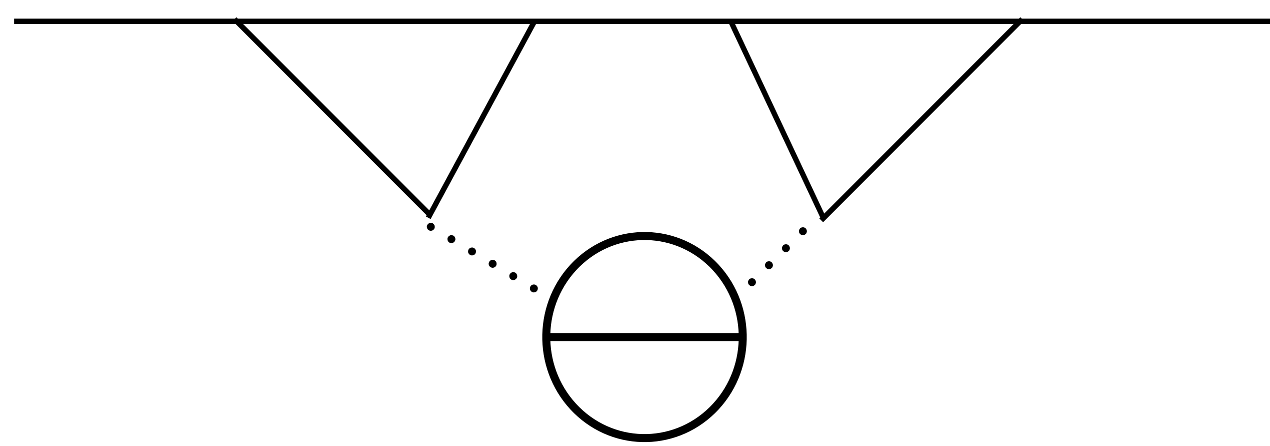
The Cosmological Bootstrap



Centro Brasileiro de Pesquisas Físicas

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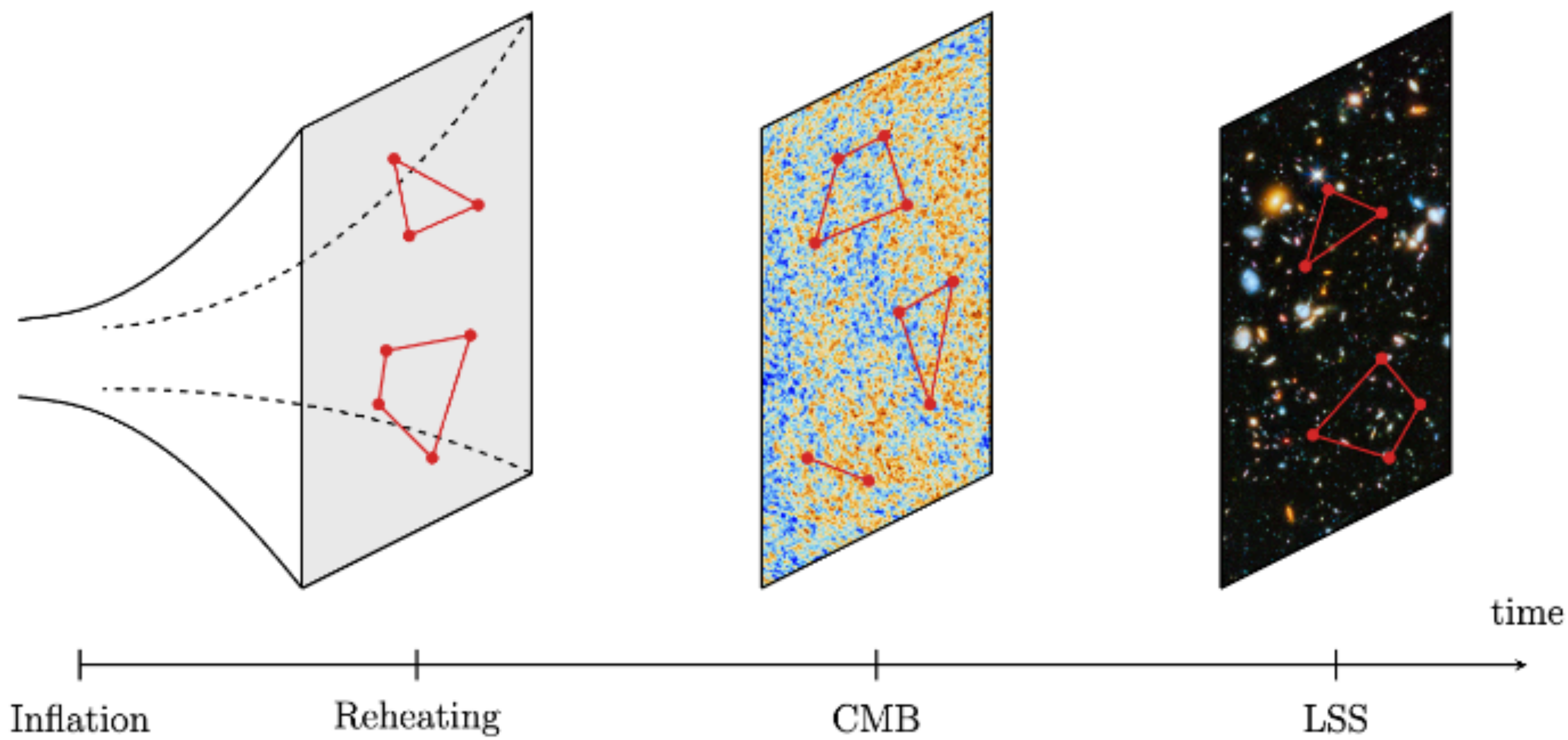
08/04/2025

Cosmological Bootstrap



Non trivial deep UV physics –> Particle creation

Cosmological Bootstrap

- Primordial Non-Gaussianity

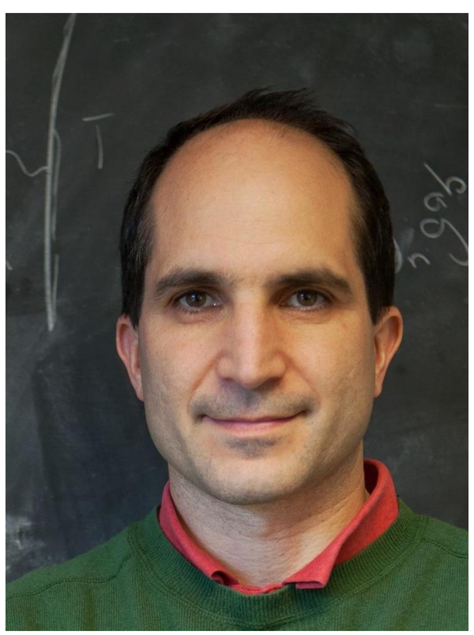
$$B(k,p,q)_\sigma \equiv \langle \zeta_{\vec{k}} \zeta_{\vec{p}} \zeta_{\vec{q}} \rangle'$$

- Bispectrum

$$B_\zeta(k_1,k_2,k_3) \approx \frac{18}{5} f_{NL} \times S\left(\frac{k_2}{k_1},\frac{k_3}{k_1}\right) \times P_\zeta^2$$

Cosmological Bootstrap

“ The Big Bang is the poor man's particle accelerator.”
- Y. Zeldovich



“The Hubble scale, H, during inflation could be as high as 10^14 GeV. This is much larger than any energy we can expect to achieve with particle accelerators in the foreseeable future. It is therefore interesting to understand how to extract information about the laws of physics at those scales. We can view inflation as a particle accelerator.” - Maldacena & Arkani-Hammed (2015)

Cosmological Bootstrap

- Planck 2018 $f_{NL} \sim O(10)$

- SPHEREx $f_{NL} \sim O(1)$

- 21cm $f_{NL} \sim O(0.01)$

Non trivial deep UV physics —> Particle creation

Cosmological Bootstrap

- **What is the Cosmological Bootstrap?!**
- Cosmological Correlators from symmetries
- Should (in principle) work for any cosmology

- **What was done??!!**
- Inflationary Correlators for (quasi-)dS ST.
- Three and four - point functions.

Cosmological Bootstrap

- How is done?!

- Inflationary Correlators for (quasi-)dS ST
- Two, three and four - point functions.

Seeds
+
Symmetries of ST and Fields

-Skip all the perturbative computations

-Solve hard time integrals

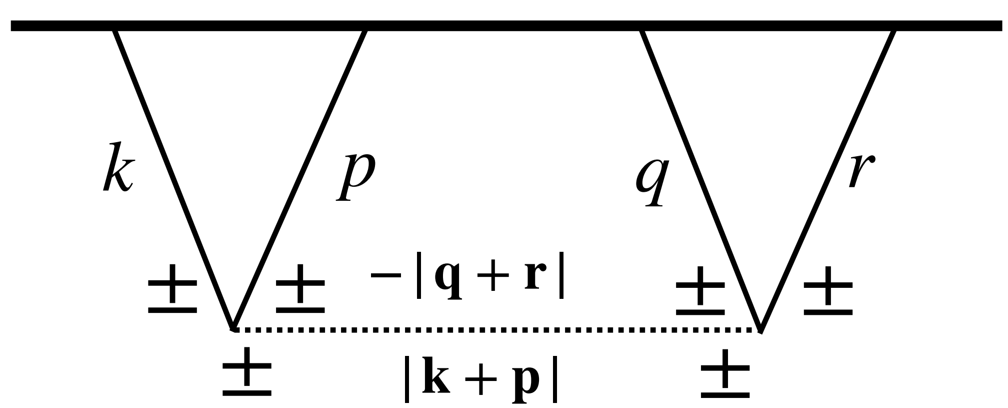
Correlation Functions

Cosmological Bootstrap

- Seed
- Inflationary Correlators for dS ST
- four - point function.

In - In

$$\mathscr{L} \supset \varphi^2 \sigma$$



S - Matrix Ansatz

<u>Conformal Limit of Inflation</u>

$$B_{\zeta\zeta\zeta} \sim \left(\frac{H^2}{4\epsilon M_{\mathrm{Pl}}^2}\right)^2 \frac{1}{e_3^3}\left(\frac{\dot{\eta}}{2H}\log(-k_T\tau_*)\right.$$

$$+(3e_3 + C_1 k_T e_2 + C_3 k_T^3)$$

$$\left. + D_0 e_3 + D_1 k_T e_2 + D_3 k_T^3 \right)$$

Cosmological Bootstrap

- Ward Identities

$$\left[-\sum_{r=1}^{n-1}\left(p_{r\mu}\frac{\partial}{\partial p_{r\mu}}+d\right)+\sum_{r=1}^{n}\Delta_r\right]\langle\mathcal{O}_1^{i_1}(p_1)\ldots\mathcal{O}_r^{i_r}(p_r)\ldots\mathcal{O}_n^{i_n}(p_n)\rangle=0,$$

- Integral representation

$$\langle\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\rangle=c_{123}\prod_i k_i^{\Delta_i-\frac{d}{2}}\int_0^\infty dx x^{\frac{d}{2}-1}\prod_i K_{\Delta_i-\frac{d}{2}}(k_i x),$$

Cosmological Bootstrap

- Integrating

$$\langle \varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\rangle = C_{local}\frac{\left(k_1^3+k_2^3+k_3^3\right)}{k_1^3k_2^3k_3^3}$$

$$+C_{conf}\frac{\left(-\sum_{i\neq j}^{3}k_i^2k_j+k_1k_2k_3-\left(\sum_i k_i^3\right)\left(\log\left(\sum_i k_i/k*\right)\right)\right)}{k_1^3k_2^3k_3^3}.$$

- Squeeze Limit

$$\lim_{k_1\ll k_2,k_3}\langle\varphi(k_1,\eta)\varphi(k_2,\eta)\varphi(k_3,\eta)\rangle \approx \frac{V'''}{6}\frac{[-2+\gamma_E+\ln(-2k_s\tau)]}{k_s^3k_l^3}$$

Cosmological Bootstrap

- Bispectrum up to some constants

$$\langle \varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\rangle = \frac{H^2}{k_1^2k_2^2k_3^2}\frac{V'''}{12}\Bigg[-\sum_{i\neq j}^{3} k_i^2 k_j + k_1k_2k_3$$

$$-\sum_i k_i^3\left(\log(\sum_i k_i/k^*) + \gamma - 1\right)\Bigg]$$

- Flat gauge

$$\zeta = \varphi/\sqrt{2\epsilon}$$

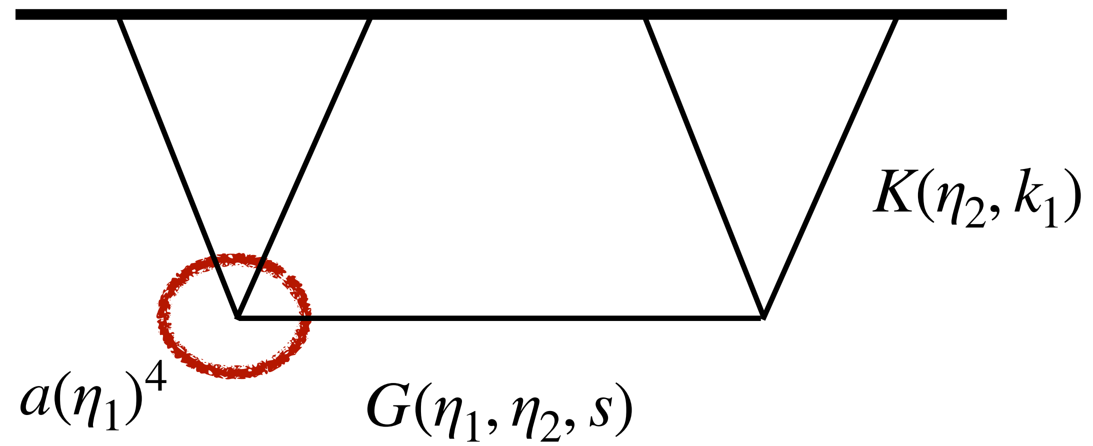
Cosmological Bootstrap

- Seed
- Inflationary Correlators for dS ST
- four - point function.

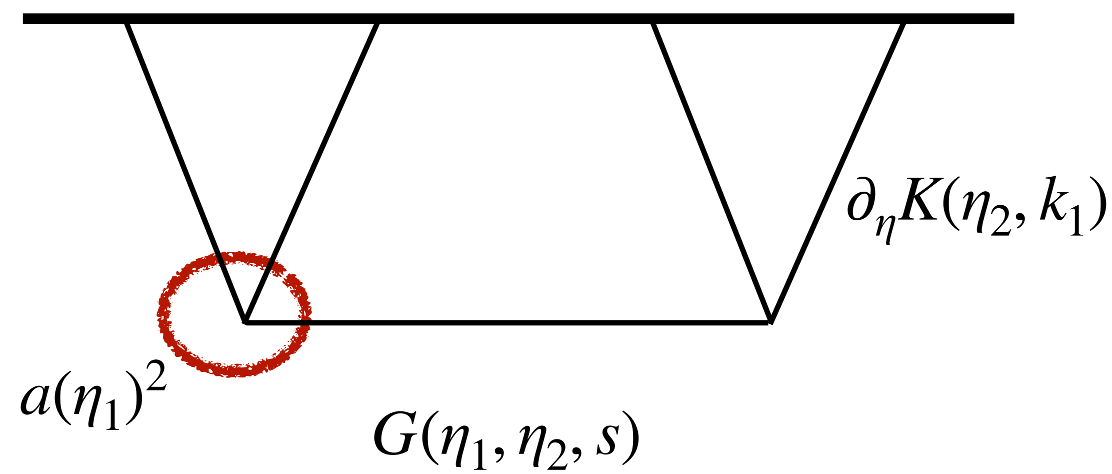
In - In	S - Matrix Ansatz
1811.00024 - Arkani Hamed et al.	1609.06993 - Pajer & Pimentel
2205.00013 - Pimentel & Wang	2010.12818 - Pajer
1910.14051 - Baumann et al.	2010.12818 - Pajer
2312.09642 - Aoki et al.	2106.15468 - Bonifacio
2309.05244 - Yuan Yin	

Cosmological Bootstrap

$\lambda \varphi^2 \sigma \rightarrow$



$\lambda (\partial_\eta \varphi)^2 \sigma \rightarrow$

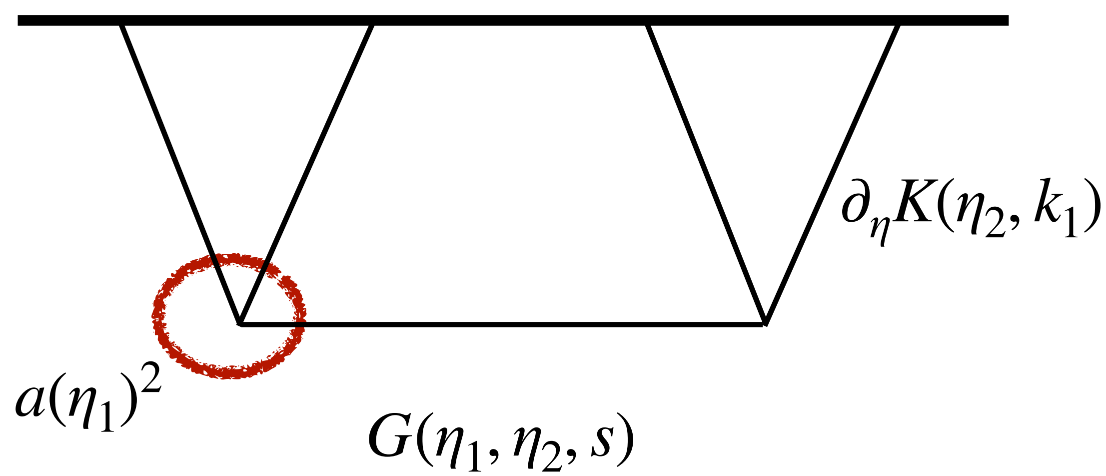


$\partial_\eta \sim a(\eta)^{-1}$

$G_{\pm\pm}(\eta_1, \eta_2, s) = \sigma_\pm(\eta_1, s)\sigma^*_\pm(\eta_2, s)$ $\qquad$ $K_\pm(\eta, , k) = \varphi_\pm(\eta, k)\varphi^*(\eta_0, k)$

Cosmological Bootstrap

$$\lambda(\partial_\eta \varphi)^2 \sigma \to$$



$$\partial_\eta \sim a(\eta)^{-1}$$

$$K_\pm(\eta, , k) = \varphi_\pm(\eta, k)\varphi^*(\eta_0, k)$$

$$G_{\pm\pm}(\eta_1, \eta_2, s) = \sigma_\pm(\eta_1, s)\sigma^*_\pm(\eta_2, s)$$

$$F_{+-}(k, p, q, r)'_\sigma = (-ig)^2 \sum_{a,b=\pm} ab \int_{-\infty}^{0} d\eta_1 a(\eta_1)^2 \partial_{\eta_1} K_+(\eta_1, k) \partial_{\eta_1} K_+(\eta_1, q)$$
$$\times \int_{-\infty}^{0} d\eta_2 a(\eta_2)^2 \partial_{\eta_2} K_-(\eta_2, q) \partial_{\eta_2} K_-(\eta_2, r) \; G_{+-}(s, \eta_1, \eta_2)$$

Cosmological Bootstrap

- Seed
- Inflationary Correlators for dS ST
- four - point function

$$F_{ab}(k,p,q,r)'_\sigma = \frac{-g^2}{4kpqr}\sum_{a,b=\pm} ab \int_{-\infty}^{0} d\eta_1 e^{ai(k+p)\eta_1} \int_{-\infty}^{0} d\eta_2 e^{bi(q+r)\eta_2}\, G(s,\eta_1,\eta_2)$$

- general seed

$$\hat{\mathcal{I}}^{l_1,l_2}_{ab} \equiv -ab\ s^{5+l_1+l_2} \int_{-\infty}^{0} d\eta_1 d\eta_2 (-\eta_1)^{l_1}(-\eta_2)^{l_2} e^{ai(k+p)\eta_1 + bi(q+r)\eta_2}\ \hat{G}_{ab}(s,\eta_1,\eta_2)$$

2301.07047 - Qin & Xianyu

Cosmological Bootstrap

- Seed

$$\hat{\mathcal{I}}^{l_1,l_2}_{ab} \equiv -ab\ s^{5+l_1+l_2}\int_{-\infty}^{0} d\eta_1 d\eta_2 (-\eta_1)^{l_1}(-\eta_2)^{l_2} e^{ai(k+p)\eta_1+bi(q+r)\eta_2}\ \hat{G}_{ab}(s,\eta_1,\eta_2)$$

- Bulk EoM

$$\left(\eta_1^2\partial_{\eta_1}^2 - 2\eta_1\partial_{\eta_1} + s^2\eta_1^2 - \frac{m^2}{H^2}\right)G_{\pm\pm}(s,\eta_1,\eta_2) = \mp\, iH^2\eta_1^2\eta_2^2\delta(\eta_1-\eta_2),$$

- Kinematical Variables

$$s = |\vec{k}+\vec{p}\,| = -\,|\vec{q}+\vec{r}\,|, \quad u = \frac{s}{k+p}, \quad v = \frac{s}{q+r}\,.$$

Cosmological Bootstrap

- Bootstrap Eqs

$$\left[(1-u^2)u^2\partial_u^2 + 4(1-u^2)u\partial_u + m^2/H^2\right]\hat{\mathcal{I}}(u,v) = \frac{uv}{u+v}$$

- Homogeneous Solution

$$\hat{\mathcal{I}}(u,v)_{\pm\mp} = \sum_{a,b=\pm} \alpha_{\pm\mp|ab} I_a^{l_1}(u) I_b^{l_2}(v)$$

$$I_\pm(u) = \left(\frac{iu}{2\mu}\right)^{\frac{1}{2}\pm i\mu} {}_2F_1\left[\frac{1}{4}\pm\frac{i\mu}{2}, \frac{3}{4}\pm\frac{i\mu}{2}, 1\pm i\mu; u^2\right],$$

$$l_1 = l_2 = -2$$

Cosmological Bootstrap

- Squeezed Limit

$$\lim_{s\to 0}\hat{\mathscr{I}}^{l_{1,2}}_{\pm\mp}(u,v) = \frac{e^{\mp i(l_1+l_2)\pi/2}}{4\pi}\left[\tilde{I}^{l_1}_{+}(u)+\tilde{I}^{l_1}_{-}(u)\right]\left[\tilde{I}^{l_2}_{+}(v)+\tilde{I}^{l_2}_{-}(v)\right]$$

$$\tilde{I}^{l}_{\pm}(u) = 2^{\mp i\mu}\left(\frac{u}{2}\right)^{5/2+l\pm i\mu}\Gamma\left[5/2+l+i\mu,-i\mu\right]$$

- Fixing coefficients

$$\hat{\mathscr{I}}^{l_{1,2}}_{\pm\mp}(u,v) = \frac{e^{\mp i(l_1+l_2)\pi/2}}{4\pi}\left[I^{l_1}_{+}(u)+I^{l_1}_{-}(u)\right]\left[I^{l_2}_{+}(v)+I^{l_2}_{-}(v)\right]$$

Cosmological Bootstrap

- Inhomogeneous Solution

$$\hat{\mathcal{I}}(u,v)_{\pm\pm} = \mathcal{H}^{l_1,l_2}_{\pm\pm}(u,v) + \sum_{a,b=\pm} \alpha_{\pm\pm|ab} I^{l_1}_a(u) I^{l_2}_b(v)$$

- Taylor series ansatz

$$\mathcal{H}^{l_1,l_2}_{\pm\pm}(u,1) = 2^{-5-(l_1+l_2)} e^{\mp i(l_1+l_2)\pi/2}\Gamma(5+l_1+l_2)\sum_{n=0}^{\infty} c_n u^{5+l_1+l_2+n}$$

- Source term

$$D^{l_1}_u \mathcal{H}^{l_1,l_2}_{\pm\pm}(u,1) = e^{\mp i(l_1+l_2)\pi/2}\Gamma(5+l_1+l_2)\left(\frac{u}{2}\right)^{5+l_1+l_2}$$

Cosmological Bootstrap

- From dS to inflation

$$\left[-\sum_{r=1}^{n-1}\left(p_{r\mu}\frac{\partial}{\partial p_{r\mu}}+d\right)+\sum_{r=1}^{n}\Delta_r\right]\langle\mathcal{O}_1^{i_1}(p_1)\ldots\mathcal{O}_r^{i_r}(p_r)\ldots\mathcal{O}_n^{i_n}(p_n)\rangle=0,$$

$$\sum_{r=1}^{n-1}\left(p_{r\mu}\frac{\partial^2}{\partial p_r^{\nu}\partial p_{r\nu}}-2p_{r\nu}\frac{\partial^2}{\partial p_r^{\mu}\partial p_{r\nu}}+2(\Delta_r-d)\frac{\partial}{\partial p_r^{\mu}}\right)\langle\mathcal{O}_1^{i_1}(p_1)\ldots\mathcal{O}_r^{i_r}(p_r)\ldots\mathcal{O}_n^{i_n}(p_n)\rangle=0.$$

Consistency relations

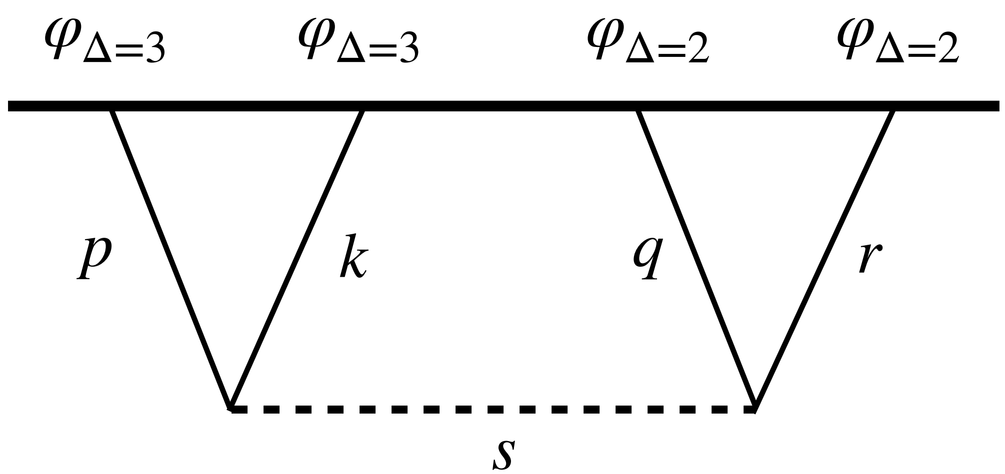
1503.08043 - P. Creminelli, J. Noreña, M. Simonović

1811.00024 - Creminelli et al.

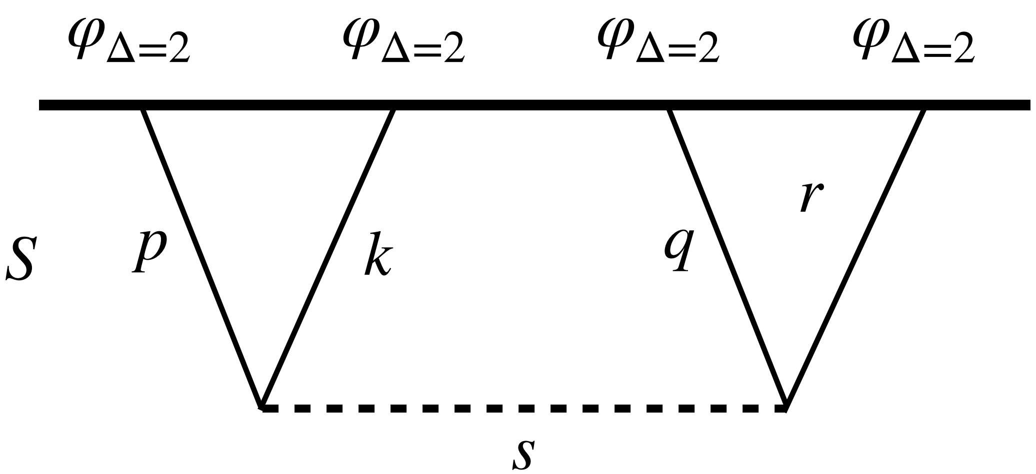
1304.5527 - K. Hinterbichler, L. Hui, J. Khoury

Cosmological Bootstrap

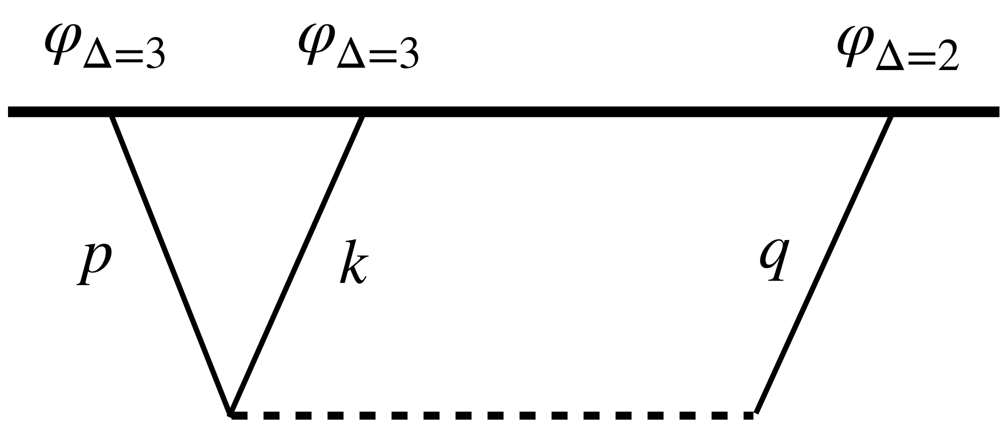
- From dS to inflation



$= \quad W\,S$



$G^{(S)}(s, \eta_1, \eta_2)$ $\qquad$ $G(s, \eta_1, \eta_2)$



$\hat{\mathcal{J}}^{l_1,l_2}_{\pm\pm}(u,1) \sim$ **Soft Limit** $r \to 0$

Cosmological Bootstrap

- From dS to inflation

$$\int (\partial_a \zeta)^2 \sigma = \int \frac{d\eta}{\eta^2}\left[-(\partial_\eta \zeta)^2 + (\nabla \zeta)^2\right]\sigma = \int \frac{d\eta}{\eta^2}\left[-k^2 p^2 \eta^2 + \vec{k}\cdot\vec{p}\left(1 - ik\eta\right)\left(1 - ip\eta\right)\right] e^{i(p+k)\eta}\sigma$$

$$\int (\partial_a \zeta)^2 \sigma = W_{12} \int \frac{d\eta}{\eta^2} e^{i(p+k)\eta}\sigma$$

$$W_{12} \equiv -\frac{1}{2}kp[r^2 - (k+p)^2]\partial^2_{p+k} + \frac{1}{2}(p^2 + k^2 + r^2)[1 - (k+p)\partial_{k+p}]$$

15'- 19'

1503.08043 - Nima & Maldacena

1811.00024 - Arkani Hamed et al.

1910.14051 - Baumann et al.

Cosmological Bootstrap

- From dS to inflation

$$\hat{\mathcal{I}}^{l_1,l_2}_{ab} \equiv -ab\ s^{5+l_1+l_2}\int_{-\infty}^{0} d\eta_1 d\eta_2 (-\eta_1)^{l_1}(-\eta_2)^{l_2} e^{ai(k+p)\eta_1+bi(q+r)\eta_2}\ \hat{G}_{ab}(s,\eta_1,\eta_2)$$

$$\langle \varphi_k\varphi_p\varphi_q\varphi_r\rangle = \frac{H^4}{4kpqrs^5}\sum_{a,b=\pm}\left[\mathcal{I}^{0,0}_{ab} + \textbf{5 perm}\right].$$

$$\langle \varphi_k\varphi_p\varphi_q\rangle = -\frac{H}{4kpq^4}\lim_{r\to 0}\sum_{a,b=\pm}\left[\mathcal{I}^{0,-2}_{ab} + \textbf{2 perm}\right].$$

$$\langle \varphi_k\varphi_q\rangle = \frac{1}{4k^3H^2}\lim_{q,r\to 0}\sum_{a,b=\pm}\left[\mathcal{I}^{-2,-2}_{ab}\right].$$

2301.07047 - Qin & Xianyu

2312.09642 - Aoki et al.

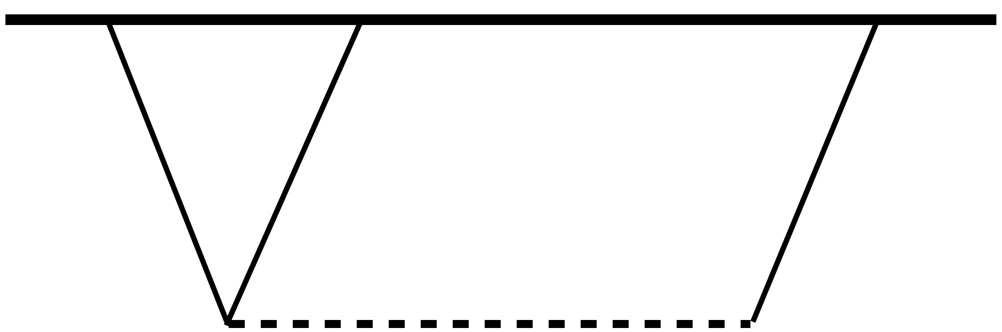
2309.05244 - Yuan Yin

24'

Cosmological Bootstrap

- Cosmic inflation

$\hat{\mathcal{I}}^{l_1,l_2}_{\pm\pm}(u,1) \sim$ **Soft Limit** $r \to 0$



$G(\eta_1, \eta_2, s)$

$\zeta = \varphi/\sqrt{2\epsilon}m_{pl} \qquad B_{ab}(k,p,q)_\sigma \equiv \langle \zeta_{\vec{k}}\zeta_{\vec{p}}\zeta_{\vec{q}}\rangle'$

$$B_{ab}(k,p,q)_\sigma \equiv (2\pi)^4 \frac{P_\zeta^2}{(kpq)^2} S$$

$$B_{ab}(k,p,q)'_\sigma = -(2\epsilon M_{pl}^2)^{-\frac{3}{2}} \frac{2H}{kpq^4} \lim_{r\to 0} \sum_{a,b=\pm} \left[\mathcal{I}^{0,-2}_{ab} + \textbf{2 perm} \right].$$

$$S = \frac{-1}{(2\pi)^4} \frac{1}{P_\zeta^2} (2\epsilon M_{pl}^2)^{-\frac{3}{2}} \frac{2H}{kpq^4} \lim_{r\to 0} \sum_{a,b=\pm} \left[\frac{kp}{q^2} \mathcal{I}^{0,-2}_{ab}\big(u,1\big) + \textbf{2 perm} \right].$$

2312.09642 - Aoki et al

Cosmological Bootstrap

- Beyond deSitter?
- Bootstrap Eqs?

- EoM deformations (time dependent mass)
- Interactions (Boostless, new seed definition)
- Modifying the vacuum (non-Bunch Davies)
- New matter content (Gravitons, Photons…)

2312.09642 - Aoki et al

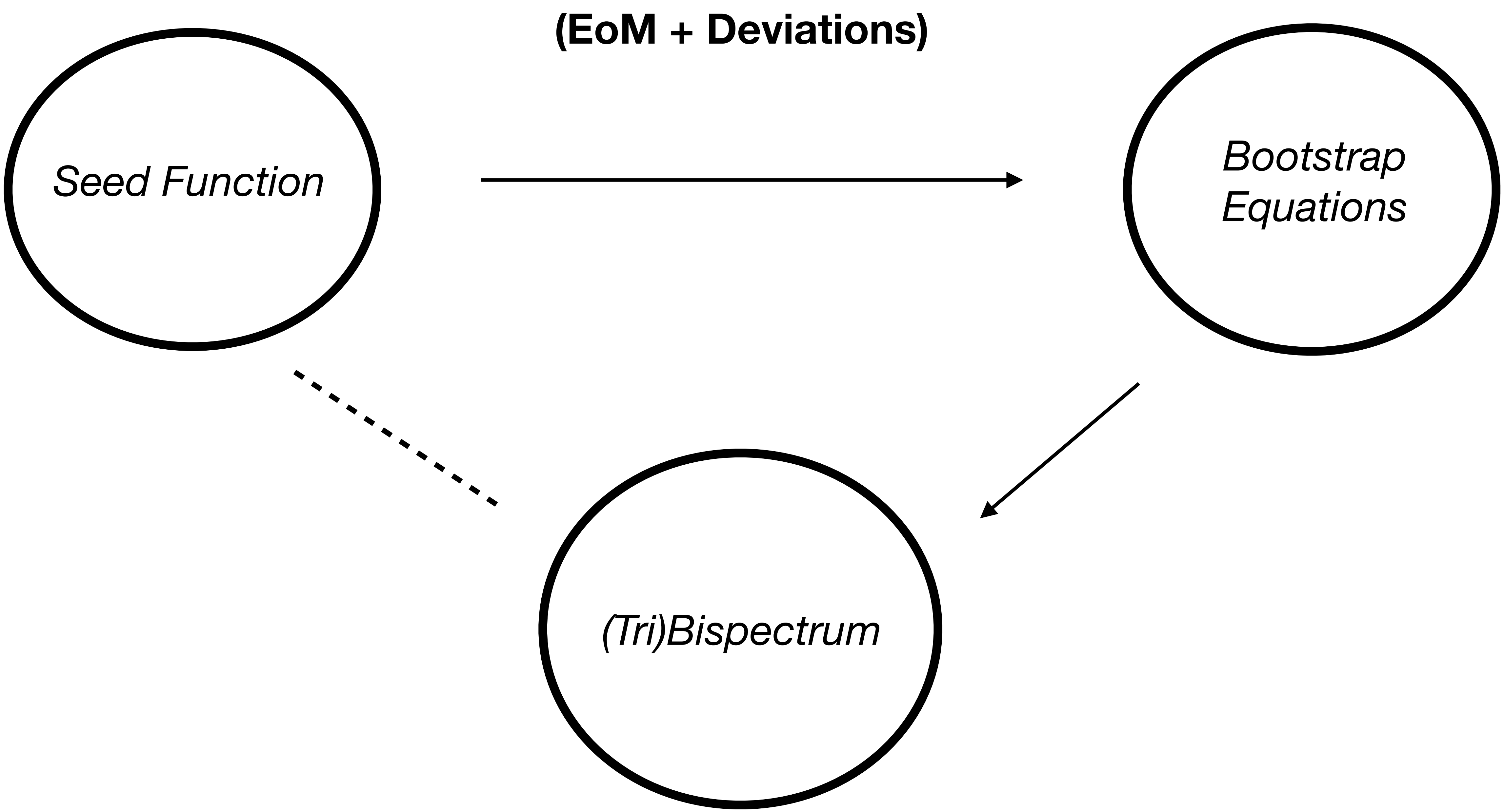
2205.00013 - Pimentel & Wang

2309.05244 - Yuan Yin

1811.00024 - Arkani Hamed et al.

Cosmological Bootstrap

- Procedure



Cosmological Bootstrap

- Projects

$$\mathscr{L} \supset -\frac{\alpha^2}{2M^2}(\nabla^2\pi)^2$$

Ghost Inflation

$$\bar{H} \equiv \frac{1}{3}\left(2H_a + H_c\right)$$

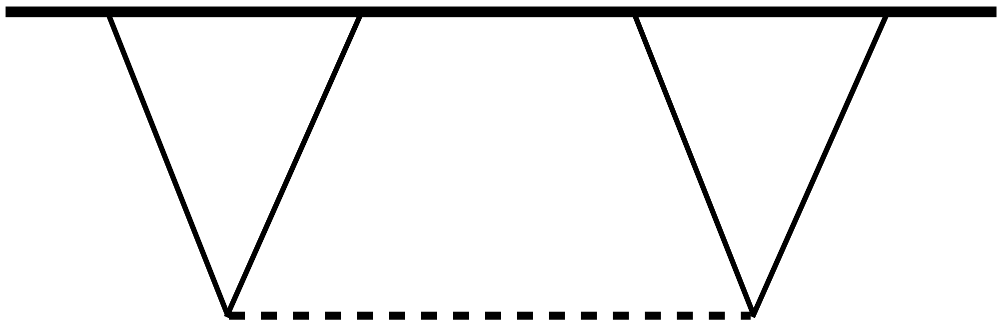
ACW Inflation

$$\mathscr{L} \supset \alpha R^2$$

Starobinsky Inflation

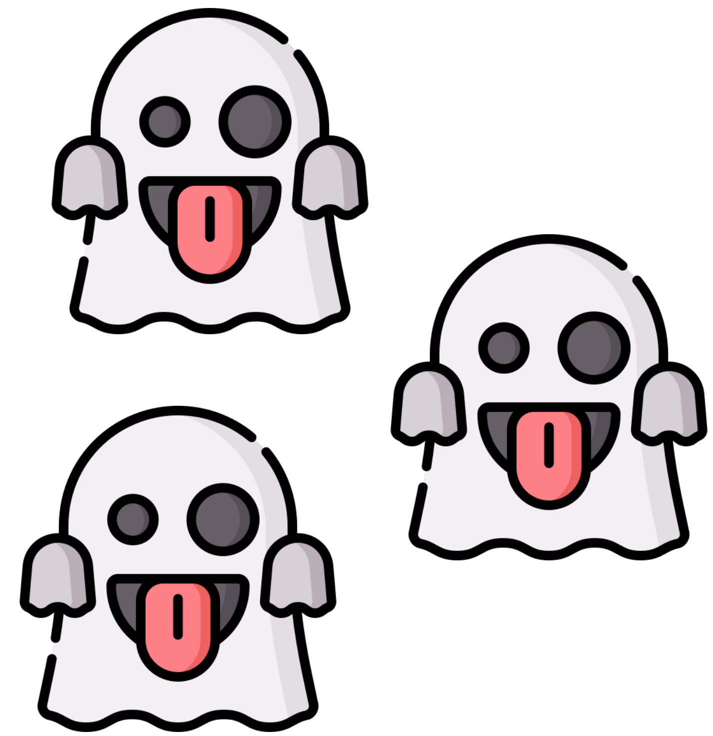
Bootstrap

- What about ghosts?!

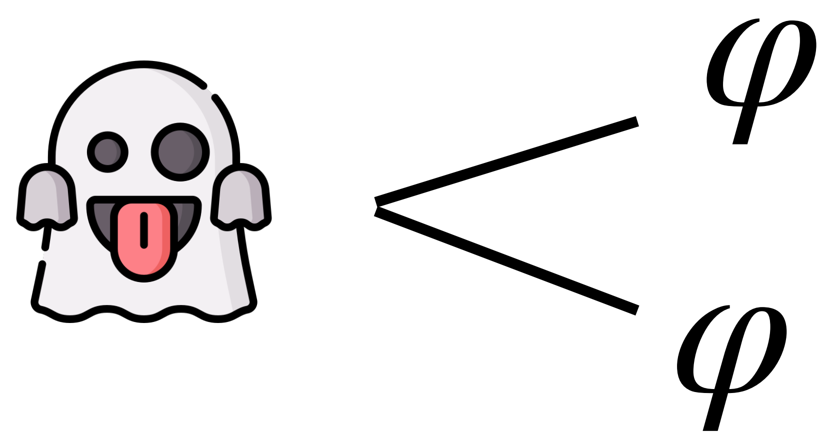


$G(\eta_1, \eta_2, s)$

$$G(\eta_1, \eta_2, s)\Bigg\{$$

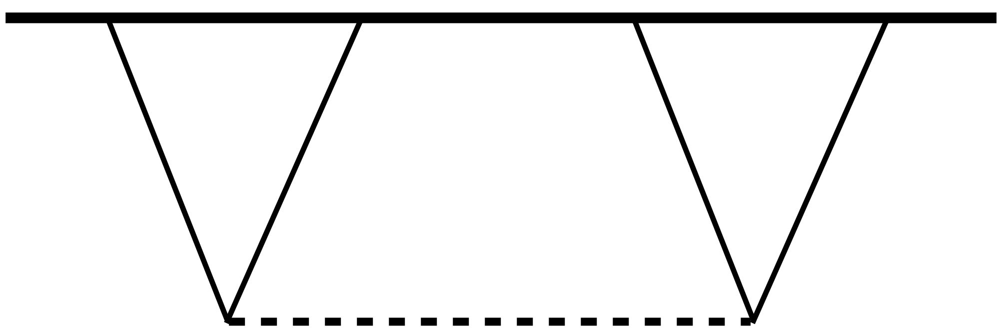


- Ghost inflation revisited!
    - Scale invariant
    - No tilt of the spectral index
    - No gravitational waves
    - UV completion



Bootstrap Ghost Collider

- Ghosts Themselves



$G(\eta_1, \eta_2, s)$

- Quantum gravitational relics?!

$$\mathscr{L} \supset -\frac{\alpha^2}{2M^2}(\nabla^2 \pi)^2$$

- Ghost EOM

$$\left[\eta^2 \partial_\eta^2 - 2\eta\partial_\eta + \boxed{\gamma^2 k^4 \eta^4} + \frac{m^2}{H^2}\right] G_{\pm\pm}(k, \eta_1, \eta_2) = \mp i \eta_1^2 \eta_2^2 \delta(\eta_1 - \eta_2)$$

Bootstrap Ghost Collider

- Ghosts Bootstrap equations

$$\mathcal{G}_u^{l_1}\mathcal{I}_{\pm\mp}^{l_1 l_2}(u,v) = 0$$

$$\mathcal{G}_u^{l_1}\mathcal{I}_{\pm\pm}^{l_1 l_2}(u,v) = H^2 e^{\mp \mathrm{i} l_{12}\pi/2}\Gamma\left(5+l_{12}\right)\left(\frac{uv}{u+v}\right)^{5+l_{12}}$$

- Ghosts are not so friendly!

$$\begin{aligned}\mathcal{G}_u^{l} \equiv (\nu^2+9/4) + (3+l)(4+l)(2+l)(1+l)\gamma^2 u^4 \\ +[4(4+l)(3+l)(2+l)\gamma^2u^5 - 2u]\partial_u + [6(4+l)(3+l) \\ \times\gamma^2u^6 + u^2]\partial_u^2 + 4(4+l)\gamma^2u^7\partial_u^3 + \gamma^2u^8\partial_u^4\end{aligned}$$

Anisotropic Collider

- Anisotropies

$$ds^2 = -\, dt^2 + a(t)dx^2 + b(t)dy^2 + c(t)dz^2,$$

$$a(t) = b(t) = e^{\bar{H}_a t},\; c(t) = e^{\bar{H}_c t}\,.$$

- Hubble constants

$$ds^2 = \frac{1}{\bar{H}^2\eta^2}\left[-d\eta^2 + \left(-\bar{H}\eta\right)^{\gamma}(dx^2 + dy^2) + \left(-\bar{H}\eta\right)^{-2\gamma} dz^2\right],$$

$$\bar{H} \equiv \frac{1}{3}\left(2H_a + H_c\right); \quad \gamma = \frac{2(H_a - H_c)}{3\bar{H}}\,.$$

Anisotropic Collider

- ACW inflation

$$S_\sigma = \int d\eta d^3x \frac{1}{\bar{H}^2\eta^2}\left\{\frac{1}{2}(\partial_\eta\sigma)^2 - \frac{1}{2}\sum_i^3 (-\bar{H}\eta)^{2\gamma_i}(\partial_i\sigma)^2 + \frac{m^2}{2\bar{H}^2\eta^2}\sigma^2\right\}$$

$$\gamma_1 = \gamma_2 = \gamma, \quad \gamma_3 = -2\gamma\,.$$

- EoM

$$\sigma'' - \frac{2}{\eta}\sigma' + \left\{|k|^2 + 2\sum_i^3 \gamma_i k_i^2 \ln(-\bar{H}\eta_*) - \frac{m^2}{\bar{H}^2\eta^2}\right\}\sigma = 0,$$

Anisotropic Collider

- Modes

$$\phi_k(\eta) = \frac{\bar{H}}{\sqrt{2k^3}} - \frac{3\bar{H}\gamma \ln|\bar{H}/k|}{4\sqrt{2k^3}} P_2(k_3/k),$$

- Slow-roll driven by an anisotropy

$$P_\phi = \frac{\bar{H}^2}{2k^3}\left[1 - 3\gamma P_2\left(\frac{k_3}{k}\right)\ln\left(\frac{\bar{H}}{k}\right)\right].$$

$$n_s^A \approx 1 + 3\gamma \frac{d\big[P_2(k_3/k)\ln|\bar{H}/k|\big]}{d\ln k}$$

Starobinsky Collider

- f(R)

$$S = \frac{m_{pl}^2}{2} \int d^4x \sqrt{-g} \left[R + R^2/6M\right]$$

- Einstein Frame

$$S^{(g)} = \int d^4x \sqrt{-\tilde{g}} \left[\frac{m_{pl}^2 \tilde{R}}{2} - \frac{1}{2} \tilde{g}^{\alpha\beta} \partial_\beta \phi \partial_\alpha \phi - V(\phi) \right],$$

$$V(\phi) = \frac{3 m_{pl}^2 M^2}{4} \left(1 - e^{-\sqrt{2/3}\frac{\phi}{m_{pl}}} \right)^2 .$$

Starobinsky Collider

- Modes in the Jordan frame

$$v_k^S(\eta) = i\frac{\sqrt{\pi\eta}}{2}e^{i(1+2\nu)\pi/4}H_\nu^{(1)}(-k\eta)$$

$$m_{eff}^2 = -\frac{2}{\eta^2}\left[1 + \frac{3}{2\ln\left(A\eta\right)}\right], \quad z^S = -\frac{\sqrt{6}}{\eta}\frac{1}{\ln\left(A\eta\right)}$$

- Einstein Frame

$$\frac{\partial_\eta^2\tilde{z}}{\tilde{z}} \approx \frac{2}{\eta^2}\left[1 + \frac{1}{\ln(A\eta)} + O\left(\ln(A\eta)^{-2}\right)\right].$$

Starobinsky Collider

- EoM deformed

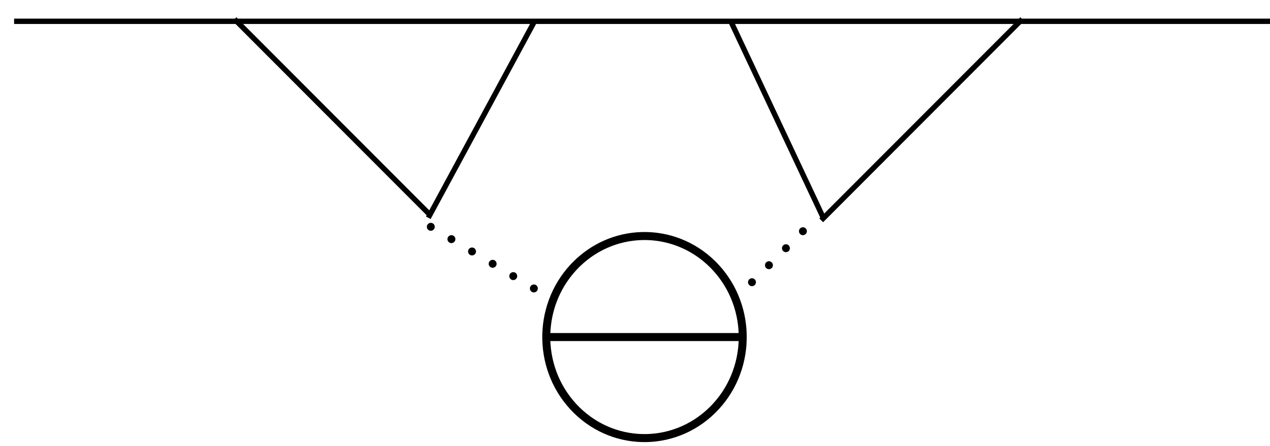
$$\left[\eta^2\partial_\eta^2 - 2\eta\partial_\eta + \eta^2 k^2 - [\ln(A\eta)]^{-1}(\eta\partial_\eta + 1/2)\right]\varphi = 0.$$

- What is the analog of the running of the spectral index in the CCP?!
- For massive bulk fields it is possible to provide a running, what about the boundary fields?!
- Modified Bulk-to-Boundary propagator?

Final Remarks

- General Solutions for the Ghost Collider.
- Weight-shifting with ghost on the boundary.
- Lorentz violation? Boostless?
- Quadratic gravity?
- ACW Inflation is still viable?
- Bounds on the anisotropic parameters…
- Can we improve the general seed to include the running?
- Wands Map holds?!

Thank you



CBPF
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Cosmological Bootstrap

- **How is done?!**
- Inflationary Correlators for (quasi-)dS ST

$$ds^2 = \frac{1}{H^2\eta^2}\Big[-d\eta^2 + dx_i dx^i\Big]$$

- **dS**

$$D_i = -\eta\partial_\eta - x^i\partial_i$$

$$R_i^j = -2x_i\eta\partial_\eta - (2x^j x_i + (\eta^2 - x^2)\delta_j^i)\partial_j$$

Cosmological Bootstrap

- **Near the Super-horizon**

$$\lim_{\eta \to 0} \phi \sim O^{+}(k)\eta^{\Delta +} + O^{-}(k)\eta^{\Delta -}$$

- **Anomalous Dimension**

$$\Delta = \frac{d}{2} \pm i\sqrt{\frac{m^2}{H^2} - \frac{9}{4}}, \quad \Delta + \overline{\Delta} = d\,.$$

Cosmological Bootstrap

- **Ward Identities**

$$\left[-\sum_{r=1}^{n-1}\left(p_{r\mu}\frac{\partial}{\partial p_{r\mu}}+d\right)+\sum_{r=1}^{n}\Delta_r\right]\langle \mathcal{O}_1^{i_1}(p_1)\ldots\mathcal{O}_r^{i_r}(p_r)\ldots\mathcal{O}_n^{i_n}(p_n)\rangle=0,$$

$$\sum_{r=1}^{n-1}\left(p_{r\mu}\frac{\partial^2}{\partial p_r^{\nu}\partial p_{r\nu}}-2p_{r\nu}\frac{\partial^2}{\partial p_r^{\mu}\partial p_{r\nu}}+2(\Delta_r-d)\frac{\partial}{\partial p_r^{\mu}}\right)\langle \mathcal{O}_1^{i_1}(p_1)\ldots\mathcal{O}_r^{i_r}(p_r)\ldots\mathcal{O}_n^{i_n}(p_n)\rangle=0.$$

Cosmological Bootstrap

- **Ward Identities**

$$\left[-\sum_{r=1}^{n-1}\left(p_{r\mu}\frac{\partial}{\partial p_{r\mu}}+d\right)+\sum_{r=1}^{n}\Delta_r\right]\langle\mathcal{O}_1^{i_1}(p_1)\ldots\mathcal{O}_r^{i_r}(p_r)\ldots\mathcal{O}_n^{i_n}(p_n)\rangle=0,$$

$$\langle\phi_i(p_1)\phi_j(p_2)\phi_k(p_3)\rangle=p_3^{\Delta_i+\Delta_j+\Delta_k-2d}f\left(\frac{p_1}{p_3},\frac{p_2}{p_3}\right),$$

Cosmological Bootstrap

$$K_x \equiv x(1-x)\frac{\partial^2}{\partial x^2} - y^2\frac{\partial^2}{\partial y^2} - 2xy\frac{\partial^2}{\partial x \partial y} + \left[\frac{d}{2} - \Delta + 1 - \big(\frac{3d}{2} - 2\Delta_1 + 1\big)x\right]\frac{\partial}{\partial x}$$

$$-\big(\frac{3d}{2} - 2\Delta_1 + 1\big)y\frac{\partial}{\partial y} - (\frac{d}{2} - \Delta_1 + \frac{\Delta_k}{2})(d - \Delta_1 + \frac{\Delta_k}{2}),$$

$$K_y \equiv y(1-y)\frac{\partial^2}{\partial y^2} - x^2\frac{\partial^2}{\partial x^2} - 2xy\frac{\partial^2}{\partial x \partial y} + \left[\frac{d}{2} - \Delta + 1 - \big(\frac{3d}{2} - 2\Delta_1 + 1\big)y\right]\frac{\partial}{\partial y}$$

$$-\big(\frac{3d}{2} - 2\Delta_1 + 1\big)x\frac{\partial}{\partial x} - (\frac{d}{2} - \Delta_1 + \frac{\Delta_k}{2})(d - \Delta_1 + \frac{\Delta_k}{2}),$$

Cosmological Bootstrap

- **Appell Hypergeometric functions**

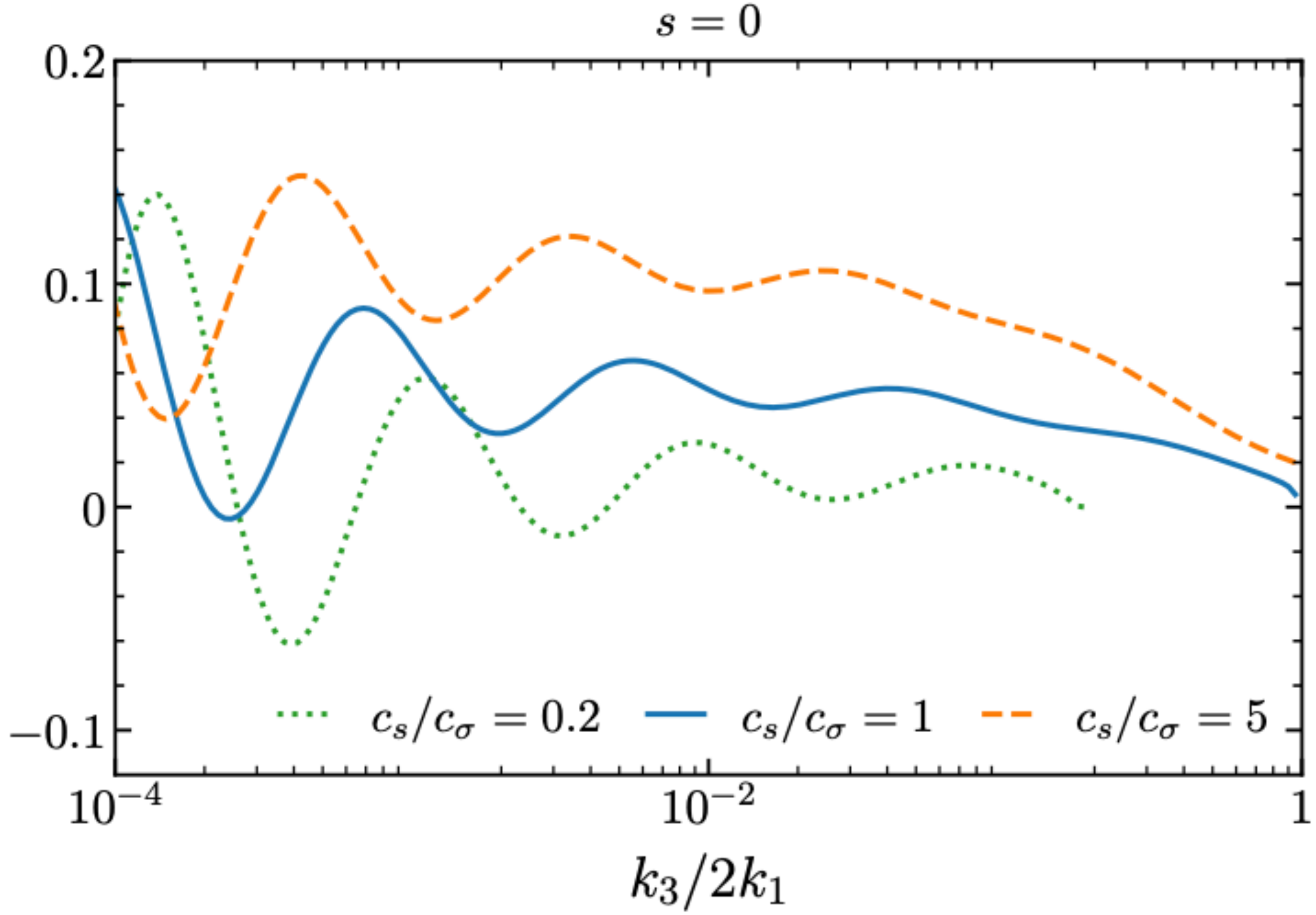
$$\langle \phi_\Delta(p_1)\phi_\Delta(p_2)\phi_2(p_3)\rangle \propto p_3^{\Delta-2}\ {}_2F_1\left(2-\Delta, \Delta-1; 1; (1-p_3)/2\right)$$

- **Slow-roll — Breaking the SO(d+1,1)**

$$\Delta \to \Delta + \epsilon_H$$

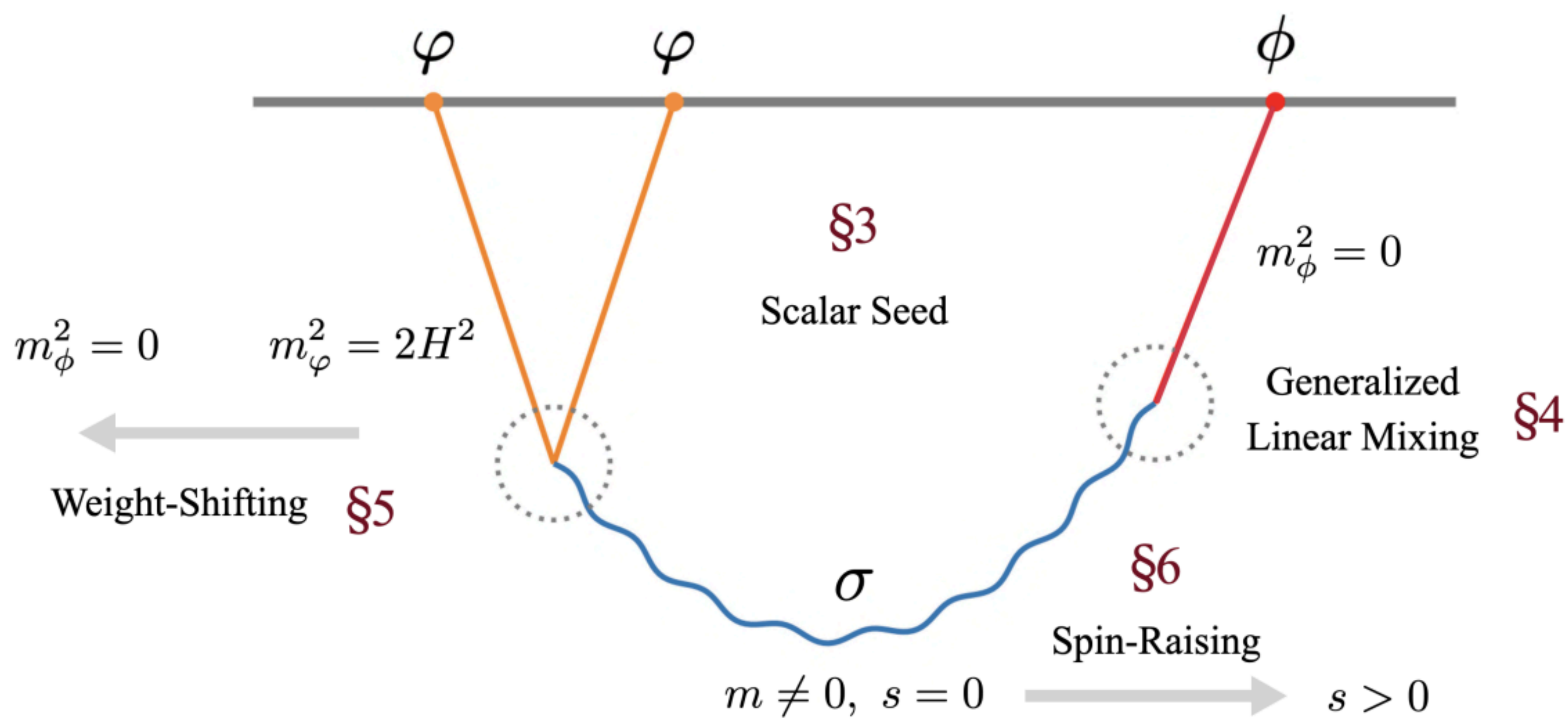
Cosmological Bootstrap

- **Bispectrum**



Cosmological Bootstrap

- **Bispectrum**



Cosmological Bootstrap

Full dS

- *Inflation*
- *CFT's*
- *< Non-Gaussianities*
- *dS/CFT (holography)*
- *Less phenomenology*

Boostless

- *Inflation*
- *CFT’s ?*
- *> Non-Gaussianities*
- *Hidden structures*
- *More Phenomenology*

Cosmological Bootstrap

- How can we deform the Conformal symmetry?
- Map the first order solutions.
- Multi-field and \delta N ?!
- Reheating?!
- Second order?!
- Is it inflation scheme independent?!

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