

O que nós (não) sabemos de energia escura

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Cosmology

- ▶ **Universe as physical system**
- ▶ **Presently observed Universe** - result of the evolution starting from a **hot early state**
- ▶ **Dynamics of the Universe: Gravitation**
- ▶ **General Theory of Relativity** (**GR**, Einstein 1915)
- ▶ **Dynamics of space-time**
(Newton: fixed space time structure)

GR: main new features

- ▶ **Physical cosmology**
- ▶ **Black holes**
- ▶ **Gravitational waves**

Cosmology: basic facts

- ▶ Hubble expansion
- ▶ Abundance of light elements
- ▶ Cosmic Microwave Background (CMB)
- ▶ Observed structures through gravitational instability

Outline

- ▶ **Introduction**
- ▶ **Basic cosmology**
- ▶ **Dark energy (DE)**
 - ▶ Cosmological constant
 - ▶ Scalar field
 - ▶ Generalized Chaplygin gas
 - ▶ Interacting and time varying DE
- ▶ **Conclusions and outlook**

Brief summary of GR

Spacetime: four-dimensional manifold, Lorentzian metric $g_{\alpha\beta}$

The **curvature** of $g_{\alpha\beta}$ is **related** to the **matter distribution** in spacetime by **Einstein's equations**

$$G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

$R_{\alpha\beta}$: **Ricci tensor**, $R^{\mu}_{\alpha\mu\beta}$: **Riemann tensor**

$$R_{\alpha\beta} = R^{\mu}_{\alpha\mu\beta}$$

R : **Curvature scalar**

$$R = R^{\mu}_{\mu}$$

$T_{\alpha\beta}$: **Energy momentum tensor**

Curvature tensor

$$R^\rho_{\mu\sigma\kappa} = \Gamma^\rho_{\mu\kappa,\sigma} - \Gamma^\rho_{\mu\sigma,\kappa} + \Gamma^\rho_{\nu\sigma}\Gamma^\nu_{\mu\kappa} - \Gamma^\rho_{\nu\kappa}\Gamma^\nu_{\mu\sigma}$$

with

$$\Gamma^\mu_{\alpha\gamma} = \frac{1}{2}g^{\mu\beta} (g_{\alpha\beta,\gamma} + g_{\gamma\beta,\alpha} - g_{\alpha\gamma,\beta}) .$$

⇒ **Field equations: nonlinear partial differential equations of 2nd order** for $g_{\alpha\beta}(x^\gamma)$.

The curvature tensor manifests itself, e.g., if the **second covariant derivatives** of a four vector W^α are interchanged,

$$W^\alpha_{;\beta;\gamma} - W^\alpha_{;\gamma;\beta} = R^\alpha_{\nu\gamma\beta} W^\nu$$

Parallel transport of a vector **depends on the path**

Components of the energy momentum tensor

T_{00}	—	energy density
T_{0a}	—	energy flux density $\cdot \frac{1}{c}$
T_{a0}	—	momentum density $\cdot c$
T_{ab}	—	negative stress tensor

Normal stresses: diagonal stress tensor

Isotropic pressure (Pascal, 1659),

$$\sigma^{ab} = -T^{ab} = -p \delta^{ab}$$

Perfect fluids: no viscous processes and no heat conductivity

Basic fluid dynamics

Timelike vector field u^μ

$$u^\mu u_\mu = -1$$

Projection tensor

$$h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

Orthogonality

$$h^{\mu\nu} u_\nu = 0$$

Matter 4-velocity: **observer's rest frame**

(Timelike) **gradient of a scalar field**

$$u_\mu \equiv -\frac{\varphi_{,\mu}}{\sqrt{-g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta}}}, \quad \Rightarrow \quad u_\mu u^\mu = -1$$

General decomposition

Energy-momentum tensor (EMT)

$$T_{\mu\nu} = \rho u_\mu u_\nu + P h_{\mu\nu} + \Pi_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu$$

Energy density

$$\rho = T_{\mu\nu} u^\mu u^\nu$$

Isotropic pressure

$$P = p + p_v = \frac{1}{3} h^{\mu\nu} T_{\mu\nu}$$

Equilibrium pressure p , viscous pressure p_v

Heat flux and anisotropic pressure

EMT

$$T_{\mu\nu} = \rho u_\mu u_\nu + P h_{\mu\nu} + \Pi_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu$$

Heat flux

$$q_\mu = -h_\mu^\nu T_{\nu\sigma} u^\sigma$$

Anisotropic pressure

$$\Pi_{\mu\nu} = h_{\langle\mu}^\sigma h_{\nu\rangle}^\tau T_{\sigma\tau} \equiv h_{(\mu}^\sigma h_{\nu)}^\tau T_{\sigma\tau} - \frac{1}{3} h_{\mu\nu} h^{\sigma\tau} T_{\sigma\tau}$$

Orthogonality

$$h^{\mu\nu} u_\nu = q_\mu u^\mu = \Pi_{\mu\nu} u^\mu = \Pi_\mu^\mu = 0$$

Perfect fluid

Preferred matter model

$$T_{\mu\nu} = \rho u_\mu u_\nu + P h_{\mu\nu}$$

Energy density ρ

Pressure P

Cosmological principle:

No spatial point and no spatial direction singled out

Universe is assumed to be **spatially homogeneous and isotropic**

Robertson-Walker metric

$$d s^2 = -c^2 d t^2 + a^2 \left[\frac{d r^2}{1 - k r^2} + r^2 (d \vartheta^2 + \sin^2 \vartheta d \varphi^2) \right]$$

Cosmic scale factor $a(t)$

Constant curvature parameter k

Comments on the cosmological principle

There is a **“mean motion”** of the matter in the Universe:
Cosmic rest frame

Our galaxy is approximately at rest with respect to this frame.
Observers: **comoving** or **“fundamental” observers**

Spacetime allows for a slicing into a one parameter family of hypersurfaces $t = \text{const}$, orthogonal to the fundamental observers' worldlines:

Maximally symmetric subspaces of the entire spacetime

Universal time coordinate

Robertson-Walker metric

Recall: **Symmetry requirements:**

Robertson-Walker metric

$$d s^2 = -c^2 d t^2 + a^2(t) \left[\frac{d r^2}{1 - k r^2} + r^2 (d \vartheta^2 + \sin^2 \vartheta d \varphi^2) \right]$$

Einstein's equations: dynamics of the scale factor

Dynamics of the scale factor

Friedmann equation (units with $c^2 = 1$)

$$3\frac{\dot{a}^2}{a^2} \equiv 3H^2 = 8\pi G\rho - \frac{3k}{a^2} + \Lambda$$

$\cdot \equiv \frac{d}{dt}$, **H - Hubble function**

and

$$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p) + \frac{\Lambda}{3}$$

Static solution $a = \text{const}$ possible for $\Lambda > 0$ (Einstein's motivation)

Simple world models

Equation of state $f(p, \rho) = 0$ necessary

- ▶ $p = \frac{\rho}{3}$, relativistic matter, radiation
- ▶ $p \ll \rho$, non-relativistic matter, “dust”

For $k = 0$ and $\Lambda = 0$:

- ▶ $a \propto t^{1/2}$ $\rho \propto a^{-4}$ ($p = \frac{\rho}{3}$)
- ▶ $a \propto t^{2/3}$ $\rho \propto a^{-3}$ ($p \ll \rho$)

Decelerated expansion: $\ddot{a} < 0$

de Sitter solution

Friedmann equation

$$3\frac{\dot{a}^2}{a^2} = 3H^2 = 8\pi G\rho - \frac{3k}{a^2} + \Lambda$$

If Λ dominates:

$$\Rightarrow \quad 3H^2 = \Lambda = \text{const} \Rightarrow \quad a \propto \exp[Ht]$$

Accelerated expansion: $\ddot{a} > 0$

Periods of accelerated Expansion

Early Universe

INFLATION

Late Universe

DARK ENERGY

Big bang expansion

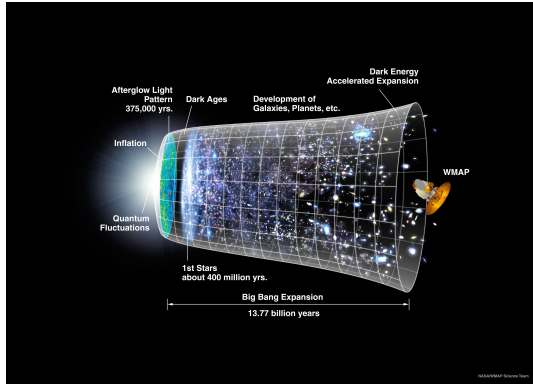


Figure: Evolution of the Universe

Observations: luminosity distance

$$d_L : \quad f = \frac{L}{4\pi d_L^2}$$

L - luminosity

f - apparent luminosity ($\sim$ observed flux)

Redshift z :

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = z + 1 = \frac{a_0}{a}$$

(a_0 - present value)

$\Rightarrow$ **Relation luminosity distance - redshift ?**

Luminosity distance vs. redshift

$$d_L = \frac{z}{H_0} + (1 - q_0) \frac{z^2}{2H_0} + \dots$$

Deceleration parameter:

$$q_0 \equiv -\frac{\ddot{a}}{\dot{a}^2} \Big|_0$$

Observational result from SCP and HZT 1998:

SN Ia fainter (larger d_L) than compatible with $p \ll \rho$

Hint on $\ddot{a}_0 > 0 \Rightarrow q_0 < 0!$ **Dynamics dominated by Λ !?**

(Recall: domination of $\Lambda \Rightarrow$ accelerated expansion)

The cosmic substratum

PREVAILING VIEW:

Two dynamically dominating components:

Dark Matter (DM), energy density ρ_M , pressure $p_M \ll \rho_M$

Dark Energy (DE), energy density ρ_X , pressure $p_X = w_X \rho_X$

Equation of state parameter w_X ?

Preferred model: Λ CDM: $\rho_X = \rho_\Lambda$, $p_X = p_\Lambda = -\rho_\Lambda$

Late-time acceleration of the scale factor

“Matter” content

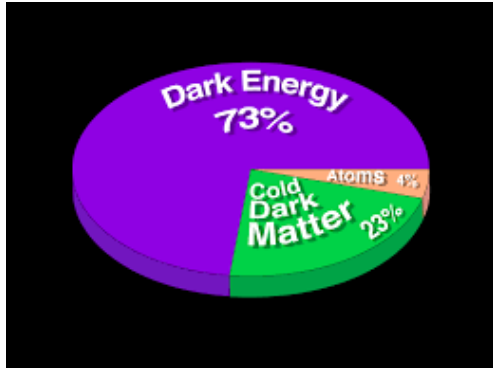


Figure: Composition of our present Universe

Friedmann equation rewritten

$$3H^2 = 8\pi G\rho_M - \frac{3k}{a^2} + \Lambda$$

Critical density

$$\rho_c \equiv \frac{3H^2}{8\pi G}$$

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} \quad \Omega_\Lambda \equiv \frac{\Lambda}{3H^2} \quad \Omega_k \equiv -\frac{k}{a^2 H^2}$$

⇒ **Friedmann equation:**

$$\Omega_M + \Omega_k + \Omega_\Lambda = 1$$

Observations combined

SN Ia: $q_0 < 0 \Leftrightarrow \Omega_\Lambda > \Omega_M$

Cosmic Microwave Background (CMB): $|\Omega_k| \ll 1$

Large Scale Structure (LSS): $\Omega_M \approx \frac{1}{4}$

Recall

$$\Omega_\Lambda + \Omega_M + \Omega_k = 1$$

Direct and indirect evidence for accelerated expansion

Planck 2018 results for cosmological parameters

Hubble constant

$$H_0 = (67.4 \pm 0.5) \text{ km/s/Mpc}$$

Matter density parameter

$$\Omega_M = 0.315 \pm 0.007$$

Curvature parameter

$$\Omega_k = 0.001 \pm 0.002$$

DE equation-of-state parameter

$$w_x = -1.03 \pm 0.03$$

Cosmological constant as “fluid” with negative pressure

$$3\frac{\dot{a}^2}{a^2} = 8\pi G\rho_M - \frac{3k}{a^2} + \Lambda$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_M + 3p_M) + \frac{\Lambda}{3}$$

Define $\Lambda \equiv 8\pi G\rho_\Lambda$ and $p_\Lambda \equiv -\rho_\Lambda$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G(\rho_M + \rho_\Lambda) - \frac{3k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_M + \rho_\Lambda + 3(p_M + p_\Lambda)]$$

“Fluid” with negative pressure

Accelerated expansion requires negative pressure

$$\rho = \rho_M + \rho_\Lambda, \quad p = p_M + p_\Lambda$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho + 3p]$$

$$\ddot{a} > 0 \quad \Leftrightarrow \quad p < -\frac{1}{3}\rho$$

Accelerated expansion $\Leftrightarrow$ negative pressure

Negative pressure as “anti-gravitation”

Newtonian potential ϕ_N :

$$\Delta\phi_N = 4\pi G [\rho_M + \rho_\Lambda + 3 (p_M + p_\Lambda)]$$

$$p_M \approx 0, \quad p_\Lambda = -\rho_\Lambda$$

$$\Delta\phi_N = 4\pi G (\rho - 2\rho_\Lambda) \quad \text{Negative sign!}$$

Cosmological constant Λ - one possibility: **other options?**

Scalar fields

$$L = -\frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - V(\varphi) \Rightarrow T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} + V(\varphi)\right)$$

Lowest energy state

$$T_{\mu\nu}^{\text{vac}} = -g_{\mu\nu}V(\varphi_{\min}) = -\rho_{\text{vac}}g_{\mu\nu}$$

Field equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$\Rightarrow$ **Effective cosmological “constant”**

$$\Lambda_{\text{eff}} = \Lambda + 8\pi G \rho_{\text{vac}}$$

Vacuum fluctuations

Zero point energy: $E_{vac} = \frac{1}{2} \sum \hbar \omega$; assume: $\omega = \sqrt{k^2 + m^2}$

$$\Rightarrow \rho_{vac} = \frac{1}{2} \frac{1}{(2\pi)^3} \int_0^{k_c} 4\pi k^2 dk = \frac{k_c^4}{16\pi^2} \quad (\hbar = 1, m = 0)$$

Expected: $\rho_{vac} \sim 10^{76} (\text{GeV})^4$ $(\approx 10^{114} \frac{\text{erg}}{\text{cm}^3})$
(Planck scale)

Observed: $\rho_{\Lambda_{eff}} \sim 10^{-47} (\text{GeV})^4$ $(\approx 10^{-9} \frac{\text{erg}}{\text{cm}^3})$

Why is the observed $\rho_{\Lambda_{eff}}$ so small?

123 orders of magnitude!!

Cosmological constant problem

Currently interesting topics

- ▶ Value of the Hubble constant
- ▶ Understanding cosmic structure formation
- ▶ Does dark energy cluster?

Problem: Hubble constant

W.L. Freedman et al. arXiv:1907.05922

Type Ia supernovae (SNe Ia)

$$H_0 = 69.8 \pm 0.8 \text{ km/s/Mpc}$$

Planck Collaboration, Aghanim, N., Akrami, Y., et al.,
arXiv:1807.06209

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc}$$

Riess, A.G., Casertano, S., Yuan, W., Macri, L.M., & Scolnic, D.
2019,

Hubble Space Telescope (HST) arXiv:1903.07603

$$H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$$

Cosmic structure formation

Inhomogeneities: **Cosmological perturbation theory**

$$\rho(t) \Rightarrow \rho(t) + \hat{\rho}(\mathbf{r}, t), \quad \delta \equiv \frac{\hat{\rho}}{\rho}$$

Evolution equation for δ

$$\delta'' + \frac{3}{a} \left[1 - \frac{\Omega_{m0} a^{-3}}{2(\Omega_{m0} a^{-3} + \Omega_{\Lambda})} \right] \delta' - \frac{3}{2a^2} \frac{\Omega_{m0} a^{-3}}{(\Omega_{m0} a^{-3} + \Omega_{\Lambda})} \delta = 0.$$

Einstein - de Sitter universe: $\Omega_{\Lambda} = 0$

$$\delta'' + \frac{3}{2a} \delta' - \frac{3}{2a^2} \delta = 0 \quad \Rightarrow \quad \delta \propto a$$

Different models predict different growth rates

Gravitational instability

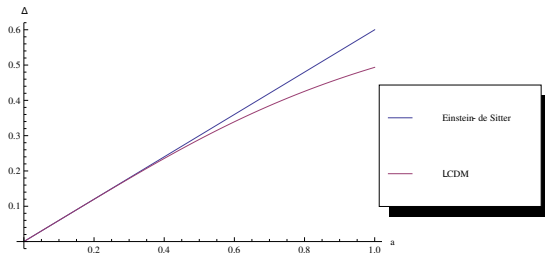


Figure: Comparação entre as taxas da crescimento de flutuações da matéria para os modelos de Einstein-de Sitter e Λ CDM.

Models beyond Λ CDM

Generalized Chaplygin gas (GCG)

Equation of state (EoS)

$$p = -\frac{A}{\rho^\alpha}$$

Energy density

$$\rho = \left[A + B a^{-3(1+\alpha)} \right]^{1/(1+\alpha)}$$

Early Universe: **(almost) pressureless matter**, $\rho \propto a^{-3}$

Late Universe: **(almost) cosmological constant**, $\rho \propto A^{1/(1+\alpha)}$

Unified dark-sector model

Does a generalized Chaplygin gas correctly describe the cosmological dark sector?

$$p = -\frac{A}{\rho^\alpha}$$

Answer: Yes, but only for a parameter value α that makes it almost coincide with the standard model ($\alpha = 0$).

Why?

Strategy: Split into pressureless matter and “vacuum”

$$\rho = \rho_{DM} + \rho_X$$

$$p = p_{DM} + p_X = p_X = w_X \rho_X$$

$$w_X = -1$$

Analysis

SN Ia analysis (JLA sample):

Broad range of α data possible

(including $\alpha = -1$ and $\alpha = 0$ (at the 2σ confidence level))

Planck data for the CMB anisotropy spectrum:

Range substantially narrowed

$$|\alpha| \lesssim 0.05$$

R.F. vom Marttens, L. Casarini, W. Z., W.S. Hipólito-Ricaldi, D.F. Mota,
Physics of the Dark Universe 15 (2017) 114 - 124

Interacting models

Cosmic medium

Radiation ρ_r , **EoS** $w_r = \frac{1}{3}$

Baryons ρ_b , **EoS** $w_b = 0$

Cold dark matter (CDM) ρ_m , **EoS** $w_m = 0$

Dark energy (DE) ρ_x , **EoS** $w_x = -1$

Total energy density

$$\rho = \rho_r + \rho_b + \rho_m + \rho_x$$

Total pressure

$$p = p_r + p_x$$

Interactions in the dark sector

Total conservation

$$T^{\mu\nu}_{;\nu} = 0, \quad T^{\mu\nu} = T^{\mu\nu}_{(r)} + T^{\mu\nu}_{(b)} + T^{\mu\nu}_{(m)} + T^{\mu\nu}_{(x)}$$

Radiation and Baryons separately conserved

$$T^{\mu\nu}_{(r);\nu} = T^{\mu\nu}_{(b);\nu} = 0$$

Interacting components

$$T^{\mu\nu}_{(m);\nu} = Q^\mu, \quad T^{\mu\nu}_{(x);\nu} = -Q^\mu$$

Interaction term

General split into parts parallel and perpendicular to u^μ

$$Q^\mu = Qu^\mu + F^\mu, \quad F^\mu u_\mu = 0$$

Background

$$\dot{\rho}_r + 4H\rho_r = 0 \quad \Rightarrow \quad \rho_r = \rho_{r0} a^{-4}$$

$$\dot{\rho}_b + 3H\rho_b = 0 \quad \Rightarrow \quad \rho_b = \rho_{b0} a^{-3}$$

$$\dot{\rho}_m + 3H\rho_m = -Q$$

$$\dot{\rho}_x = Q$$

Class of interaction terms

$$Q = 3H\gamma\rho_m^\alpha\rho_x^\beta(\rho_m + \rho_x)^\sigma$$

Density parameters for interacting models

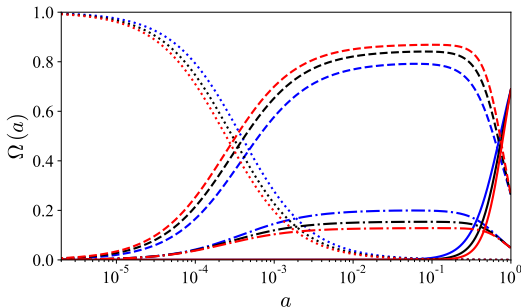


Figure: Density parameters for all components of the Universe. Solid: DE, dashed: CDM, dot-dashed: baryons, dotted: radiation. Blue: $\gamma = -0.2$, black: $\gamma = 0$ (non-interacting case), red: $\gamma = +0.2$.

Perturbations

Standard Boltzmann equations for radiation and baryons
(Ma, Bertschinger, 1995)

Interacting fluid description of CDM and DE

Specific models

► $Q = 3H\gamma \frac{\rho_m \rho_x}{\rho_m + \rho_x}$

► $Q = 3H\gamma \frac{\rho_x^2}{\rho_m + \rho_x}$

► $Q = 3H\gamma \frac{\rho_m^2}{\rho_m + \rho_x}$

► $Q = 3H\gamma\rho_x$

► $Q = 3H\gamma\rho_m$

Data base

CLASS code combined with MontePython

(Blas, Lesgourgues, Tram, 2011; Audren, Lesgourgues, Benabed, Prunet 2013; Brinckmann, Lesgourgues 2018)

Data from

- ▶ Supernova of type SNe Ia (JLA)
- ▶ Baryon acoustic oscillations (BAO)
- ▶ Hubble constant H_0
- ▶ Cosmic chronometers (CC)
- ▶ Planck satellite (Planck TT)

General message

Data from the late Universe (H_0 , SNe Ia and CC) allow for an interaction in the dark sector.

Data related to the early Universe (BAO and Planck TT) constrain this interaction substantially.

Conclusion

An interaction in the dark sector is not excluded but the range for the still admissible interaction parameter is very narrow, being always consistent in 1σ CL with the zero coupling case ($\gamma = 0$).

(R. von Marttens, L. Casarini, D.F. Mota, W.Z., Physics of the Dark Universe 23 (2019) 100248)

Results

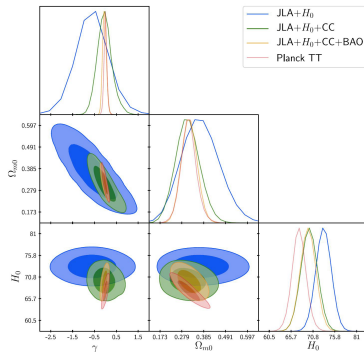


Fig. 17. Statistical analysis (IDM 1).

Figure: Statistical analysis model $Q = 3H\gamma \frac{\rho_m \rho_x}{\rho_m + \rho_x}$

Statistical analysis

Table 3Result of the statistical analysis with 2σ CL for all IDEMs.

Model	Data	H_0	Ω_{m0}	γ
IDEM1	SNe Ia+ H_0	$73.37^{+3.63}_{-3.61}$	$0.354^{+0.109}_{-0.162}$	$-0.53^{+1.02}_{-0.91}$
	SNe Ia+ H_0 +CC	$70.78^{+3.62}_{-3.61}$	$0.307^{+0.108}_{-0.122}$	$-0.07^{+0.58}_{-0.74}$
	SNe Ia+ H_0 +CC+BAO	$69.44^{+3.62}_{-3.61}$	$0.321^{+0.072}_{-0.078}$	$-0.06^{+0.16}_{-0.18}$
	Planck TT	$68.13^{+2.86}_{-2.96}$	$0.3143^{+0.0636}_{-0.0685}$	$-0.010^{+0.108}_{-0.140}$
IDEM2	SNe Ia+ H_0	$73.29^{+3.60}_{-3.60}$	$0.371^{+0.189}_{-0.136}$	$-0.40^{+0.58}_{-0.94}$
	SNe Ia+ H_0 +CC	$69.75^{+3.60}_{-3.60}$	$0.307^{+0.186}_{-0.148}$	$-0.04^{+0.60}_{-0.70}$
	SNe Ia+ H_0 +CC+BAO	$69.72^{+3.60}_{-3.60}$	$0.326^{+0.110}_{-0.104}$	$-0.08^{+0.28}_{-0.24}$
	Planck TT	$68.00^{+2.28}_{-2.47}$	$0.3054^{+0.054}_{-0.050}$	$-0.0024^{+0.104}_{-0.105}$

Figure: Models $Q = 3H\gamma \frac{\rho_m \rho_x}{\rho_m + \rho_x}$ and $Q = 3H\gamma \frac{\rho_x^2}{\rho_m + \rho_x}$.

Matter growth rate

Baryonic perturbations

Three-component description

Deviations from $w_S = -1$ admitted

(S. del Campo, J.C. Fabris, R. Herrera, W.Z.,
Eur. Phys. J. C 77 (2017)479;
W.Z., J.C. Fabris, H.E.S. Velten, R. Herrera, work in progress)

Cosmic medium

Small redshift: radiation negligible

Baryons ρ_B , $p_B = 0$

Dark matter ρ_D , $p_D = w_D \rho_D$

Scalar field ρ_S , $p_S = w_S \rho_S$

Total energy density

$$\rho = \rho_B + \rho_D + \rho_S$$

Total pressure

$$p = p_D + p_S$$

Baryons and dark matter

Baryons

$$p_B = 0$$

Energy density

$$\rho_B = \rho_{B0} a^{-3}$$

Dark matter

$$p_D = w_D \rho_D$$

Energy density

$$\rho_D = \rho_{D0} a^{-3(1+w_D)}$$

Scalar field

Equation of state

$$p_S = w_S \rho_S$$

Parametrization CPL (Chevallier, Polarski, Linder)

$$w_S = w_0 + w_1 (1 - a)$$

Energy density

$$\rho_S = \rho_{S0} a^{-3(1+w_0+w_1)} \exp[3w_1(a-1)]$$

Density parameters

Dark matter

$$\Omega_D = \frac{8\pi G \rho_D}{3H^2}$$

Scalar field (SF, fluid representation)

$$\Omega_S = \frac{8\pi G \rho_S}{3H^2}$$

Baryons

$$\Omega_B = \frac{8\pi G \rho_B}{3H^2}$$

First-order perturbations

Total energy-density perturbations δ

$$\rho = \rho_D + \rho_S + \rho_B, \quad \hat{\rho} = \hat{\rho}_D + \hat{\rho}_S + \hat{\rho}_B$$

$$\delta \equiv \frac{\hat{\rho}}{\rho}, \quad \delta_B \equiv \frac{\hat{\rho}_B}{\rho_B}, \quad \delta_D \equiv \frac{\hat{\rho}_D}{\rho_D}, \quad \delta_S \equiv \frac{\hat{\rho}_S}{\rho_S}$$

Relative perturbations S_{BD} and S_{BS}

$$S_{BD} \equiv \delta_B - \frac{\delta_D}{1 + w_D}$$

and

$$S_{BS} \equiv \delta_B - \frac{\delta_S}{1 + w_S}$$

Basic perturbation variables

Total energy-density perturbations δ

Relative perturbations S_{BD} and S_{BS}

Strategy

Coupled system of total energy-density perturbations δ and relative perturbations S_{BD} and S_{BS}

Final aim

Baryonic perturbations

$$\delta_B = \frac{\delta + (1 + w_D) \Omega_D S_{BD} + (1 + w_S) \Omega_S S_{BS}}{1 + w_D \Omega_D + w_S \Omega_S}$$

First-order perturbations

Metric

$$ds^2 = -(1 + 2\phi) dt^2 + a^2 (1 - 2\psi) \delta_{ab} dx^a dx^b$$

Perturbed time components of the four-velocity

$$\hat{u}_0 = \hat{u}^0 = \frac{1}{2} \hat{g}_{00} = -\phi$$

Velocity potential

$$a^2 \hat{u}^a = \hat{u}_a \equiv \mathbf{v}_{,a}$$

Comoving fractional density perturbations δ^c

$$\delta^c \equiv \delta - 3H(1+w)v, \quad \delta \equiv \frac{\hat{\rho}}{\rho}, \quad w \equiv \frac{p}{\rho}$$

Determines gravitational potential

Poisson-type equation:

$$-\frac{2}{3} \frac{k^2}{H^2 a^2} \psi = \delta^c$$

(Combination of 00 and 0a field equations)

Equation for δ^c

Influence of pressure perturbations

$$\delta^{c''} + \left[\frac{3}{2} - \frac{15}{2} \frac{p}{\rho} + 3 \frac{p'}{\rho'} \right] \frac{\delta^{c'}}{a} - \left[\frac{3}{2} + 12 \frac{p}{\rho} - \frac{9}{2} \frac{p^2}{\rho^2} - 9 \frac{p'}{\rho'} \right] \frac{\delta^c}{a^2} + \frac{k^2}{a^2 H^2} \frac{\hat{p}^c}{\rho a^2} = 0$$

Comoving pressure perturbations $\hat{p}^c$

In general: $p \neq p(\rho)$

$\Rightarrow$ **Nonadiabatic perturbations**

Phenomenological sound speed parameters

Assumptions about (rest-frame) pressure perturbations

“Silent” DM

$$\hat{p}_D = 0$$

“Stiff” SF

$$\hat{p}_S = \hat{\rho}_S$$

Subhorizon scales $\frac{k^2}{a^2 H^2} \gg 1$

Growth for a typical scale

Growth function

$$f = \frac{d \ln \delta_B}{d \ln a}$$

Observed quantity $f\sigma_8$

σ_8 : rms mass fluctuation in spheres with radius $8h^{-1}\text{Mpc}$

Data sets of RSD measurements of $f\sigma_8$
(SDSS, BOSS, WiggleZ, Vipers, 6dFGS ...)

Typical scale: $k = 0.1h\text{Mpc}^{-1}$: $\frac{k^2}{a^2 H^2} \approx 1.8 \cdot 10^5$

Growth function for $w_D = 0$ and $w_S = -1 \pm 10^{-3}$

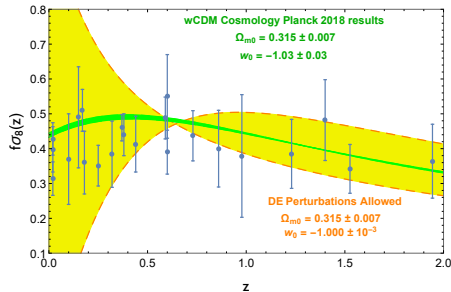


Figure: Green stripe: standard cosmology with Planck 2018 parameter values. Yellow region: interval $w_S = -1 \pm 10^{-3}$ with the same Ω_{m0} values as in the standard case. (W.Z., J.C. Fabris, H.E.S. Velten, R. Herrera, work in progress)

Result

Extension of the standard model

Matter growth rate for $w_x \neq -1$

Tiny modifications - unacceptably large consequences

No indication for deviations from Λ CDM

Conclusion

Is dark energy a dynamical quantity?

Three different extension of the standard model

- ▶ Cosmic medium as generalized Chaplygin gas
- ▶ Interactions between dark matter and dark energy
- ▶ Matter growth rate

No indication for deviations from Λ CDM

Very likely, dark energy does not clump.

Outlook

More general models?

More substantial deviation from standard cosmology?

General fluid dynamics applicable for

- ▶ **Intrinsically inhomogeneous models**
- ▶ **Backreaction models**
- ▶ **Effective fluid models for modified gravity**

Inhomogeneous models

Inhomogeneities only from perturbations?

Most research: **cosmological principle**

Perturbations on a homogeneous background

Gravitational instability $\Rightarrow$ nonlinear structures

Homogeneity scale?

Homogeneous solution: adequate starting point?

Inhomogeneous cosmological models?

Inhomogeneous models

K. Tomita astro-ph/9906027

“Cosmological void models may explain the observed deviation of high-redshift supernovas (SNIa) from the relation in homogeneous Friedmann models, independently of the cosmological constant.”

M.-N. Célérier astro-ph/9907206

“Do we really see a cosmological constant in the supernovae data?” “The presently published SNIa data can be interpreted as implying either a strictly positive cosmological constant in an homogeneous universe or large scale inhomogeneity with no constraint on Λ .”

Light propagation different from that in homogeneous models

Lemaître-Tolman-Bondi (LTB) solution

Simplest inhomogeneous cosmological solution of GR

Dust solution

Spherical symmetry

FLRW solution as a well defined limit

Mimics effects attributed to dark energy

Distance - redshift relation for SNIa reproduced without Λ

Applicable as long as radiation is dynamically irrelevant

Current situation

Difficult to reconcile all observations on the LTB basis

Less activity than on Λ CDM

LTB ruled out?

However: spherical symmetry too simple?

Useful toy models

Observational difference to Λ CDM?

Measure redshift drift Chul-Moon Yoo, T. Kai, Ken-ichi Nakao 2011

Redshift drift for LTB and Λ CDM

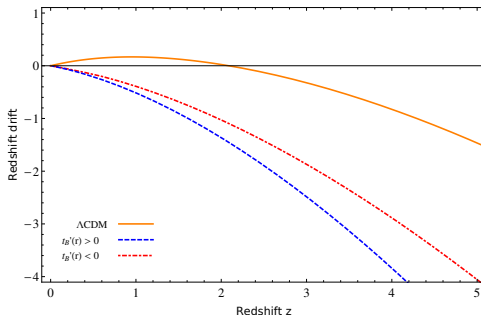


Figure: Redshift drift for LTB models compared with the Λ CDM model ($\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$). (E. G. Chirinos Isidro, C. Zuñiga Vargas, WZ, JCAP 1605 (2016) 003)

Backreaction models

Cosmic backreaction

“Accelerated expansion” sets in at about the time at which the largest structures are forming.

Influence of cosmic structures on the (homogeneous) large-scale dynamics?

Difference between averaged inhomogeneous dynamics and the standard model dynamics?

Can backreaction mimic a cosmological constant?

Curvature fluctuations due to inhomogeneities?

Curvature fluctuation?

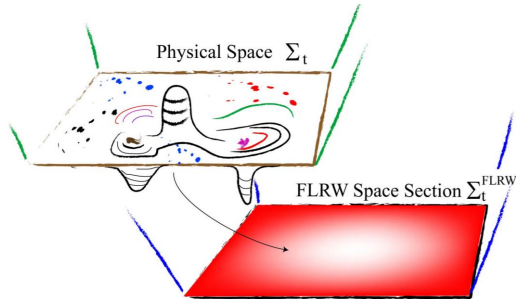


Figure 2. The picture that the scalar curvature in the physical space would average out on some large scale of homogeneity is naive in a number of ways. There is no equipartition law for the scalar curvature that would be dynamically preserved.

Figure: Buchert, Carfora CQG 25:195001,2008

Length scales of cosmic structures

Solar system scale

Galactic scale

Scale of galaxy clusters

Homogeneity scale

Dynamics on a larger scale as average over the dynamics of the underlying smaller scale?

Averaged Einstein equations?

Einstein equations on which scale?

Nonlinearity

$$\langle G_{\alpha\beta}(g) \rangle \neq G_{\alpha\beta}(\langle g \rangle)$$

Homogeneous dynamics as an average over an inhomogeneous configuration?

Cosmological principle?

Homogeneous and isotropic background

Background fictitious

Statistically homogeneous and isotropic?

Suitable average?

Backreaction of inhomogeneities?

Backreaction from spatial inhomogeneities on the average large-scale homogeneous cosmological dynamics

Backreaction equivalent to an effective negative pressure?

Dust universe without additional DE component?

Problem: **Averaging in GR**

Buchert 2000:

Average of scalar quantities over spatial hypersurfaces

Status of backreaction models?

- ▶ Averaging in GR not yet well established
- ▶ Averaging seems to imply backreaction effects
- ▶ Quantitatively not sufficient to replace Λ
- ▶ Emergence of curvature

Curvature growth through growth of inhomogeneities

Nonlinear evolution and backreaction closely associated with spatial curvature

Growth of inhomogeneities cannot be separated from growth of the spatial curvature

Szekeres solution

(K. Bolejko, Emergence of spatial curvature, arXiv:1707.01800)

Notice: low-redshift data alone (without combining with CMB) implies large spatial curvature $\Omega_k \approx 0.2$.

Only after inclusion of the CMB: $\Omega_k \approx 0.005 \pm 0.009$

Effective fluid models for modified gravity

Example: Jordan-Brans-Dicke type scalar-tensor theory

Field equations

Modified Einstein equations

$$\Phi \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = \kappa^2 T_{(m)\mu\nu} + \frac{\omega(\Phi)}{\Phi} \left(\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} (\nabla \Phi)^2 \right) \\ + \nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \square \Phi - \frac{1}{2} g_{\mu\nu} U$$

Scalar-field equation

$$\square \Phi = \frac{1}{2\omega(\Phi) + 3} \left(\kappa^2 T - \frac{d\omega(\Phi)}{d\Phi} (\nabla \Phi)^2 + \Phi \frac{dU}{d\Phi} - 2U \right)$$

Matter conservation

$$T_{(m)}^{\mu\nu}{}_{;\nu} = 0$$

Method

- ▶ Map additional (compared with GR) geometrical degrees of freedom onto an effective fluid component
- ▶ Effective two-fluid system within GR
- ▶ Gravitational dynamics does not depend on the details of the (underlying) scalar-field dynamics
- ▶ Phenomenological fluid dynamical equations

Effective Einstein equations

Rewrite scalar-tensor equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 T_{\mu\nu}$$

Total energy-momentum tensor

$$T_{\mu\nu} = T_{(m)\mu\nu} + T_{(x)\mu\nu}$$

Effective fluid

$$\begin{aligned} T_{(x)\mu\nu} \equiv & \left(\frac{1}{\Phi} - 1 \right) T_{(m)\mu\nu} + \frac{1}{\kappa^2 \Phi} \frac{\omega(\Phi)}{\Phi} \left[\partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu} (\nabla \Phi)^2 \right] \\ & + \frac{1}{\kappa^2 \Phi} \left[\nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \square \Phi - \frac{1}{2} g_{\mu\nu} U \right] \end{aligned}$$

Two-component cosmic medium

- ▶ Pressureless perfect-fluid type matter $\mathbf{T}_{(m)\mu\nu}$
- ▶ General imperfect fluid $\mathbf{T}_{(x)\mu\nu}$

Total energy-momentum tensor (EMT)

$$\mathbf{T}_{\mu\nu} = \mathbf{T}_{(m)\mu\nu} + \mathbf{T}_{(x)\mu\nu}$$

Effective anisotropic pressure

EMT

$$T_{(x)\mu\nu} = \rho_x u_{(x)\mu} u_{(x)\nu} + p_x h_{(x)\mu\nu} + \Pi_{(x)\mu\nu} + q_{(x)\mu} u_{(x)\nu} + q_{(x)\nu} u_{(x)\mu}$$

Anisotropic pressure of component x

$$\Pi_{(x)\mu\nu} = h_{(x)(\mu}^{\sigma} h_{(x)\nu)}^{\tau} T_{(x)\sigma\tau} - \frac{1}{3} h_{(x)\mu\nu} h_{(x)}^{\sigma\tau} T_{(x)\sigma\tau}$$

Generality of the (effective) fluid approach

Relevant variable

Scalar part Π of the anisotropic stress tensor

$$\Pi_{ab} = \left(h_a^m h_b^n - \frac{1}{3} h_{ab} h^{mn} \right) \Pi_{,m,n}$$

Gravitational slip

Difference of gravitational potentials

$$\psi - \phi = 8\pi G\Pi$$

Anisotropic stress generates gravitational slip

Potential signature of deviations from the standard model

Deviations quantified by a suitable parametrization of the anisotropic stress

Example

Extended Λ CDM model inspired by Jordan-Brans-Dicke theory

Fluid dynamical approach

$e_\phi \Lambda$ CDM model: modified background dynamics

Hubble rate explicitly

$$\frac{H^2}{H_0^2} = \frac{A\Omega_{m0}a^{-3}}{\Phi} + [1 - A\Omega_{m0}]\Phi$$

$$\Phi = a^{\frac{-6m}{1+3m}} \quad A \equiv \frac{(1+3m)^2}{1-m}$$

Consequence of the fluid dynamical equations

Scalar Φ modifies the cosmological dynamics

$$\Phi = 1 \Leftrightarrow m = 0: \Lambda\text{CDM}$$

W.C. Algoner, H.E.S. Velten, WZ, JCAP 11 (2016) 034

Conservation equations

Separate conservation

$$T^{\mu\nu}_{;\nu} = 0, \quad T^{\mu\nu}_{(m);\nu} = 0, \quad T^{\mu\nu}_{(x);\nu} = 0$$

Aim: Energy-density perturbations

$$\delta_x = \frac{\hat{\rho}_x}{\rho_x}, \quad \delta = \frac{\hat{\rho}}{\rho}, \quad \delta_m = \frac{\hat{\rho}_m}{\rho_m}$$

Matter growth rate determined by δ_m

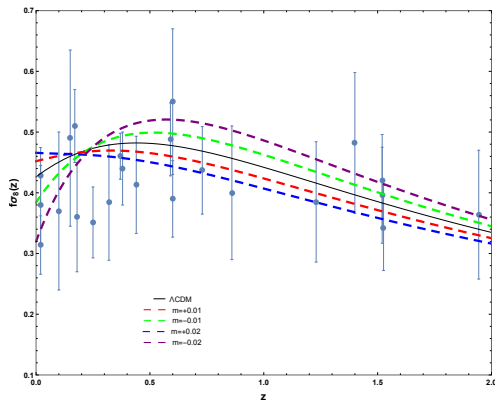


Figure: Dependence of $f\sigma_8(z)$ on z if only the background dynamics is changed.

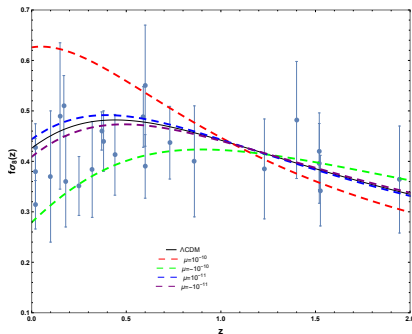


Figure: Dependence of $f\sigma_8(z)$ on z in the presence of anisotropic stresses.

W.Z., H.E.S. Velten, W.C. Algoner, Universe 2019, 5, 68;
arXiv:1903.03383

Discussion: JBD inspired model

- ▶ Minimal extension of the standard model: $e_\phi \Lambda \text{CDM}$
- ▶ Modified background dynamics analytically
- ▶ Phenomenological fluid dynamics: matter growth rate
- ▶ Observational constraints on anisotropic stresses

Outlook

More general approaches on a fluid dynamical basis

Potential areas of interest:

- ▶ **Inhomogeneous models**
- ▶ **Backreaction models**
- ▶ **Effective fluid models for modified gravity**