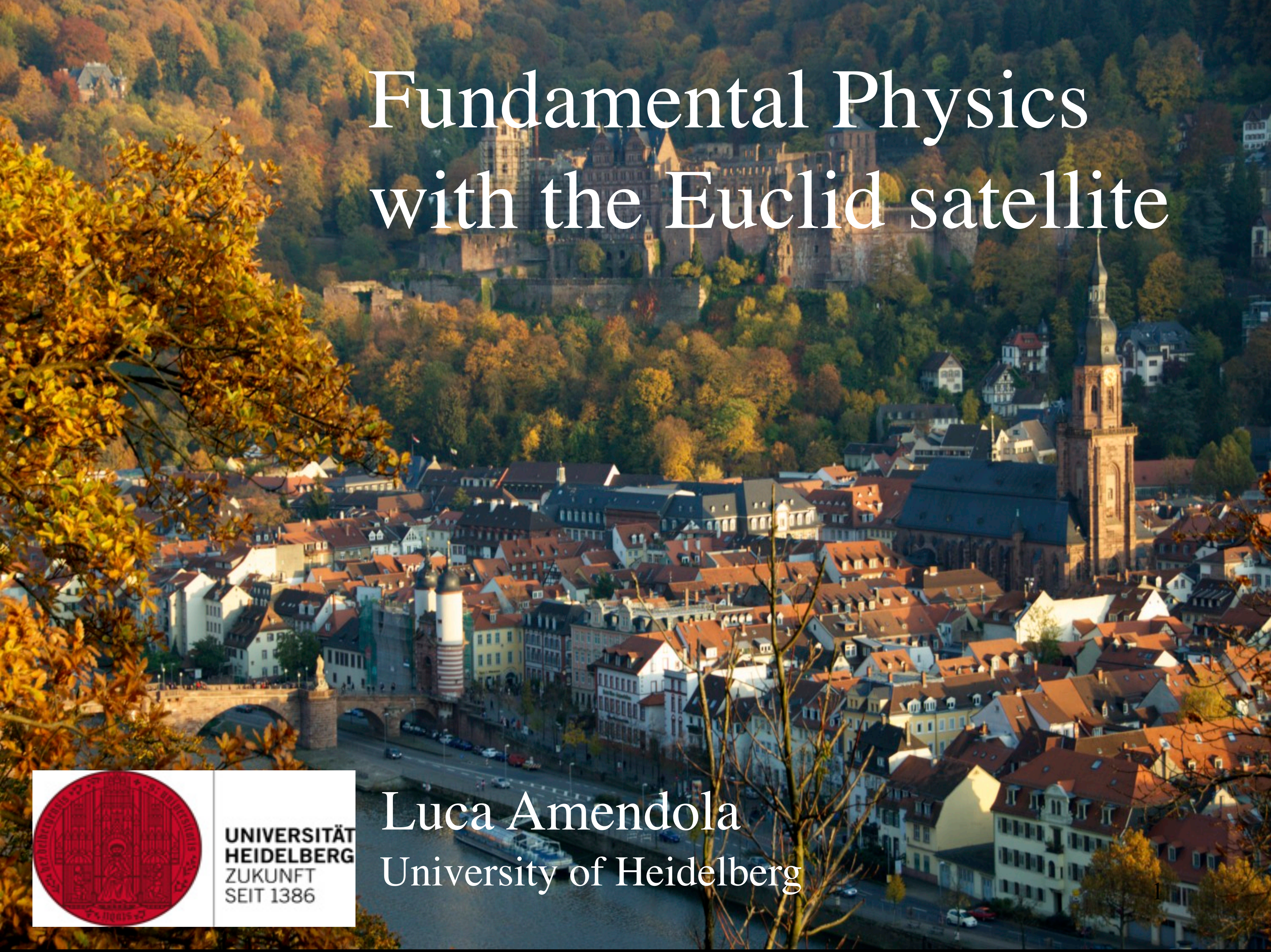

Fundamental Physics with the Euclid satellite

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Outline

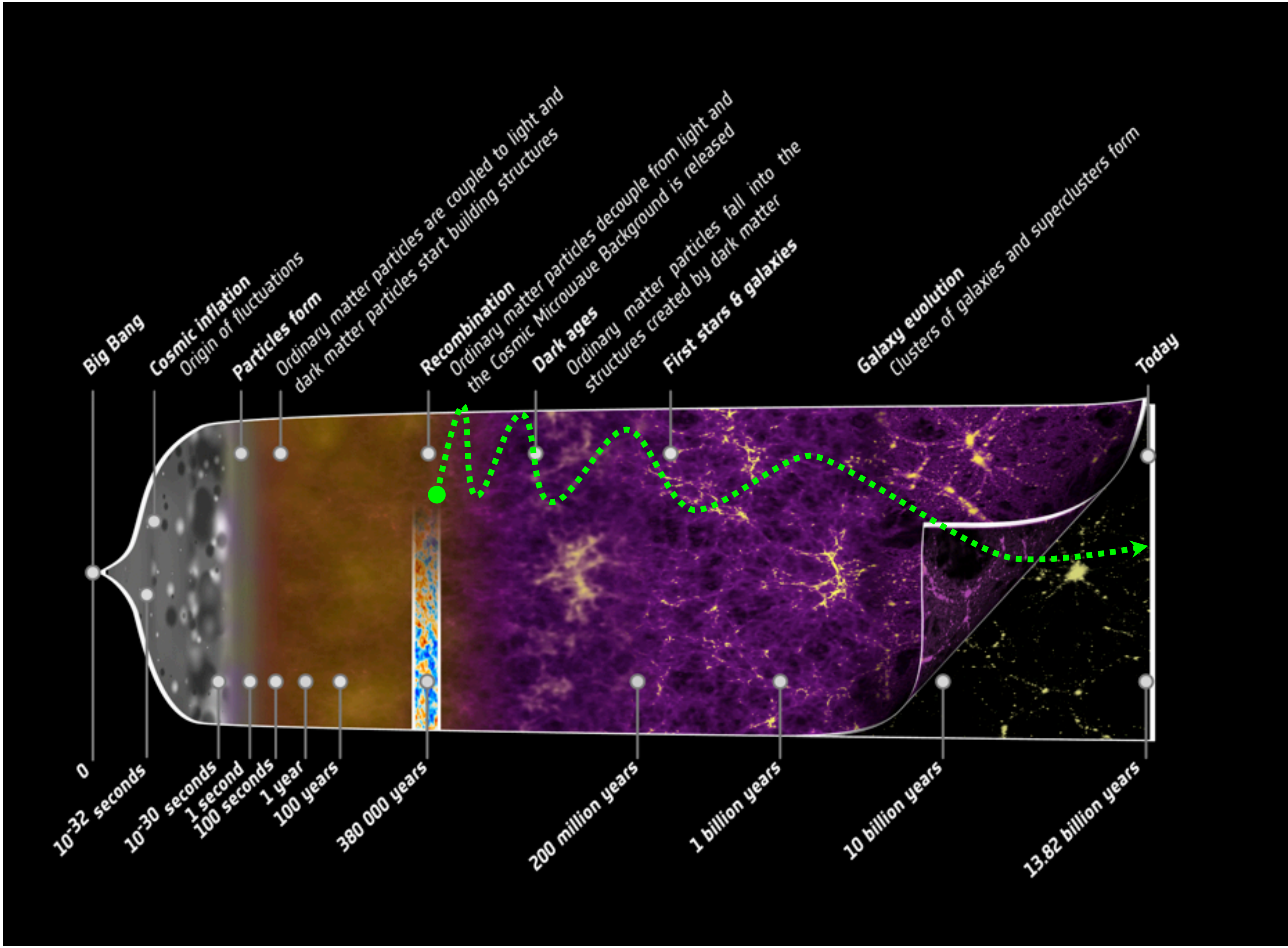
- Lecture 1 The Euclid satellite

- Lecture 2 Testing gravity with Euclid

Suggested texts

- L.A. et al arXiv:1606.00180 Living Rev.Rel. 21 (2018) no.1, 2
- L.A. and S. Tsujikawa, *Dark Energy. Theory and Observations*, Cambridge U. P.
- Proceedings of the Pedra Azul school 2017

Disclaimer: some material in these slides has been taken from other authors and is believed to be in the public domain.
Whenever known, I hope I have always correctly credited the authors. I apologize if I have missed some.


Big Bang
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and
dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and
the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the
structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

Friedman equation

GR+Homogeneity+Isotropy:
Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi}{3}\rho$$

$$\Omega_{m,0} = \frac{8\pi\rho_{m0}}{3H_0^{\ 2}}, \quad \Omega_{rad,0} = \frac{8\pi\rho_{rad,0}}{3H_0^{\ 2}}, \quad \Omega_{\Lambda,0} = \frac{8\pi\rho_{\Lambda,0}}{3H_0^{\ 2}}, \quad a = (1+z)^{-1}$$

$$H^2 = H_0^2\left[\Omega_{m,0}(1+z)^3 + \Omega_{rad,0}(1+z)^4 + \Omega_{\Lambda,0}\right]$$

$$1 = \Omega_{m,0} + \Omega_{rad,0} + \Omega_{\Lambda,0}$$

$$H^2 = H_0^2 \sum \Omega_{i,0}(1+z)^{3(1+w_i)}$$

Cosmology Executive Summary

Dark matter 27%

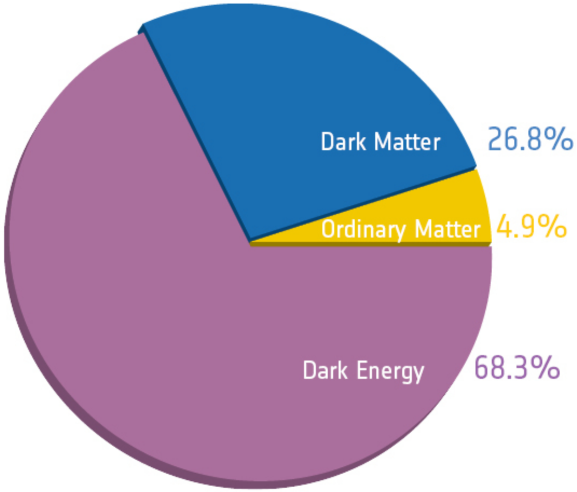
Baryons 5%

Massive neutrinos: 0.1%

Photons: 0.01%

Humans: 10^{-39} %

Spatial curvature: very close to 0



Something else: ≈ 70%

Cosmic inventory

name	density	EOS w
baryons	0.05	≈ 0
CDM	0.27	≈ 0
radiation	0.0001	1/3
Massive neutrinos	<0.05	≈ 0
Cosm. const.	0.68	-1
curvature	<0.01	-1/3
Other ?	?	?

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

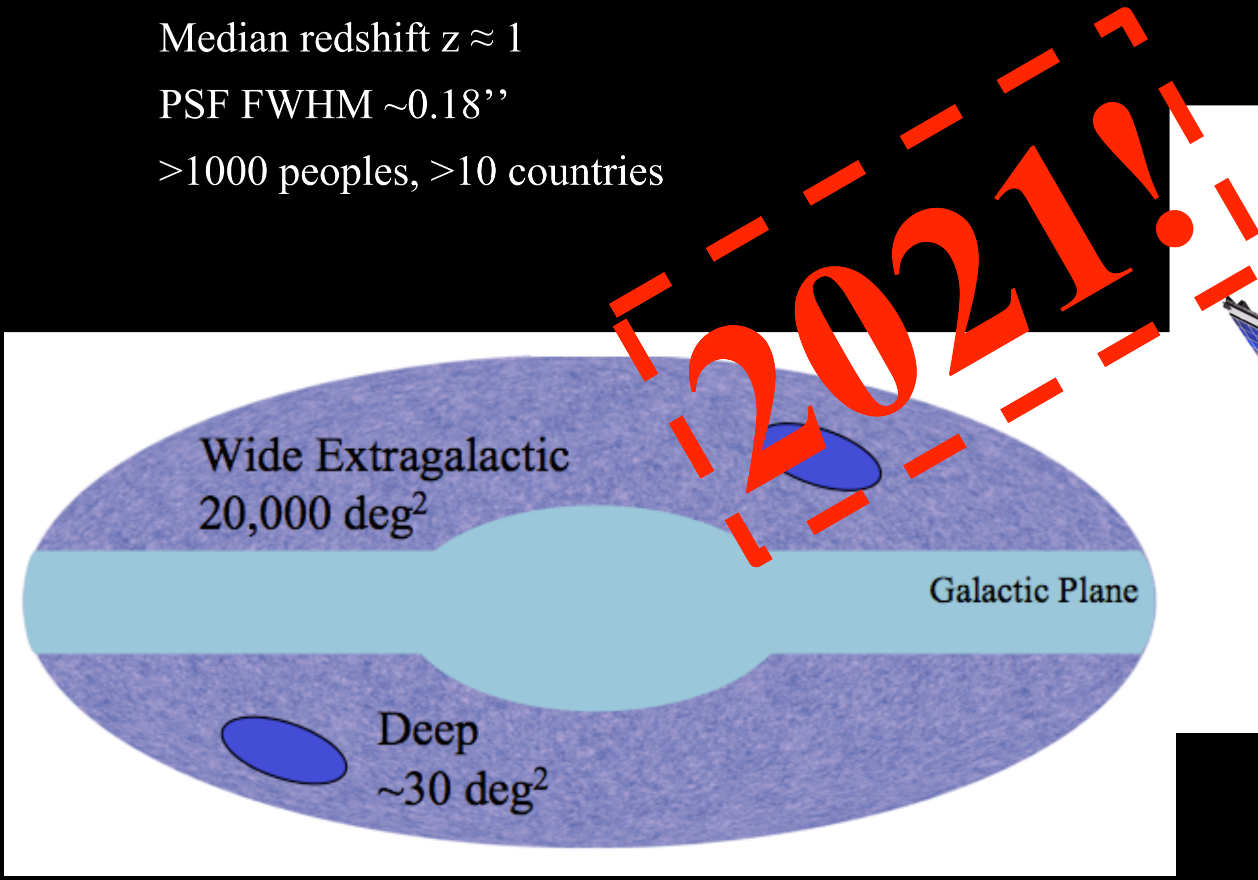
15,000 square degrees

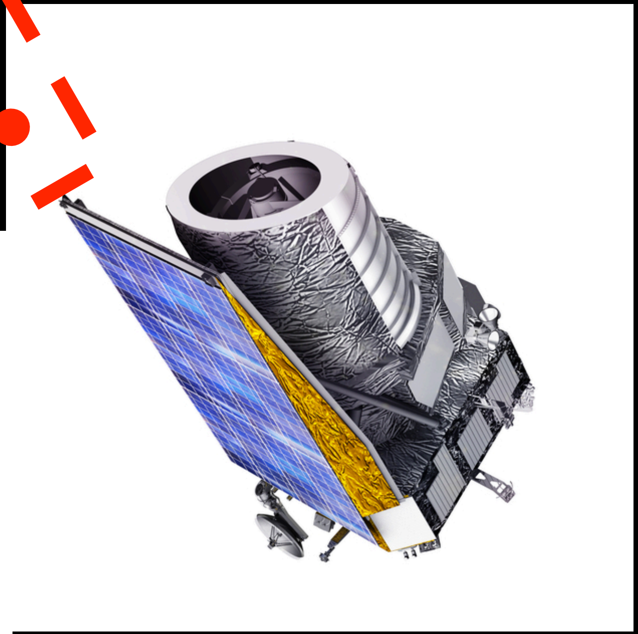
40 million redshifts, 2 billion images

Median redshift z ≈ 1

PSF FWHM ~0.18’’

>1000 peoples, >10 countries





Euclid satellite

arXiv Red Book 1110.3193

Euclid's goals

- Nature of dark energy (DE), test of gravity (MG) and of dark matter (DM)
- Distinguish DE, MG, DM effects
    - on the geometry of the Universe:

      Weak Lensing (WL), Galaxy Clustering (GC),
    - on structure formation:

      WL, Redshift-Space Distortion, Clusters of Galaxies
- controlling systematic residuals to a very high level of accuracy.

(elaboration from a talk by Y. Mellier)

Quantifying precision

Parameterising our ignorance:

DE equation of state: $P/\rho = w$ and $w(a) = w_p + w_a(a_p\text{-}a)$

What precision should we aim to?

From Euclid data alone, get $FoM=1/(Dw_a x\ Dw_p) > 400$:

if data consistent with Λ, and $FoM > 400$ then :

Λ is favoured with odds of more than 100:1 = a "decisive" statistical evidence.

(elaboration from a talk by Y. Mellier)

WL and GC

• **Weak Lensing (WL)**
3-D cosmic shear measurements (tomography) over $0<z<3$
probes distrib. of matter (D+L), expansion history, growth factor .
photo-z sufficient

• **Galaxy Clustering (GC)**
3-D position measurements over $0.5<z<2$
probes clustering history of galaxies induced by gravity
spectroscopic redshifts needed.

(elaboration from a talk by Y. Mellier)

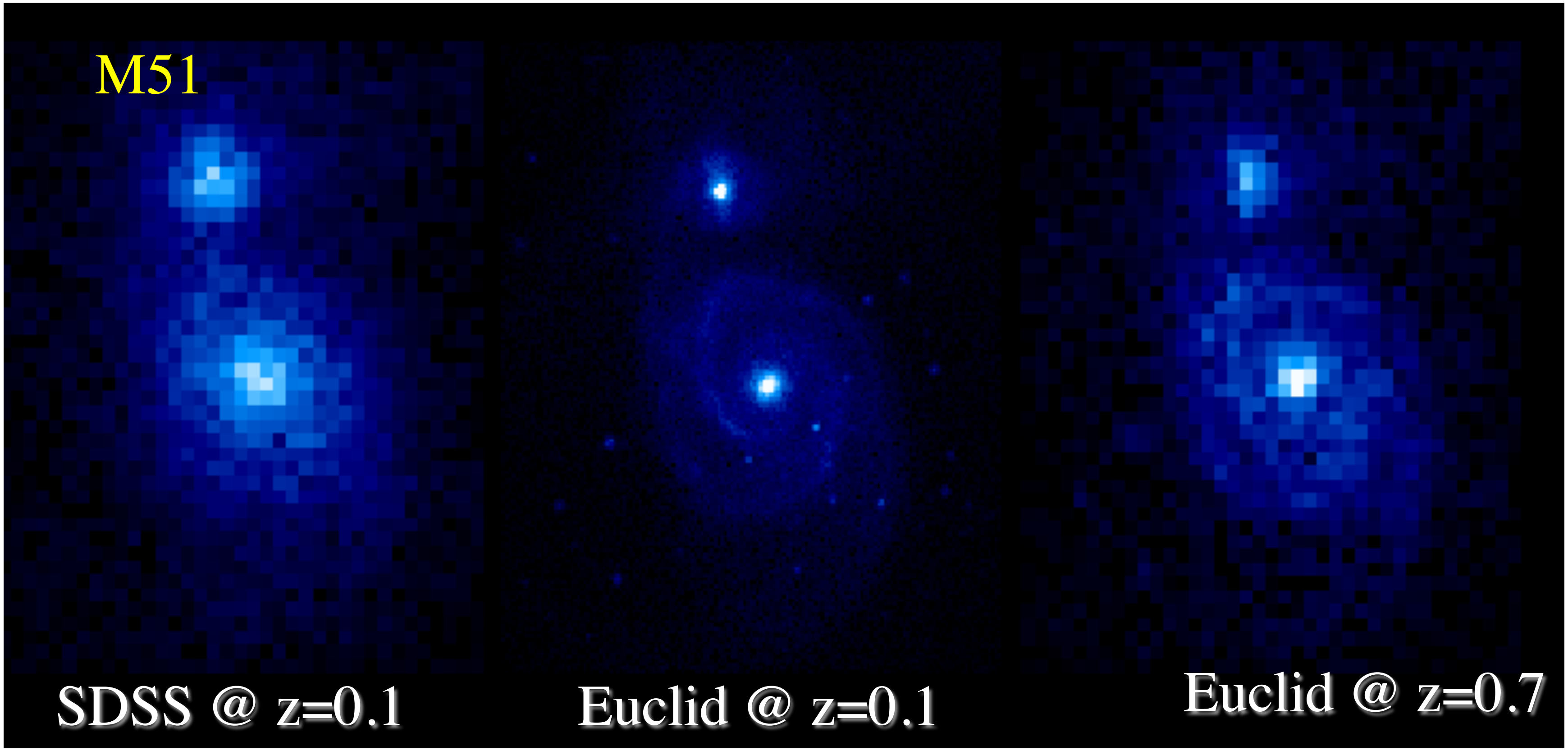
SURVEYS	In ~5.5 years	
	Area (deg2)	Description
Wide Survey	**15,000 deg^2**	Step and stare with 4 dither pointings per step.
Deep Survey	**40 deg^2**	In at least 2 patches of > 10 deg^2 2 magnitudes deeper than wide survey

PAYLOAD					
Telescope	1.2 m Korsch, 3 mirror anastigmat, f=24.5 m				
Instrument	VIS	NISP			
Field-of-View	0.787×0.709 deg^2	0.763×0.722 deg^2			
Capability	Visual Imaging	NIR Imaging Photometry			NIR Spectroscopy
Wavelength range	550– 900 nm	Y (920-1146nm),	J (1146-1372 nm)	H (1372-2000nm)	1100-2000 nm
Sensitivity	24.5 mag 10σ extended source	24 mag 5σ point source	24 mag 5σ point source	24 mag 5σ point source	3 10^{-16} erg cm-2 s-1 3.5σ unresolved line flux
	Shapes + Photo-z of *n* = 1.5 x10^9 galaxies ?			***z* of *n*=5x10^7 galaxies**	
Detector Technology	36 arrays 4k×4k CCD	16 arrays 2k×2k NIR sensitive HgCdTe detectors			
Pixel Size Spectral resolution	0.1 arcsec	0.3 arcsec			0.3 arcsec R=250

Possibility to propose other surveys: SN and/or m-lens surveys, Milky Way ?

Ref: Euclid RB arXiv:1110.3193

Euclid: optimised for shape measurements



Euclid images of z~1 galaxies: same resolution as SDSS images at z~0.1
and at least 3 magnitudes deeper.
Space imaging of Euclid will outperform any other surveys of weak lensing.

(elaboration from a talk by Y. Mellier)

Third Euclid probe: Clusters of galaxies

Clusters of galaxies: probe of peaks in density distribution

number density of high mass, high redshift clusters very sensitive to primordial non-Gaussianity and deviations from standard DE models

Euclid data

- 60,000 clusters with a S/N>3 between $0.2<z<2$ (obtained for free).
- more than 10^4 of these will be at $z>1$.
- ~ 5000 giant gravitational arcs

very accurate masses for the whole sample of clusters (WL)

dark matter density profiles on scales >100 kpc

direct constraints on numerical simulations.

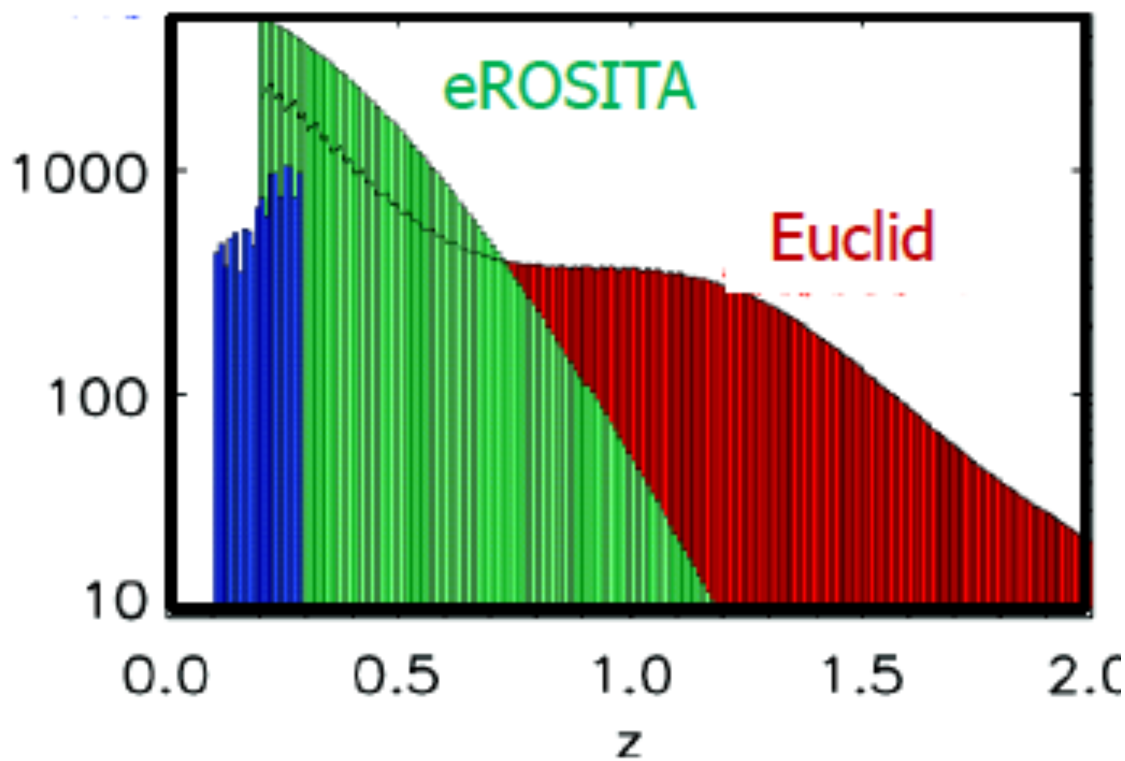
300000 strong galaxy lensing + 5000 giant arcs

test of CDM : probe substructure and small scale density profile.

(elaboration from a talk by Y. Mellier)

Third Euclid probe: Clusters of galaxies

Clusters of galaxies: probe of peaks in density distribution
number density of high mass, high redshift clusters very sensitive to primordial non-Gaussianity and deviations from standard DE models



(elaboration from a talk by Y. Mellier)

Predicted FoM of the Euclid mission

	Modified Gravity	Dark Matter	Initial Conditions	Dark Energy		
Parameter	g	m_n /eV	f_{NL}	w_p	w_a	*FoM*
Euclid primary (WL+GC)	0.010	0.027	5.5	0.015	0.150	430
Euclid All	0.009	0.020	2.0	0.013	0.048	1540
Euclid+Planck	0.007	0.019	2.0	0.007	0.035	4020
Current (2009)	0.200	0.580	100	0.100	1.500	~10
Improvement Factor	**30**	**30**	**50**	**>10**	**>40**	**>400**

Ref: Euclid RB arXiv:1110.3193
More detailled forecasts given in **Amendola et al arXiv:1606.00180, Living Rev.Rel. 21 (2018) no.1, 2**

The two pillars of Euclid

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

Motion of a particle in a perturbed metric

$$\gamma^2\dot{\mathbf{v}}(1+\hat{\varepsilon}_1) = -\tilde{H}\mathbf{v}(1-2\Psi) + \gamma^2[2\mathbf{v}\mathbf{v}\cdot\nabla(\Psi-\Phi) - v^2\nabla(\Psi-\Phi)]$$

$$\gamma^2 = (1-v^2)^{-1}$$

for v<<1, ***v*** depends only on Ψ

for v≈1, ***v*** depends on Ψ - Φ

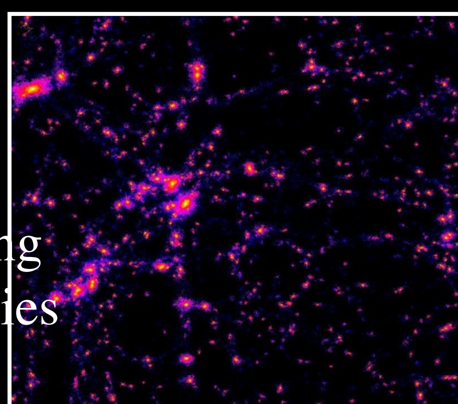
The two pillars of Euclid

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

Non-relativistic particles respond to Ψ

$$\delta'' + (1 + \frac{H'}{H})\delta' = k^2\Psi$$

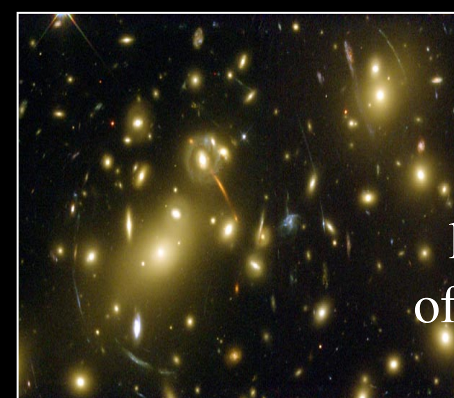
$$\delta = \frac{\rho - \overset{\frown}{\rho}}{\overset{\frown}{\rho}}$$



clustering of galaxies

Relativistic particles respond to Φ-Ψ

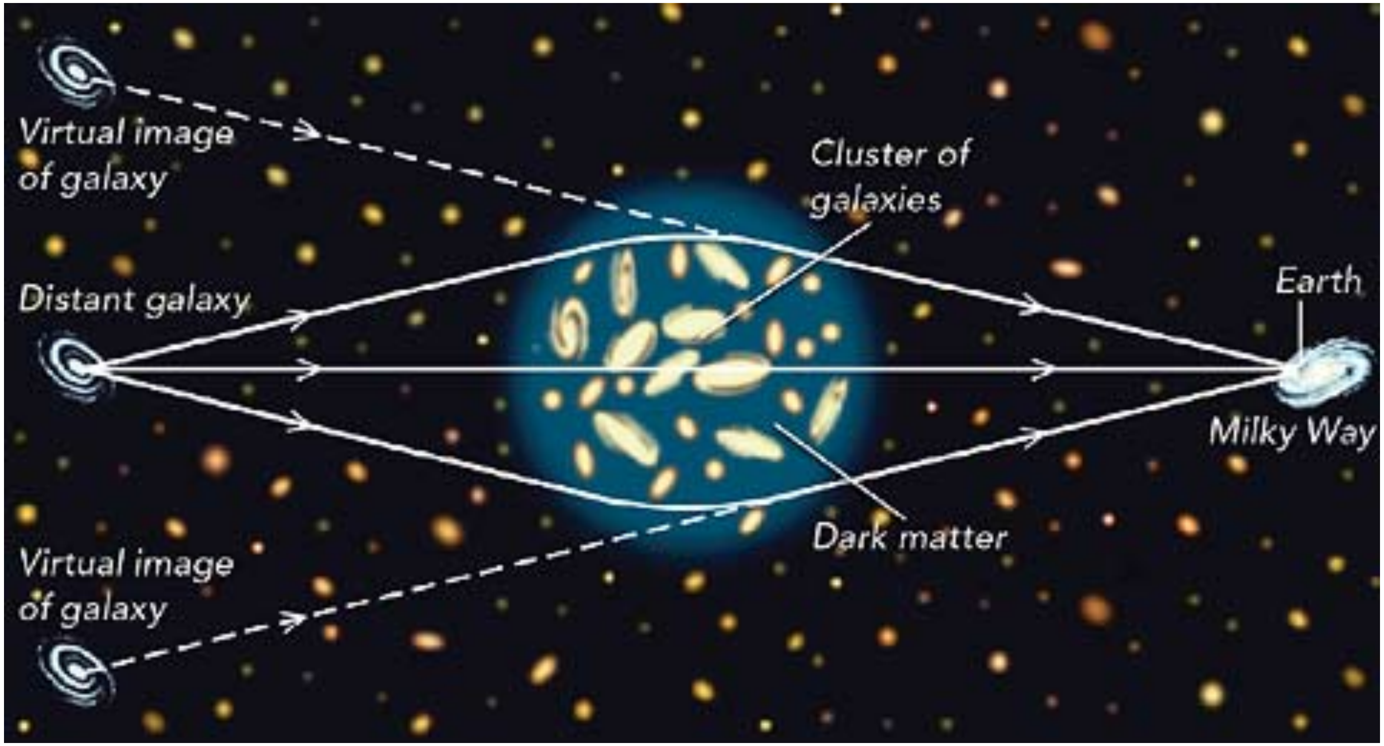
$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$



lensing of galaxies

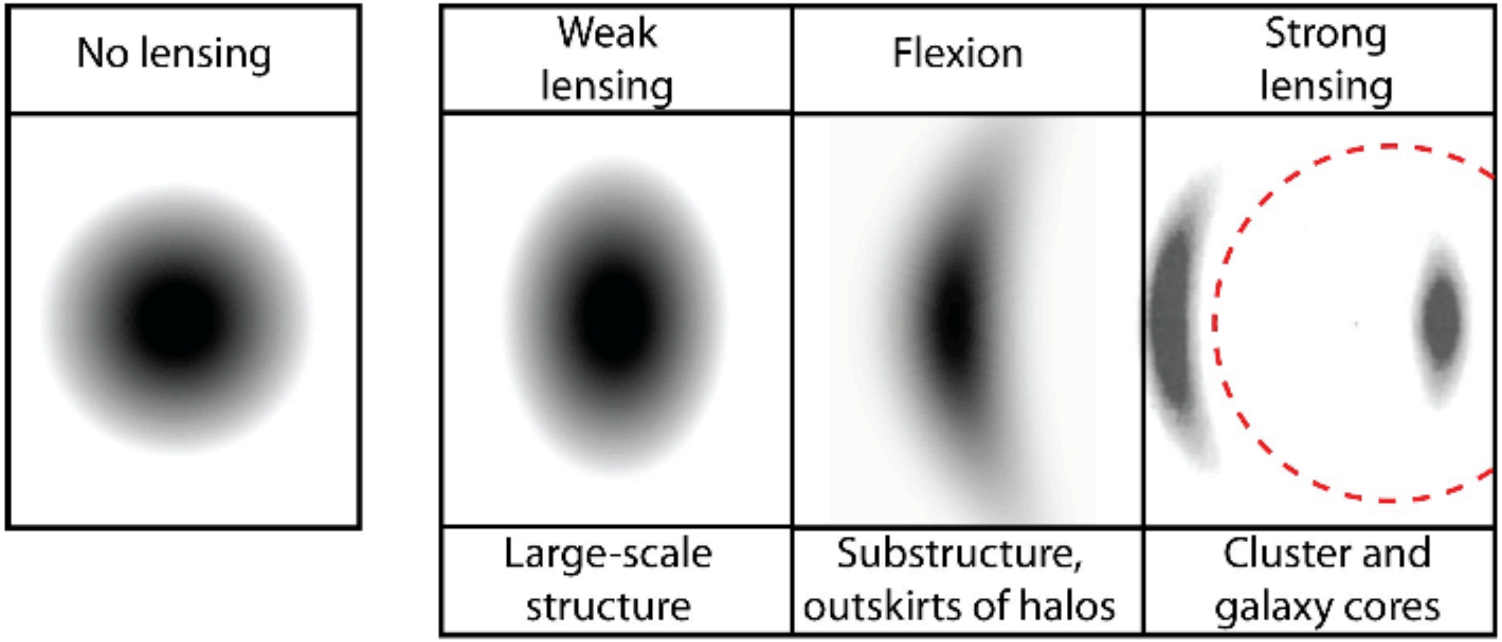
The first pillar: Weak Lensing

Light deflection



Short intro to Weak Lensing

Lensing, weak lensing, strong lensing…



Short intro to Weak Lensing

Abell 2218

Credit: NASA, ESA,
and Johan Richard (Caltech, USA)

Weak Shear Lensing

The Lens equation

Propagation of light in a given metric

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j \,.$$

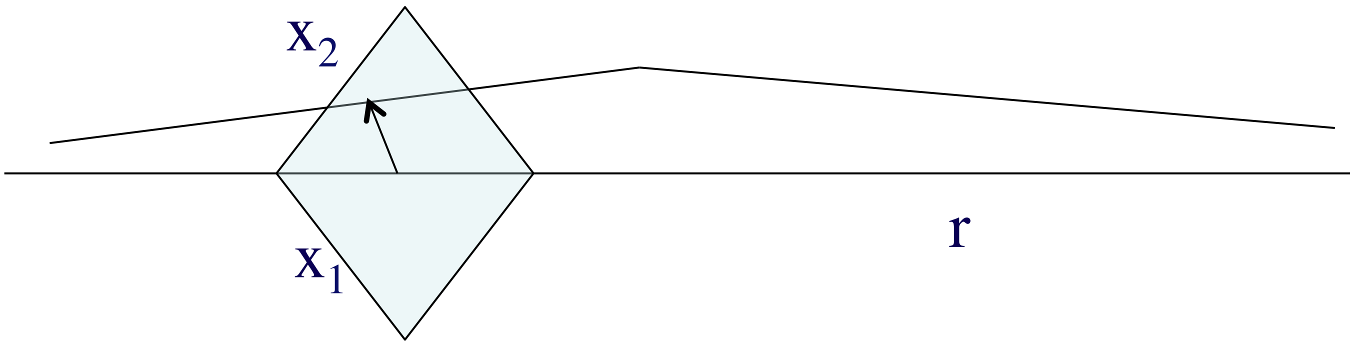
$k^\mu = dx^\mu/d\lambda_s$ is the photon momentum

General equations of motion

$$k^\mu k_\mu = 0 \,,$$
$$\frac{dk^\mu}{d\lambda_s} + \Gamma^\mu_{\alpha\beta}k^\alpha k^\beta = 0 \,,$$

The Lens equation

$$k^\mu k_\mu = 0\,,$$
$$\frac{\mathrm{d}k^\mu}{\mathrm{d}\lambda_s} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0\,,$$



Equations of motion on the plane in the linearly perturbed metric

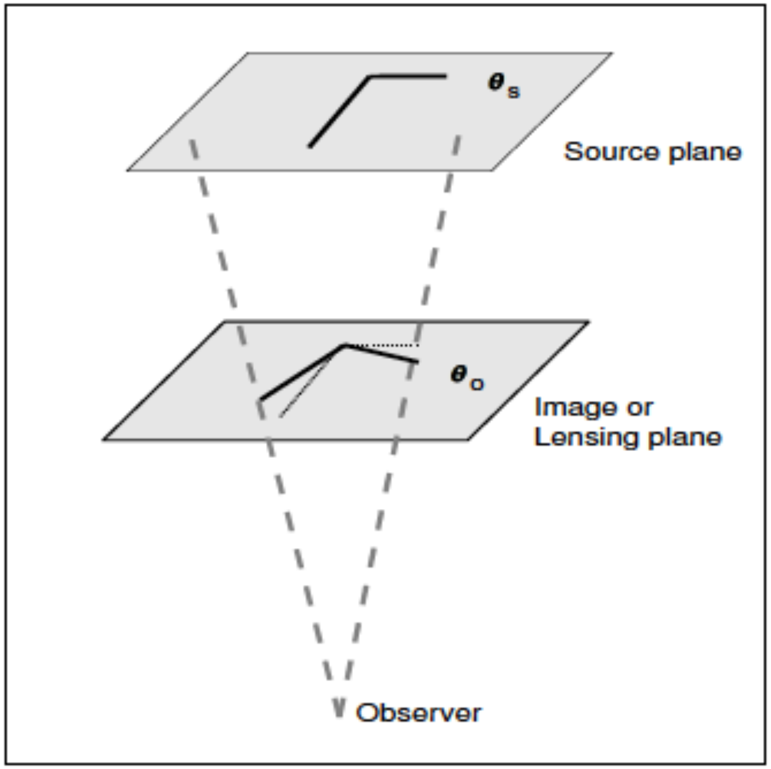
$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}r^2} = \psi_{,i}\,, \qquad \psi \equiv \Phi - \Psi\,.$$ lensing potential

(see my book)

The distortion matrix

$$x_i = r\theta_i$$

$$A_{ij} \equiv \frac{\partial \theta_s^i}{\partial \theta_0^j} = \delta_{ij} + D_{ij} \, ,$$



Mapping of the position in the **source plane** to the observed position in the **lens plane**

The distortion matrix

$$A_{ij} \equiv \frac{\partial \theta_s^i}{\partial \theta_0^j} = \delta_{ij} + D_{ij} \, ,$$

$$D_{ij} = \int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r' \psi_{,ij} = \begin{pmatrix} -\kappa_{\mathrm{wl}} - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa_{\mathrm{wl}} + \gamma_1 \end{pmatrix} ,$$

$$= -\kappa \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

= magn +shear

The distortion matrix

$$\kappa_{\mathrm{wl}} = -\frac{1}{2}\int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r'(\psi_{,11} + \psi_{,22}) \,,$$

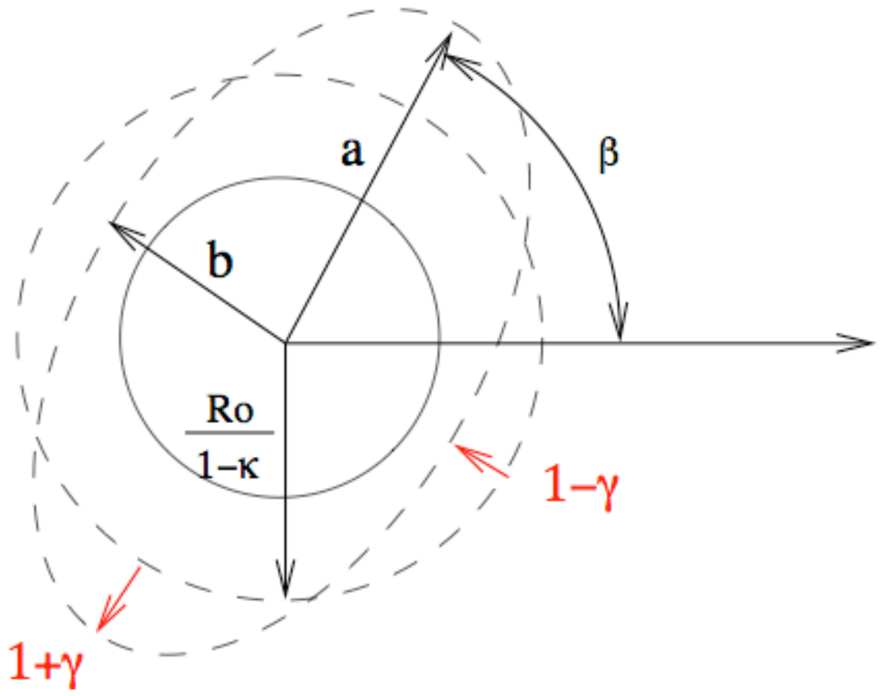
Magnification

$$\gamma_1 = -\frac{1}{2}\int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r'(\psi_{,11} - \psi_{,22}) \,,$$

$$\gamma_2 = -\int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r'\psi_{,12} \,,$$

Shear

An elliptical distortion



Measuring the shear

$I(\theta_x, \theta_y)$ intensity of light as a function of angles on the sky

$$q_{ij} = \int \mathrm{d}^2\theta \, I(\boldsymbol{\theta})\theta_i\theta_j \, ,$$ quadrupole

$$\varepsilon_1 = \frac{q_{xx} - q_{yy}}{q_{xx} + q_{yy}} \, , \qquad \varepsilon_2 = \frac{2q_{xy}}{q_{xx} + q_{yy}} \, .$$ ellipticity

it turns out…

$$\varepsilon_1 \approx 2\gamma_1 \, , \qquad \varepsilon_2 \approx 2\gamma_2 \, .$$

The power spectrum of ellipticity

$$\gamma_1 = -\frac{1}{2}\int_0^{r_s} \mathrm{d}r' \left(1-\frac{r'}{r_s}\right) r'(\psi_{,11} - \psi_{,22}),$$

$$\gamma_2 = -\int_0^{r_s} \mathrm{d}r' \left(1-\frac{r'}{r_s}\right) r'\psi_{,12} ,$$

Now, the ellipticity depends linearly on $\gamma_{1,2}$.
These depend linearly on the metric potential Ψ,Φ.
The metric potential depend linearly on the matter density contrast along the line of sight.
Therefore**, the power spectrum of the ellipticity depends linearly on the power spectrum of matter**!

It turns out…

The power spectrum of ellipticity depends linearly on the power spectrum of (dark+luminous) matter!

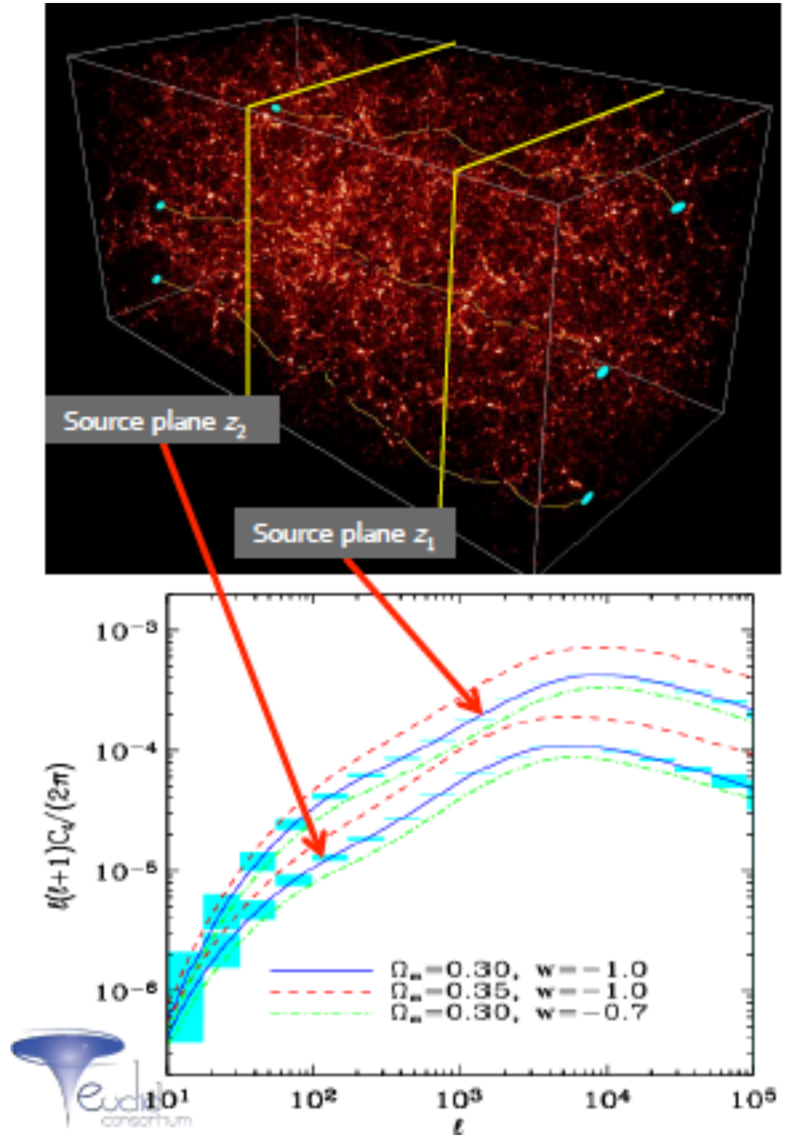
$$P_{\kappa_{\rm wl}}(q) = \frac{9H_0^3}{4}\int_0^\infty {\rm d}z \frac{W(z)^2 E^3(z)\Omega_m^2(z)}{(1+z)^4} P_{\delta_m}\left(\frac{q}{r(z)}\right),$$

depends only on background quantities, i.e., on distances, H(z), and on the source distribution

matter power spectrum

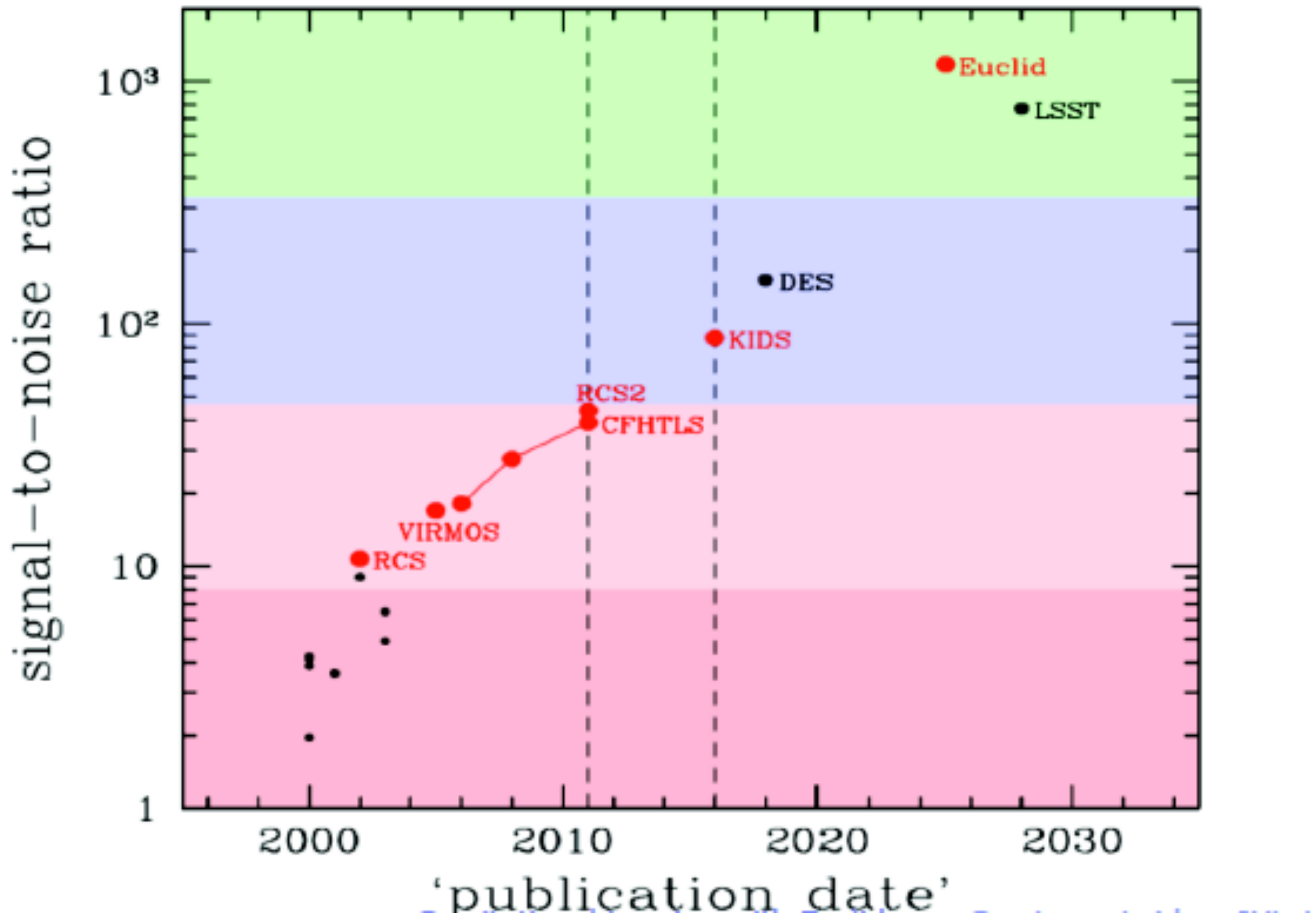
WL surveys

$$P_{\kappa_{\mathrm{wl}}}(q) = \frac{9H_0^3}{4}\int_0^\infty \mathrm{d}z\,\frac{W(z)^2E^3(z)\Omega_m^2(z)}{(1+z)^4}P_{\delta_m}\left(\frac{q}{r(z)}\right),$$



(elaboration from a talk by Y. Mellier)

WL surveys



(elaboration from a talk by Y. Mellier)

The second pillar: Galaxy clustering

Modified-gravity
sub-horizon linear matter equations

$$\theta = ik_j v^j$$

$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$

$$\boxed{\begin{aligned} \dot{\delta} &= -\theta \\ \dot{\theta} &= -\mathcal{H}\theta + k^2\Psi \\ k^2\Phi &= 4\pi G a^2 \rho\delta \end{aligned}}$$

continuity

Euler

Poisson

This can be written as a single equation:

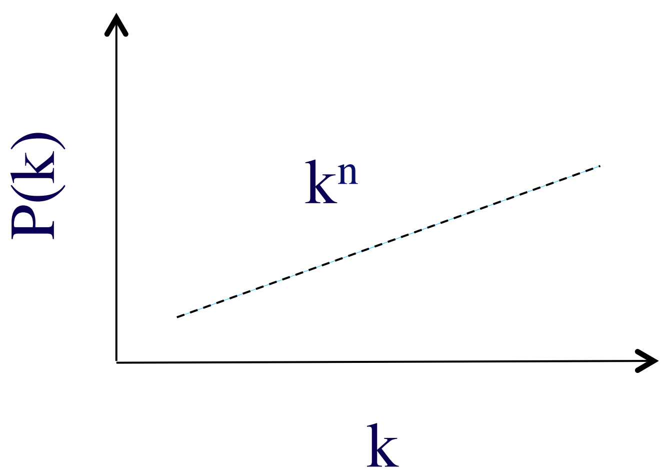
$$\delta'' + (1 + \frac{H'}{H})\delta' - \frac{3}{2}\Omega_m \delta = 0$$

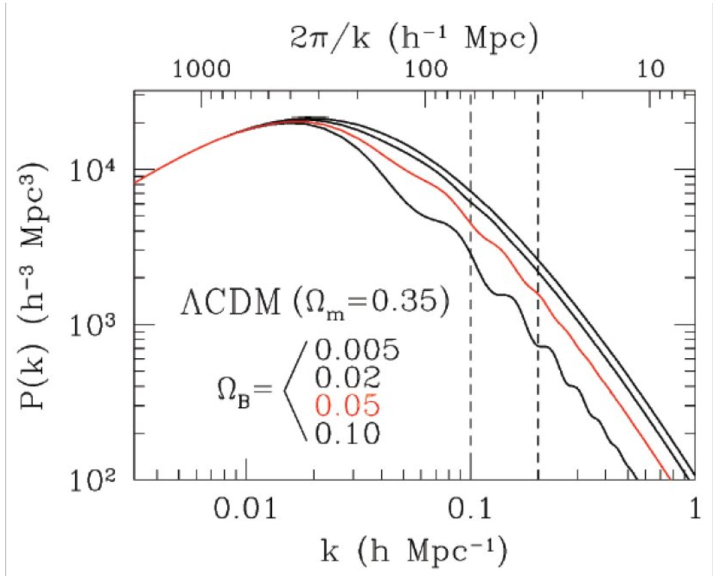
prime is d/dlog(a)

Galaxy clustering

$$\delta''+(1+\frac{H'}{H})\delta'-\frac{3}{2}\Omega_m\delta=0$$

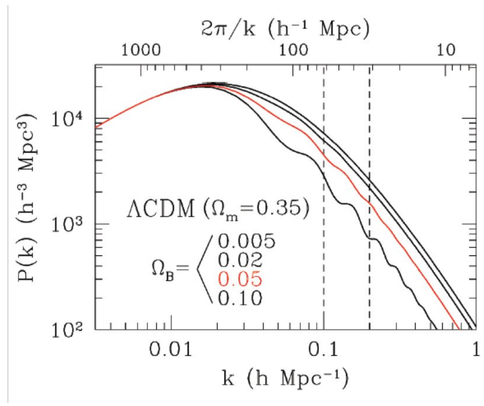
With this evolution equation, one can in principle predict the power spectrum of fluctuations today given the initial (inflationary) power spectrum

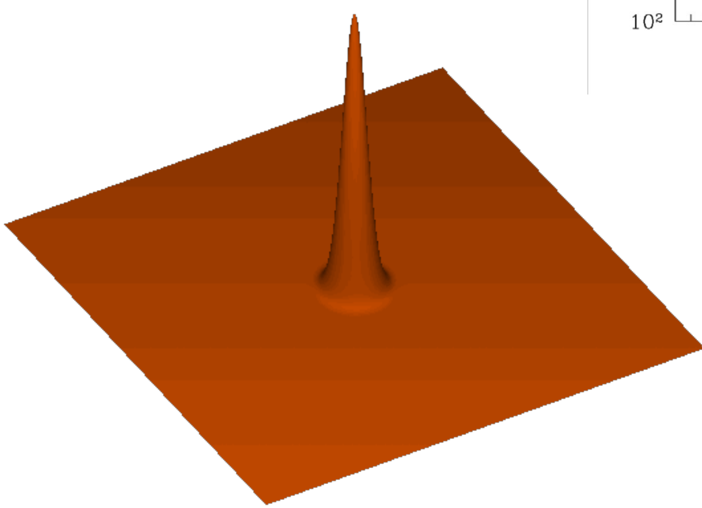


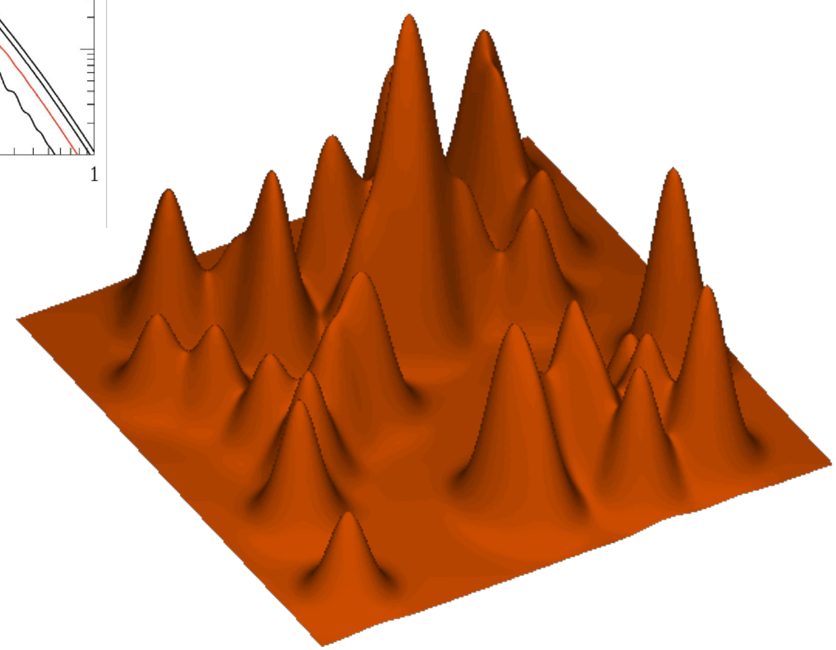


Galaxy clustering: BAO

Baryon acoustic oscillations cause peaks in the power spectrum



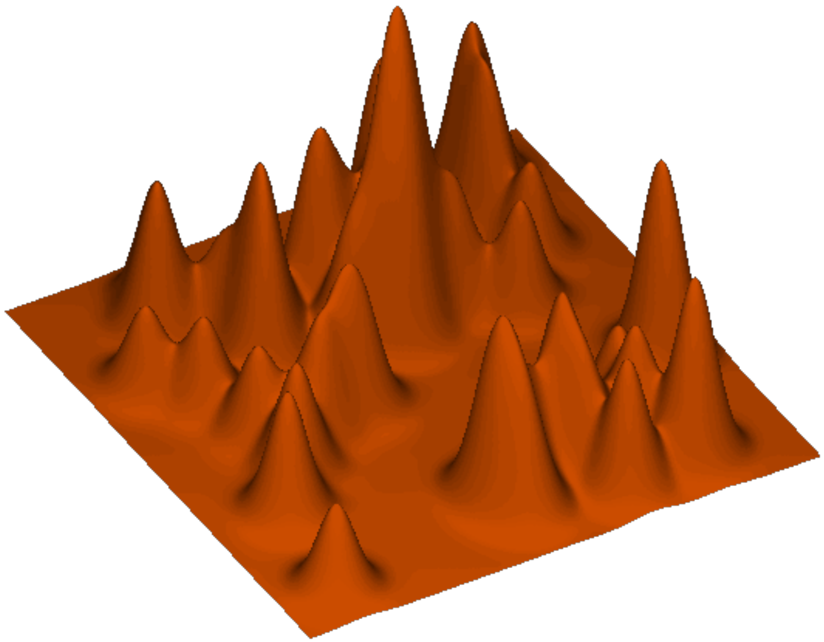


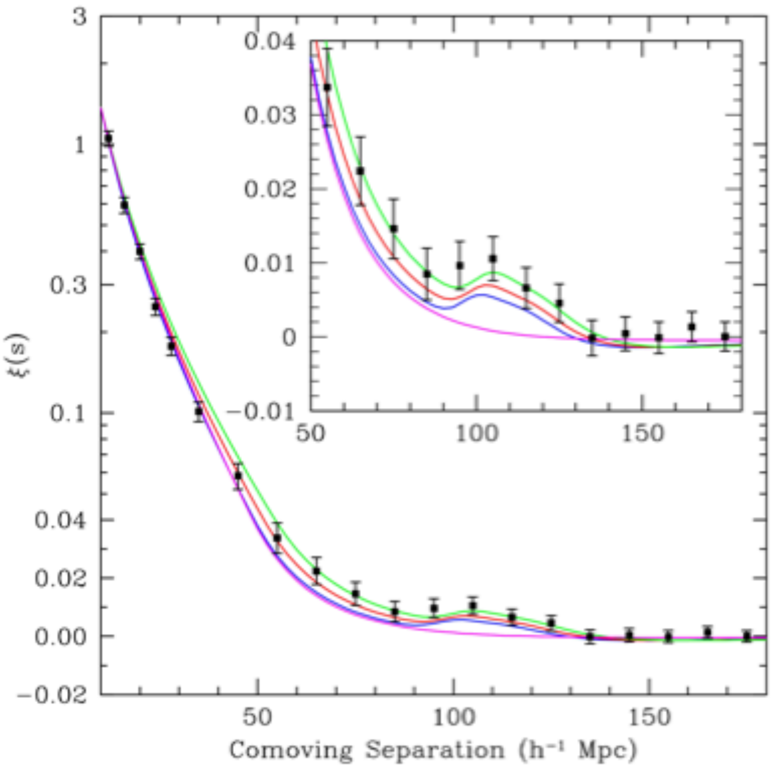


Bassett, Hlozek 2009

Galaxy clustering: BAO

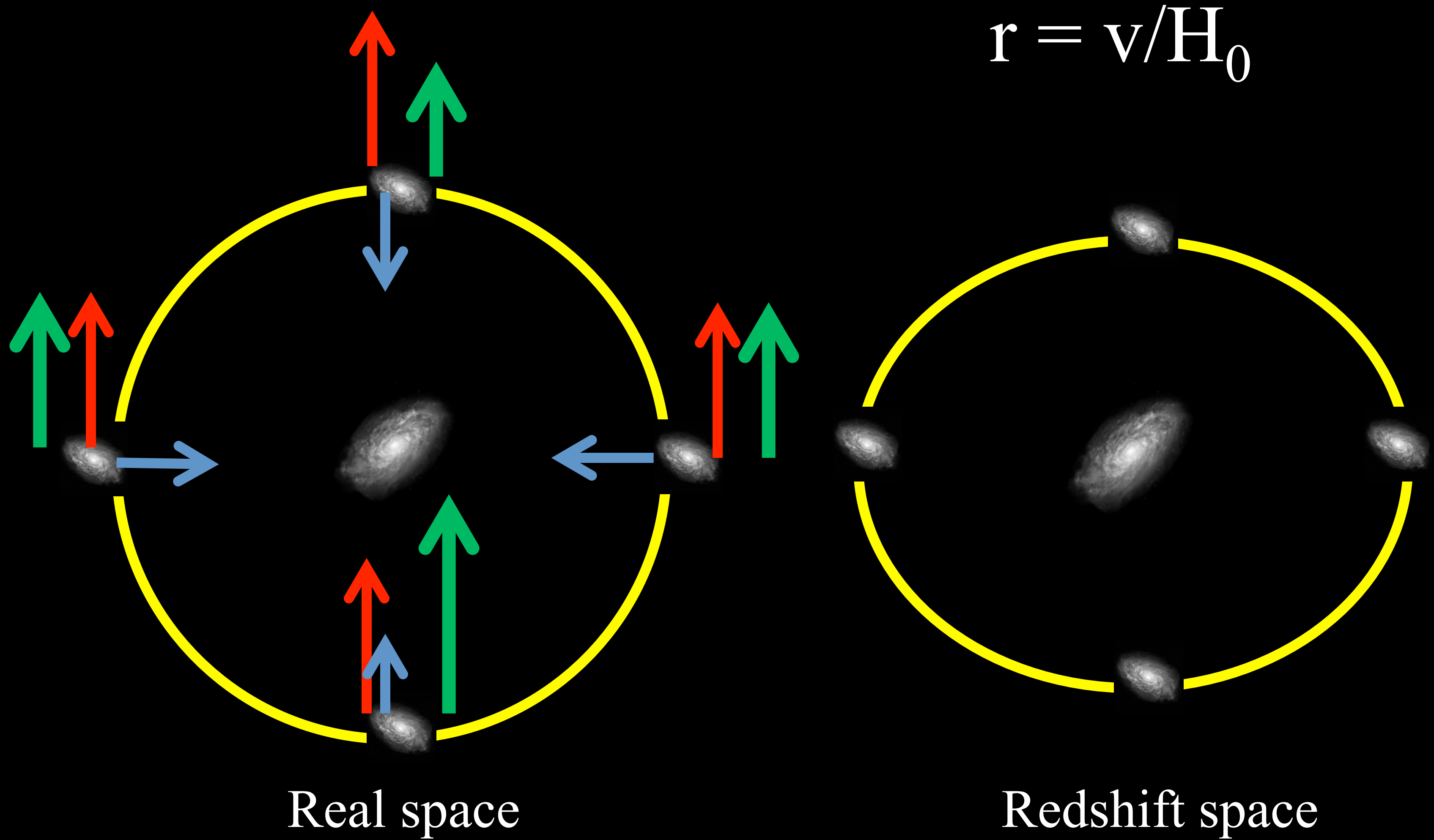
BAO peak in the galaxy correlation function



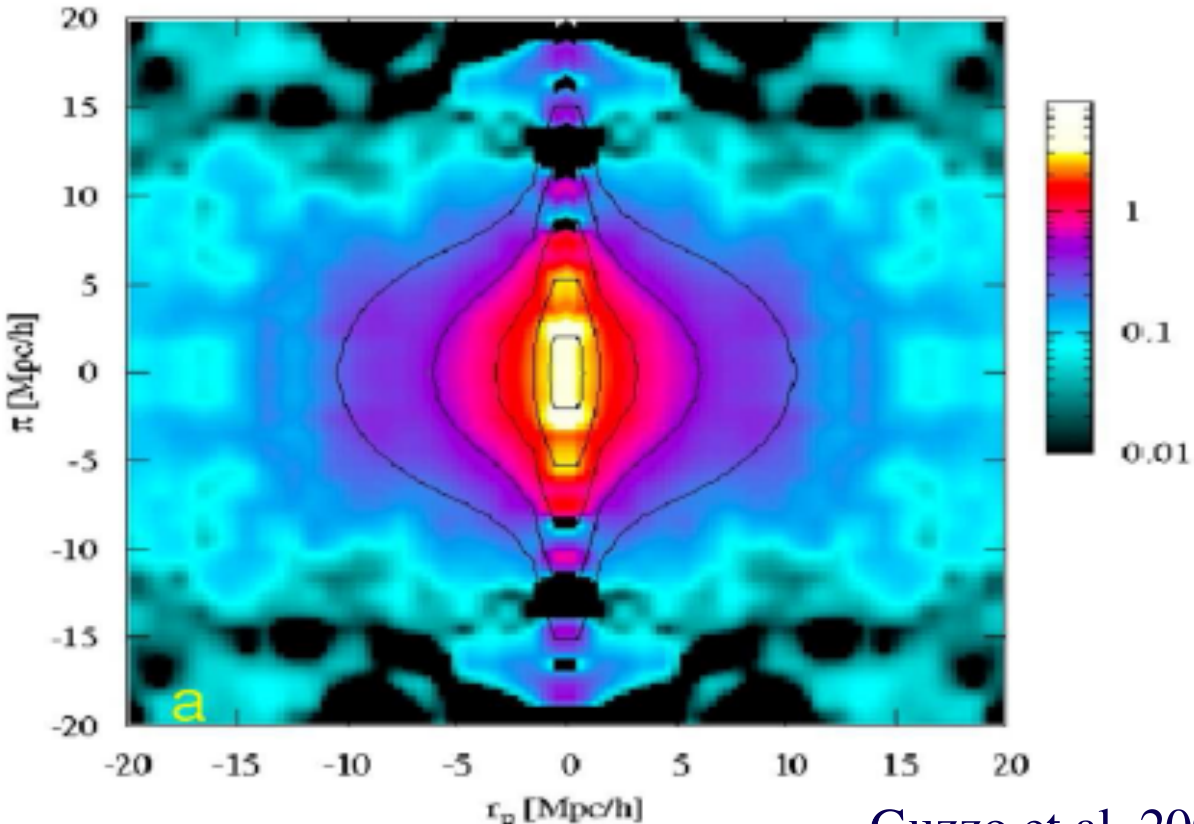


Eisenstein+ 05

Galaxy clustering: Redshift Space Distortions

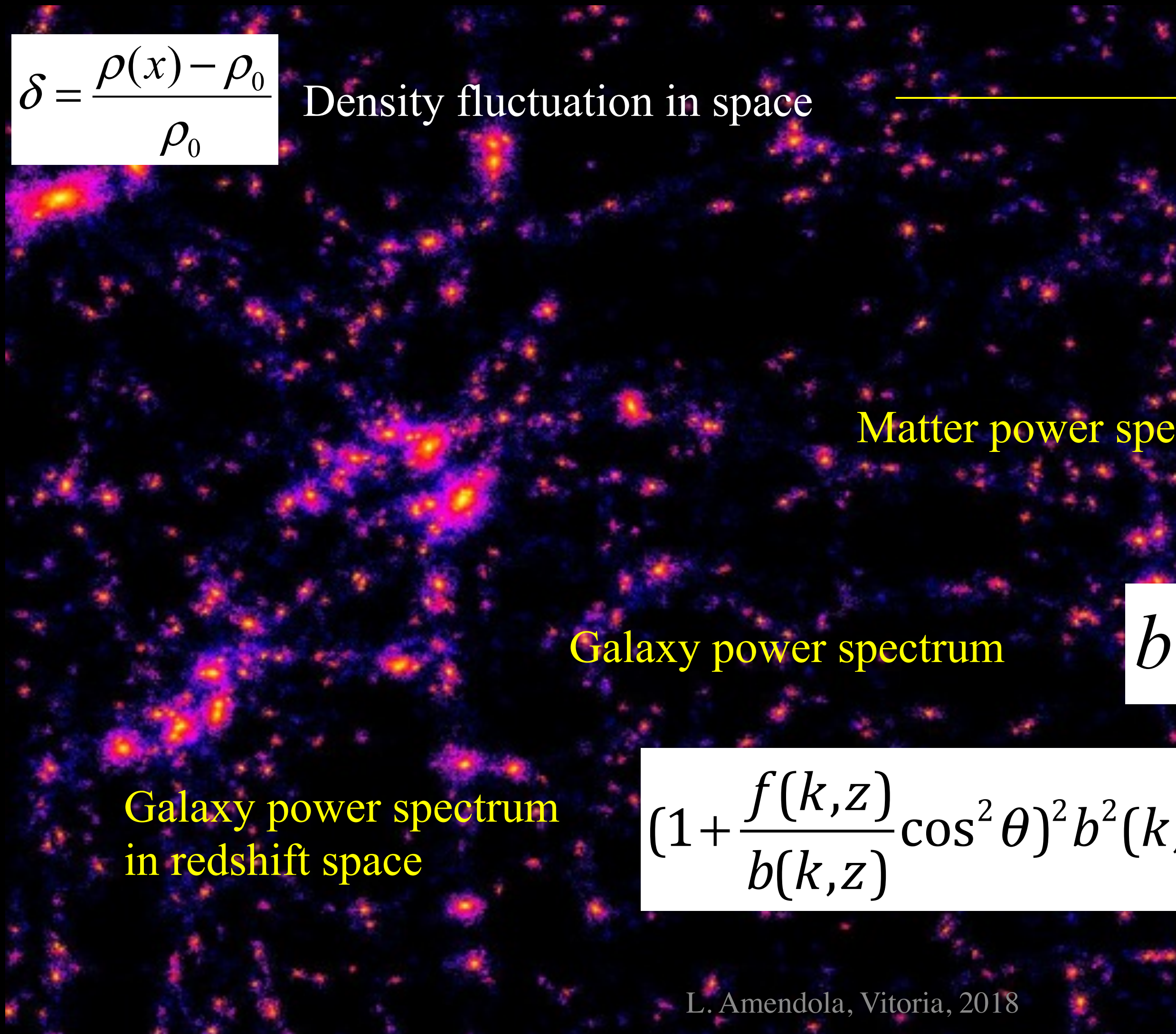


RSD on the correlation function



Guzzo et al. 2008

From theory to observations



$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$ Density fluctuation in space $\longrightarrow$ $$\left\langle \delta_k^{\ 2} \right\rangle = P(k,z)$$

Matter power spectrum $$P_{matter}(k,z)$$

Galaxy power spectrum $$b^2(k,z)P_{matter}(k,z)$$

Galaxy power spectrum
in redshift space
$$\left(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta\right)^2 b^2(k,z)G^2(k,z)P^{matter}_{initial}(k,z_{in})$$

Deconstructing the galaxy power spectrum

$$P_{galaxy}(k,z,\mu=\cos\theta)=(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z)G^2(k,z)P^{matter}_{initial}(k,z_{in})$$

$b(k,z)$ depend on local, non-cosmological physics

$P^{matter}_{initial}(k,z_{in})$ depend on initial conditions (inflation)

$G(k,z)$

$f(k,z)$

depend on dark energy/gravity

First way to go: Parametric method

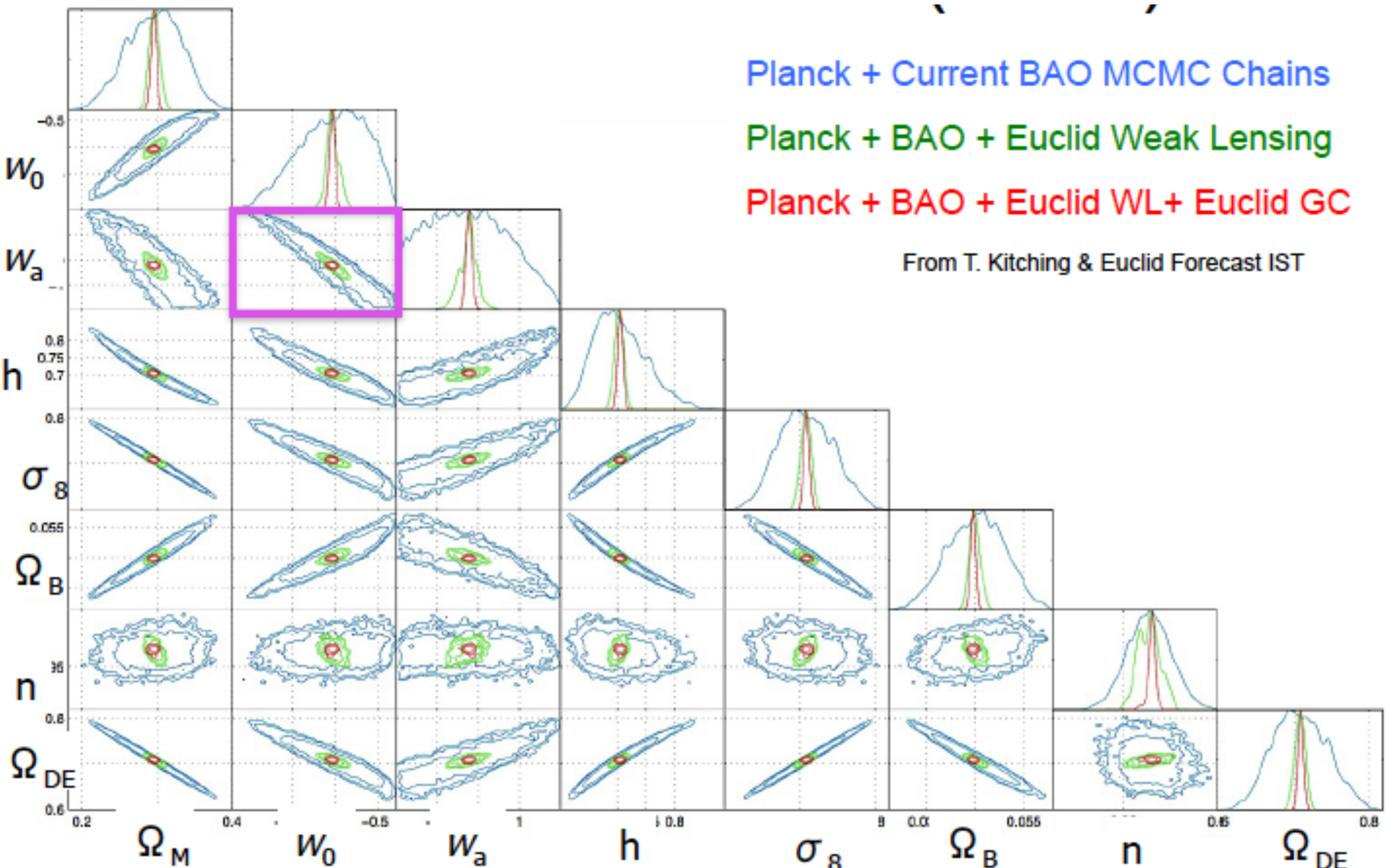
$$P_{galaxy}(k,z,\mu=\cos\theta)=(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z)G^2(k,z)P_{initial}^{matter}(k,z_{in})$$

$b(k,z)$ depend on local, non-cosmological physics

$$\left.\begin{array}{l} P_{initial}^{matter}(k,z_{in}) \\ G(k,z) \\ f(k,z) \end{array}\right\}$$

Given a model, eg. inflation+ΛCDM, the power spectrum can be parametrized by a small number of parameters: n_s,Λ,Ω_m,h,w, etc

Euclid parameter constraints



Second way to go: non-parametric method

$$P_{galaxy}(k,z,\mu=\cos\theta)=(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z)G^2(k,z)P_{initial}^{matter}(k,z_{in})$$

$b(k,z)$ depend on local, non-cosmological physics

$P_{initial}^{matter}(k,z_{in})$ depend on initial conditions (inflation)

$G(k,z)$

$f(k,z)$

depend on dark energy/gravity

Deconstructing the galaxy power spectrum

Line-of-sight decomposition

$$\mu \equiv \cos\theta$$

$$\delta_{galaxy}(k,z,\mu) = G(k,z)\left(1+\frac{f(k,z)}{b(k,z)}\mu^2\right)\ b(k,z)\delta^{matter}_{initial}(k,z_{in})$$

$$\equiv A + R\mu^2$$

$$\delta_{\text{lensing}}(k,z) = -\frac{3}{2}G(k,z)\Omega_m\delta^{matter}_{initial}(k,z_{in}) \equiv L$$

Three linear observables: A, R, L

galaxy clustering

$$A = Gb\delta_{m,0}(k)$$

$$R = Gf\delta_{m,0}(k)$$

weak gravitational lensing

$$L = -\frac{3}{2}G\Omega_m\delta_{m,0}(k)$$

The model-independent observables

Redshift distortion/Amplitude $$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion $$P_2 = \frac{L}{R} = \frac{\Omega_{m0}}{f}$$

Redshift distortion rate $$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate $$E = \frac{H}{H_0}$$

All we know about present-day cosmology

A compilation of all available datasets of lensing and growth

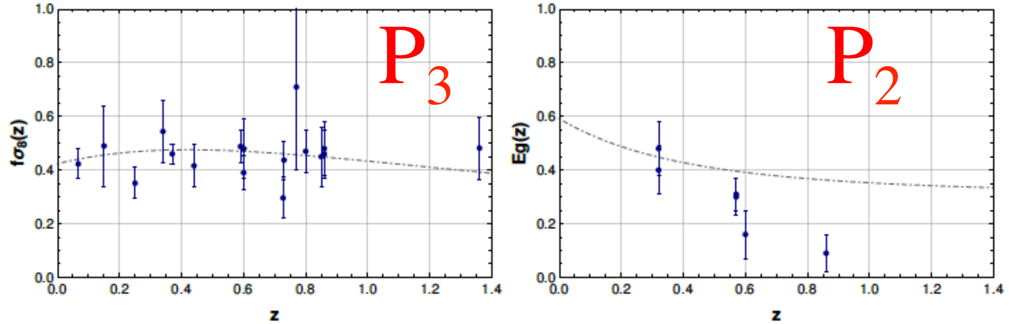


Figure 1. Left: Plot of the full $f\sigma_8$ data (table II) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full E_g data (table III) as a function of z with the theoretical curve in dashed dot.

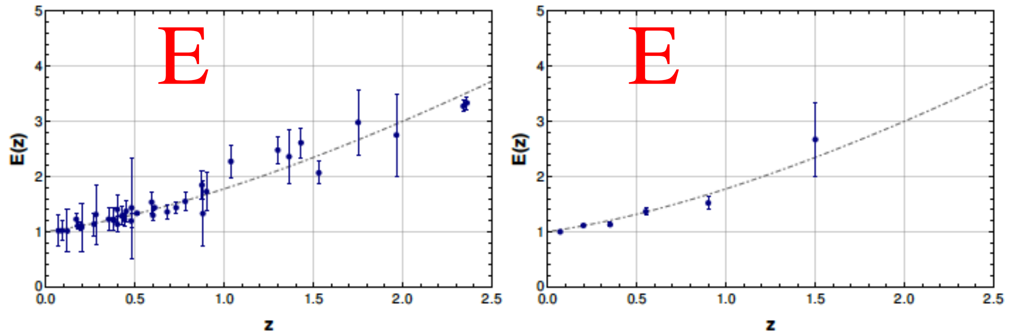
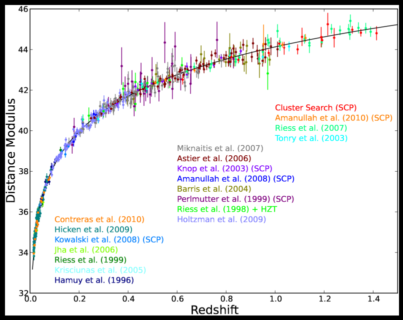
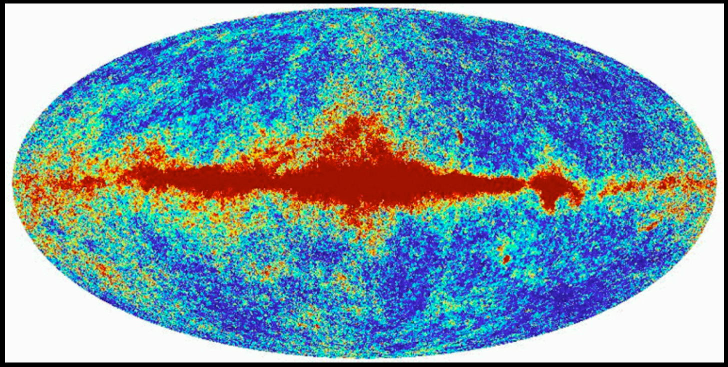


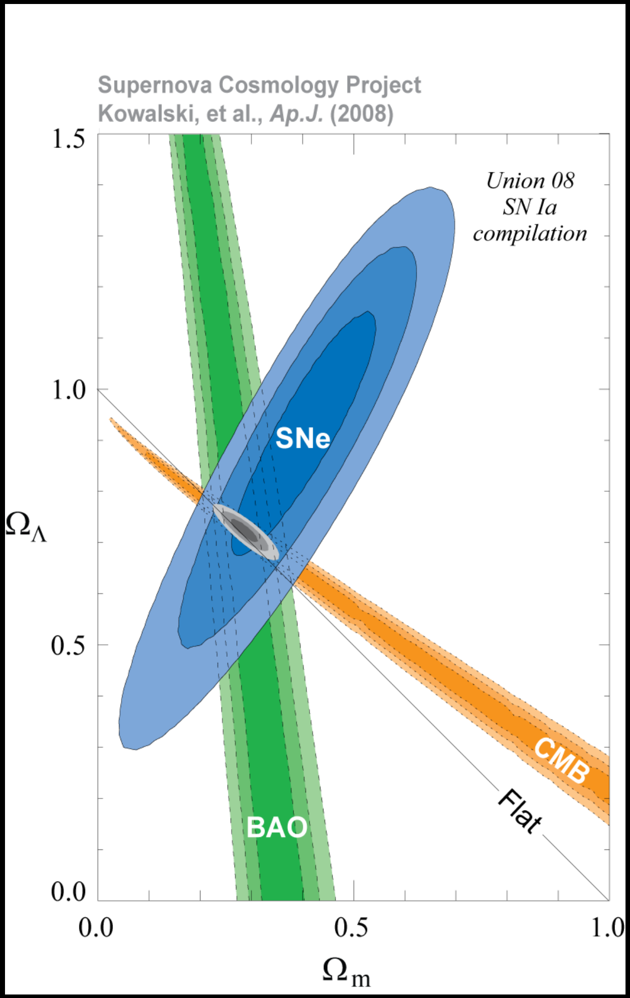
Figure 2. Right: Plot of the full $H(z)$ data (table IV) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full $E(z)$ data (table II) as a function of z with the theoretical curve in dashed dot.

Collab. with Santiago Casas and Ana M. Pinho arXiv:1805.00025

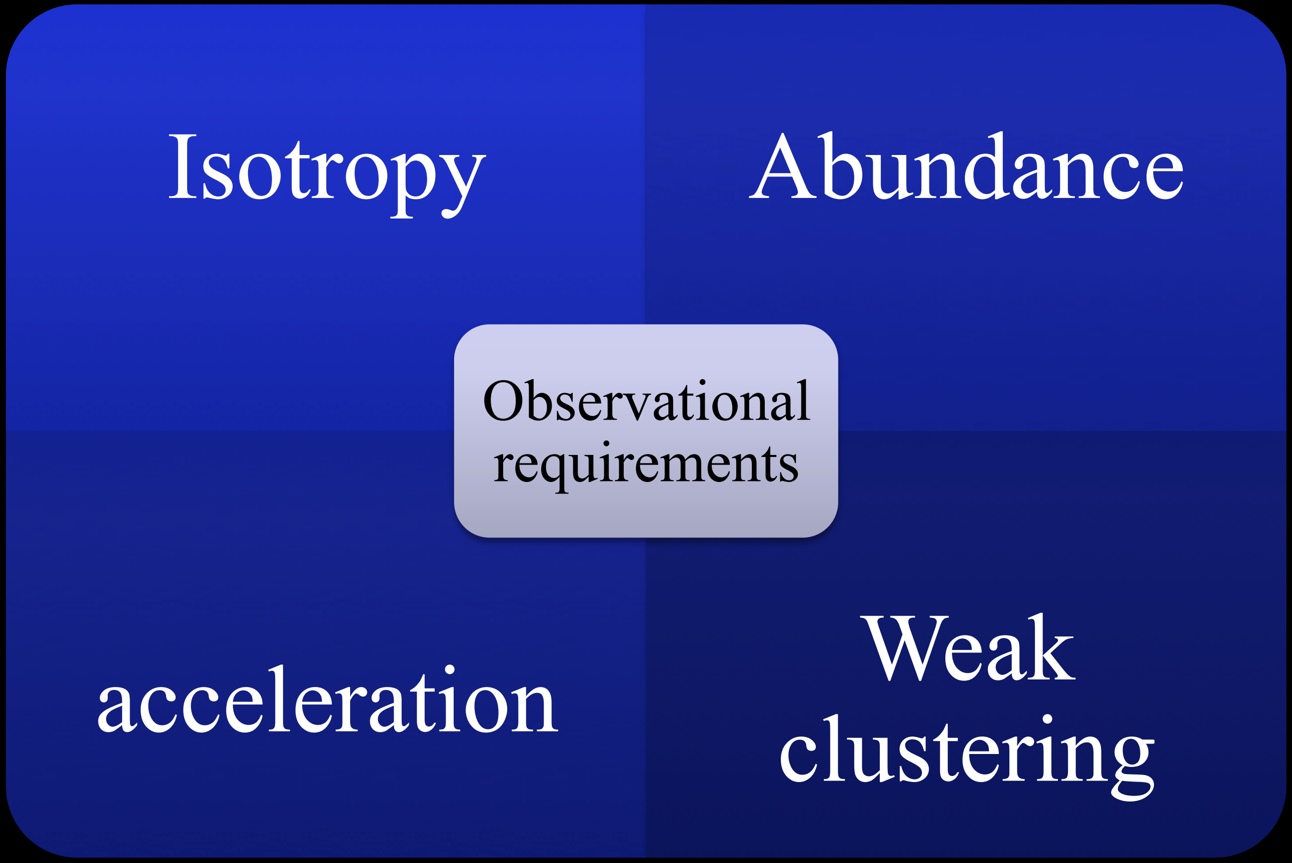
Properties of Dark Energy







Properties of Dark Energy



Properties of Dark Energy

Observations:

- Isotropy
- Large abundance
- Acceleration
- Weak clustering

Theory:

- Scalar field?
- $\Omega_{DE} \approx \Omega_m$
- $w_{eff} \approx -1$
- $c_s \approx 1$

The magic of Lagrangians

$$\int dx^4 \sqrt{-g}\left[R + L_{matter}\right] \xrightarrow{\text{variation}} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

The two laws of Lagrangian-cooking:
a) form a scalar: Eqs are covariant
b) no explicit functions of coords: Energy-momentum tensor is conserved

Random example:

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + R_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + V(\phi) + R_{\mu\nu\sigma\tau}\phi^{,\nu}\phi^{,\mu}\phi^{,\sigma}\phi^{,\tau} + R_{\mu\nu\sigma\tau}R^{,\sigma\tau}\phi^{,\nu\mu} + ... + L_{matter}\right]$$

The magic of Fourier space

Fourier space

$$\phi(x,y,z,t) \rightarrow \phi(t)\exp(i\vec{k}\,\vec{x})$$

therefore

$$\frac{\partial}{\partial x}\phi(x,y,z,t) \rightarrow \phi(t)ik_x \exp(i\vec{k}\,\vec{x})$$

$$\nabla\phi(x,y,z,t) \rightarrow \phi(t)i\vec{k}\exp(i\vec{k}\,\vec{x})$$

$$\nabla\nabla\phi(x,y,z,t) \rightarrow -\phi(t)k^2\exp(i\vec{k}\,\vec{x})$$

Very useful for linear equations because the space part drops out!

The past ten years of DE research

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi,\frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

Cosmological constant, Dark energy w=const, Dark energy w=w(z),quintessence, scalar-tensor model, coupled quintessence, k-essence, f(R), Gauss-Bonnet, Galileons, KGB,

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\begin{aligned}
\mathcal{L}_2 =& K(\phi, X)\,, \\
\mathcal{L}_3 =& -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}$$

$$X = \frac{1}{2}\phi_{,\mu}\phi^{,\mu}$$

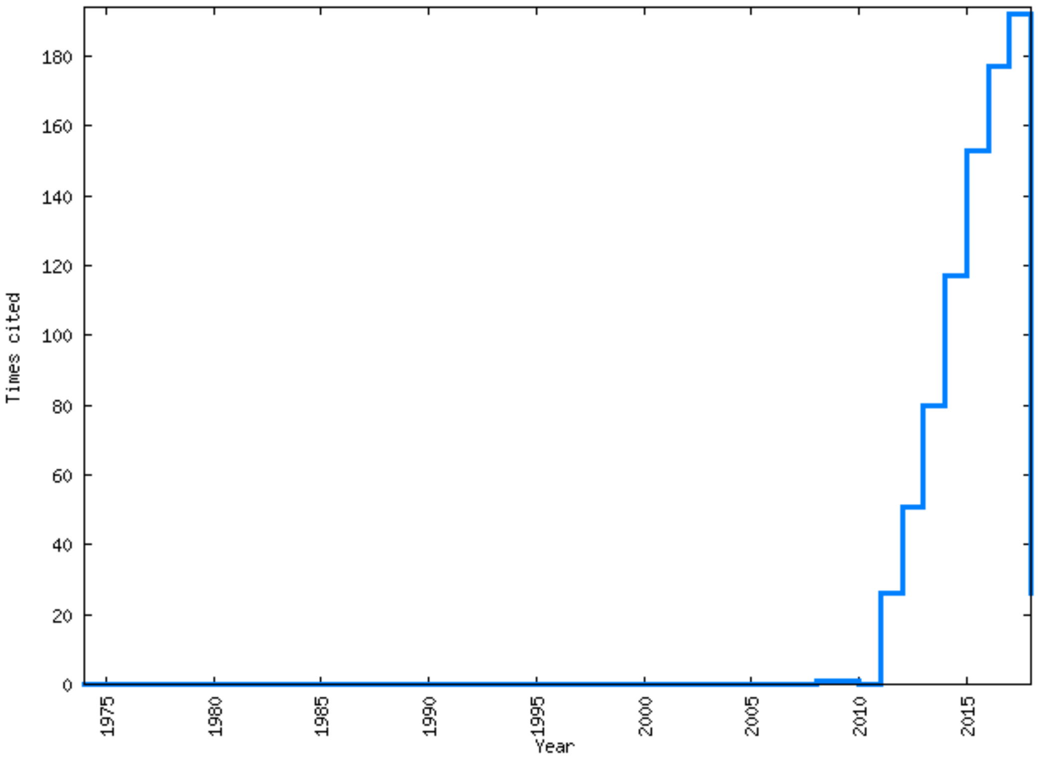
- ✓ First found by Horndeski in 1975
- ✓ rediscovered by Deffayet et al. in 2011
- ✓ no ghosts, no classical instabilities
- ✓ it modifies gravity!
- ✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc
- ✓ Invariant under conformal and disformal transformations

Horndeski…

Second-order scalar-tensor field equations in a four-dimensional space

Gregory Walter Horndeski (Waterloo U.)

Int.J.Theor.Phys. 10 (1974) 363-384



Limits of the Horndeski Lagrangian

- If $G_4 = 1/2$ and $G_5 = 0$ (it is actually sufficient $G_5 = const$) the HL reduces to ordinary gravity with a scalar field having a non-canonical kinetic sector given by $\mathcal{L}_2, \mathcal{L}_3$. The canonical form is obtained for $K = X - V(\phi)$ and $G_3 = 0$ ($G_3 = const$ is sufficient). ΛCDM is recovered for $K = -2\Lambda$.

- The "minimal" form of modified gravity within the HL is provided by $G_4 = G_4(\phi)$ and $G_5 = const$: this is then equivalent to a Brans-Dicke scalar-tensor model, again with a non-canonical kinetic sector.

- The original Brans-Dicke model is recovered assuming a kinetic sector, $K = (\omega_{BD}/\phi)X, G_3 = 0$, and $G_4(\phi) = \phi/2$.

- If the kinetic sector vanishes, $K_{,X} = G_3 = 0$, then we reduce ourselves to a $f(R)$ model [13], whose Lagrangian is $\mathcal{L}_R = (R + f(R))/2$. In fact, this model is equivalent to a scalar-tensor theory with $G_4(\phi) = e^{2\phi/\sqrt{6}}/2$ and a potential $K(\phi) = -(Rf_{,R} - f)/2$ where $\phi = \sqrt{6}/2 \log(1 + f_{,R})$. This relation should then be inverted to get $R = R(\phi)$ and used to replace R with ϕ in $K(\phi)$.

- If one sets $G_i(\phi, X) = G_i(X)$ then the Lagrangian is invariant under the shift $\phi \to \phi + c$ with $c = const$. This shift-symmetric version of the HL is connected to the Covariant Galileon when the functional dependence of the G_i is fixed [5] and is able to produce the accelerated expansion without a potential that makes the field slow roll.

The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

The Great Horndeski Hunt

Let us assume we have only

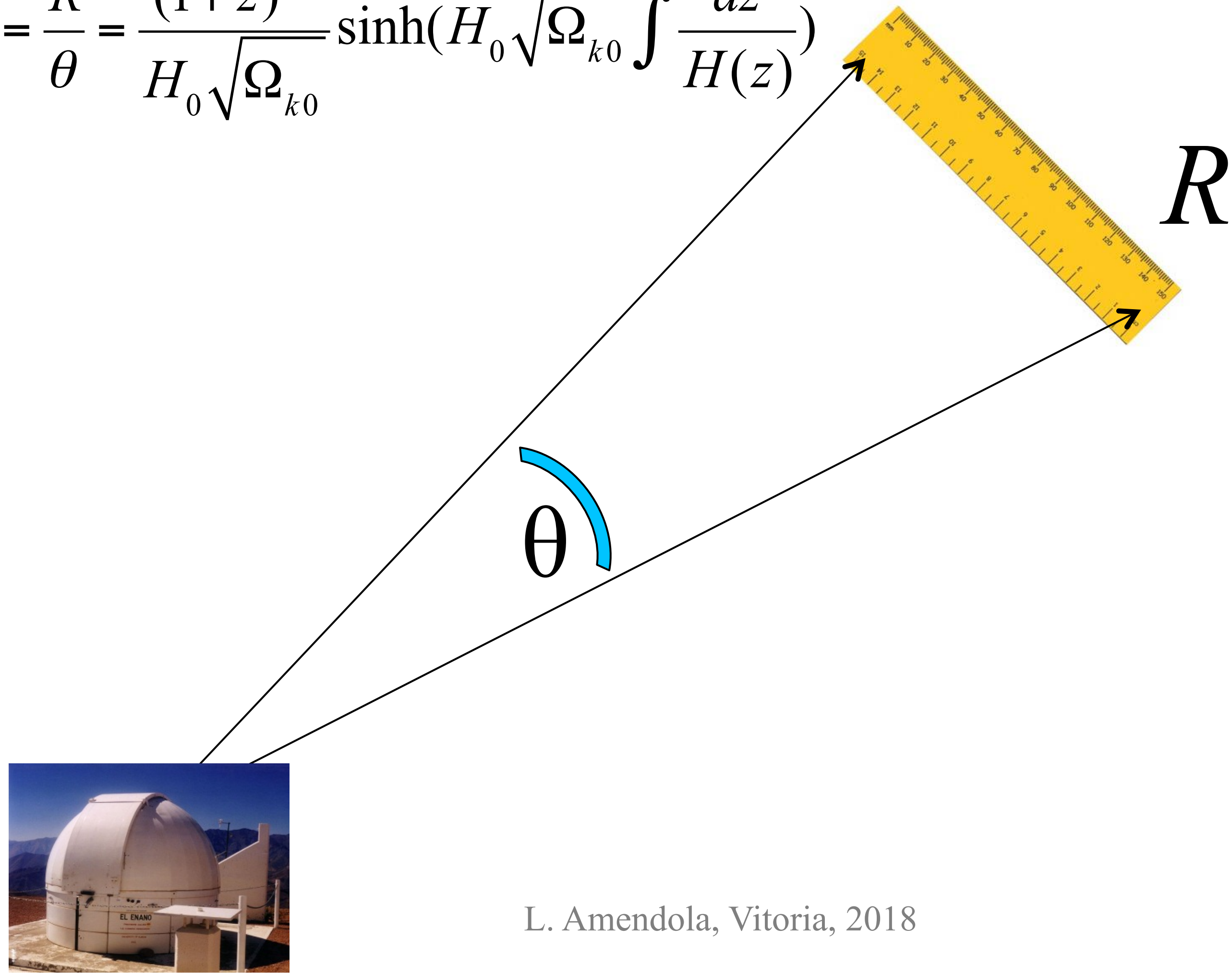
1) a perturbed FRW metric

2) pressureless matter

3) the Horndeski field

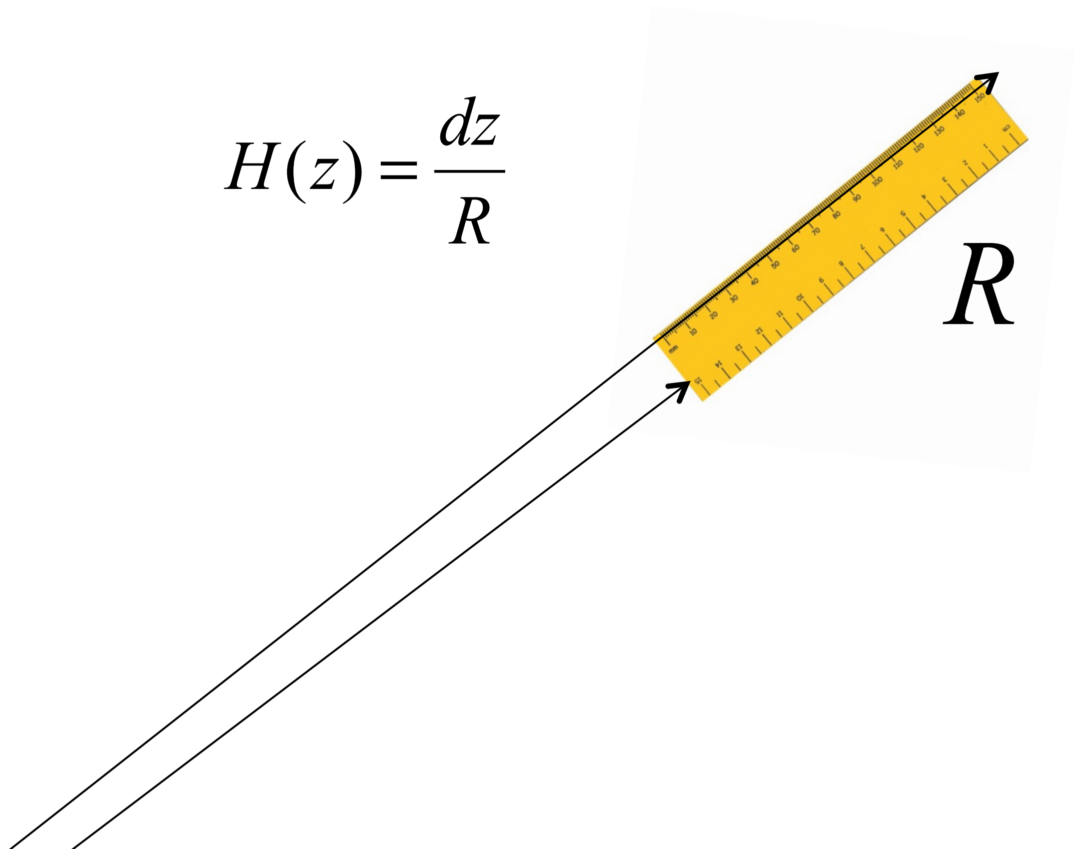
Standard rulers

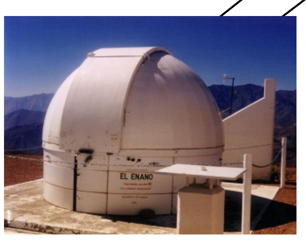
$$D(z)=\frac{R}{\theta}=\frac{(1+z)^{-1}}{H_0\sqrt{\Omega_{k0}}}\sinh\left(H_0\sqrt{\Omega_{k0}}\int\frac{dz}{H(z)}\right)$$



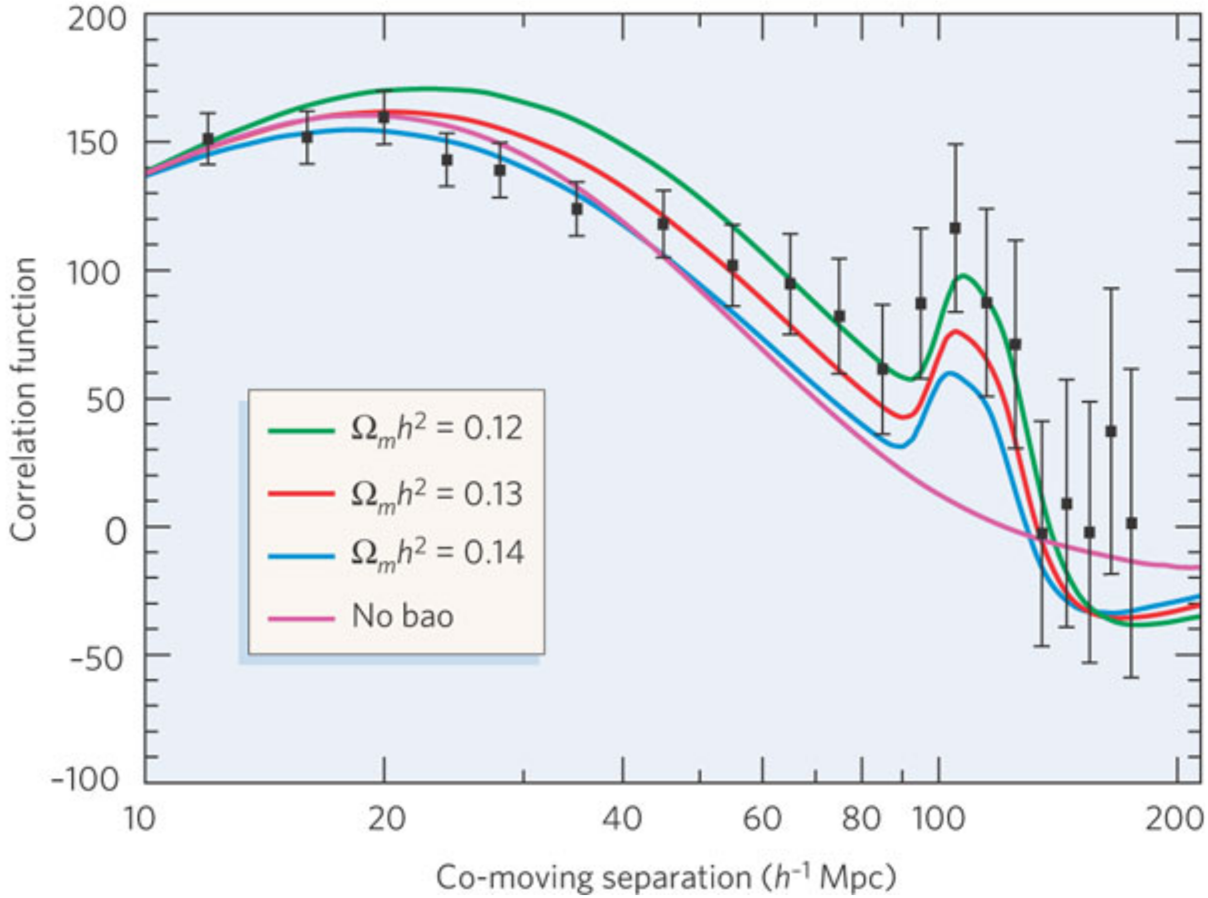
Standard rulers

$$H(z) = \frac{dz}{R}$$





BAO ruler



Charles L. Bennett

Nature 440, 1126-1131(27 April 2006)

Background: SNIa, BAO, …

Then we can measure H(z) and

$$D(z) = \frac{1}{H_0\sqrt{\Omega_{k0}}}\sinh\left(H_0\sqrt{\Omega_{k0}}\int\frac{dz}{H(z)}\right)$$

and therefore we can reconstruct the full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1-\frac{\Omega_{k0}}{4}r^2\right)^2}(dx^2+dy^2+dz^2)$$

Two free functions

The most general linear, scalar metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

- Poisson's equation $$\nabla^2\Psi = 4\pi G\rho_m\delta_m$$
- anisotropic stress $$1 = -\frac{\Psi}{\Phi}$$

Warning: all the perturbation variables in this talk are root mean squares!

Two free functions

The most general parametrization of gravity at linear level

- Poisson's equation $$\nabla^2\Psi = 4\pi G\, Y(k,a)\rho_m\delta_m$$

- anisotropic stress $$\eta(k,a) = -\frac{\Phi}{\Psi}$$

Modified Gravity at the linear level

- standard gravity

$$Y(k,a)=1$$
$$\eta(k,a)=1$$

- scalar-tensor models

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$$
$$\eta(a)=1+\frac{F'^2}{F+F'^2}$$

Boisseau et al. 2000
Acquaviva et al. 2004
Schimd et al. 2004
L.A., Kunz &Sapone 2007

- f(R)

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$$

Bean et al. 2006
Hu et al. 2006
Tsujikawa 2007

- DGP

$$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_c w_{DE}$$
$$\eta(a)=1+\frac{2}{3\beta-1}$$

Lue et al. 2004;
Koyama et al. 2006

- massive bi-gravity

$$Y(a)=....$$
$$\eta(a)=....$$

F. Koennig and L. A. 2014,
Y. Akrami et al. 2014

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2 h_4}{1+k^2 h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2 h_5}{1+k^2 h_3}\right)$$

k = wavenumber

$\boldsymbol{h_i}$ **= time-dependent functions**

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_T^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_T^{-2},$$

$$h_3 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_m)}{2w_1^2},$$

$$h_4 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1(\dot{w}_2 + \rho_m)}{w_1},$$

$$h_5 \equiv \frac{H^2}{2XM^2} \frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_m)}{w_4},$$

$$\begin{aligned}
w_1 &\equiv 1 + 2(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}XHG_{5,X}), \\
w_2 &\equiv -2\dot{\phi}(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}) + 2H(w_1 - 4X(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X})) - 2\dot{\phi}XH^2(3G_{5,X} + 2XG_{5,XX}), \\
w_3 &\equiv 3X(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}) + 18\dot{\phi}XH(2G_{3,X} + XG_{3,XX}) - 18\dot{\phi}H(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX}) \\
&\quad - 18H^2(1/2 + G_4 - 7XG_{4,X} - 16X^2G_{4,XX} - 4X^3G_{4,XXX}) - 18XH^2(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX}) \\
&\quad + 6\dot{\phi}XH^3(15G_{5,X} + 13XG_{5,XX} + 2X^2G_{5,XXX}), \\
w_4 &\equiv 1 + 2(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}).
\end{aligned} \tag{A2}$$

De Felice et al. 2011; L.A. et al.,PRD, arXiv:1210.0439, 2012

Yukawa Potential

$$\eta(k,a) = h_2\left(\frac{1+k^2 h_4}{1+k^2 h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2 h_5}{1+k^2 h_3}\right)$$

Momentum space

$$k^2\Psi = 4\pi G Y(a,k)\rho_m(k)$$

$$k^2\Phi = 4\pi G Y(a,k)\eta(a,k)\rho_m(k)$$

$$\Psi = -\frac{GM}{r}h_2\left(1+\frac{h_4-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\bar{G}M}{r}\left(1+Qe^{-mr}\right)$$

$$\Phi = -\frac{GM}{r}h_1\left(1+\frac{h_3-h_5}{h_5}e^{-r/\sqrt{h_5}}\right) = -\frac{\hat{G}M}{r}\left(1+\hat{Q}e^{-mr}\right)$$

Real space

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

2nd order perturbed Lagrangians

An elegant way to derive the perturbation equations is to
write down the perturbed Lagrangian at second order:
when you differentiate it, you get first order perturbation equations!

For instance standard EH Lagrangian gives in Minkowski

$$ds^2 = -(1+2\Psi)dt^2 + 2B_{,i}dx^i dt + ((1+2\Phi)\delta_{ij} + 2E_{,ij})dx^i dx^j$$

$$S_g = \frac{1}{2}\int d^3x dt[-8B_i\Phi_i' + 4\Phi_i\Psi_i - 4\Phi'\Box E' + 2\Phi_i^2 - 6\left(\Phi'\right)^2]$$

Definition

Propagating degree of freedom: a field with two time derivatives in the 2nd order Lagrangian, **once all the constraints have been taken into account**

Identifying the degrees of freedom

Classifying theories by their dof

Standard gravity: no scalar degree of freedom, 2 tensor dofs
Horndeski: one scalar dof, 2 tensor dofs
Massive grav: one scalar dof, 2+2 tensor dofs
Bimetric models: one scalar dof, 2 vector dofs, 2+2 tensor dofs

2nd order pert. Lagrangian shows explicitly the dofs

For Horndeski:

$$S=\int d^3x dt\{Q_S[\dot{\varphi}^2-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h}_\alpha^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

De Felice & Tsujikawa 2011

Identifying the degrees of freedom

2nd order pert. Lagrangian shows explicitly the dofs

$$S=\int d^3x dt\{Q_S[\dot{\varphi}^2-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

scalar tensor

…and the minimal number of free functions: two for each dof (plus a function for the background, i.e. 5 functions for Horndeski)

The magic of 2nd order perturbed Lagrangians

2nd order pert. Lagrangian also clarifies the stability conditions

$$S = \int d^3x dt \{ Q_S [\dot{\varphi}^2 - \frac{c_s^2}{a^2} (\partial_i \varphi)^2] + \sum_{\alpha=1}^{2} Q_T [\dot{h_\alpha}^2 - \frac{c_T^2}{a^2} (\partial_i h_\alpha)^2] \}$$

Equation of motion in Fourier space $\ddot{\phi} + c_s^2 k^2 \phi = 0$

Positive squared sound speeds $c_{s,T}^2 \geq 0$

Positive kinetic terms $Q_{s,T} \geq 0$

Horndeski Lagrangian

$$\begin{aligned}
\mathcal{L}_2 =& K(\phi, X)\,, \\
\mathcal{L}_3 =& -G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,, \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\,.
\end{aligned}$$

Linearized Horndeski: Bellini-Sawicki parametrization

$$
\begin{aligned}
M_*^2 \equiv & 2\left(G_4 - 2XG_{4X} + XG_{5\phi} - \dot{\phi}HXG_{5X}\right) \\
HM_*^2\alpha_{\rm M} \equiv & (\dot{M_*^2}) \\
H^2M_*^2\alpha_{\rm K} \equiv & 2X\left(K_X + 2XK_{XX} - 2G_{3\phi} - 2XG_{3\phi X}\right) + \\
& + 12\dot{\phi}XH\left(G_{3X} + XG_{3XX} - 3G_{4\phi X} - 2XG_{4\phi XX}\right) + \\
& + 12XH^2\left(G_{4X} + 8XG_{4XX} + 4X^2G_{4XXX}\right) - \\
& - 12XH^2\left(G_{5\phi} + 5XG_{5\phi X} + 2X^2G_{5\phi XX}\right) + \\
& + 4\dot{\phi}XH^3\left(3G_{5X} + 7XG_{5XX} + 2X^2G_{5XXX}\right) \\
HM_*^2\alpha_{\rm B} \equiv & 2\dot{\phi}\left(XG_{3X} - G_{4\phi} - 2XG_{4\phi X}\right) + \\
& + 8XH\left(G_{4X} + 2XG_{4XX} - G_{5\phi} - XG_{5\phi X}\right) + \\
& + 2\dot{\phi}XH^2\left(3G_{5X} + 2XG_{5XX}\right) \\
M_*^2\alpha_{\rm T} \equiv & 2X\left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5X}\right)
\end{aligned}
$$

sound speeds

$$S=\int d^3x dt\{Q_S[\dot{\varphi}^2-\frac{c_s^2}{a^2}(\partial_i\varphi)^2]+\sum_{\alpha=1}^{2}Q_T[\dot{h_\alpha}^2-\frac{c_T^2}{a^2}(\partial_i h_\alpha)^2]\}$$

$$Q_{\rm S}=\frac{M_*^2(2\alpha_K+3\alpha_B^2)}{(2-\alpha_{\rm B})^2},$$

$$c_{\rm S}^2=\frac{(2-\alpha_{\rm B})\,\alpha_1+2\alpha_2}{2\alpha_K+3\alpha_B^2},$$

$$Q_{\rm T}=\frac{M_*^2}{8},$$

$$c_{\rm T}^2=1+\alpha_{\rm T}$$

$$\alpha_1\equiv\alpha_B+(\alpha_B-2)\,\alpha_T+2\alpha_M$$

$$\alpha_2\equiv\alpha_B\xi+\alpha_B'-2\xi-3\tilde{\Omega}_m$$

Bellini-Sawicki parametrization

Model Class		α_K	α_B	α_M	α_T
ΛCDM		***0***	***0***	***0***	***0***
cuscuton ($w_X \neq -1$)	[71]	0	0	0	0
quintessence	[1, 2]	$(1-\Omega_m)(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_m)(1+w_X)}{c_s^2}$	0	0	0
kinetic gravity braiding	[47–49]	$\frac{m^2(n_m+\kappa_\phi)}{H^2 M_{Pl}^2}$	$\frac{m\kappa}{H M_{Pl}^2}$	0	0
galileon cosmology	[57]	$-\frac{3}{2}\alpha_M^3 H^2 r_c^2 e^{2\phi/M}$	$\frac{\alpha_K}{6} - \alpha_M$	$\frac{-2\dot{\phi}}{HM}$	0
BDK	[26]	$\frac{\dot{\phi}^2 K_{,\dot{\phi}\dot{\phi}} e^{-\kappa}}{H^2 M^2}$	$-\alpha_M$	$\frac{\dot{\kappa}}{H}$	0
metric $f(R)$	[3, 72]	0	$-\alpha_M$	$\frac{B\dot{H}}{H^2}$	0
MSG/Palatini $f(R)$	[73, 74]	$-\frac{3}{2}\alpha_M^2$	$-\alpha_M$	$\frac{2\dot{\phi}}{H}$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot{\xi}}{M^2+H\dot{\xi}}$	$\frac{\dot{H}\dot{\xi}+H\ddot{\xi}}{H(M^2+H\dot{\xi})}$	$\frac{\ddot{\xi}-H\dot{\xi}}{M^2+H\dot{\xi}}$

Beyond Horndeski, I

Look now at the matter equations of motion

$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

Standard sub-horizon matter equations

$$\theta = ik_j v^j$$

$$\boxed{\begin{aligned} \dot{\delta} &= -\theta \\ \dot{\theta} &= -\mathcal{H}\theta + k^2\Psi \\ k^2\Phi &= 4\pi G a^2 \rho\delta \end{aligned}}$$

continuity

Euler

Poisson

Modifying gravity

$$\boxed{k^2\Phi = 4\pi G a^2 \rho\delta Y\eta}$$

Modifying continuity equation

$$\boxed{\dot{\delta} = -\theta + \Delta}$$

$$\Delta = 4\alpha'_H \frac{\mathcal{H}M_P^2}{\rho_m a^2} k^2\Phi$$

Gleyzes et al 2014;
Lombriser et al 2015

Beyond Horndeski, II

Or, we can add several Horndeski fields!

$$\int dx^4 \sqrt{-g}\left[L_{H1} + L_{H2} + \ldots + L_{matter}\right]$$

$$Y \equiv A_1 \frac{1 + A_2 k^2 + A_3 k^4}{1 + A_4 k^2 + A_5 k^4},$$

$$\eta \equiv -\frac{\Phi}{\Psi} = B_1 \frac{1 + B_2 k^2 + B_3 k^4}{1 + B_4 k^2 + B_5 k^4},$$

A. Silvestri et al. 2013
T. Baker et al. 2013
V. Vardanyan & L.A., 2015

Beyond Horndeski, III

- Torsion
- Non-metricity
- Non-universal coupling
- Palatini
- Vectors
- Tensors

Beyond Horndeski, IV

Pauli-Fierz (1939) action: the only ghost-free quadratic action for a massive spin two field

$$\int d^4x\sqrt{g}R_g + m^2\int d^4x h_{\mu\nu}h_{\alpha\beta}\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

The three deadly sins of Pauli-Fierz theory:

- It does not reduce to massless gravity for $m \to 0$ (vDVZ disc.)
- It violates diffeomorphism invariance
- It contains a ghost when extended to non-linear level (Boulware-Deser ghost)

Ghost-free Bigravity

- The first problem was solved by Vainshtein (1972): there exists a radius below which the linear theory cannot be applied;
  - For the Sun, this radius is larger than the solar system!

- The second and third problems can be solved introducing a second metric:

$$
\begin{aligned}
S \;=\; & -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f) \\
& + \; m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n(\sqrt{g^{\alpha\beta}f_{\beta\gamma}}) + \int d^4x\sqrt{-\det g}L_m(g,\Phi)
\end{aligned}
$$

The only ghost-free local non-linear massive gravity theory!

deRham, Gabadadze, Tolley 2010
Hassan & Rosen, 2011

Modified Gravity in bimetric gravity

In the quasi-static limit, every Horndeski model
and bimetric gravity model is
characterized at linear scales
by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

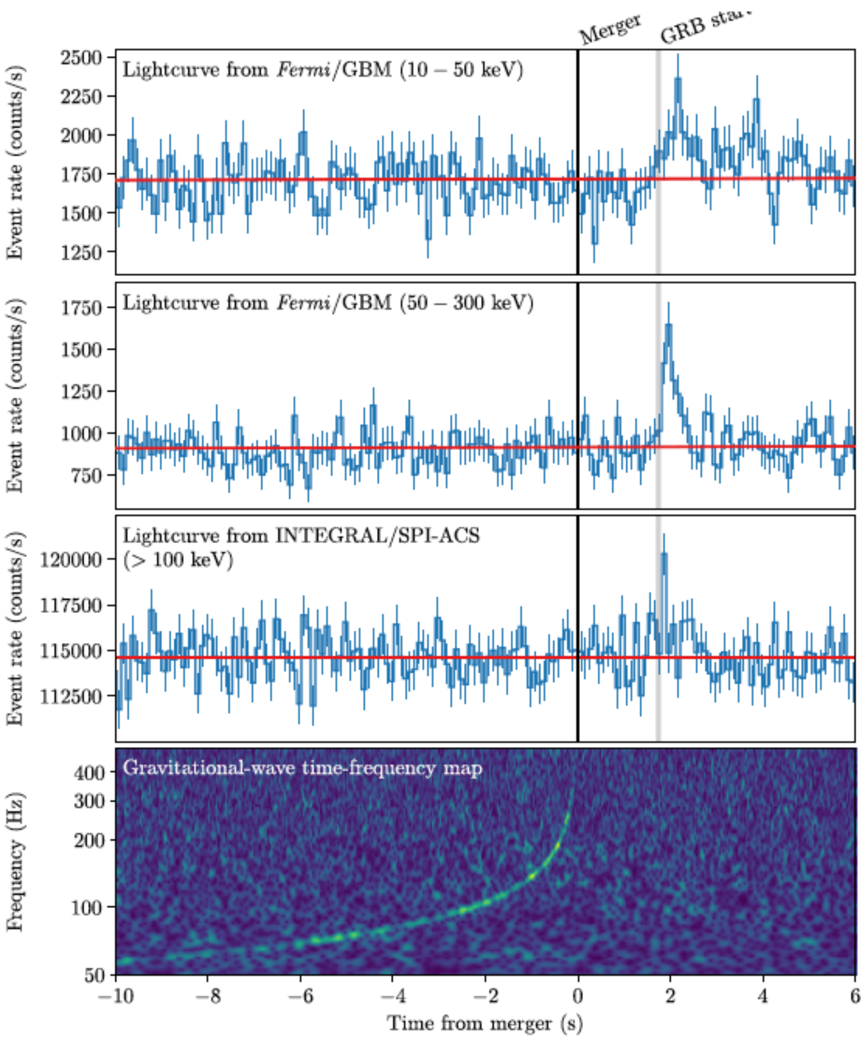
k = wavenumber

$\boldsymbol{h_i}$ **= time-dependent functions**

De Felice et al. 2011; L.A. et al.PRD, arXiv:1210.0439, 2012

The speed of gravitational waves

2017:
GW and gamma-ray simultaneous detection



LIGO, Integral, Fermi coll.

The speed of gravitational waves

$$S = \int d^3x dt \{ Q_S [\dot{\varphi} - \frac{c_s^2}{a^2}(\partial_i \varphi)^2] + \sum_{\alpha=1}^{2} Q_T [\dot{h_\alpha}^2 - \frac{c_T^2}{a^2}(\partial_i h_\alpha)^2] \}$$

$$Q_\mathrm{S} = \frac{M_*^2(2\alpha_K + 3\alpha_B^2)}{(2-\alpha_\mathrm{B})^2},$$

$$c_\mathrm{S}^2 = \frac{(2-\alpha_\mathrm{B})\,\alpha_1 + 2\alpha_2}{2\alpha_K + 3\alpha_B^2},$$

$$Q_\mathrm{T} = \frac{M_*^2}{8},$$

$$c_\mathrm{T}^2 = 1 + \alpha_\mathrm{T}$$

$$\alpha_1 \equiv \alpha_B + (\alpha_B - 2)\,\alpha_T + 2\alpha_M$$

$$\alpha_2 \equiv \alpha_B \xi + \alpha_B' - 2\xi - 3\tilde{\Omega}_m$$

The speed of gravitational waves

$$c^2_T = 1 + \alpha_T$$

$$M_*^2 \alpha_{\mathrm{T}} \equiv 2X\left(2G_{4X} - 2G_{5\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5X}\right)$$

Only way to have $c_T=1$ is

$$G_{4,X} = 0$$

$$G_5 = 0$$

Conformal coupling!

Caveats...

The speed of GW has been measured only at $z = 0$!

Observing η

$$\eta(k,a) = -\frac{\Phi}{\Psi}$$

Deconstructing the galaxy power spectrum

$$P_{galaxy}(k,z,\mu=\cos\theta)=(1+\frac{f(k,z)}{b(k,z)}\cos^2\theta)^2 b^2(k,z)G^2(k,z)P^{matter}_{initial}(k,z_{in})$$

$b(k,z)$

$P^{matter}_{initial}(k,z_{in})$

do not depend on gravity

$G(k,z)$

$f(k,z)$

depend on gravity

Deconstructing the galaxy power spectrum

Line-of-sight decomposition

$$\mu \equiv \cos\theta$$

$$\delta_{galaxy}(k,z,\mu) = G(k,z)\left(1+\frac{f(k,z)}{b(k,z)}\mu^2\right)\ b(k,z)\delta_{initial}^{matter}(k,z_{in})$$

$$\equiv A + R\mu^2$$

$$\delta_{\text{lensing}}(k,z) = -\frac{3}{2}Y(1+\eta)G(k,z)\Omega_m\delta_{initial}^{matter}(k,z_{in}) \equiv L$$

Three linear observables: A, R, L

galaxy clustering

$$A = Gb\delta_{m,0}(k)$$

$$R = Gf\delta_{m,0}(k)$$

weak gravitational lensing

$$L = -\frac{3}{2}Y(1+\eta)G\Omega_m\delta_{m,0}(k)$$

The only model-independent ratios

Redshift distortion/Amplitude $P_1 = \frac{R}{A} = \frac{f}{b}$

Lensing/Redshift distortion $P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y (1+\eta)}{f}$

Redshift distortion rate $P_3 = \frac{R'}{R} = \frac{f'}{f} + f$

Expansion rate $E = \frac{H}{H_0}$

How to combine them to test the theory?

Summarizing....

Matter conservation equation independent of gravity theory

$$\delta''+(1+\frac{H'}{H})\delta'-\frac{3}{2}\Omega_m Y\delta=0$$

Observables

$$P_2=\frac{L}{R}=\frac{\Omega_{m0}Y(1+\eta)}{f} \qquad P_3=\frac{R'}{R}=\frac{f'}{f}+f \qquad E=\frac{H}{H_0}$$

Testing the entire Horndeski Lagrangian

A unique combination of observables

Independent of the bias, of the initial conditions, of the specific model

$$\frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Testing the entire Horndeski Lagrangian

$$\eta_{obs} \equiv \frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})} - 1$$

$\eta_{obs} \neq 1$ → gravity is modified

$\eta_{obs}(k) \neq \eta(Horndeski)$ → The entire Horndeski model is falsified

L.A., M. Motta, I. Sawicki, M. Kunz, I. Saltas, 1210.0439 1305.0008

Can we measure it?

The first measurement ever

A compilation of all available datasets of lensing and growth

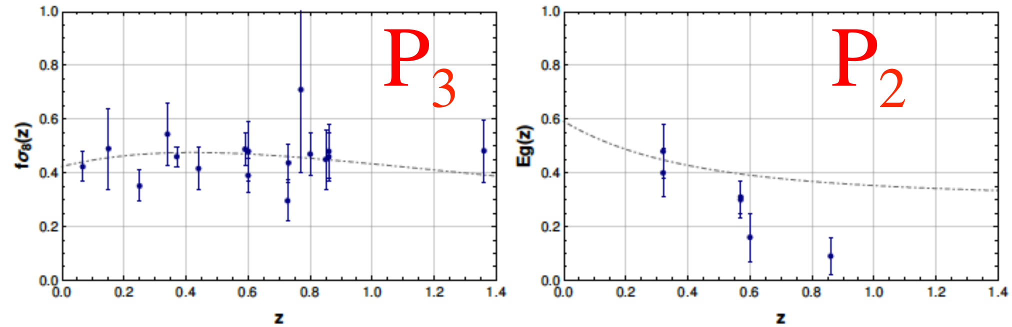


Figure 1. Left: Plot of the full $f\sigma_8$ data (table II) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full E_g data (table III) as a function of z with the theoretical curve in dashed dot.

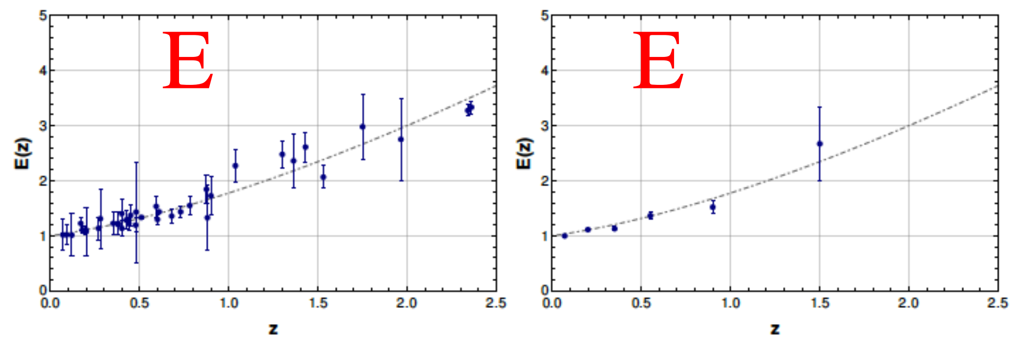
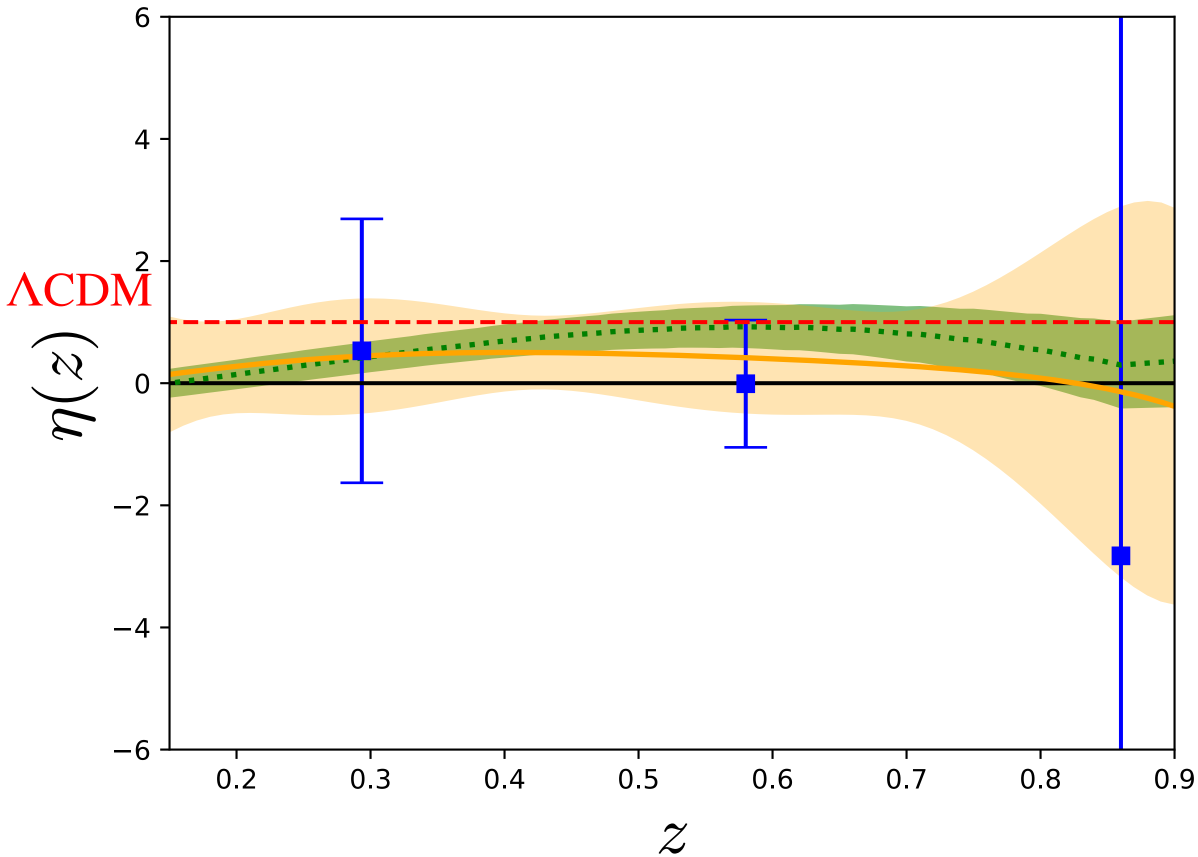


Figure 2. Right: Plot of the full $H(z)$ data (table IV) as a function of z with the theoretical curve in dashed dot. Right: Plot of the full $E(z)$ data (table II) as a function of z with the theoretical curve in dashed dot.

The first model-independent measurement ever

Result compressed
in a single central bin

$$\eta = 0.40 \pm 0.60$$



Collab. with Santiago Casas
and Ana M. Pinho
arXiv:1805.00025

Results...

$$\eta(k,a) = H_2\left(\frac{1+k^2H_4}{1+k^2H_5}\right)$$

Model 1: η constant for all z, k
Error on η around 2%

η=1/2 for f(R)!
(at small scales)

Model 2: η varies in z
Error on η

TABLE X. Fiducial values and errors for the parameters P_1, P_2, P_3, E'/E and $\bar{\eta}$ for every bin. The last bin has been omitted since R' is not defined there.

$\bar{z}$	P_1	ΔP_1	$\Delta P_1(\%)$	P_2	ΔP_2	$\Delta P_2(\%)$	P_3	ΔP_3	$\Delta P_3(\%)$	(E'/E)	$\Delta E'/E$	$\Delta E'/E(\%)$	$\bar{\eta}$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
0.6	0.766	0.012	1.6	0.729	0.013	1.8	0.134	0.13	99	-0.920	0.022	2.4	1	0.11	11
0.8	0.819	0.010	1.2	0.682	0.011	1.6	0.317	0.12	38	-1.04	0.046	4.4	1	0.091	9.1
1.0	0.859	0.0093	1.1	0.650	0.011	1.7	0.460	0.12	26	-1.13	0.099	8.7	1	0.090	9.0
1.2	0.888	0.0092	1.0	0.628	0.014	2.3	0.569	0.13	23	-1.21	0.12	10	1	0.097	9.7
1.4	0.911	0.010	1.1	0.613	0.020	3.3	0.654	0.11	16	-1.26	0.09	7.1	1	0.073	7.3

L.A, M. Kunz, A. Vollmer,
A. Guarnizo, S. Fogli, 1311.4765, 2013

Conclusions

- Euclid will combine weak lensing, galaxy clustering, galaxy clusters, to determine the cosmic expansion and the growth of fluctuations
- There is a way to systematically classify all models of gravity plus a single scalar field, the Horndeski Lagrangian
- The GW speed already kills a sector of this mod. grav. theory
- The surviving freedom can be constrained in a model-independent way