

Quantum Gravity: Bounce models as probes

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Outline

- 1 Brief discussion on quantum gravity
 - Why quantum gravity?
 - Quantum gravity problems
 - Paths for quantum gravity
- 2 Bounce as a cosmological probe
 - A complete bounce model
 - Bounce initial conditions
 - Contraction phase
 - The bounce
 - Particle creation
- 3 Quantum non-equilibrium
 - Introduction
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Quantum gravity: why?

No question about quantum gravity is more difficult than the question: "What is the question?" (John Wheeler 1984)

- All interactions and matter fields are described by Quantum Field Theories (QFT). Unification?
- The initial conditions theory for cosmology may involve a quantum gravity phase. Inflation shifts the initial condition problem to before inflation but it is still there!
- Singularity issues: big bang and black holes evaporation.
- Semi-classical approach has inconsistencies, e.g.,

$$G_{ab} = \frac{8\pi G}{c^4} \langle \Psi | T_{ab} | \Psi \rangle ,$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|\rho_1\rangle + \frac{1}{\sqrt{2}}|\rho_2\rangle \Rightarrow \langle \Psi | T_{00} | \Psi \rangle = \frac{\rho_1 + \rho_2}{2}$$

but

$$\left\langle \frac{\rho_1 + \rho_2}{2} \middle| \Psi \right\rangle = 0!$$

(see C. Kiefer, *Quantum Gravity*, International Series of Monographs on Physics

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Quantum gravity: problems

General problems not related to specific approaches to quantum gravity.

- Time problem: in Quantum Mechanics (QM) the time behaves as an external global entity and in QFT the whole geometry is a given global stage. In General Relativity (GR) time and space have a dynamical and local meaning through $g_{\mu\nu}$.
- Fitting problem, what is the right scale for GR? Averaging procedures modify the dynamics.
- Quantum gravity will describe the universe as a whole. What is the meaning of the universe wave-function?
- Each approach for quantum gravity potentially brings additional problems.

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Paths for quantum gravity

Direct approaches:

- Starts from a classical theory, GR, Brans-Dicke, etc. Then applies the quantization rules.
- Canonical method starts from a classical 3+1 decomposition and proceeds to quantize the system (quantum GR: γ_{ab} , K_{ab} , loop quantum gravity, etc).
- Covariant method keeps the covariance but usually requires approximation steps (e.g., perturbative approach $g_{ab} = \eta_{ab} + h_{ab}$).
- Straightforward but does not help with the unification business...

Indirect approaches:

- Starts with a theory that contains quantum gravity as some limit of the full theory (e.g., String theory).
- Very speculative.
- No general guiding principles.

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Quantum cosmology: our approach

Quantum bouncing cosmology summary:

- 1 Universe starts large, homogeneous, isotropic and contracting. (Initial conditions **not** addressed! A quantum start?¹)
- 2 Quantum background: mini-super-space with well defined classical regions using Wheeler-DeWitt equation and Bohmian trajectories.
- 3 All inhomogeneities result from quantum fluctuations. Scalar sector probes the matter QFT, and tensor probes the quantum gravity perturbatively.
- 4 Contraction phase (classical and quantum) amplifies the quantum fluctuations.
- 5 Quantum bounce.
- 6 Universe is left in the expanding phase with the *right* initial conditions.

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Initial conditions: background

- Assuming an initial Friedmann metric as a working hypothesis. This assumption, as in Inflationary models, sets an infinite number of degrees of freedom.
- Using an anisotropic metric would decrease the arbitrariness of the initial conditions by 2, we would be setting only $(\infty - 2)$ degrees of freedom.
- All initial inhomogeneity and anisotropy are left for the perturbation sector.
- Bohmian trajectory for the scale-factor: no-quantum effects on the initial evolution of the background!

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Initial conditions: perturbations

- All perturbations start in the adiabatic vacuum. Simple and well know for single component. However, there is a unique canonical representation where Unitary evolution can be achieved.²
- Non-trivial for several linearly coupled fields (quadratic coupling with time dependent coefficients), solved in Ref.³.
- The initial anisotropy generated does not grow too large and dominates.⁴ No BKL singularity happens, the bounce takes place before.
- If there is a dark-energy like fluid ($w < -1/3$) in the contraction branch the adiabatic vacuum is non-longer well defined. All modes appear to be dominated by the background evolution. Simultaneous quantization? (see⁵ for a workaround)

²S. D. P. Vitenti, *ArXiv e-prints* (2015), arXiv: 1505.01541 [gr-qc].

³P. Peter, N. Pinto-Neto, and S. D. P. Vitenti, *Phys. Rev. D* 93.2, 023520 (2016), 023520, arXiv: 1510.06628 [gr-qc].

⁴S. D. P. Vitenti and N. Pinto-Neto, *Phys. Rev. D* 85 (2012), 023524, arXiv: 1111.0888 [astro-ph.CO]; N. Pinto-Neto and S. D. P. Vitenti, *Phys. Rev. D* 89 (2014), 028301, arXiv: 1312.7790 [astro-ph.CO].

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Contraction phase: background

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Contraction phase: perturbations

Numerical problems

- System of equations includes many fluid components with radically different frequencies $c_1^2 k^2, c_2^2 k^2, \dots$. For instance $_1$ begin a very cold dark matter $c_1^2 < 10^{-12}$ (air sound speed in unit of c) and $_2$ begin radiation $c_2^2 = 1/3$.
- Since $c_1^2 \ll c_2^2$, one mode is deep in the adiabatic regime $u_1 \approx \exp(c_1 k \eta) \left[\sum_n \mathcal{O}\left(\frac{1}{k^{2n}}\right) \right]$ while the second mode is already dominated by the potential $u_2 \approx \sum_n \left[(k^{2n}) \right]$.
- Initial conditions must be set in the far past where all modes are in the adiabatic regime. However, doing so puts mode $_1$ with a high frequency impossible to evolve numerically! Mixing WKB and numerics seems difficult, action angle variables may solve it.

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Quantum bounce: background

- We use the Wheeler-DeWitt equation $H|\Psi\rangle = 0$;
- For a single barotropic fluid with equation of state w and a flat Friedmann metric

$$L = \rho \dot{T} + \Pi_X \dot{X} - NH, \quad H = \rho - \Pi_X^2, \quad X \equiv \frac{2a^{\frac{3(1-w)}{2}}}{3(1-w)}.$$

- Using the representation $\Psi(x) = \langle x|\Psi\rangle$ for the states and $X\Psi(x) = x\Psi(x)$ for the “position” operator, $\Pi_X\Psi(x) = -i\partial\Psi(x)/\partial x$ for the “momentum” operator and $\rho\Psi(x) = i\frac{\partial}{\partial T}\Psi(x)$, **time problem**.
- To circumvent the time problem we chose T as a parameter, Ψ will not be normalized in T !
- In Ref.⁶ we use the following wave-function:

$$\Psi_0(x) = \frac{2^{(1-2\alpha)/4} x^\alpha}{\sigma^{\alpha+1/2} \sqrt{\Gamma(\alpha + \frac{1}{2})}} \exp \left[-\frac{1}{2} x^2 \left(\frac{1}{2\sigma^2} - i\mathcal{H}_{\text{ini}} \right) \right],$$

⁶P. Peter and S. D. P. Viti, *Mod. Phys. Lett. A* 31 (2016), 1640006, URL: <http://dx.doi.org/10.1142/S021773231640006X>.

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- To circumvent the time problem we chose T as a parameter, Ψ will not be normalized in T !
- In Ref.⁶ we use the following wave-function:

$$\Psi_0(x) = \frac{2^{(1-2\alpha)/4} x^\alpha}{\sigma^{\alpha+1/2} \sqrt{\Gamma(\alpha + \frac{1}{2})}} \exp \left[-\frac{1}{2} x^2 \left(\frac{1}{2\sigma^2} - i\mathcal{H}_{\text{ini}} \right) \right],$$

⁶Peter and Vitenti, “The simplest possible bouncing quantum cosmological model”

Quantum bounce: background

- We use the Wheeler-DeWitt equation $H|\Psi\rangle = 0$;
- For a single barotropic fluid with equation of state w and a flat Friedmann metric

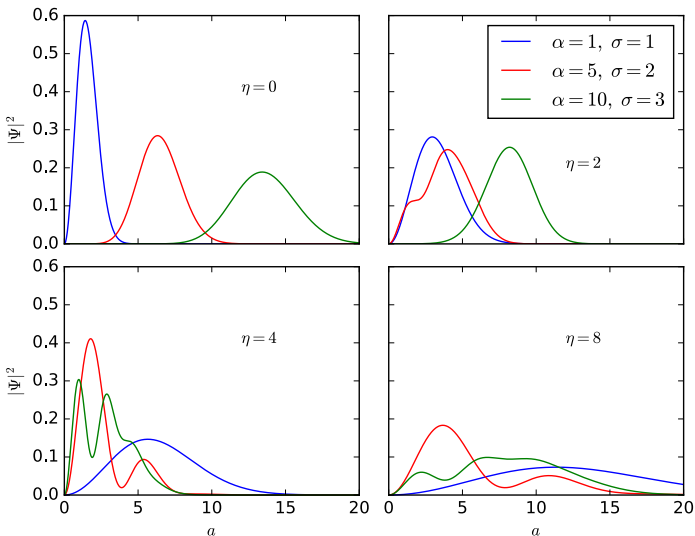
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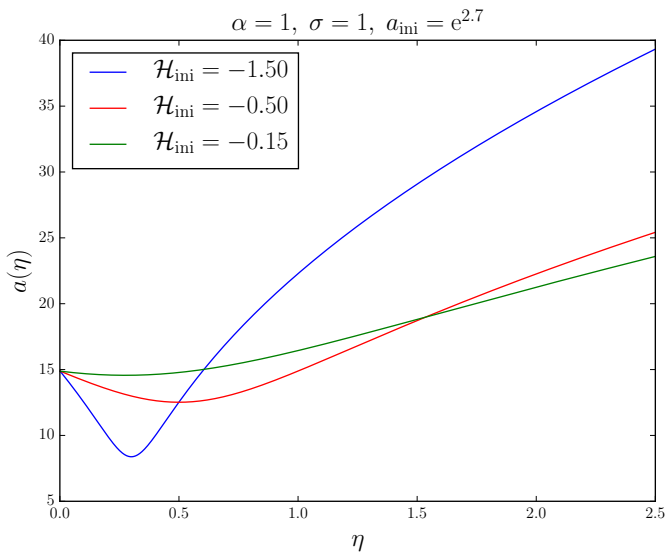
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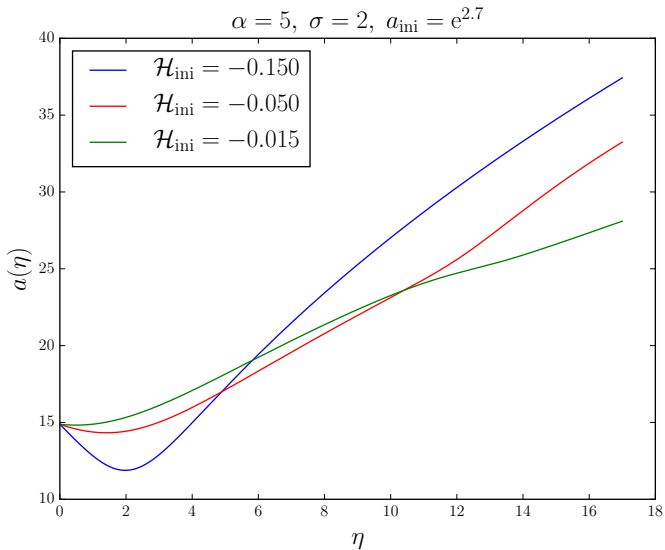
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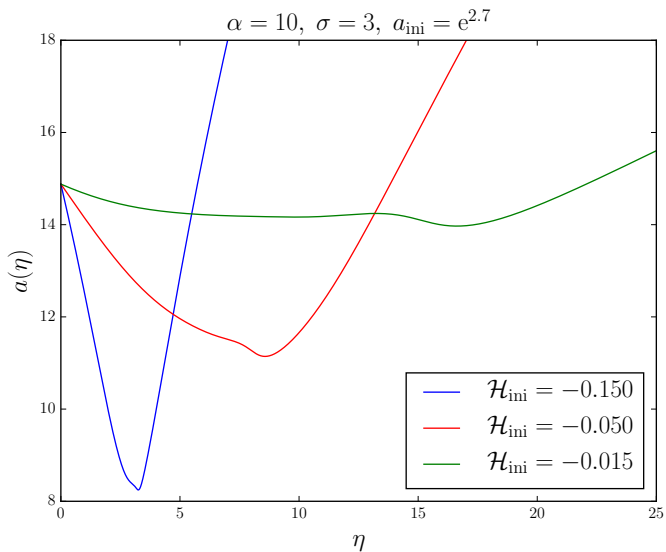
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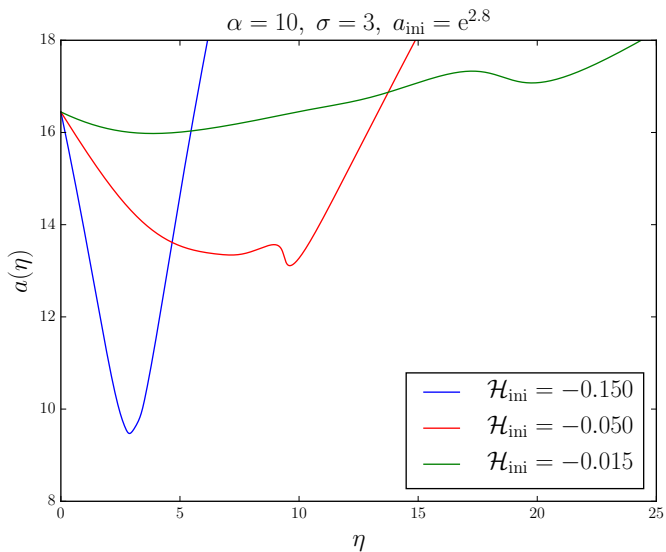
Quantum bounce: background



Quantum bounce: background



Quantum bounce: background



Quantum bounce: perturbations

- Passing through the bouncing phase, all second order action must not use background equations of motion.
 - Single fluid Ref.⁷, single scalar field Ref.⁸, multiple fluids Ref.⁹
- This chooses preferred gauge invariant variables, e.g.,

$$\Pi_V \propto \delta\rho - 3H(\rho + p)V,$$

that must be used instead of

$$\Pi_V \propto \delta\rho + \dot{\rho}V,$$

for a quantum background $\dot{\rho} \neq -3H(\rho + p)$ (considering the Bohmian trajectory of ρ).

- For a single fluid perturbation the bounce controls the amplitude but leaves the form of the spectrum unchanged.
- Many fluids: does the bounce leave imprints in the adiabatic/entropy modes mixing?

⁷S. D. P. Vitenti, F. T. Falciano, and N. Pinto-Neto, *Phys. Rev. D* 87 (2013), 103503, arXiv: 1206.4374 [gr-qc].

⁸F. T. Falciano, N. Pinto-Neto, and S. D. P. Vitenti, *Phys. Rev. D* 87 (2013), 103514, arXiv: 1305.4664 [astro-ph.CO].

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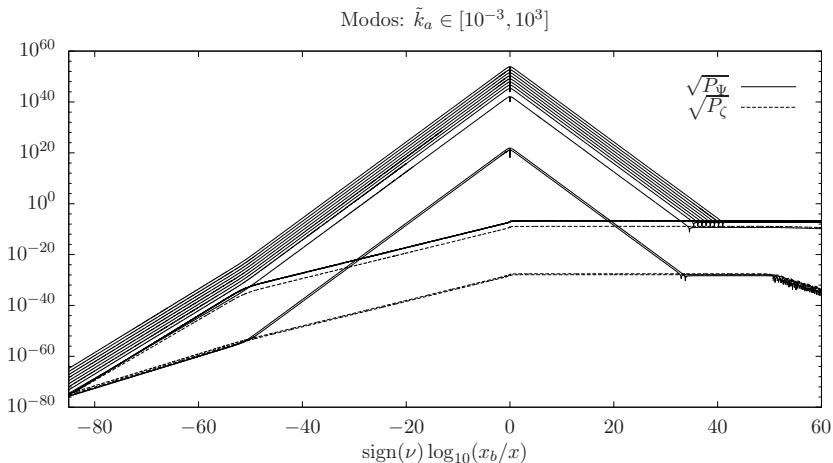
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Quantum bounce: perturbations

Example: two-fluids adiabatic mode only.



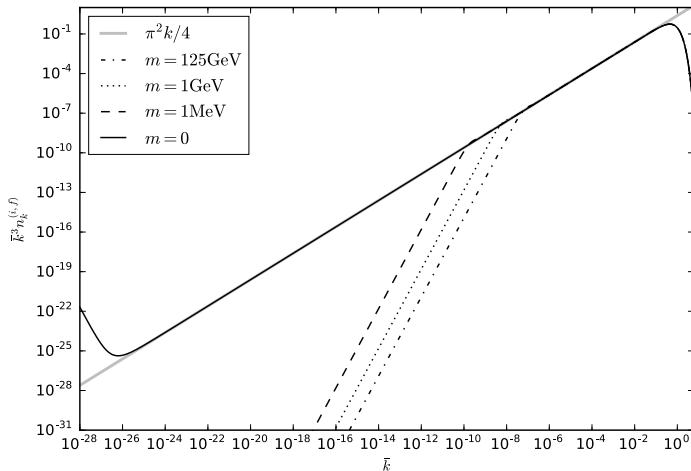
Quantum bounce: particle creation

- In Ref.¹⁰ we show that the quantum bounce can produce significant density of particles.
- The particle number density is controlled by the bounce scale and does not depend on the particle mass (most of the particle production takes place at the bounce where most particles $m \ll M_{\text{planck}}$ are ultra-relativistic).
- What is the effect on fermion fields? Can we use it to reheat the universe?
- Interactions, loop corrections?

¹⁰D. C. F. Celani, N. Pinto-Neto, and S. D. P. Viteni, *Phys. Rev. D* 95.2, 023523 (2017), 023523, arXiv:1610.04933 [gr-qc].

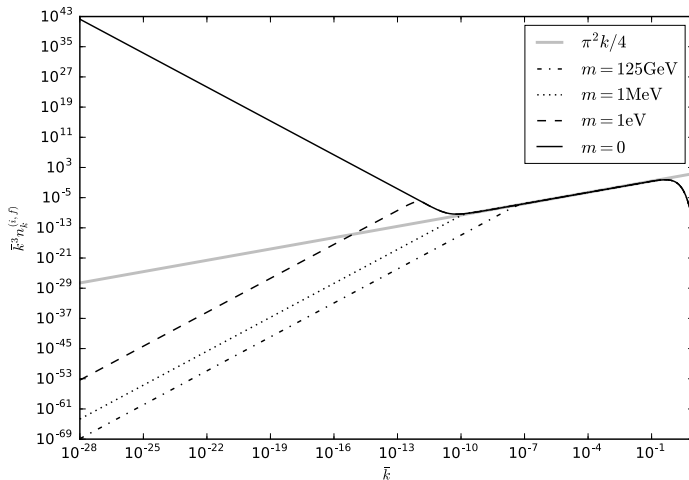
Quantum bounce: particle creation

Example: minimal coupling, radiation and low dust content.



Quantum bounce: particle creation

Example: minimal coupling, radiation and high dust content.



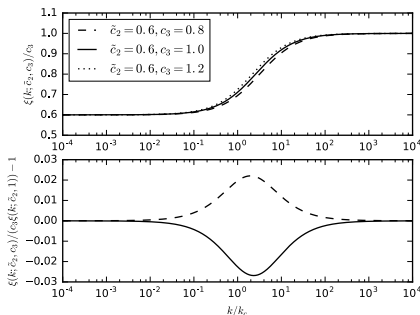
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Quantum non-equilibrium

Quantum non-equilibrium and the primordial power spectrum P_k , see Ref.¹¹.

- Probability distribution begins different from the quantum distribution $\rho(t_{\text{ini}}) \neq |\Psi(t_{\text{ini}})|^2$.
- For $t > t_{\text{ini}}$, $\rho \rightarrow |\Psi(t_{\text{ini}})|^2$ but it may leave imprints on large scales.
- These imprints are measured by $P_k = \xi(k) A_s \left(\frac{k}{k_{\text{pivot}}} \right)^{n_s - 1}$.



¹¹S. Colin and A. Valentini, *Phys. Rev. D* **92**, 043520 (2015), arXiv:1407.8262 [astro-ph.CO].

Quantum non-equilibrium: results from PlanckTT

$$\xi_{\text{plaw}} = 1, \quad \xi_{\text{atan}} = \arctan \left(\frac{k}{k_c} + c_2 \right) - \frac{\pi}{2} + c_3, \quad \Gamma = \Delta\chi^2$$

$$\text{low} \Rightarrow \ell \in [2, 29], \quad \text{high} \Rightarrow \ell \in [30, 2500].$$

Table: PlanckTT.

$\xi(k)$	$k_c^{-1}[\text{Gpc}]$	$\tilde{c}_2$	c_3	Γ	Γ_{low}	Γ_{high}	Γ_{priors}	p-value
plaw	—	—	—	0.00	0.00	0.00	0.00	—
bpl	2.851	—	—	2.70	2.83	−0.01	−0.11	25.91%[2]
atan1	3.695	0.000	1.000	5.39	4.26	0.95	0.17	2.03%[1]
atan2	1.548	0.481	1.000	5.71	5.05	0.47	0.19	5.74%[2]
atan3	1.978	0.472	0.929	5.79	4.94	0.64	0.20	12.25%[3]
expc1	2.262	0.000	—	5.51	4.55	0.78	0.18	1.89%[1]
expc2	2.922	0.000	—	5.71	4.85	0.77	0.09	5.75%[2]
expc3	2.922	0.000	—	5.71	4.85	0.77	0.09	12.65%[3]

Quantum non-equilibrium: results from PlanckTTTEEE+lowP

Table: PlanckTTTEEE+lowP.

$\xi(k)$	$k_c^{-1}[\text{Gpc}]$	$\tilde{c}_2$	c_3	Γ	Γ_{low}	Γ_{high}	Γ_{priors}	p-value
plaw	—	—	—	0.00	0.00	0.00	0.00	—
bpl	0.225	—	—	4.53	2.60	1.78	0.15	10.41%[2]
atan1	6.561	0.000	1.000	3.08	2.97	0.35	−0.24	7.91%[1]
atan2	6.645	0.000	1.000	3.07	3.02	0.25	−0.20	21.56%[2]
atan3	6.645	0.000	1.000	3.07	3.02	0.25	−0.20	38.12%[3]
expc1	3.314	0.000	—	3.86	2.53	1.33	−0.00	4.93%[1]
expc2	2.904	0.000	—	3.92	2.48	1.42	0.01	14.11%[2]
expc3	1.940	0.259	—	3.94	2.63	1.29	0.02	26.82%[3]

Quantum non-equilibrium: results

- Significant evidence for TT only.
- Significance drops when adding polarization data.
- Different non-equilibrium on scalar and tensor modes can account for worse fit when using polarization.
- Adding large scale structure data seems to improve evidence for TT only (when adding $H_0 = 73.24 \pm 1.74 \text{ ms}^{-1}\text{Mpc}^{-1}$ from Riess 2016). But does not improve when adding polarization.

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Using the quantum bounce cosmology program we can prove quantum gravity regimes through:

- Perturbations amplitude connected to the bounce scale.
- Particle creation also connected to the bounce scale.
- Multi-component bounce, further imprints in the primordial power spectrum?
- Quantum non-equilibrium, measure the initial probability distribution by probing large scales on CMB. Can it be mimicked by features in the inflationary potential or bounce model?
- Different aspects of quantum gravity can be investigated and used in different contexts:
 - Quantization of mini-super-spaces.
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