Reissner-Nordström, Kerr-Newman, and non-singular black holes in Palatini theories of gravity

Diego Rubiera-Garcia
Institute of Astrophysics and Space Sciences, Lisbon University
and
Complutense University of Madrid

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  - Gravitational waves propagate at the same speed as electromagnetic radiation. Experimental constraint: one part in $\sim 10^{-16}$.
  - Slaughter on modified theories of gravity (mainly those motivated by cosmological considerations)
    - Viable: GR, quintaessence/K-essence, Brans-Dicke/$f(R)$, Kinetic Gravity Braiding, Derivative Conformal, Disformal Tuning, some DHOST, ...
    - Non-viable: quartic/quintic Galileons, Fab Four, de Sitter Hordenski, Gauss-Bonnet, quartic/quintic GLPV, some DHOST, ...

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  - Purely tensor polarizations strongly favoured over purely scalar/vector polarizations (LIGO, PRL, September 27th).
  - Other polarizations to be tested in the future.

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  - Further compact objects may exist with similar/different GW signatures.
- Need to rethink fundamental principles, and understand their phenomenological consequences.
- Any guidance?.

Space-time singularities

- The theorems on singularities (Hawking, Penrose, 60's) tell us that space-time singularities are unavoidable in the context of GR. Example: the Schwarzschild solution.
- Fundamentally different from singularities on the fields living on a fixed space-time background! (e.g. Coulomb's divergence in classical electrodynamics).
- If space-time breaks down when a singularity, how can we even speak of a singularity as something ("blow up" of things) occurring at some "location"?.

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- If space-time breaks down when a singularity, how can we even speak of a singularity as something ("blow up" of things) occurring at some "location"?.
- If we see space-time singularities as indicative of a physically troublesome region, a natural guess is that something is going on ill with the geometry: blow up of curvature scalars.
- Numerous attempts to build regular space-times have focused on keeping curvature divergences at bay: non-linear electrodynamics, surgically joined space-times... (Ansoldi, arXiv:0802.0330 [gr-qc]).
- Several criticisms: Theorems on singularities still apply (pay the price of violation of energy conditions) + ad hoc removal of singularities + no reference to blow up of things in the singularity theorems.

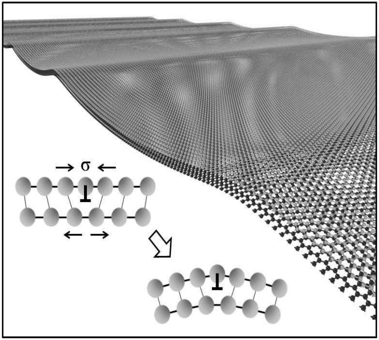
- A more powerful characterization of space-times containing singularities is provided by the notion of geodesic completeness (Geroch, 68).
- This is so because time-like geodesics are associated to physical observers and null geodesics to light rays and transmission of information.
- Such geodesics should be complete: in a physically consistent theory nothing should cease to exist suddenly or “emerge” from nowhere.

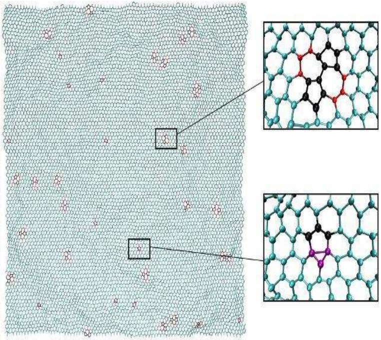
- This concept precludes any idea of “things” blowing up at some point!.

- The widespread identification between curvature divergences and space-time singularities comes from the fact that in many cases of interest, those space-times showing curvature divergences are also geodesically incomplete.
- The possibility of creating/destruction of particles/information poses a treat to predictability and determinism.
- Is there any sensible approach to deal with holes in the geometry?.

The geometry of crystalline structures

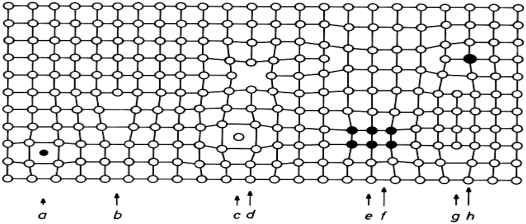
- A microstructure with a macroscopic continuum limit is found in condensed matter systems such as graphene or Bravais crystals.





- Perfect crystals admit a macroscopic Riemannian geometrical description.

- Real crystals have defects of different kinds and have dynamics.



a) Interstitial impurity atom, b) Edge dislocation, c) Self interstitial atom
d) Vacancy, e) Precipitate of impurity atoms, f) Vacancy type dislocation loop,
g) Interstitial type dislocation loop, h) Substitutional impurity atom

- The effective geometry in the limit to the continuum depend on the type of defects present.
- Point-like defects ("holes") are associated to non-metricity: $Q_{\alpha\mu\nu} = \nabla^{\Gamma}_{\alpha} g_{\mu\nu} = 0$.
- Dislocations are associated to torsion: $\Gamma^{\alpha}_{\mu\nu} \neq \Gamma^{\alpha}_{\nu\mu}$.

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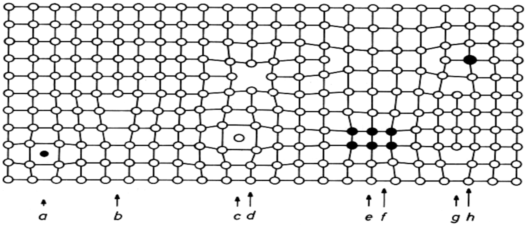

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- Could the metric-affine formulation of gravitational theories help to understand the transition from quantum gravity to classical space-time (and the other way round)? (Lobo, Olmo, DRG, arXiv:1412.4499 (PRD), arXiv:1507.07777 (IJMPD)) .
- What about space-time singularities?.

A quick digression...

- In general, an affine connection can be split into three independent pieces:

$$\Gamma^{\lambda}{}_{\mu\nu} = \left\{ {}^{\lambda}{}_{\mu\nu} \right\} + K^{\lambda}{}_{\mu\nu} + L^{\lambda}{}_{\mu\nu}$$

  - Levi-Civita connection of the metric $g_{\mu\nu}$ (associated to curvature) : $\left\{ {}^{\lambda}{}_{\mu\nu} \right\}$.
  - Contortion (associated to torsion $T^{\lambda}{}_{\mu\nu} \equiv 2\Gamma^{\lambda}{}_{[\mu\nu]}$)

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  - Disformation (associated to nonmetricity $Q_{\rho\mu\nu} \equiv \nabla_{\rho} g_{\mu\nu}$)

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- Three equivalent (modulo boundary terms technicalities) formulations of GR (see e.g. Jimenez et al. arXiv:1710.03116 [gr-qc])
  - Riemannian-based GR: $R \neq 0, T = 0, Q = 0$.
  - Teleparallel equivalent of GR: $R = 0, T \neq 0, Q = 0$.
  - Symmetric (or coincident) teleparallel GR: $R = 0, T = 0, Q \neq 0$.

Metric-affine formulation

- Consider a family of Ricci-based extensions of GR (RBG) defined by the action:

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}L_G[g_{\mu\nu},R_{(\mu\nu)}(\Gamma)]+S_M[g_{\mu\nu},\psi_m]$$

Examples: GR, $f(R)$, $f(R,R_{(\mu\nu)}R^{(\mu\nu)})$, Born-Infeld inspired theories of gravity...

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  Examples: GR, $f(R)$, $f(R, R_{(\mu\nu)}R^{(\mu\nu)})$, Born-Infeld inspired theories of gravity...
- If connection is metric-compatible, $\nabla^{\Gamma}_{\mu}(\sqrt{-g}g^{\alpha\beta}) = 0$ (Metric approach) then EOM:

$$\delta g^{\mu\nu} \Rightarrow \left(\frac{\delta L_G}{\delta g^{\mu\nu}} - \frac{L_G}{2}g_{\mu\nu}\right) + \nabla_\lambda\left[g_{\gamma\nu}\frac{\delta L_G}{\delta \Gamma^{\mu}_{\lambda\gamma}} - g_{\beta\mu}g_{\gamma\nu}g^{\alpha\lambda}\frac{\delta L_G}{\delta \Gamma^{\alpha}_{\beta\gamma}}\right] = \kappa^2 T_{\mu\nu}$$

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- Difficulties: higher-order EOM?, ghosts?, $c_g\neq c$?, compatibility with solar system experiments? viable as effective models of quantized GR?...
- If $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\beta\gamma}$ are independent (Palatini/metric-affine approach) then EOM:

$$\delta S = \int d^4x\left[\sqrt{-g}\left(\frac{\delta L_G}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}\right)\delta g^{\mu\nu}+\sqrt{-g}\frac{\delta L_G}{\delta\Gamma^{\alpha}_{\beta\gamma}}\delta\Gamma^{\alpha}_{\beta\gamma}\right]+\delta S_M$$

$$\delta g^{\mu\nu} \Rightarrow \frac{\delta L_G}{\delta g^{\mu\nu}}-\frac{L_G}{2}g_{\mu\nu}=\kappa^2T_{\mu\nu}$$

$$\delta\Gamma^{\alpha}_{\beta\gamma} \Rightarrow \frac{\delta L_G}{\delta\Gamma^{\alpha}_{\beta\gamma}}=0$$

- If we introduce an auxiliary metric $\nabla^{\Gamma}_{\mu}(\sqrt{-q}q^{\alpha\beta}) = 0$, then the metric-affine RBG system of equations can always be recast as

$$G^{\mu}{}_{\nu}(q) = \frac{\kappa^2}{|\hat{\Omega}|^{1/2}}\left[T^{\mu}{}_{\nu} - \delta^{\mu}{}_{\nu}\left(L_G + \frac{T}{2}\right)\right]$$

  where $G^{\mu}{}_{\nu}(q) \equiv q^{\mu\alpha}R_{\alpha\nu}(q) - \frac{1}{2}\delta^{\mu}{}_{\nu}R(q)$ is the Einstein tensor of $q_{\mu\nu}$, related to the spacetime metric as

$$q_{\mu\nu} = g_{\mu\alpha}\Omega^{\alpha}{}_{\nu}$$

  where $\Omega^{\alpha}{}_{\nu}$ depends on the particular L_G chosen, but it can always be written on-shell as a function of the energy-momentum tensor, $T^{\mu}{}_{\nu}$.
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- In GR, $L_G=R$, then $G_{\mu\nu}(q)=\kappa^2 T_{\mu\nu}$ and $q_{\mu\nu}=g_{\mu\nu}$.
- Nice features:
  - Second-order field equations.
  - Vacuum solutions are those of GR.
  - No ghost-like instabilities.
  - $c_g=c$ and two tensorial polarizations.
  - Compatibility with solar system experiments and with GW observations so far.

Spherically symmetric solutions

- Consider Ricci-squared gravity coupled to an electrostatic (Maxwell) field

$$L_G = R + l_\varepsilon^2 (R^2 + aR_{(\mu\nu)} R^{(\mu\nu)}) - \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F_{\alpha\beta} F^{\alpha\beta}$$

where l_ε is some new scale and the energy-momentum tensor reads $T_\mu{}^\nu = \frac{q^2}{8\pi r^4} \mathrm{diag}(-1,-1,1,1)$ with q the electric charge.

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- Electrostatic, spherically symmetric solutions (Olmo, DRG, Sanchez, arXiv: 1311.0815 (EPJC)). Relevant scales: $r_q^2 = \kappa^2 q^2/4\pi$, $r_c^4 = l_P^2 r_q^2$, $z = r/r_c$

$$\begin{aligned} ds^2 &= A(x)dt^2 + A(x)^{-1}dx^2 + r^2(x)d\Omega^2 \\ A(x) &= \frac{1}{\sigma_+}\left[1 - \frac{[1+\delta_1 G(z)]}{\delta_2 z \sigma_-^{1/2}}\right]; \left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2} \end{aligned}$$

where $\delta_1 = \frac{1}{2r_S}\sqrt{r_q^3/l_P}$, $\delta_2 = \frac{r_c}{r_S}$, $r_S = 2M_0$, $\sigma_\pm = 1 \mp \frac{1}{z^4}$ and $dG/dz = \sigma_+/(z^2\sigma_-^{1/2})$
- For $z = r/r_c \gg 1 \rightarrow$ Reissner-Nordström space-time: $A(x) \approx 1 - \frac{r_S}{r} + \frac{r_q^2}{2r^2} + O(r^{-4})$.

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- For $z \simeq 1$ (N_q: number of charges, $\delta_c \simeq 0.572069$):

$$A(x) \simeq \frac{N_q}{4N_c}\frac{(\delta_1 - \delta_c)}{\delta_1\delta_c}\sqrt{\frac{r_c}{r - r_c}} + \frac{N_c - N_q}{2N_c} + \ldots$$

- Horizons:
  - If $\delta_1 > \delta_c$: RN-like solutions (two, one -extreme-, or none horizons).
  - If $\delta_1 < \delta_c$: Schwarzschild-like solutions (a single horizon).
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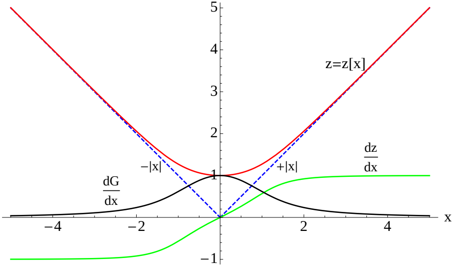
  - If $\delta_1 = \delta_c$: A single horizon or none (below a number of charges $N_q < N_c \simeq 16.55$).
- From the relation of radial coordinates: $\left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2}$ we find $r^2(x) = \frac{x^2+\sqrt{x^4+4r_c^4}}{2}$, which has a minimum at $x = 0$. A wormhole geometry!.



- Double cover the entire manifold in terms of z, while $x \in (-\infty, +\infty)$ covers it alone.
- Curvature scalars blow up at the wormhole throat $x = 0$, excepting for $\delta_1 = \delta_c$ (mass-to-charge ratio).

Geodesics

- A geodesic curve $\gamma^\mu = x^\mu(\lambda)$ with tangent vector $u^\mu = \frac{dx^\mu}{d\lambda}$ and affine parameter λ satisfies (e.g. Chandrasekhar's book):

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

- Comments:
  - The metric defines a natural connection (Christoffel) and defines a set of geodesics.
  - The independent connection can be used to define a different set of geodesics.
  - Assuming the EEP, matter is not coupled directly to the independent connection, so geodesics are those associated to the metric (but GWs couple to $q_{\mu\nu}$! Beltran, Heisenberg, Olmo and DRG, arXiv:1707.08953 (JCAP)).

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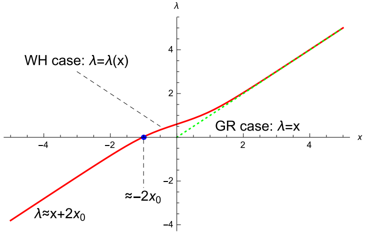
- By spherical symmetry there are two conserved quantities: $L = r^2 d\varphi/d\lambda$ (angular momentum per unit mass) and $E = Adt/d\lambda$ (total energy per unit mass).
- Rewrite the geodesic equation in terms of the geodesic tangent vector (Olmo, DRG, Sanchez-Puente, arXiv:1504.07015 (EPJC))

$$\frac{1}{\sigma_+^2}\left(\frac{dx}{d\lambda}\right)^2 = E^2 - A(x)\left(\kappa + \frac{L^2}{r^2(x)}\right)$$

where $k = 0(1)$ for null (time-like) geodesics.

- Radial null geodesics ($k=0, L=0$) integrate the geodesic equation as:

$$\pm E \cdot \lambda(x) = \begin{cases} {}_2F_1[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{r_c^4}{r^4}]r & \text{if } x \geq 0 \\ 2x_0 - {}_2F_1[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{r_c^4}{r^4}]r & \text{if } x \leq 0 \end{cases}$$



- Complete! $\lambda(x)$ extends over the whole real axis for any value of δ_1 (i.e. no matter the behaviour of curvature scalars). In GR $r(\lambda) = \pm E\lambda$ incomplete!.
- Furthermore, completeness of all time-like and null geodesics for all values of mass and charge (Olmo, DRG, Sanchez-Puente, arXiv:1508.03272 (PRD)).

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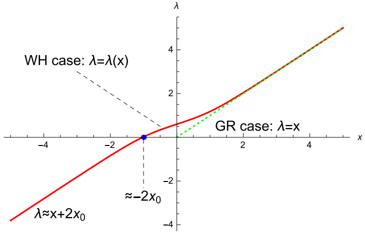

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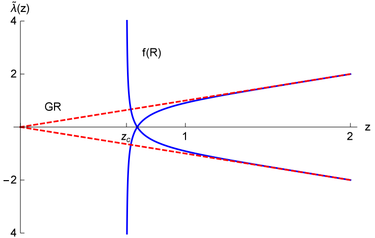
- Furthermore, completeness of all time-like and null geodesics for all values of mass and charge (Olmo, DRG, Sanchez-Puente, arXiv:1508.03272 (PRD)).
- No unavoidable destruction of physical observers by large tidal forces (Olmo, DRG, Sanchez, arXiv:1602.01798 (CQG)) :
- Scattering of waves off the wormhole well posed (Olmo, DRG, Sanchez, arXiv:1504.07015 (EPJC)).
- Accelerated motion well behaved (Olmo, DRG, Sanchez, arXiv:1710.08712 (CQG)).

Further compact objects

- Couple different models, $f(R)$, $f(R_{\mu\nu}R^{\mu\nu})$, EiBI, etc, + anisotropic fluids satisfying energy conditions to find:
  - Regular naked cores.
  - Wormholes whose throat can only be reached in infinite affine time in $f(R) = R + \alpha R^2$ (Bambi, Cardenas-Avendano, Olmo, DRG, arXiv:1511.03755 (PRD)))



  - Asymmetric wormholes (when sourced by scalar fields).
  - de Sitter cores $R_{\mu\nu} \approx \Lambda g_{\mu\nu}$ (Bejarano, Olmo and DRG, arXiv:1702.01292 (PRD))
- Black hole mimickers?. To be explored.

The mapping procedure

- Let us recall the structure of the RBG field equations in $q_{\mu\nu}$ frame

$$G^{\mu}{}_{\nu}(q) = \frac{\kappa^2}{|\hat{\Omega}|^{1/2}}\left[T^{\mu}{}_{\nu} - \delta^{\mu}{}_{\nu}\left(L_G + \frac{T}{2}\right)\right]$$

- Fundamental difficulty:
  - Note that $q_{\alpha\beta} = g_{\alpha\rho}\Omega^{\rho}{}_{\beta}$ and $\Omega^{\rho}{}_{\beta}$ is a nonlinear function of $T^{\mu}{}_{\nu}$, which itself depends on $g^{\mu\nu}$.
  - There are certain configurations with high symmetry (cosmology, BHs, ...) in which specific models can be treated (as we just showed).
  - Dynamical scenarios with less symmetry are plagued by technical difficulties.
  - The application of numerical methods must be strongly model dependent and computationally expensive because of the need to invert the relation between the metrics at each step.

- Rewrite the EOM as Einstein field equations $G_{\mu\nu}(q) = \kappa^2 \tilde{T}_{\mu\nu}$, where

$$\tilde{T}_{\mu\nu}(q) = \frac{1}{|\hat{\Omega}|^{1/2}} \left[T^{\mu}{}_{\nu}(g) - \delta^{\mu}{}_{\nu} \left(L_G + \frac{T(g)}{2} \right) \right]$$

  $\nabla_{\mu}^{q} \tilde{T}^{\mu\nu}(q) = 0$ (because Bianchi identities). $\tilde{T}_{\mu\nu}(q)$ fields of the same type as $T^{\mu}{}_{\nu}(g)$.
- Recast this energy-momentum tensor as a standard (but non-canonical) for some fields $\tilde{\psi}_m$. If the matter field equations on both frames are compatible, a correspondence between the spaces of solutions of RBGs$+\psi_m$ (modified gravity sector) and GR$+\tilde{\psi}_m$ (modified matter sector) should be possible.

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- It works!:
  - Electromagnetic fields: Afonso, Olmo, Orazi, DRG, arXiv:1807.06385 (EPJC).
  - Scalar fields: Afonso, Olmo, Orazi, DRG, arXiv: 1810.04239 [gr-qc].
  - Anisotropic fluids: Afonso, Olmo, Orazi, DRG, arXiv:1801.10406 (PRD).

Rotating black holes in EiBI gravity

- GR + Maxwell maps into Born-Infeld gravity with Born-Infeld electrodynamics!.

$$S = \frac{1}{\kappa^2 \varepsilon} \int d^4x \left[\sqrt{-|g_{\mu\nu} + \varepsilon R_{\mu\nu}|} - \sqrt{-g} \right] + \frac{1}{8\pi} \int d^4x \sqrt{-g} \left(1 - \sqrt{1 - \frac{\kappa^2 \varepsilon}{2\pi} X} \right)$$

- Electrostatic spherically symmetric solutions yield black holes with similar properties as those studied before.

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- Electrostatic spherically symmetric solutions yield black holes with similar properties as those studied before.
- For axisymmetric solutions (Kerr-Newman like) the mapping provides a closed solution (details to be published elsewhere):

$$\begin{aligned}
ds^2 &= -\left(1-f+\varepsilon\kappa^2\rho^q\frac{(\Delta+a^2\sin^2\theta)}{\Sigma}\right)dt^2-2a\left(f-\varepsilon\kappa^2\rho^q\frac{(\Delta+x^2+a^2)}{\Sigma}\right)\sin^2\theta dtd\phi\\
&+ \frac{(1+\varepsilon\kappa^2\rho^q)\Sigma}{\Delta}dr^2+(1-\varepsilon\kappa^2\rho^q)\Sigma d\theta^2\\
&+ \left[\left(x^2+a^2+fa^2\sin^2\theta\right)-\varepsilon\kappa^2\rho^q\frac{(x^2+a^2)^2+a^2\Delta\sin^2\theta}{\Sigma}\right]\sin^2\theta d\phi^2\\
f &= \frac{r_Sx+r_q^2/2}{\Sigma};\Sigma=x^2+a^2\cos^2\theta;\Delta=x^2-r_Sx+a^2+r_q^2/2
\end{aligned}$$

- ε-corrections induce qualitative and quantitative changes as compared to GR: horizons, ergospheres, disk-type wormholes?. Work in progress.

Conclusions

- Metric-affine RBGs have unique features:
  - Second-order field equations, no ghosts.
  - Compatible with $c = c_g$ and two tensor polarizations.
  - Explicit scenarios (for compact objects and cosmological settings) may be analytically worked out.
  - Regular solutions are tested according to various criteria.
  - Cosmological bouncing/loitering solutions also arise naturally.

Conclusions

- Metric-affine RBGs have unique features:
  - Second-order field equations, no ghosts.
  - Compatible with $c = c_g$ and two tensor polarizations.
  - Explicit scenarios (for compact objects and cosmological settings) may be analytically worked out.
  - Regular solutions are tested according to various criteria.
  - Cosmological bouncing/loitering solutions also arise naturally.
- Mapping procedure:
  - An Einstein frame representation is always possible in the Palatini formulation. Exploit the duality with GR to find new solutions in RBGs out of GR ones!.
  - Important technical progress $\rightarrow$ established numerical methods applicable. Degeneracies btwn modified gravity and GR with exotic sources may arise.
  - Rotating BHs: normal modes, perturbations of all kinds, black hole shadows, ringdown physics, echoes, ...

- A field plenty of opportunities!:
  - Dynamical scenarios?. Numerical expertise is welcome (any offer?...).
  - Astrophysical significance of horizonless compact objects disguised as black holes: new GW signatures such as (echoes? [Cardoso and Pani, 1709.01525[gr-qc]]).
  - Stellar structure scenarios and observational discriminators with respect to GR (see Aneta Wojnar's talk coming next).
  - Unified (bouncing) early-time cosmologies and late-time cosmologies?.

THANK YOU VERY MUCH FOR YOUR ATTENTION!