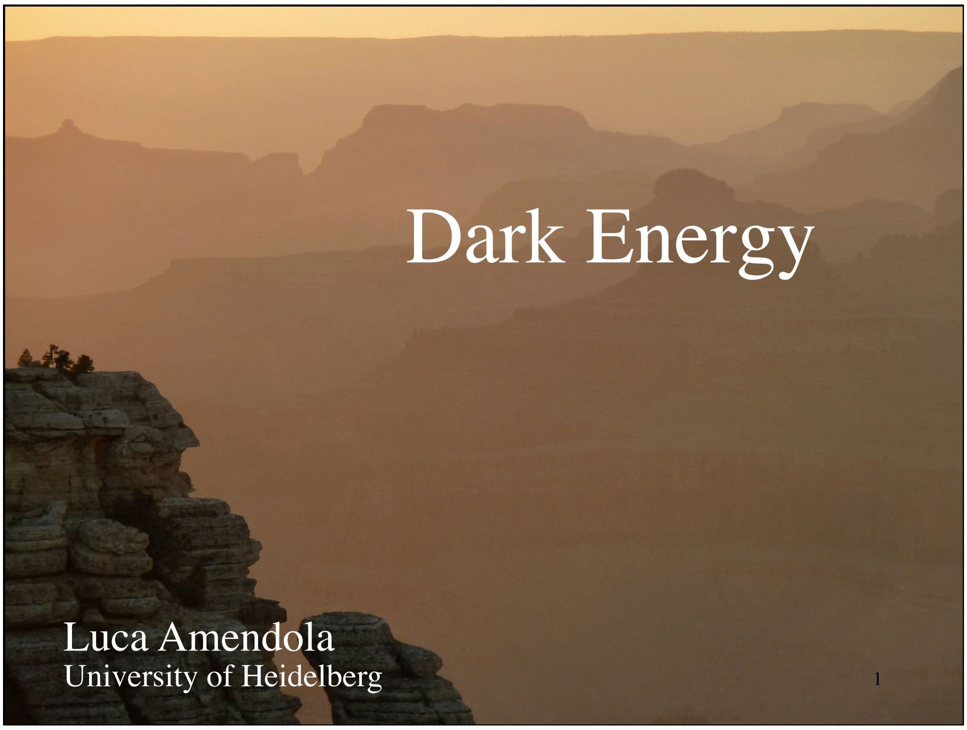


Outline

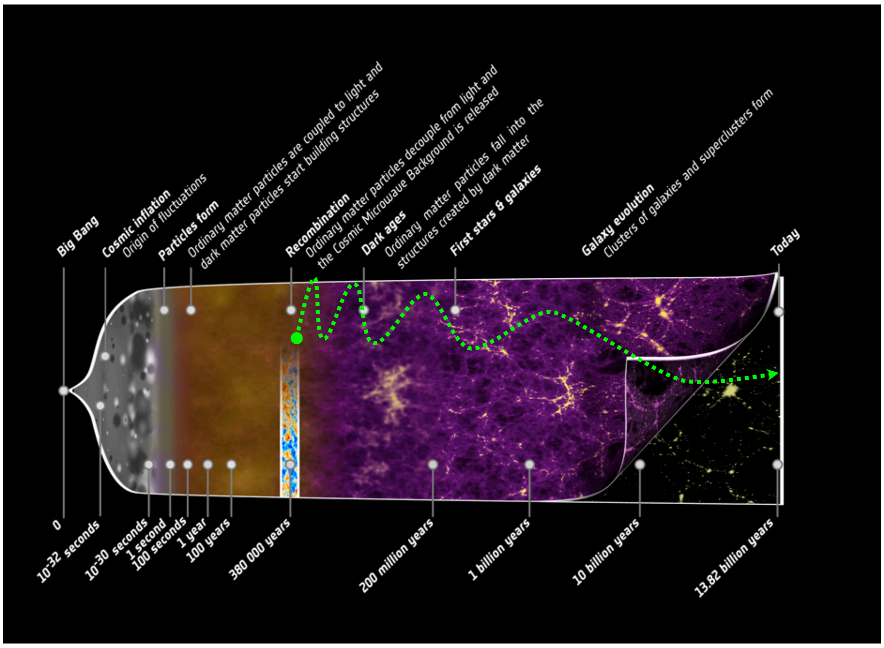
- Lecture 1 Introduction to Dark Energy
- Lecture 2 Testing Dark Energy, 1
- Lecture 3 Testing Dark Energy, 2
- Lecture 4 Further probes of DE
- Lecture 5 Euclid

Some collaborators

Luciano Casarini, Adalto Gomes, Valerio Marra, Miguel Quartin, Rogerio Rosenfeld, Ioav Waga, ...

Texts

- **Dodelson**, Modern Cosmology
- **Amendola & Tsujikawa**, Dark Energy. Theory and Observations, Cambridge UP (rev. ed. 2015)
- **Euclid Theory WG**, Cosmology and Fundamental Physics with the Euclid Satellite, arXiv 1206.1225 +1606.00180



Cosmology Executive Summary

Dark matter 27%

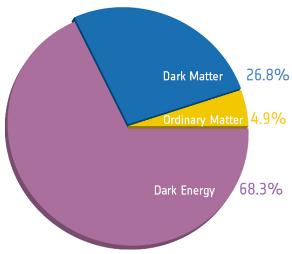
Baryons 5%

Massive neutrinos: 0.1%

Photons: 0.01%

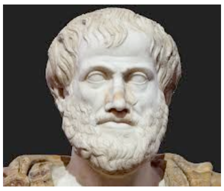
Spatial curvature: very close to 0

Something else: ≈ 70%



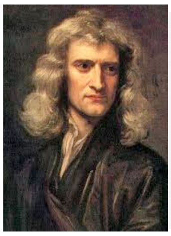
Back to the classics

Historical perspective, circa 350 BCE



- Gravity is always attractive: how to avoid that the **sky** falls on our head?
- **Aristotle**'s answer: quintessence

Historical perspective, circa 1700



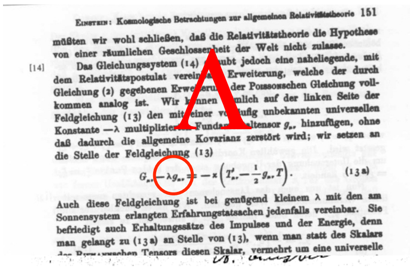
- Gravity is always attractive: how to avoid that the **stars** fall on our head?
- **Newton**'s answer: initial conditions

Historical perspective, circa 1900

- Gravity is always attractive: how to avoid that the **Universe** fall on our head?
- **Einstein**'s answer: to avoid collapse (to make the universe stable) it is necessary to introduce a form of repulsive gravity, by modifying the equations of General Relativity.

Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



EINSTEIN: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie 151

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

[14] Das Gleichungssystem (14) [illegible]ubt jedoch eine naheliegende, mit dem Relativitätspostulat verein[illegible] Erweiterung, welche der durch Gleichung (2) gegebenen Erwe[illegible] der Poissonschen Gleichung vollkommen analog ist. Wir k[illegible]nen [illegible]mlich auf der linken Seite der Feldgleichung (13) den mit einer vo[illegible]ufig unbekannten universellen Konstante $-\lambda$ multiplizier[illegible] Funda[illegible]altensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

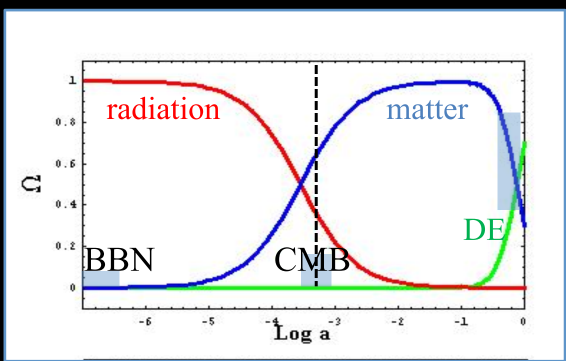
$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa \left(T'_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle

Einstein 1917

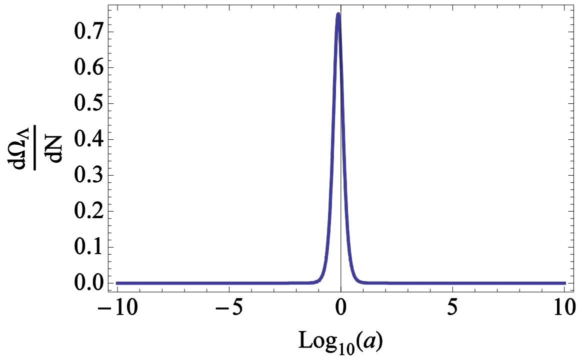
Time view

We know so little about the evolution of the universe!



Why now?

The coincidence problem



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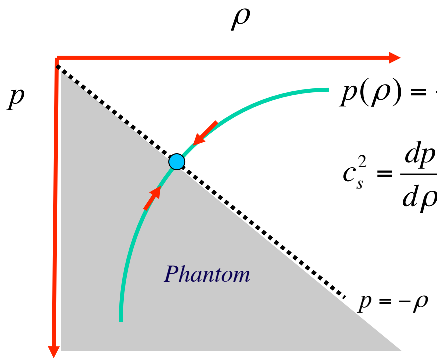
Beyond the cosmological constant



$$p(\rho) = -\frac{A}{\rho}$$

$$c_s^2 = \frac{dp}{d\rho} > 0$$

$$\dot{\rho} = -3H(p+\rho)$$

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From observations to theory

- What we really observe in cosmology is light from **sources** and from **backgrounds**.
- How do we connect these observables to cosmological quantities like $\rho_m, \rho_\gamma, k, a(t), H_0$ etc?
- First, define

$$\Omega_M = \frac{8\pi\rho_0}{3H_0^2}, \quad \Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3H_0^2}, \quad \Omega_k = \frac{8\pi k}{3H_0^2}$$

and note that

$$1 = \Omega_M + \Omega_\Lambda + \Omega_k$$

so rewrite Friedman equation as ($a_0 = 1$)

$$H^2 = H_0^2(\Omega_m a^{-3} + \Omega_\Lambda a^0 + \Omega_k a^{-2})$$

- Then, generalize it to several components:

$$\begin{aligned} H^2 &= H_0^2(\Omega_m a^{-3(1+w_m)} + \Omega_\Lambda a^{-3(1+w_\Lambda)} + \ldots) \\ &= H_0^2 \sum_i \Omega_i a^{-3(1+w_i)} = H_0^2 E(a)^2 \end{aligned}$$

Cosmic Inventory

name	density	EOS w
baryons	0.05	≈ 0
CDM	0.27	≈ 0
radiation	0.0001	1/3
Massive neutrinos	<0.05	≈ 0
Cosm. const.	0.68	-1
curvature	<0.01	-1/3
Other ?	?	?

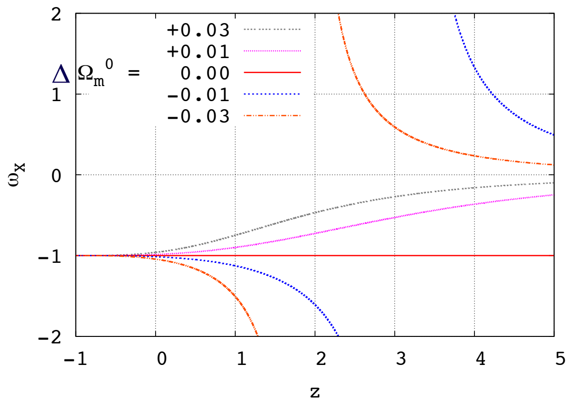
Warning 1

- Not everything we insert in our equations is necessarily an observable!
- Consider only two components, pressureless uncoupled matter and "something else".
- Then the EOS of this "something else" is related to the expansion E(z) as

$$w_{DE} = \frac{(1+z)(E^2)' - 3E^2}{3[E^2 - \Omega_{m0}(1+z)^3]}$$

- Even a perfect knowledge of E(z) is not sufficient to obtain w(z): one needs also Ω_{m0}
- But all you get from distance indicators is E(z), not Ω_{m0}
- Conclusion: either you "know" Ω_{m0} and obtain w(z) or you "know" w(z) and obtain Ω_{m0}

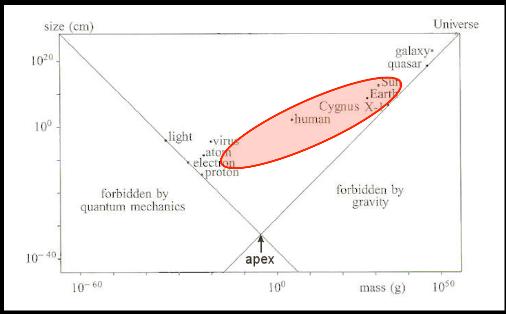
Example



LCDM with different Ω

Warning 2

- We only directly test gravity within the solar system, at the present time, and with "baryons"



On Space and Time, Edited by Shahn Majid

Testing Gravity

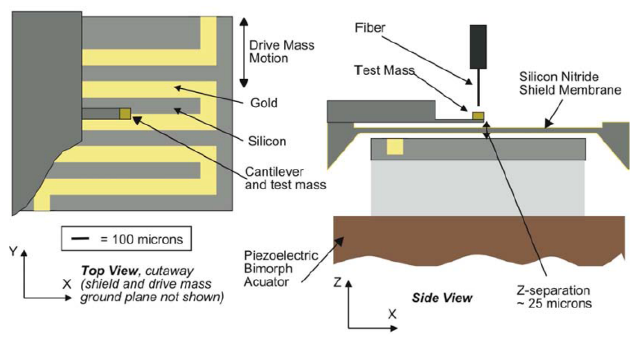
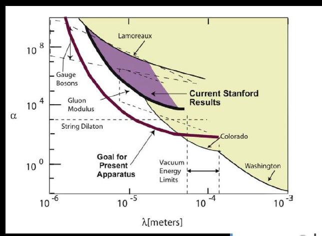


Figure 2: Schematic of the masses and the cantilevers. Geometry in the $x-y$ plane is to scale; the $z-$separation and the thicknesses of the cantilever and shield are not to scale.

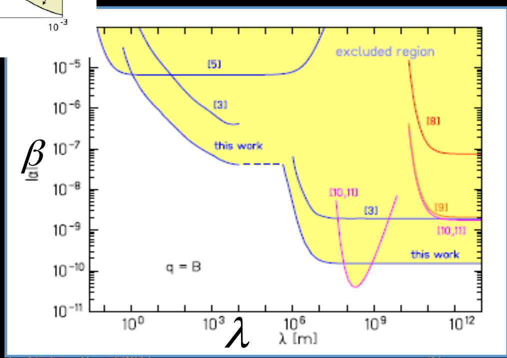
Smullin et al. 2004 (SLAC)

Testing Gravity



Smullin et al. 2004

$$\frac{GM}{r} \rightarrow \frac{GM}{r}(1+\beta e^{-r/\lambda})$$



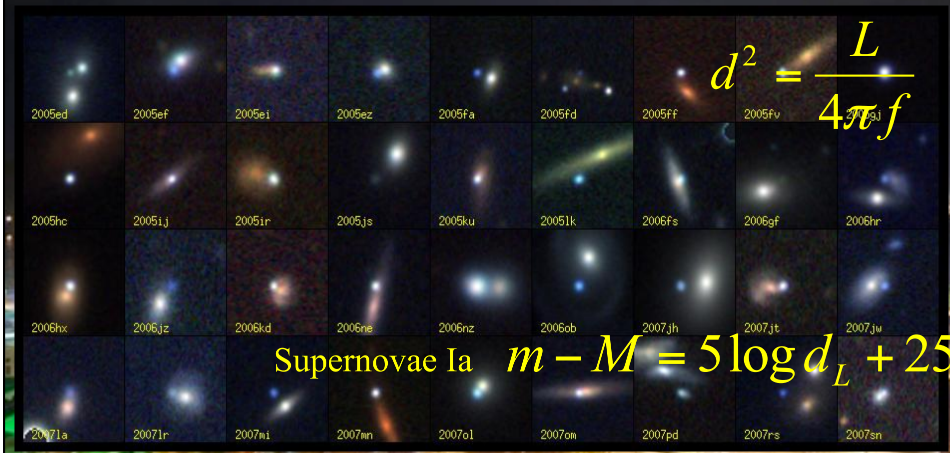
Schlamminger et al 2008

The fourfold way out from local gravity

$$\Psi = -\frac{MG^*}{r}(1+\beta\ e^{-m_\varphi r})$$

m_ϕ, β {
- depends on time
- depends on space
- depends on local density
- depends on species

Lighthouses in the dark



$$d^2 = \frac{L}{4\pi f}$$

Supernovae Ia $\quad m - M = 5\log d_L + 25$

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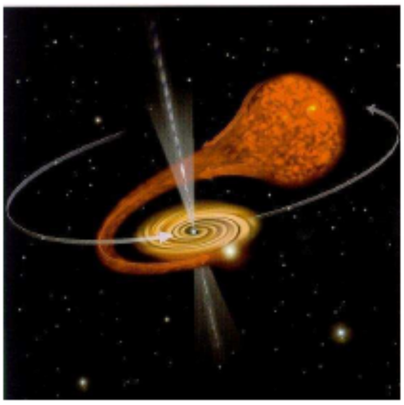

- This hypothesis can be tested and calibrated through a local sample whose distance we know by other means.

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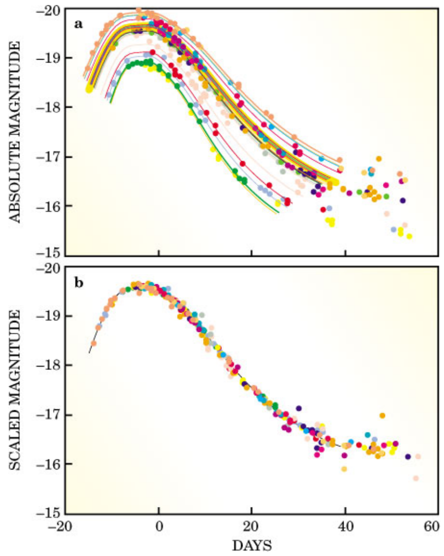

Phillips, Hamuy, et al. 1993, 1995

- Then, we compare $m_{obs}(z)$ with

$$m_{theor}(z) = M + 25 + \log d(z; \Omega_M, \Omega_\Lambda, ..)$$

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Luminosity distance

$$f = \frac{L}{4\pi r^2(z)(1+z)^2}$$

$$d_L = r(z)(1+z)$$

$$ds^2 = 0 \quad \Longrightarrow \quad r(z)$$

$$d_L(z) = \frac{1+z}{H_0\sqrt{-\Omega_{k0}}}\sinh(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)})$$

$$H(z) = H_0(\Omega_{m0}a^{-3} + \Omega_\Lambda + \Omega_{k0}a^{-2})^{1/2}$$

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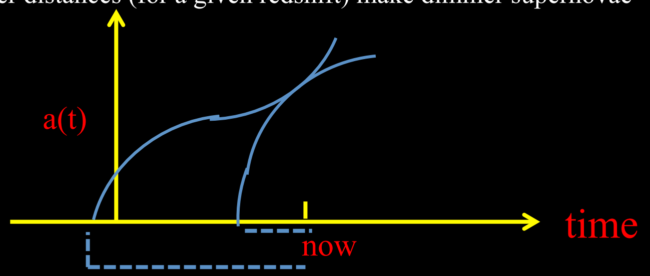
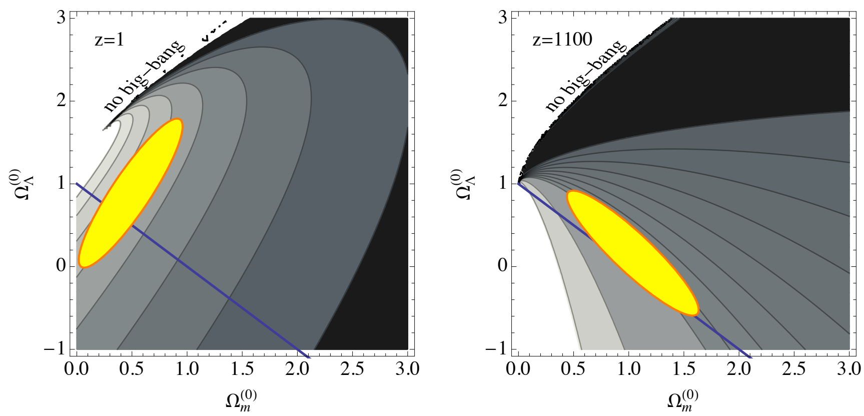
Basic property 1

Local Hubble law $r(z) = \frac{z}{H_0}$ $\longrightarrow$ $r(z) = \int \frac{dz}{H(z)}$ Global Hubble law

If H(z) in the past is smaller (i.e. acceleration), then r(z) is larger: larger distances (for a given redshift) make dimmer supernovae

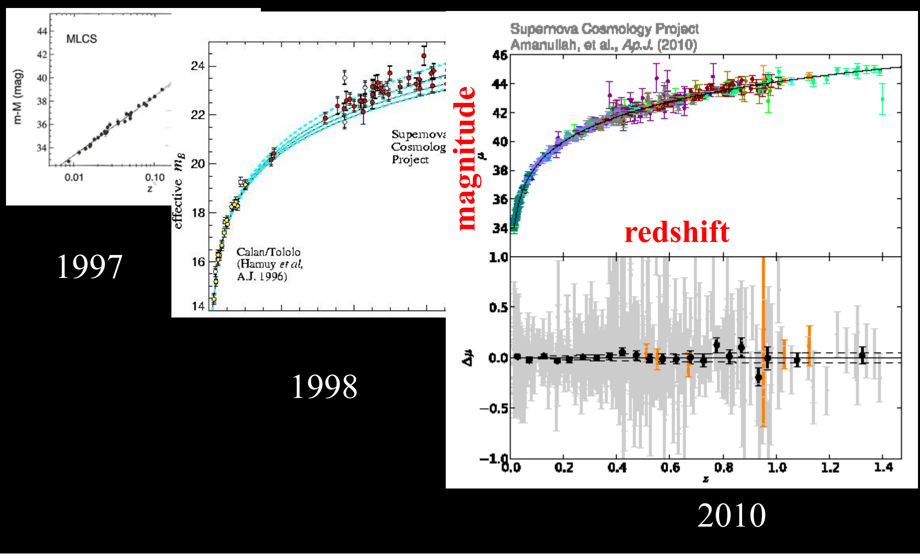


Basic property 2

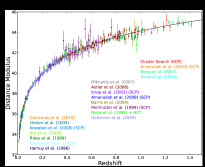


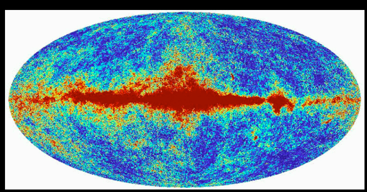
Curves of constant luminosity distance

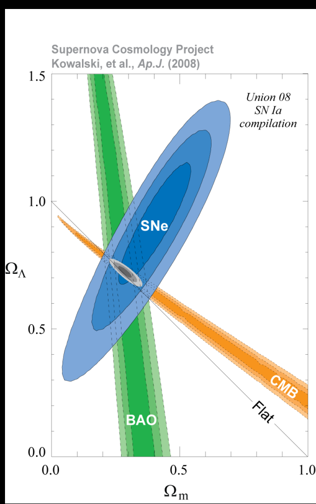
Hubble diagram



Properties of Dark Energy







Properties of Dark Energy

Isotropy

Abundance

Observational requirements

Slow evolution

Weak clustering

Properties of Dark Energy

Observations:

- Isotropy
- Large abundance
- Slow evolution
- Weak clustering

Theory:

- Scalar field?
- $\Omega_{DE} \approx \Omega_m$
- $w_{eff} \approx -1$
- $c_s \approx 1$

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The magic of Lagrangians

$$\int dx^4 \sqrt{-g}\left[R + L_{matter}\right] \xrightarrow{\text{variation}} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

The two laws of Lagrangian-cooking:
a) form a scalar: Eqs are covariant
b) No explicit functions of coords: Eqs are conserved

Random example:

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + R_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + V(\phi) + R_{\mu\nu\sigma\tau}\phi^{,\nu}\phi^{,\mu}\phi^{,\sigma}\phi^{,\tau} + R_{\mu\nu\sigma\tau}R^{,\sigma\tau}\phi^{,\nu\mu} + ... + L_{matter}\right]$$

The magic of Fourier space

Fourier space

$$\phi(x,y,z,t) \rightarrow \phi(t)\exp(i\vec{k}\vec{x})$$

therefore

$$\frac{\partial}{\partial x}\phi(x,y,z,t) \rightarrow \phi(t) i k_x \exp(i\vec{k}\vec{x})$$

$$\nabla\phi(x,y,z,t) \rightarrow \phi(t) i\vec{k} \exp(i\vec{k}\vec{x})$$

$$\nabla\nabla\phi(x,y,z,t) \rightarrow -\phi(t) k^2 \exp(i\vec{k}\vec{x})$$

Very useful for linear equations because then you can drop the space part!

The past ten years of DE research

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi,\frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

Cosmological constant, Dark energy w=const, Dark energy w=w(z),quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

define $X \equiv \frac{1}{2}\phi_{;\mu}\phi_{;\nu}$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\,[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)]\,, \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\,[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)]\,.
\end{aligned}$$

- ✓ First found by Horndeski in 1975
- ✓ rediscovered by Deffayet et al. in 2011
- ✓ no ghosts, no classical instabilities
- ✓ it modifies gravity!
- ✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc

A riddle for you all

The most general 4D scalar field theory with second order equations of motion

$$\int dx^4 \sqrt{-g} \left[\sum_i L_i + L_{matter} \right]$$

$$\begin{aligned}
\mathcal{L}_2 &= \underline{K(\phi, X)}, \\
\mathcal{L}_3 &= -\underline{G_3(\phi, X)}\Box\phi, \\
\mathcal{L}_4 &= \underline{G_4(\phi, X)}\, R + G_{4,X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \right], \\
\mathcal{L}_5 &= \underline{G_5(\phi, X)}\, G_{\mu\nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} \left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi) \right].
\end{aligned}$$

It turns out that

$$\sum_i L_i = p(\phi, X)$$

Why?

The Ostrogradski theorem

Higher-than-second order equations of motion are unstable

$$L = L(q, \dot{q}, \ddot{q})$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} = 0$$

The Hamiltonian contains a term linear in a canonical momentum

$$H = P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, a)$$

Energy states are unbounded from below!

Woodard astro-ph/0601672

The Ostrogradski theorem: proof

$$L = L(q,\dot{q},\ddot{q}) \qquad \frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} = 0$$

$$Q_1 = q, \quad P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}$$

$$Q_2 = \dot{q}, \quad P_2 = \frac{\partial L}{\partial \ddot{q}}$$ ← this has to be invertible $\quad \ddot{q} = a(Q_1, Q_2, P_2)$

$$H(Q_1,Q_2,P_1,P_2) = \sum P_i q^{(i)} - L = P_1 Q_2 + P_2 a(Q_1,Q_2,P_2) - L(Q_1,Q_2,a)$$

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Caveats…

- The dangerous term in the Hamiltonian can actually be absent in special degenerate cases
- The instability can manifest itself in very long time scales
- The proof assumes locality

The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

The Great Horndeski Hunt

Let us assume we have only

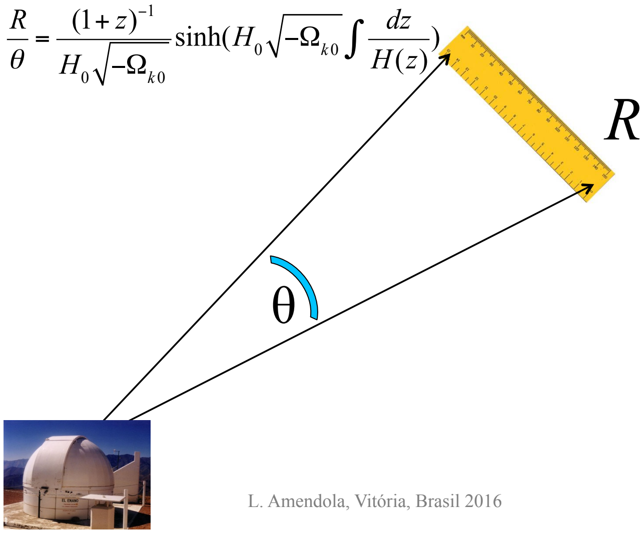
1) a perturbed FRW metric

2) pressureless matter

3) the Horndeski field

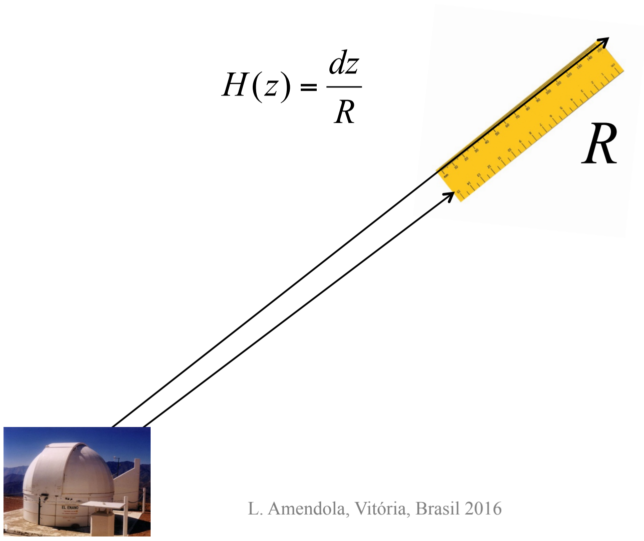
Standard rulers

$$D(z) = \frac{R}{\theta} = \frac{(1+z)^{-1}}{H_0\sqrt{-\Omega_{k0}}}\sinh(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)})$$

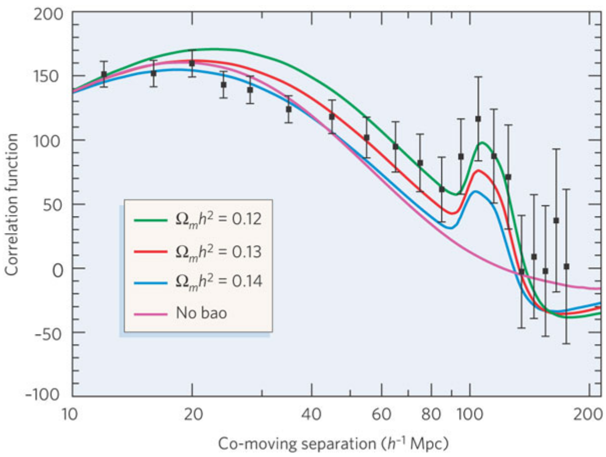


Standard rulers

$$H(z) = \frac{dz}{R}$$



BAO ruler



Charles L. Bennett
Nature 440, 1126-1131(27 April 2006)

Background: SNIa, BAO, ...

Then we can measure H(z) and

$$D(z) = \frac{1}{H_0\sqrt{-\Omega_{k0}}}\sinh(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)})$$

and therefore we can reconstruct the full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1-\frac{\Omega_{k0}}{4}r^2\right)^2}(dx^2+dy^2+dz^2)$$

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $\nabla^2\Psi = 4\pi G\rho_m\delta_m$
- anisotropic stress $1 = -\frac{\Psi}{\Phi}$

Warning: all the perturbation variables in this talk are root mean squares!

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Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m\delta_m$
- anisotropic stress $\eta(k,a) = -\frac{\Phi}{\Psi}$

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Modified Gravity at the linear level

▪ standard gravity	$Y(k,a)=1$ $\eta(k,a)=1$	
▪ scalar-tensor models	$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$ $\eta(a)=1+\frac{F'^2}{F+F'^2}$	Boisseau et al. 2000 Acquaviva et al. 2004 Schimd et al. 2004 L.A., Kunz &Sapone 2007
▪ f(R)	$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$	Bean et al. 2006 Hu et al. 2006 Tsujikawa 2007
▪ DGP	$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_c w_{DE}$ $\eta(a)=1+\frac{2}{3\beta-1}$	Lue et al. 2004; Koyama et al. 2006
▪ massive bi-gravity	$Y(a)=....$ $\eta(a)=....$	F. Koennig and L. A. 2014, Y. Akrami et al. 2014

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a)=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a)=h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

k = wavenumber

h_i = time-dependent functions

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_\mathrm{T}^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_\mathrm{T}^{-2},$$
$$h_3 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_\mathrm{m})}{2w_1^2},$$
$$h_4 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1(\dot{w}_2 + \rho_\mathrm{m})}{w_1},$$
$$h_5 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_\mathrm{m})}{w_4},$$

$$\begin{aligned}
w_1 &\equiv 1 + 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}),\\
w_2 &\equiv -2\dot{\phi}(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}) + 2H(w_1 - 4X(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X})) - 2\dot{\phi}XH^2(3G_{5,X} + 2XG_{5,XX}),\\
w_3 &\equiv 3X(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}) + 18\dot{\phi}XH(2G_{3,X} + XG_{3,XX}) - 18\dot{\phi}H(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX})\\
&\quad - 18H^2(1/2 + G_4 - 7XG_{4,X} - 16X^2G_{4,XX} - 4X^3G_{4,XXX}) - 18XH^2(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX})\\
&\quad + 6\dot{\phi}XH^3(15G_{5,X} + 13XG_{5,XX} + 2X^2G_{5,XXX}),\\
w_4 &\equiv 1 + 2(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}).
\end{aligned} \tag{A2}$$

De Felice et al. 2011; L.A. et al.,PRD, arXiv:1210.0439, 2012

Yukawa Potential

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Momentum space

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

$$k^2\Psi = 4\pi GY(a,k)\rho_m(k)$$

$$k^2(\Phi+\Psi) = 8\pi GY(a,k)[1+\eta(a,k)]\rho_m(k)$$

$$\Psi = -\frac{GM}{r}h_2(1+\frac{h_4-h_5}{h_5}e^{-r/\sqrt{h_5}}) = -\frac{\bar{G}M}{r}(1+Qe^{-mr})$$

Real space

$$\Phi = -\frac{GM}{r}h_1(1+\frac{h_3-h_5}{h_5}e^{-r/\sqrt{h_5}}) = -\frac{\hat{G}M}{r}(1+\hat{Q}e^{-mr})$$

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Quasi-static approximation

$$\phi_{;\mu}\phi^{;\mu} - V' = Q\rho$$
$$\phi \to (\delta\phi)e^{ikx}$$
$$-\delta\ddot{\phi} - c_s^2 k^2 \delta\phi - V''\delta\phi = Q(\delta\rho)$$
$$-\delta\phi'' - (c_s k / aH)^2 \delta\phi - (V''/a^2H^2)\delta\phi = (Q/a^2H^2)(\delta\rho)$$

$$c_s^2 k^2 \gg a^2 H^2 \quad \Rightarrow \quad c_s k^2 \delta\phi + m^2 \delta\phi = -Q(\delta\rho)$$

Quasi-static approximation

$$c_s^2 k^2 \gg a^2 H^2$$

From a wave equation:

$$E_{\delta\phi} \equiv D_1\ddot{\Phi} + D_2\ddot{\delta\phi} + D_3\dot{\Phi} + D_4\dot{\delta\phi} + D_5\dot{\Psi} + D_6\frac{k^2}{a^2}\dot{\chi} + \left(D_7\frac{k^2}{a^2} + D_8\right)\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + \left(D_{10}\frac{k^2}{a^2} + D_{11}\right)\Psi + D_{12}\frac{k^2}{a^2}\chi = 0,$$

To a "Poisson" equation:

$$B_7\frac{k^2}{a^2}\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + A_6\frac{k^2}{a^2}\Psi \simeq 0,$$

The magic of 2nd order perturbed Lagrangians

An elegant way to derive the perturbation equations is to
write down the perturbed Lagrangian at second order:
when you differentiate it, you get first order perturbation equations!

For instance standard EH Lagrangian gives in Minkowski

$$ds^2 = -(1+2\Psi)dt^2 + 2B_{,i}dx^i dt + ((1+2\Phi)\delta_{ij} + 2E_{,ij})dx^i dx^j$$

$$S_g = \frac{1}{2}\int d^3x dt[-8B_i\Phi_i' + 4\Phi_i\Psi_i - 4\Phi'\Box E' + 2\Phi_i^2 - 6\left(\Phi'\right)^2]$$

Definition

Degree of freedom: a field with two time derivatives
in the 2nd order Lagrangian after taking into account
all the constraints

The magic of 2nd order perturbed Lagrangians

Perturbed Horndeski Lagrangian at second order

$$\begin{aligned}S^{(2)} = \int d^4x\, a^3 \Bigg[& \{2w_1\dot{\Phi} - w_2\Psi + \sum_l \rho_l(1+w_l)v_l\}\frac{\partial^2\chi}{a^2} + \left(\frac{1}{2}\sum_l \frac{(1+w_l)\rho_l}{w_l} + \frac{w_3}{3}\right)\Psi^2 + \frac{w_4}{a^2}(\partial\Phi)^2 - 3w_1\dot{\Phi}^2 \\ & + \left\{3w_2\dot{\Phi} - 2w_1\frac{\partial^2\Phi}{a^2} - \sum_l \frac{\rho_l(1+w_l)(\dot{v}_l - 3Hw_l v_l)}{w_l}\right\}\Psi + \sum_l \frac{\rho_l(1+w_l)}{2w_l}\left\{\dot{v}_l^2 - \frac{w_l}{a^2}(\partial v_l)^2\right\} \\ & + 3\Phi\sum_l \rho_l(1+w_l)(\dot{v}_l - 3Hw_l v_l) + \frac{3}{2}\dot{H}\sum_l (1+w_l)\rho_l v_l^2 \Bigg], \qquad (\end{aligned}$$

De Felice & Tsujikawa 2011

The magic of 2nd order perturbed Lagrangians

Classifying theories by their dof

Standard gravity: no scalar degree of freedom, two tensor dof (beside matter)
Horndeski: one scalar dof, two tensor dof (beside matter)
Massive grav: one scalar dof, 2+2 tensor dof (beside matter)
Bimetric models: one scalar dof, 2 vector dof, 2+2 tensor dof (beside matter)

2nd order pert. Lagrangian shows explicitly the dofs

After several simplifications:

$$S_2 = \int \mathrm{d}t \mathrm{d}^3 x a^3 \left[Q_\mathrm{S} \left(\dot{\zeta}^2 - \frac{c_\mathrm{s}^2}{a^2} (\partial_i \zeta) \right) + Q_\mathrm{T} \left(\dot{h}_{ij}^2 - \frac{c_\mathrm{T}^2}{a^2} (\partial_k h_{ij})^2 \right) \right] ,$$

De Felice & Tsujikawa 2011, Bellini & Sawicki 2014

The magic of 2nd order perturbed Lagrangians

2nd order pert. Lagrangian shows explicitly the dofs

$$S_2 = \int \mathrm{d}t \mathrm{d}^3 x a^3 \left[\underbrace{Q_\mathrm{S} \left(\dot{\zeta}^2 - \frac{c_\mathrm{s}^2}{a^2} (\partial_i \zeta) \right)}_{\text{scalar}} + \underbrace{Q_\mathrm{T} \left(\dot{h}_{ij}^2 - \frac{c_\mathrm{T}^2}{a^2} (\partial_k h_{ij})^2 \right)}_{\text{tensor}} \right] ,$$

…and the minimal number of free functions: two for each dof (plus a function for the background, i.e. 5 functions for Horndeski)

The magic of 2nd order perturbed Lagrangians

2nd order pert. Lagrangian also clarifies the stability conditions

$$S_2 = \int \mathrm{d}t\mathrm{d}^3x a^3 \left[Q_\mathrm{S} \left(\dot{\zeta}^2 - \frac{c_\mathrm{s}^2}{a^2}(\partial_i \zeta) \right) + Q_\mathrm{T} \left(\dot{h}_{ij}^2 - \frac{c_\mathrm{T}^2}{a^2}(\partial_k h_{ij})^2 \right) \right] ,$$

Equation of motion in Fourier space $\ddot{\varsigma} - c_s^2 k^2 \varsigma = 0$

Positive squared sound speeds	$c_{s,T}^2 \geq 0$
Positive kinetic terms	$Q_{s,T} \geq 0$

Parametrizations of the 5 free functions

Variable Translations		M_*^2	$M_*^2 H \alpha_\mathrm{M}$	$M_*^2 H^2 \alpha_\mathrm{K}$	$M_*^2 H \alpha_\mathrm{B}$	$M_*^2 \alpha_\mathrm{T}$
Amendola et al.	[10]	w_1	$\dot{w}_1$	$\frac{2}{3}w_3 + 6Hw_2 - 6H^2 w_1$	$-w_2 + 2Hw_1$	$w_4 - w_1$
Bloomfield et al.	[35, 37]	$m_0^2\Omega + \bar{M}_2^2$	$m_0^2\dot{\Omega} + \dot{\bar{M}}_2^2$	$2c + 4M_2^4$	$-\bar{M}_1^3 - m_0^2\dot{\Omega}$	$-\bar{M}_2^2$
De Felice et al.	[89]	$\mathcal{G}_T$	$\dot{\mathcal{G}}_T$	$2\Sigma + 12H\Theta - 6H^2\mathcal{G}_T$	$-2\Theta + 2H\mathcal{G}_T$	$\mathcal{F}_T - \mathcal{G}_T$
Gubitosi et al.	[34, 36]	$M_*^2 f + 2m_4^2$	$M_*^2 \dot{f} + 2\left(m_4^2\right)^{\cdot}$	$2c + 4M_2^4$	$-m_3^3 - M_*^2 \dot{f}$	$-2m_4^2$
Piazza et al.	[38]	$M^2(1+\epsilon_4)$	$\left(M^2(1+\epsilon_4)\right)^{\cdot}$	$2M^2\left(\mathcal{C} + 2\mu_2^2\right)$	$-M^2(\mu_3 + \mu)$	$-M^2\epsilon_4$

Bellini & Sawicki 2014

Bellini-Sawicki parametrization

Model Class		$\alpha_{\rm K}$	$\alpha_{\rm B}$	$\alpha_{\rm M}$	$\alpha_{\rm T}$
ΛCDM		***0***	***0***	***0***	***0***
cuscuton ($w_X \neq -1$)	[71]	0	0	0	0
quintessence	[1, 2]	$(1-\Omega_{\rm m})(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_{\rm m})(1+w_X)}{c_s^2}$	0	0	0
kinetic gravity braiding	[47–49]	$\frac{m^2(n_m+\kappa_\phi)}{H^2 M_{\rm Pl}^2}$	$\frac{m\kappa}{HM_{\rm Pl}^2}$	0	0
galileon cosmology	[57]	$-\frac{3}{2}\alpha_{\rm M}^3 H^2 r_{\rm c}^2 e^{2\phi/M}$	$\frac{\alpha_{\rm K}}{6} - \alpha_{\rm M}$	$\frac{-2\dot{\phi}}{HM}$	0
BDK	[26]	$\frac{\dot{\phi}^2 K_{,\dot{\phi}\dot{\phi}} e^{-\kappa}}{H^2 M^2}$	$-\alpha_{\rm M}$	$\frac{\dot{\kappa}}{H}$	0
metric $f(R)$	[3, 72]	0	$-\alpha_{\rm M}$	$\frac{B\dot{H}}{H^2}$	0
MSG/Palatini $f(R)$	[73, 74]	$-\frac{3}{2}\alpha_{\rm M}^2$	$-\alpha_{\rm M}$	$\frac{2\dot{\phi}}{H}$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot{\xi}}{M^2+H\dot{\xi}}$	$\frac{\dot{H}\dot{\xi}+H\ddot{\xi}}{H(M^2+H\dot{\xi})}$	$\frac{\ddot{\xi}-H\dot{\xi}}{M^2+H\dot{\xi}}$

Beyond Horndeski, I

Look now at the matter equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

Standard sub-horizon matter equations

$\theta = ik_j v^j$

$$\dot{\delta} = -\theta$$ continuity

$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$ Euler

$$k^2\Phi = 4\pi G a^2 \rho\delta$$ Poisson

Modifying gravity

$$k^2\Phi = 4\pi G a^2 \rho\delta \Upsilon\eta$$

Modifying continuity equation

$$\dot{\delta} = -\theta + \Delta$$

$$\Delta = 4\alpha'_H \frac{\mathcal{H}M_P^2}{\rho_m a^2} k^2\Phi$$

Gleyzes et al 2014;
Lombrisier et al 2015

Beyond Horndeski, II

Or, we can add several Horndeski fields!

$$\int dx^4 \sqrt{-g}\left[L_{H1} + L_{H2} + \ldots + L_{matter}\right]$$

$$Y \equiv A_1 \frac{1 + A_2 k^2 + A_3 k^4}{1 + A_4 k^2 + A_5 k^4},$$

$$\eta \equiv -\frac{\Phi}{\Psi} = B_1 \frac{1 + B_2 k^2 + B_3 k^4}{1 + B_4 k^2 + B_5 k^4},$$

A. Silvestri et al. 2013
T. Baker et al. 2013
V. Vardanyan & L.A., 2015

Beyond Horndeski, III

- Torsion
- Non-metricity
- Palatini
- Vectors
- Tensors

Beyond Horndeski, IV

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field

$$\int d^4x\sqrt{g}R_g + m^2\int d^4x h_{\mu\nu}h_{\alpha\beta}\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

The three deadly sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for m $\rightarrow$ 0 (vDVZ disc.)
- It violates diffeomorphism invariance
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)

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Ghost-free Bigravity

- The first problem was partially solved by Vainshtein (1972): there exis a radius below which the linear theory cannot be applied;
  - For the Sun, this radius is larger than the solar system!
- The second and third problems can be solved introducing a second metr

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f)$$
$$+ m^2M_g^2\int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n(\sqrt{g^{\alpha\beta}f_{\beta\gamma}}) + \int d^4x\sqrt{-\det g}L_m(g,\Phi)$$

The only ghost-free local non-linear massive gravity theory!

deRham, Gabadadze, Tolley 201(
Hassan & Rosen, 2011

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Observing η

$$\eta(k,a) = -\frac{\Phi}{\Psi}$$

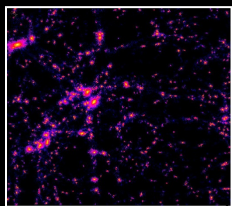
Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

massive particles respond to Ψ

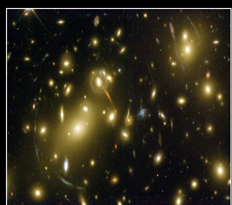
$$\delta'' + (1 + \frac{H'}{H})\delta' = \nabla^2\Psi$$

$$\delta = \frac{\delta\rho}{\rho}$$

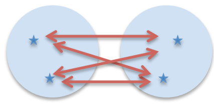


massless particles respond to Φ-Ψ

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$

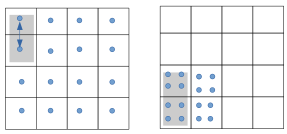


Correlation function and Power spectrum



$$dN_{ab} = \langle dN_a dN_b \rangle = \rho_0^2 dV_a dV_b (1 + \xi(r_{ab}))$$

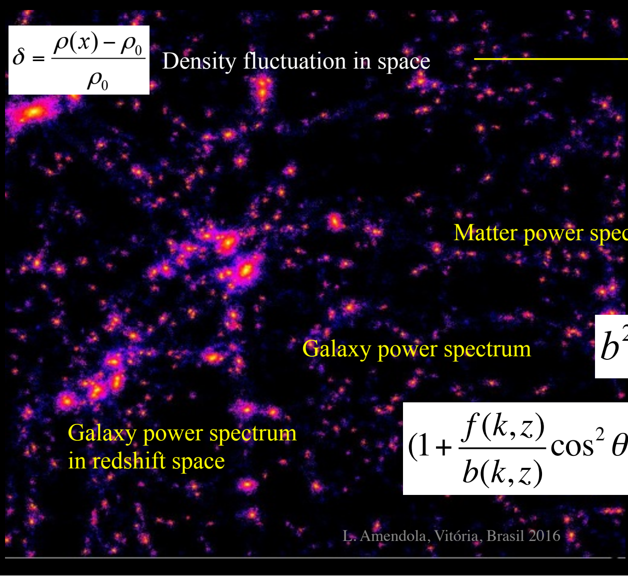
Average number of pairs = 1



Average number of pairs = 4

$$P(k) = \int \xi(r) e^{ikr} dV$$

Reality check



$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$

Density fluctuation in space

$$\langle \delta_k{}^2 \rangle = P(k,z)$$

Matter power spectrum

$$P_{matter}(k,z)$$

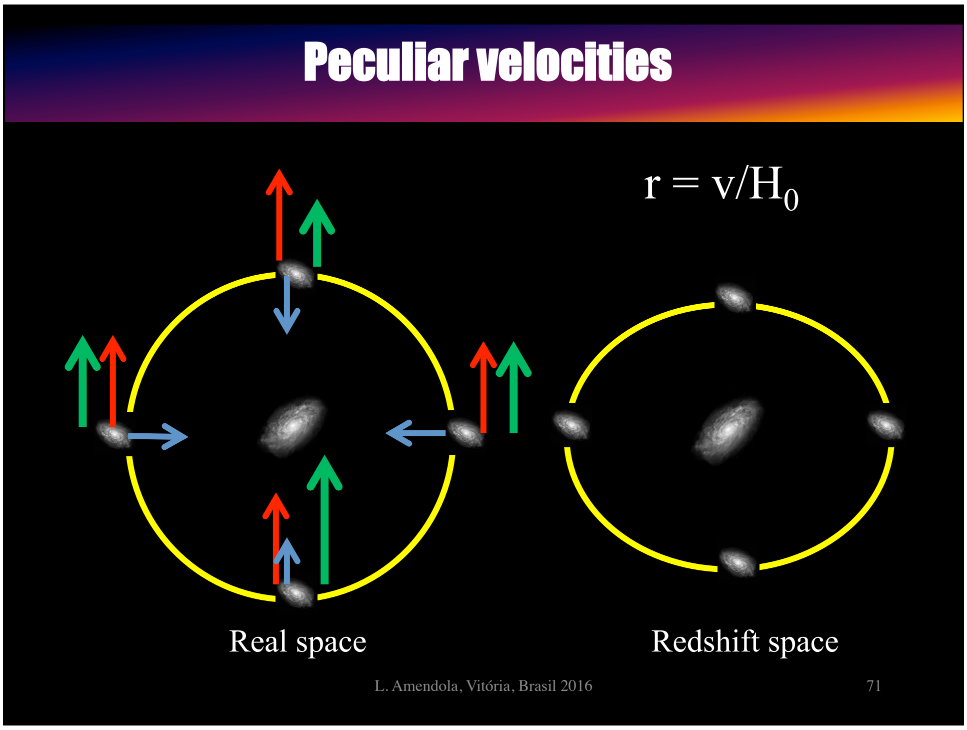
Galaxy power spectrum

$$b^2(k,z) P_{matter}(k,z)$$

Galaxy power spectrum in redshift space

$$(1 + \frac{f(k,z)}{b(k,z)} \cos^2\theta)^2 b^2(k,z) P_{matter}(k,z)$$

$$f = \frac{d\log\delta}{d\log a}$$


Peculiar velocities
r = v/H$_0$
Real space
Redshift space
L. Amendola, Vitória, Brasil 2016
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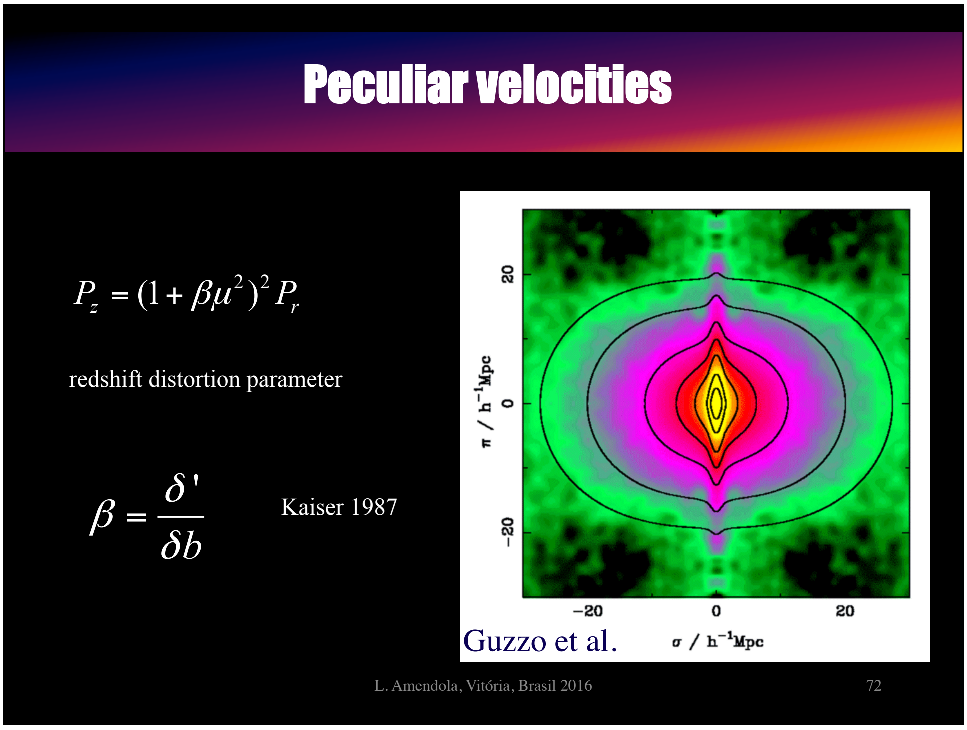
Peculiar velocities
$P_z = (1+\beta\mu^2)^2 P_r$
redshift distortion parameter
$\beta = \frac{\delta'}{\delta b}$
Kaiser 1987
20
0
−20
π / h^{-1}Mpc
−20
0
20
σ / h^{-1}Mpc
Guzzo et al.
L. Amendola, Vitória, Brasil 2016
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Deconstructing the galaxy power spectrum

$$P_{galaxy}(k,z,\cos\theta) = (1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z) P_{matter}(k,z)$$

Galaxy clustering

Redshift distortion

Line of sight angle

$$\delta_{gal}(k,z,\mu) = Gb\sigma_8(1+\frac{f}{b}\mu^2)\delta_{t,0}(k)$$

$$f = \frac{d\log\delta}{d\log a}$$

Growth function

Galaxy bias

normalization

Present mass power spectrum

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Three linear observables: A, R, L

clustering

μ=0 Amplitude A

$$\delta_{gal}(k,z,0) = Gb\sigma_8\delta_{m,0}(k) \equiv A$$

μ=1 Redshift distortion R

$$\delta_{gal}(k,z,1) = G\sigma_8 f\delta_{m,0}(k) \equiv R$$

lensing

Lensing L

$$k^2\Phi_{lens} = k^2(\Psi - \Phi) = -\frac{3}{2}\Sigma G\Omega_m\sigma_8\delta_{m,0}(k) \equiv L$$

$$\Sigma = Y(1+\eta)$$

L. Amendola, Vitória, Brasil 2016 74

The only model-independent ratios

Redshift distortion/Amplitude $$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion $$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f}$$

Redshift distortion rate $$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate $$E = \frac{H}{H_0}$$

How to combine them to test the theory?

Summarizing....

Matter conservation equation independent of gravity theory

$$\delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi$$

Observables

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f} \qquad P_3 = \frac{R'}{R} = \frac{f'}{f} + f \qquad E = \frac{H}{H_0}$$

The anisotropic stress is directly observable

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})}-1=\eta$$

Observables

Theory

Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Horndeski Lagrangian: not too big to fail

$$g(z,k) \equiv \frac{(REa^2)'}{LEa^2}$$

$$2g_{,k}g_{,kkk} - 3(g_{,kk})^2 = 0$$

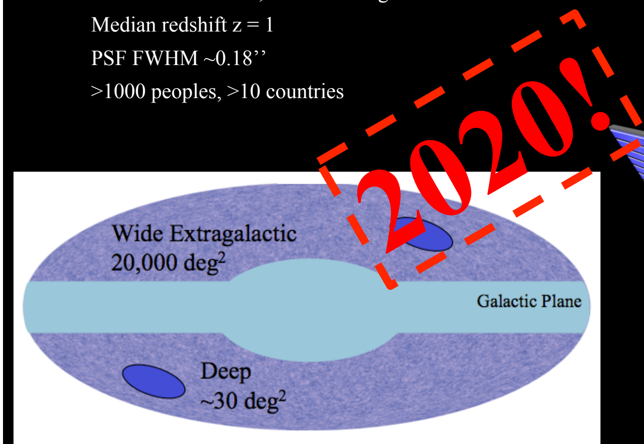
If this relation is falsified, the Horndeski theory is rejected*

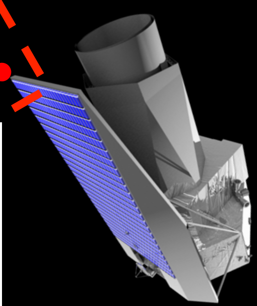
L.A., M. Motta, I. Sawicki,
M. Kunz, I. Saltas, 1210.0439

L. Amendola, Vitória, Brasil 2016 79

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy
15,000 square degrees
70 million redshifts, 2 billion images
Median redshift z = 1
PSF FWHM ~0.18''
>1000 peoples, >10 countries





Euclid
satellite

arXiv Red Book 1110.3193

Results…

$$\eta(k,a) = H_2 \left(\frac{1+k^2 H_4}{1+k^2 H_5} \right)$$

Model 1: η constant for all z, k
Error on η around 2%

η=1/2 for f(R)!

Model 2: η varies in z
Error on η

TABLE X. Fiducial values and errors for the parameters $P_1, P_2, P_3, E'/E$ and $\bar{\eta}$ for every bin. The last bin has been omitted since R' is not defined there.

$\bar{z}$	P_1	ΔP_1	$\Delta P_1(\%)$	P_2	ΔP_2	$\Delta P_2(\%)$	P_3	ΔP_3	$\Delta P_3(\%)$	(E'/E)	$\Delta E'/E$	$\Delta E'/E(\%)$	$\bar{\eta}$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
0.6	0.766	0.012	1.6	0.729	0.013	1.8	0.134	0.13	99	-0.920	0.022	2.4	1	0.11	11
0.8	0.819	0.010	1.2	0.682	0.011	1.6	0.317	0.12	38	-1.04	0.046	4.4	1	0.091	9.1
1.0	0.859	0.0093	1.1	0.650	0.011	1.7	0.460	0.12	26	-1.13	0.099	8.7	1	0.090	9.0
1.2	0.888	0.0092	1.0	0.628	0.014	2.3	0.569	0.13	23	-1.21	0.12	10	1	0.097	9.7
1.4	0.911	0.010	1.1	0.613	0.020	3.3	0.654	0.11	16	-1.26	0.09	7.1	1	0.073	7.3

L.A, M. Kunz, A. Vollmer,
A. Guarnizo, S. Fogli, 1311.4865, 2013

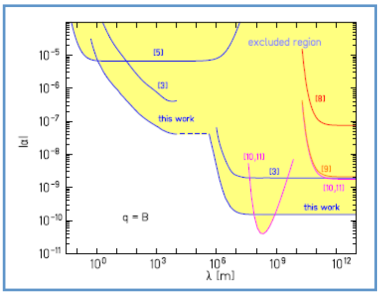
Observing Y

$$\nabla^2 \Psi = 4\pi G \, Y(k,a) \rho_m \delta_m$$

In collab. with Laura Taddei and Matteo Martinelli

Observing Y

Excluded
in the Lab



$$\Psi(r) = -\frac{GM}{r}(1+\alpha e^{-r/\lambda})$$

Observing Y

Modified-gravity
sub-horizon matter equations
$\theta = ik_j v^j$

$$\dot{\delta} = -\theta$$
$$\dot{\theta} = -\mathcal{H}\theta + k^2\Psi$$
$$k^2\Phi = 4\pi G a^2 \rho\delta Y\eta$$

continuity
Euler
Poisson

This can be written as a single equation:

$$\delta'' + (1+\frac{H'}{H})\delta' + \frac{3}{2}\Omega_m Y\delta = 0$$

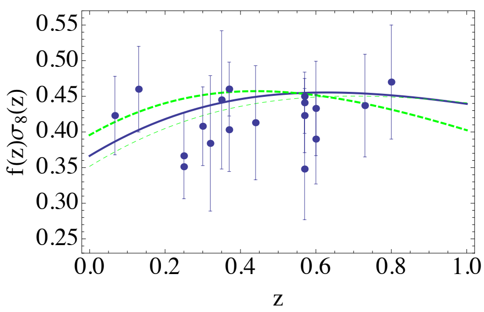
prime is d/dlog(a)

Observing Y

$$\delta'' + (2 + \frac{H'}{H})\delta' + \frac{3}{2}\Omega_m Y \delta = 0$$

$$f = \frac{d\log\delta}{d\log a}$$ Growth rate

$$f\sigma_8(z) = \sigma_8 \frac{\delta'}{\delta_0}$$



By measuring the growth of fluctuations
we can find Y!

Two problems

$$\delta'' + (2 + \frac{H'}{H})\delta' + \frac{3}{2}\Omega_m \delta Y = 0$$

Changing initial conditions
the evolution can change
considerably:
marginalize over initial conditions

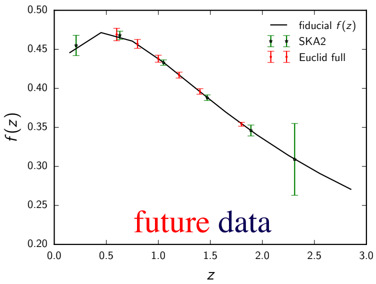
$$\delta_{gal}(k, z, 1) = G\sigma_8 f \delta_{m,0}(k) \equiv R$$

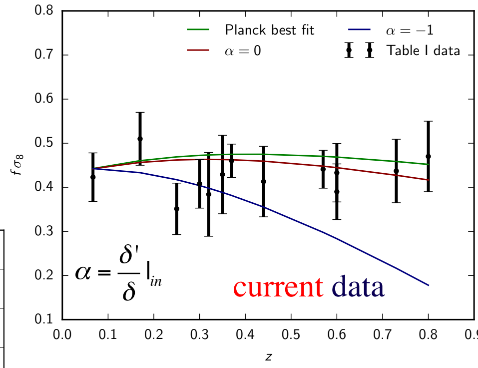
The observable is R not $f\sigma(z)$:
marginalize over an overall factor

Observing Y

$$\delta''+(2+\frac{H'}{H})\delta'+\frac{3}{2}\Omega_m\delta Y=0$$

Changing initial conditions the evolution can change considerably



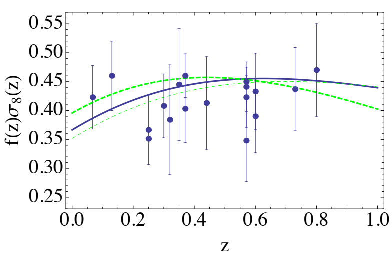


M. Martinelli, L. Taddei

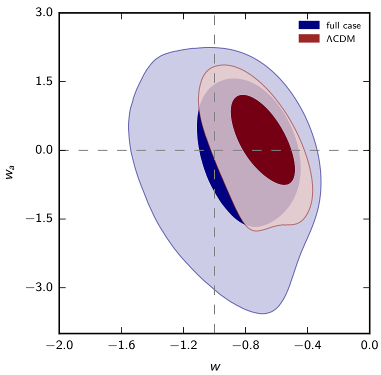
Observing Y: current constraints

Free parameters:

$$\Omega_{m,0}, w_0, w_a, Y, \sigma_8, \alpha=\frac{\delta'}{\delta}|_{in}$$



current SN, growth data



Observing Y: Euclid forecast

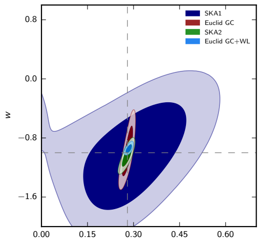
WL, SN, growth data

Free parameters:

$$\Omega_{m,0}, w_0, w_a, Y, \sigma_8, \alpha = \frac{\delta'}{\delta}|_{in}$$

Standard case Y=α=1

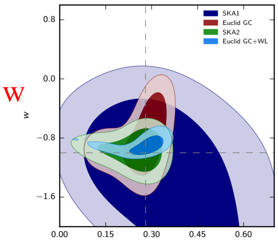
W



$\Omega_{m,0}$

General mod grav case

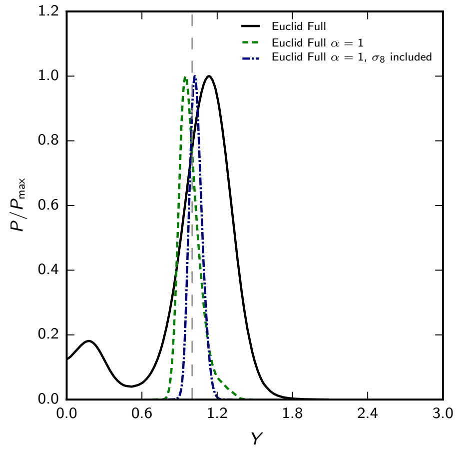
W



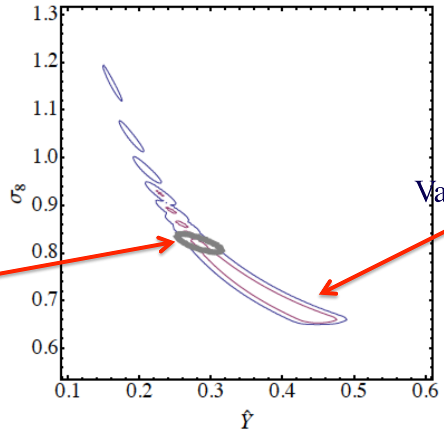
$\Omega_{m,0}$

Observing Y: Euclid forecast

WL, SN, growth data



Method 3: Cosmological exclusion plot



Fixing initial conditions

Varying initial conditions

$$\hat{Y} = \Omega_{m0} Y$$

L. Taddei, L.A., 2014

Yukawa Potential

$$Y(k,a) = h_1 \left(\frac{1 + k^2 h_5}{1 + k^2 h_3} \right)$$

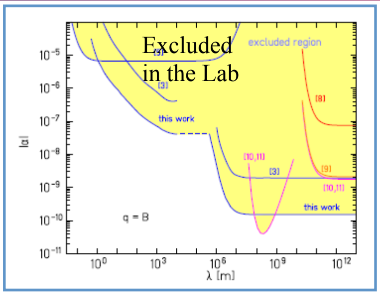
Momentum space

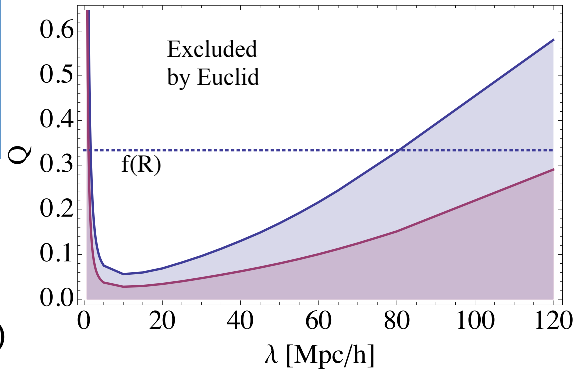
$$\nabla^2 \Psi = 4\pi G \, Y(k,a) \rho_m \delta_m$$

$$\Psi = -\frac{GM}{r} h_2 \left(1 + \frac{h_4 - h_5}{h_5} e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}\left(1 + Q e^{-mr}\right)$$

Real space

Cosmological exclusion plot





$$\Psi(r) = -\frac{GM}{r}(1 + Qe^{-r/\lambda})$$

L. Taddei, L.A., 2014

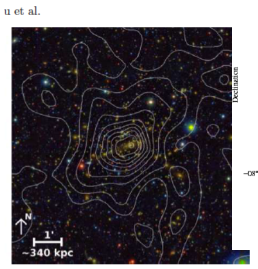
Modified Gravity and clusters

Collab. with S. Borgani. B. Sartoris, L. Pizzuti and the CLASH-VLT team

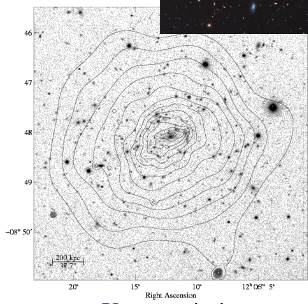
Modified Gravity and clusters

- Clusters are the largest gravitationally bound objects
- Their gravitational potential can be mapped with galaxy dynamics, X-ray intracluster gas emission, weak and strong lensing, SZ effect
- They are a bridge between linear and strongly non linear scales

MACS 1206
z=0.44



lensing



X-ray emission

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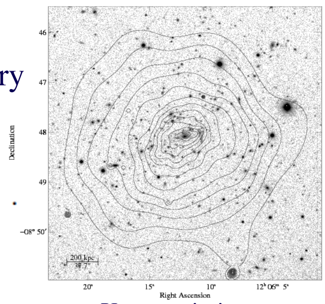
X-ray emission

Clusters contain a hot intracluster medium (10^7 K) that emits in the X-ray; if the gas is in equilibrium between its own pressure and gravity then

$$\nabla P_{\rm gas} = -\rho_{\rm gas} \nabla \Phi_N \,,$$

further, assuming spherical symmetry and ideal gas

$$M(r) = -\frac{r}{G}\frac{k_B T}{\mu m_p}\left(\frac{d\ln \rho_{\rm gas}}{d\ln r} + \frac{d\ln T}{d\ln r}\right) .$$



MACS 1206
z=0.44

X-ray emission

96

Modified Gravity and clusters

As before, lensing and galaxy dynamics are governed by different combinations of potentials

$$k^2\Psi = 4\pi G Y(k)\rho_m(k)$$

$$k^2(\Phi+\Psi) = 8\pi G Y(k)[1+\eta(k)]\rho_m(k)$$

So if we replace

$$\eta = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right),\ Y = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right).$$

and move to real space, we obtain for $\eta(r) = -\frac{\Phi(r)}{\Psi(r)}$

$$\eta(r) = h_2\frac{\int_{r_0}^{r}\frac{dr'}{r'^2}[M(r')+2\hat{Q}^2M_{mg}(r')]}{\int_{r_0}^{r}\frac{dr'}{r'^2}[M(r')+2Q^2M_{mg}(r')]}$$

$$= h_2\frac{1+2\hat{Q}^2\Sigma(r)}{1+2Q^2\Sigma(r)}$$

$$Q^2 \equiv (h_5-h_3)/2h_3\ ,\ \hat{Q}^2 \equiv (h_4-h_3)/2h_3$$

Modified Gravity and clusters

Now if we assume a NFW profile

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}(1+\frac{r}{R_s})^2}$$

we obtain

$$\Sigma(r) = \frac{2\int_0^\infty \frac{W(kR_s)}{m^2+k^2}[\frac{\sin(kr_0)}{r_0}-\frac{\sin(kr)}{r}]dk]}{\pi[\frac{1}{r_0}\log\frac{r_0+R_s}{R_s}-\frac{1}{r}\log\frac{r+R_s}{R_s}]}$$

By combining lensing and galaxy dynamics we can measure the parameters

$$h_2, Q, \hat{Q}, m, R_s$$

Modified Gravity and clusters

Galaxy dynamics obey the Jeans equation (spherical symmetry, virialized system)

$$\frac{\mathrm{d}\nu{\sigma_r}^2}{\mathrm{d}r} + 2\beta\frac{\nu\sigma_r^2}{r} = -\nu\frac{\mathrm{d}\Psi}{\mathrm{d}r}$$

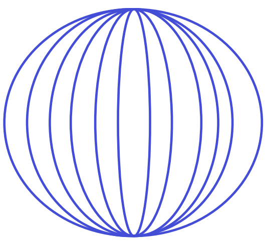
Variance radial velocity

Velocity anisotropy parameter

Density of sources

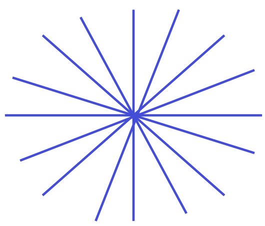
Velocity anisotropy parameter

Purely tangential orbits



β=∞

Purely radial orbits



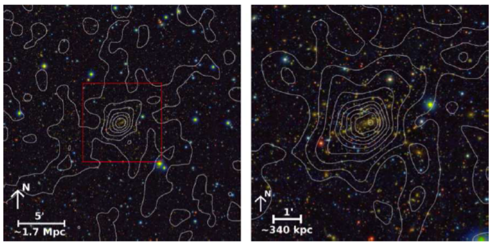
β=0

Modified Gravity and clusters

Similarly, the lensing effective mass is determined by the distortion of background galaxies

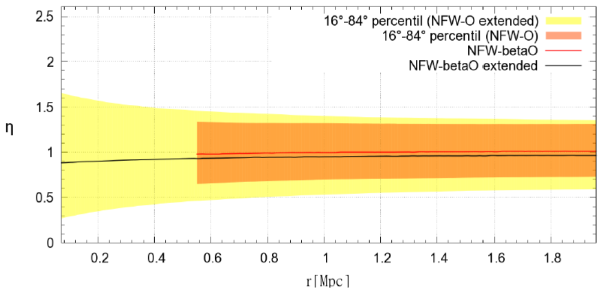
$$M_{\rm 2D}(<\theta_i) = \pi(D_l\theta)^2\Sigma_{\rm crit}\overline{\kappa}_{\rm min} + 2\pi D_l^2\Sigma_{\rm crit}\int_{\theta_{\rm min}}^{\theta_i} d\ln\theta\,\theta^2\kappa(\theta).$$

Umetsu et al.



Modified Gravity and clusters: preliminary results

Single cluster MACS1206



$$\eta(1.96\,Mpc) = 1.01^{+0.31}_{-0.28}\,(\text{stat})\ \pm 0.34\,(\text{syst}),$$

Fixing all parameters except rs and η !

L.Pizzuti, S. Borgani, B. Sartoris, L.A. and the CLASH team, 2016

Modified Gravity and clusters: preliminary results

$$\eta(1.96\,Mpc) = 1.01^{+0.31}_{-0.28}\,(\text{stat})\ \pm 0.34\,(\text{syst}),$$

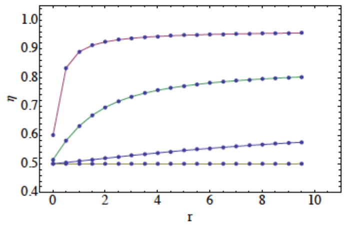
In a f(R) theory we have

$$\eta = h_2 \frac{1+2\hat{Q}^2\Sigma(r)}{1+2Q^2\Sigma(r)} \quad \text{with} \quad \boxed{\begin{array}{ll} 2Q^2 = \frac{1}{3} & m^2 = \frac{R}{3}\left(\frac{f_{,R}}{Rf_{,RR}} - 1\right). \\ 2\hat{Q}^2 = -\frac{1}{3} & h_2 = 1 + f_{,R} \approx 1 \end{array}}$$

$$\Sigma(r) \to 1 \; for \; r \to 0$$

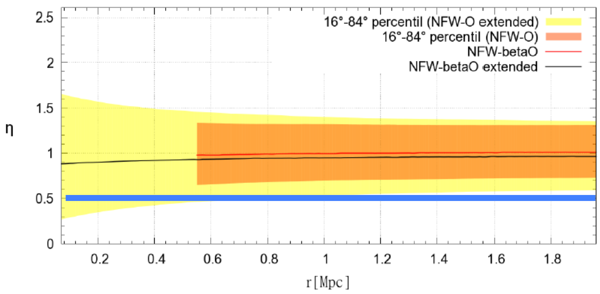
$$\Sigma(r) \to 0 \; for \; r \to \infty$$

So η goes from 0.5 to 1



Modified Gravity and clusters: preliminary results

$$\eta(1.96\,Mpc) = 1.01^{+0.31}_{-0.28}\,(\text{stat})\ \pm 0.34\,(\text{syst}),$$



f(R) for m>(2Mpc)$^{-1}$

So we can "exclude" m > (2Mpc)$^{-1}$…

Modified Gravity and Grav. Waves

Gravitational waves in GR

General perturbation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

redefinition $\bar{h}^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{1}{2}\eta^{\alpha\beta}h$

Gauge freedom $x^{\bar{\alpha}} = x^{\alpha} + \xi^{\alpha}(x^{\beta})$

Gauge choice $\bar{h}^{\mu\nu}_{\ ,\nu} = 0$

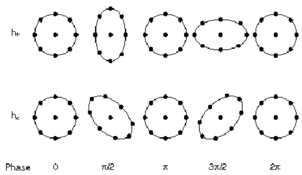
Linearized Einstein equations $\Box\bar{h}_{\alpha\beta} = -16\pi T_{\alpha\beta}$

In FRW $\ddot{h} + 3H\dot{h} + k^2 h = 0$

Gravitational waves in GR

General solution in vacuum

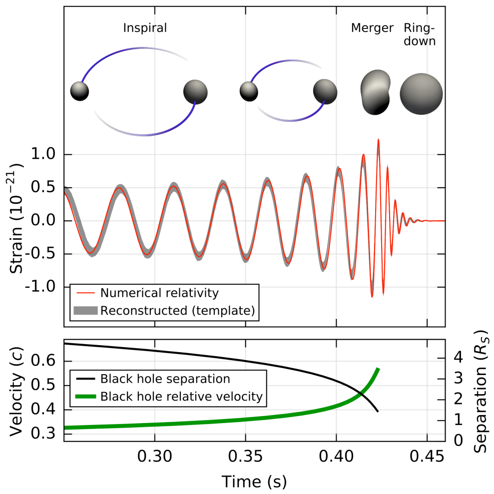
$$\bar{h}^{TT}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{ik_\alpha x^\alpha}$$



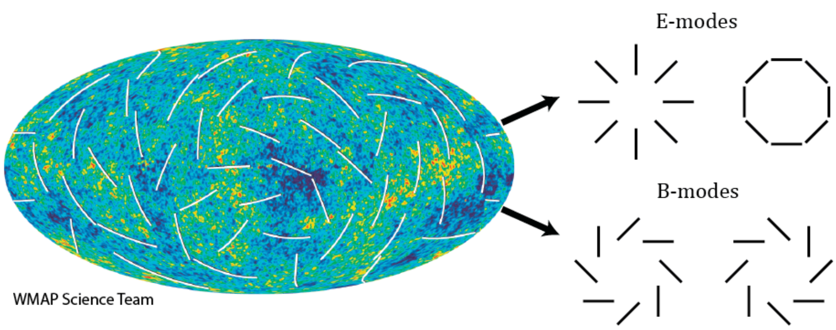
Erik Elfgren, Tomas Samuelsson

Gravitational waves in GR

Ligo collaboration



B- and E-modes



B-modes are excited only by GW and lensing!

GW in modified gravity

GW equation in GR
$$\ddot{h} + 3H\dot{h} + k^2 h = 0$$

GW equation in modified gravity
$$\ddot{h} + 3H(1+\alpha_M)\dot{h} + (1+\alpha_T)k^2 h = 0$$

Bellini & Sawicki 2013
I. Saltas, I. Sawicki, L.A., M. Kunz 1406.7139

Eta and gravitational waves

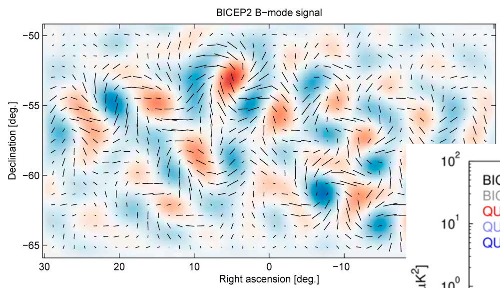
Example from Horndeski and bimetric

$$h''_{ij} + (2+\nu)Hh'_{ij} + c_T^2 k^2 h_{ij} + a^2\mu^2 h_{ij} = a^2\Gamma\gamma_{ij}\,,$$
$$\Phi - \Psi = \sigma(t)\Pi,$$

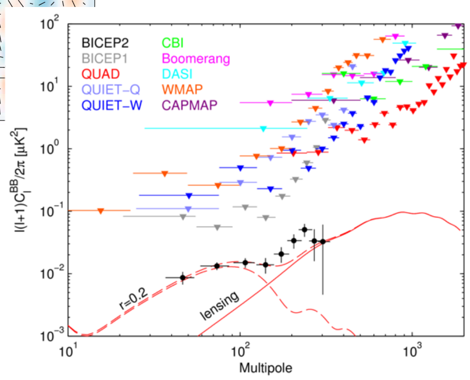
Horndeski	bimetric
$\sigma = \alpha_M - \alpha_T$ $\Pi = H\delta\phi/\dot\phi + \alpha_T/(\alpha_M - \alpha_T)\Phi\,.$ $\nu = \alpha_M\,,\quad c_T^2 = 1 + \alpha_T\,,$	$\sigma = a^2 m^2 f_1 \quad \Pi = E_2$ $\nu = 0\,,\quad c_T^2 = 1\,,$ $\mu^2 = m^2 f_1\,,\quad \Gamma = m^2 f_1\,.$

It turns out that iff η ≠ 1 then the GW equation is modified.
CMB B-polarization can be a tool to detect modified gravity!

B modes and GWs



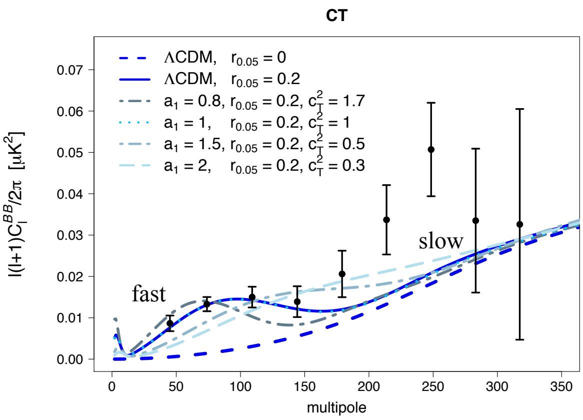
BICEP collab. 2014



Gravitational wave speed

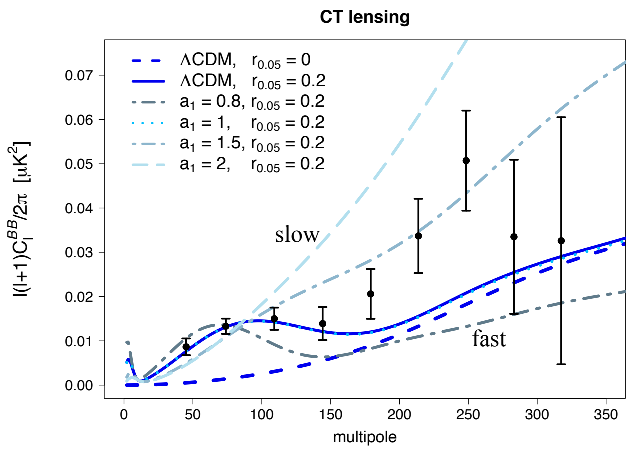
varying

$$c_T^2 = 1 + \alpha_T$$



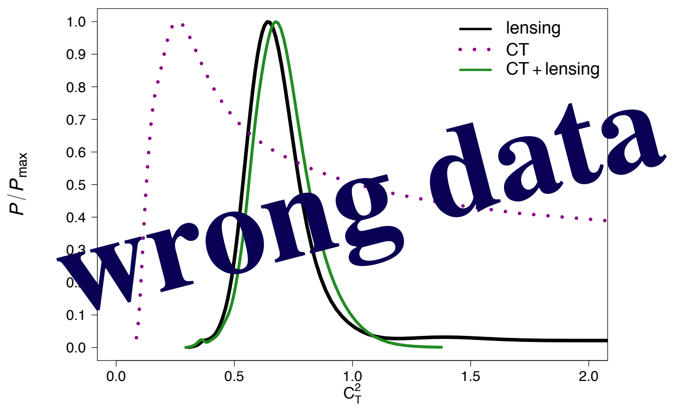
L.A., G. Ballesteros, V. Pettorino, 2014
See also Raveri, Silvestri and Zhou, 2014

GW speed and lensing



L.A., G. Ballesteros, V. Pettorino, 2014

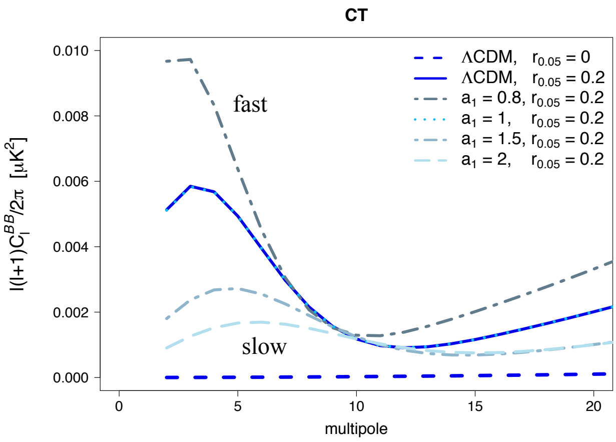
Constraints on GW speed



L.A., G. Ballesteros, V. Pettorino, 2014

GW speed at reionization

The proposed satellite LiteBIRD plans to measure r at the reionization peak to 0.001 !



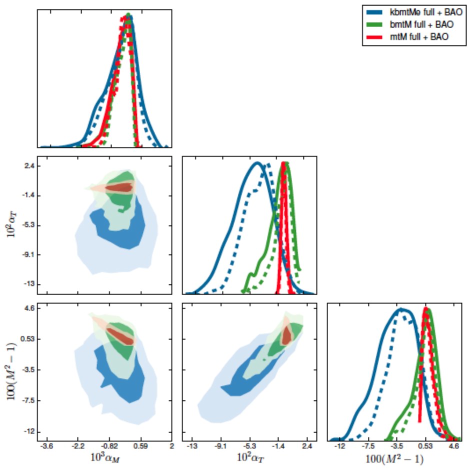
L.A., G. Ballesteros, V. Pettorino, 2014

Full analysis (in progress...)

Simultaneous CMB constraints on all modified gravity parameters

α_T

M



α_M α_T

With M. Zumalacarregui, V. Pettorino, I. Sawicki, J. Lesgourgues

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The fourfold way to gravity screening

$$G^* = G(1+\alpha e^{-m_\varphi r})$$

$$m_\varphi, \alpha \left\{ \begin{array}{l} \text{depend on time} \\ \text{depend on space} \\ \text{depend on local density} \\ \text{depend on species} \end{array} \right.$$

L. Amendola, Vitória, Brasil 2016

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Screening mechanisms

$$G^* = G(1+\alpha e^{-m_\varphi r})$$
$$m = m(\phi)$$

The field ϕ obeys a Poisson equation

$$\nabla^2\phi + m^2\phi = \alpha^{1/2}\rho\delta$$

So the solution is something like

$$\phi = \phi(\rho\delta) = \phi(\text{local density})$$

$$G^* = G(1+\alpha e^{-m_\varphi(\rho)r})$$

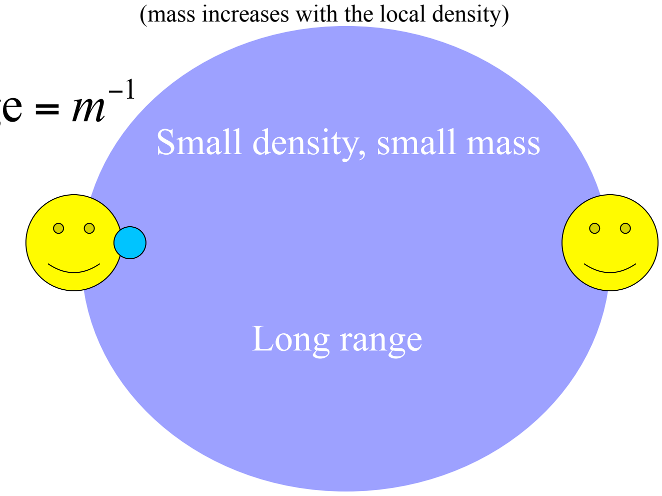
A density-dependent range!

Khoury Weltman 2003

Screening mechanisms

(mass increases with the local density)

$$\text{range} = m^{-1}$$



Screening mechanisms

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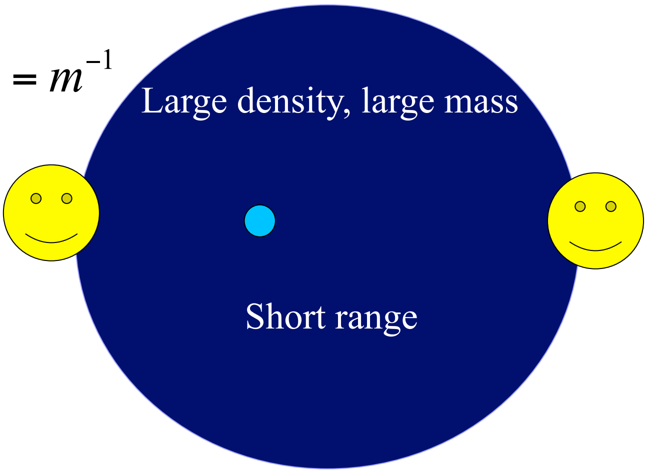
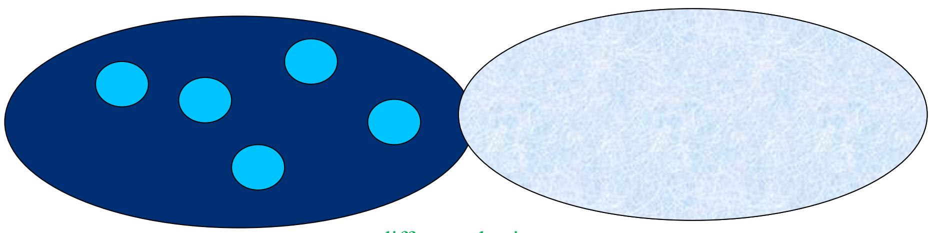


Under the carpet..

Problem of non-linearity:
screening effects mix linear and non-linear scales

same density contrast

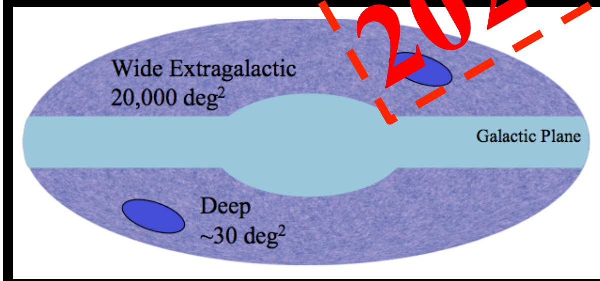


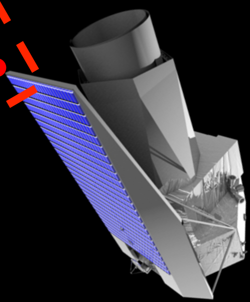
different physics

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy
15,000 square degrees
70 million redshifts, 2 billion images
Median redshift z = 1
PSF FWHM ~0.18''
>1000 peoples, >10 countries

2020!





Euclid satellite

arXiv Red Book 1110.3193