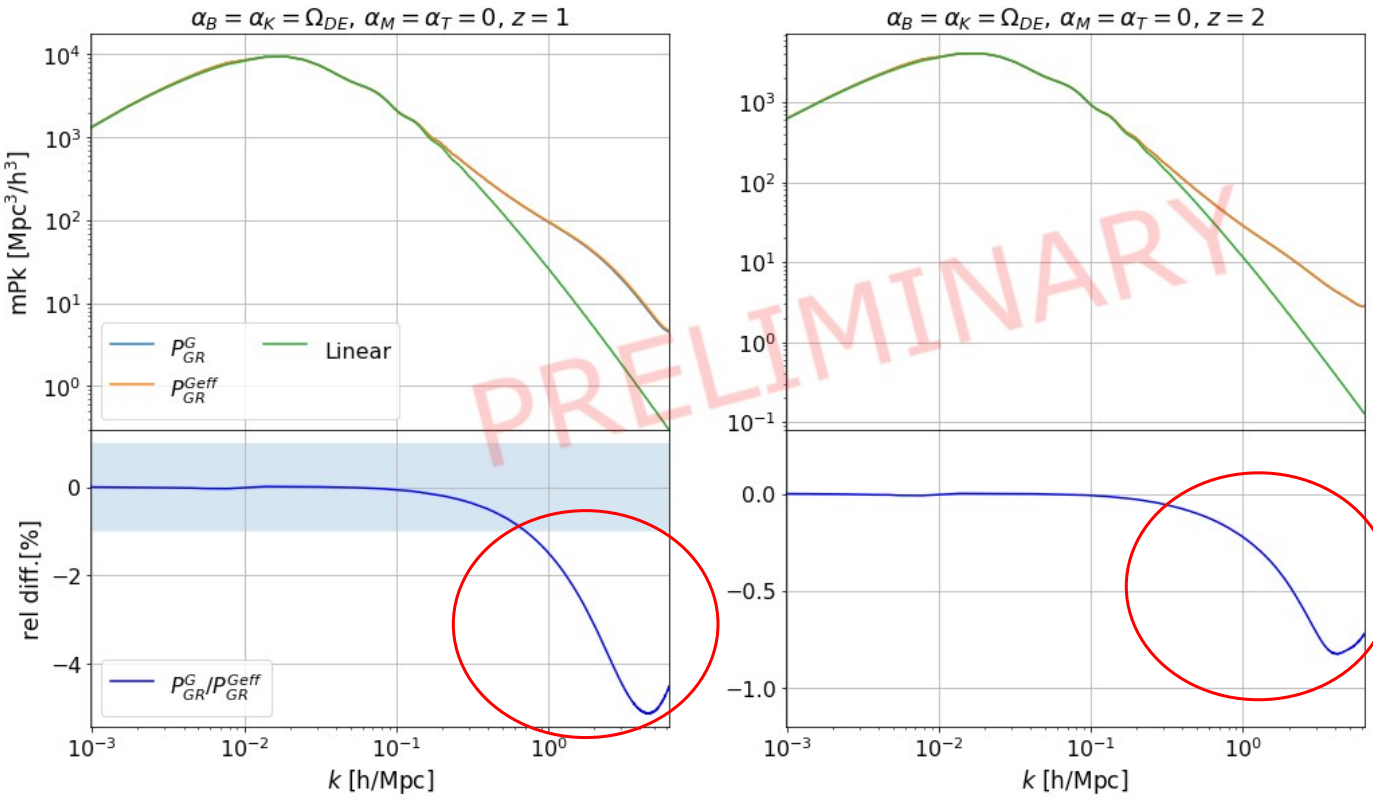
- Still difference at small scales
- Why?

$$\delta\rho_{\rm DE}^{\rm QSA} \propto \delta\rho_m$$
$$\sigma_{\rm DE}^{\rm QSA} \propto \delta\rho_m$$

$G_{\rm eff}$

$P_{\rm GR}$

Always linear



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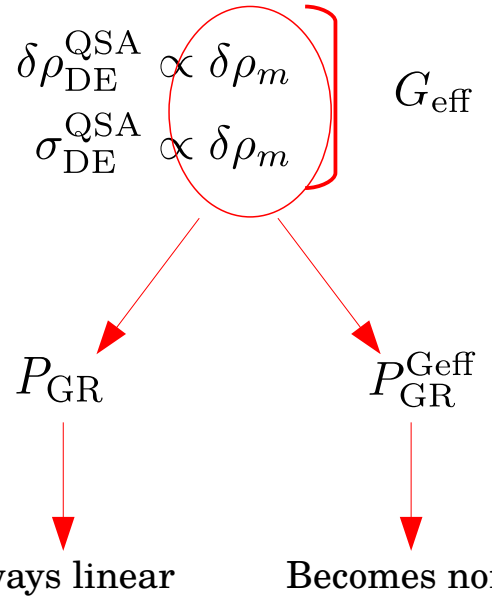

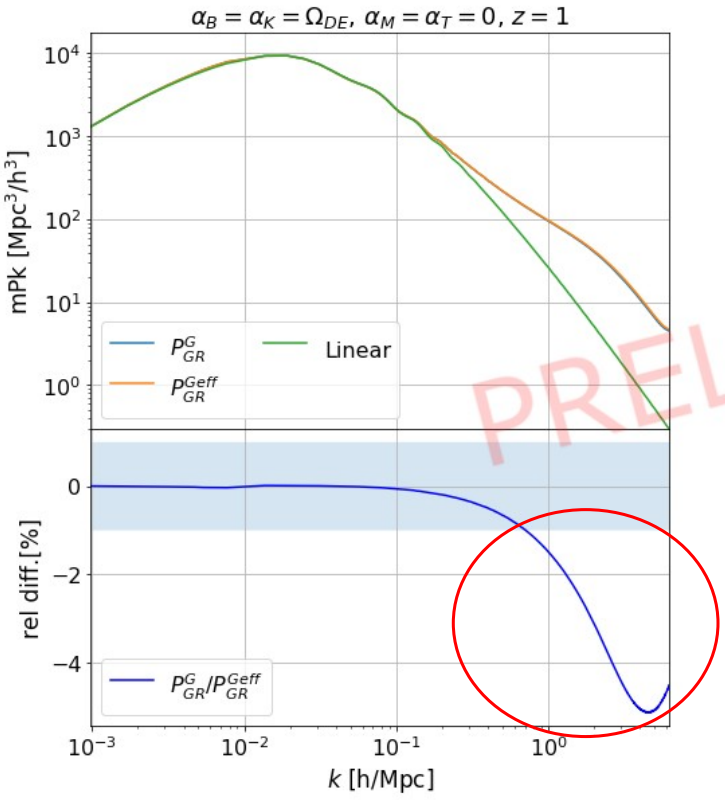

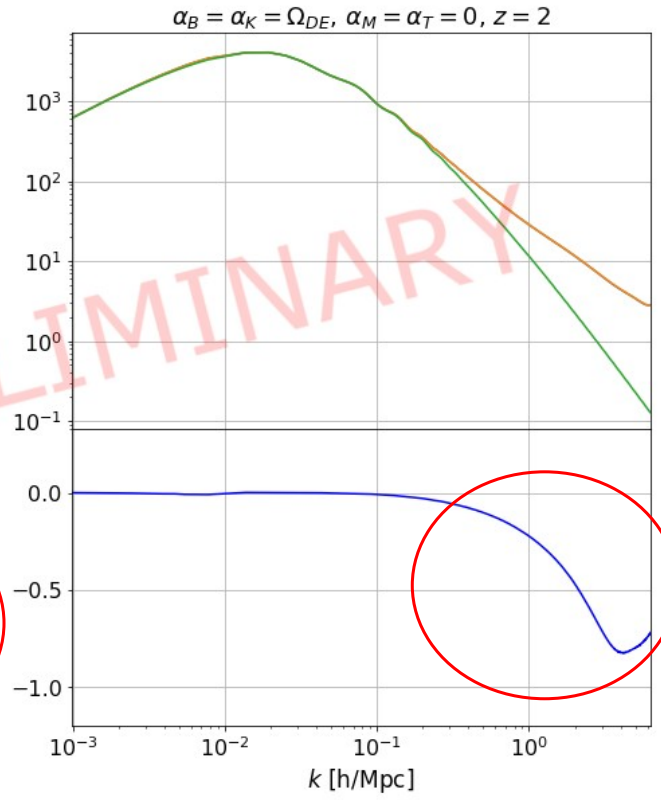

Conclusions/Future

- Relativistic effects of DE not accounted for in MG simulations can now be readily implemented with $\delta\rho_{\rm GR}$

- As stage-IV LSS surveys will aim at higher redshifts, we can treat DE linearly.

- This reduces the computational time to run simulations.

- Finish the implementation of relativistic effects on N-Body codes and validate it with other codes: gevolution, k-evolution, post-processing methods.

Thank you!

Obrigado!