Entropy Bounds and Nonlinear Electrodynamics

M. L. Peñafiel

in collaboration with F. T. Falciano, S. E. P. Bergliaffa & J. C. Fabris

V José Plínio Baptista School of Cosmology

Outline

Introduction

Black Holes (BHs) are fundamental objects in the study of General Relativity (GR). There exist several important results regarding BHs such as

- BH uniqueness theorems: Kerr-Newman is the *most general* electrovacuum solution $\rightarrow$ All BHs are characterized uniquely by three parameters (M, Q, J)

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- BH mechanics possesses several formal analogies with the laws of thermodynamics at the classical level, $S \sim A$, which are confirmed at the quantum level via Hawking radiation.
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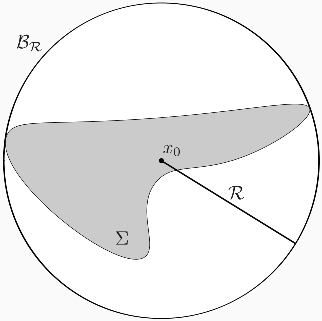
  - In order for the GSL to hold, an entropy-bearing object must respect some entropy bound. $\rightarrow$ Bekenstein's bound.

Bekenstein bounds

- Bound for the entropy of an object falling into a black hole, required for the fulfilment of the Generalized Second Law (GSL). The *most general* bound is (Bekenstein & Mayo, 1999)

$$S \leq \frac{2\pi\kappa_B}{\hbar c}\mathcal{R}\left(\sqrt{\mathcal{E}^2 - \frac{\mathcal{J}^2}{\mathcal{R}^2}} - \frac{q^2}{2\mathcal{R}}\right) \tag{1}$$

- Bound (1) is expected to be a *universal* bound. The equality is attained for the case of the Kerr-Newman black hole, and it cannot be further generalized in light of no-hair theorems.



Bekenstein bounds

- In the case where $S = 0$, (1) implies the inequalities (Dain, 2015)

$$\mathcal{E} \geq \frac{q^2}{2\mathcal{R}} \qquad ; \qquad \mathcal{E}(U) \geq \frac{|\mathcal{J}(U)|}{\mathcal{R}} \, . \tag{2}$$

  In particular, the first inequality is violated for some field configurations in Born-Infeld (BI) Electrodynamics[1].

- We intend to derive an entropy bound valid for the BI particle. In particular, analyzing the situation of a BI particle falling into a Schwarzschild[2] and a BI BH[3].

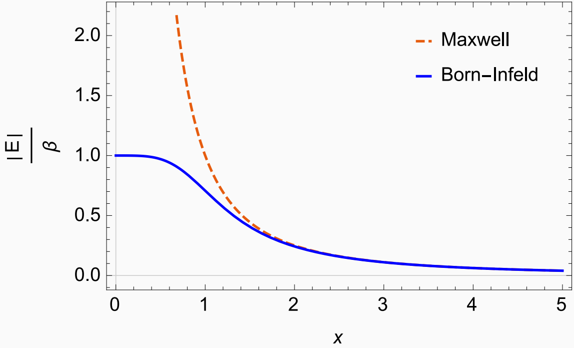
[1] MLP, F. T. Falciano, PRD, **96**, 125011, 2017.
[2] F. T. Falciano, MLP, S. E. P. Bergliaffa, PRD, **100**, 125008, 2019.
[3] F. T. Falciano, MLP, J. C. Fabris, PRD, **103**, 084046, 2021.

Born-Infeld (BI) Electrodynamics is given by the Lagrangian

$$\mathcal{L}_{\mathrm{BI}} = \frac{\beta^2}{4\pi}\left(1 - \sqrt{1 + \frac{F}{\beta^2} - \frac{G^2}{4\beta^4}}\right) . \tag{3}$$

This formulation of NLED offers two interesting results:

- Regular field and energy for the point particle.



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- Absence of birefringence.
- In the electrostatic case we have the relation

$$\mathbf{D} = \frac{\mathbf{E}}{\sqrt{1 - \frac{|\mathbf{E}|^2}{\beta^2}}} \qquad \Rightarrow \qquad \mathbf{E} = \frac{\mathbf{D}}{\sqrt{1 + \frac{|\mathbf{D}|^2}{\beta^2}}} . \tag{4}$$

- Additionally, the electrostatic potential for a point particle in flat spacetime is ($x := r/\sqrt{q/\beta}$)

$$\phi_{\mathrm{BI}}(r) = -\int_{\infty}^{r} d\mathbf{r}\cdot\mathbf{E} = \frac{\sqrt{q\beta}}{x}\ {}_2F_1\left[\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -\frac{1}{x^4}\right] .$$

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$$\phi_{\mathrm{BI}}(0) = \frac{\Gamma^2\left(\frac{1}{4}\right)}{4\sqrt{\pi}}\sqrt{q\beta} \equiv \frac{q}{\lambda_{\mathrm{BI}}} .$$

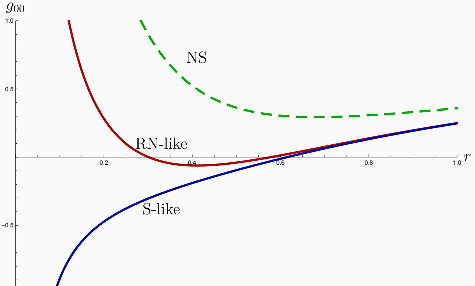
Einstein-Born-Infeld Black Hole

The EBI BH is a solution to GR + BI electrodynamics, whose time-time component is given by

$$g_{00} = 1 - \frac{r_s}{r} + \frac{r_q^2}{r^2} f\left(\frac{r}{\sqrt{r_q r_\beta}}\right) \, . \tag{5}$$

$$f(x) = \frac{2}{3}x^4\left(1 - \sqrt{1 + \frac{1}{x^4}}\right) + \frac{4}{3}\, {}_2F_1\left(\frac{1}{2}, \frac{1}{4}, \frac{5}{4}; -\frac{1}{x^4}\right)$$

Depending on the field configuration, the EBI BH can have either 2 (RN-like), 1 (S-like / extreme) or 0 (NS) horizons.



(Interlude) Maxwell particle in Schwarzschild spacetime

The electrostatic potential for a Maxwell particle located at a position $\bar{a}$, immersed in a Schwarzschild spacetime is exact, whose solution is (in isotropic coordinates)

$$\phi_L = \underbrace{\frac{e\left(\sqrt{\mu\left(\bar{r},\theta\right)} + \frac{m}{2\bar{a}}\frac{1}{\sqrt{\mu(\bar{r},\theta)}}\right)^2}{\bar{r}\left(1+\frac{m}{2\bar{a}}\right)^2\left(1+\frac{m}{2\bar{r}}\right)^2}}_{\text{Copson (1928)}} + \underbrace{\frac{em}{\bar{r}\left(1+\frac{m}{2\bar{r}}\right)^2\bar{a}\left(1+\frac{m}{2\bar{a}}\right)^2}}_{\text{Linet (1976)}} \ . \tag{6}$$

with

$$\mu\left(\bar{r},\theta\right) := \left(\frac{\left(\bar{r}+\bar{b}\right)^2 + 2\bar{r}\bar{b}\left(1-\cos\theta\right)}{\left(\bar{r}+\bar{a}\right)^2 + 2\bar{a}\bar{r}\left(1-\cos\theta\right)}\right)^{1/2} \ ; \quad \bar{b} = \frac{m^2}{2\bar{a}}$$

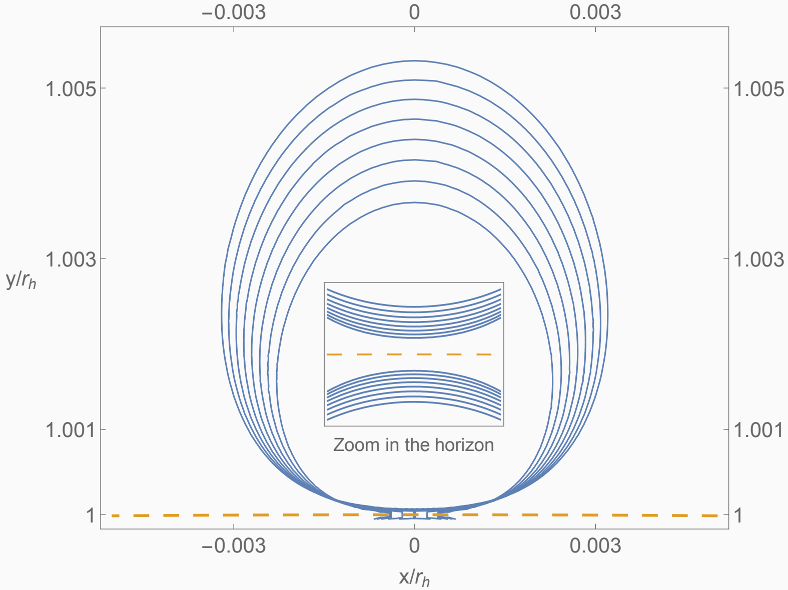
$\bar{b}$ can be thought as the location of an image particle inside the horizon. The horizon acts as a conducting surface.

Born-Infeld Electrodynamics in curved spacetime

What happens when the BI particle is in a curved background?

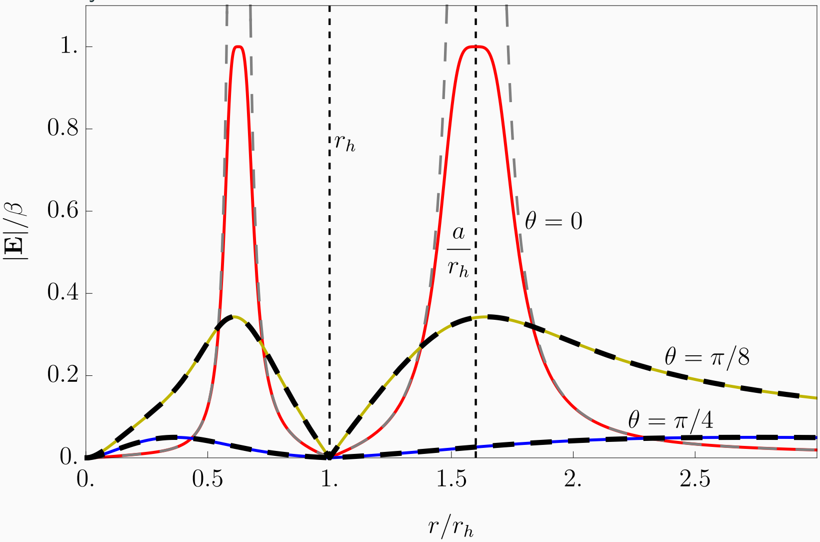
- It is not possible to find a closed analytic solution for the electrostatic potential for BI electrodynamics. However, it is possible to approximate this solution in use of the Maxwell situation and given that:
    1. It is expected that nonlinearities become effective near the particle's position.
    2. The horizon acts as a conducting surface (black hole polarization).
- Even though $\nabla \cdot \mathbf{D} = 4\pi$, $\nabla \times \mathbf{D} \neq 0$, in a spherically symmetric background $\mathbf{D}$ can be written as a gradient of an auxilliary scalar potential, $\mathbf{D} = -\nabla\psi$ (see Theorem 1 of [2,3].)

Born-Infeld Electrodynamics in curved spacetime



Born-Infeld Electrodynamics in curved spacetime

We can see the behavior of the electric field both in Maxwell and BI electrodynamics.



Bekenstein Bound for Born-Infeld Electrodynamics.

Our strategy is to lower the BI particle arbitrarily close to the horizon and find a lower bound for the conserved energy of the particle.

- Once the particle is absorbed, we calculate the change in the horizon area (defining $\lambda_\beta \equiv 4\sqrt{\pi}\Gamma\left(\frac{1}{4}\right)^2\sqrt{e/\beta}$).

$$\delta A \geq \frac{4\pi G}{c^2}\begin{cases} 2m\mathcal{R} + e^2\left(1 + \sqrt{\frac{2\mathcal{R}r_s}{\lambda_\beta^2}}\right) & \text{Deep BI (S-like)} \\ 2m\mathcal{R} - e^2\left(1 - \sqrt{\frac{2\mathcal{R}r_s}{\lambda_\beta^2}}\right) & \text{Superficial BI (RN-like)} \end{cases}$$

  which depends on the black hole's parameters!
- In order to obtain a bound that depends on the body's parameters only, we can use the inequality $r_s \geq \mathcal{R}$ and obtain

$$S_{\mathrm{BI}} \leq \frac{2\pi\kappa_B}{\hbar c}\begin{cases} mc^2\mathcal{R} + \frac{e^2}{2}\left(1 + \sqrt{2}\frac{\mathcal{R}}{\lambda_\beta}\right) & \text{Deep BI (S-like)} \\ mc^2\mathcal{R} + \frac{e^2}{2}\left(-1 + \sqrt{2}\frac{\mathcal{R}}{\lambda_\beta}\right) & \text{Superficial BI (RN-like)} \end{cases} \tag{7}$$

  Which demonstrates that nonlinear interactions can modify Bekenstein's bound.

Conclusions & Perspectives

- We have shown that in the case of NLED in a spherically symmetric background the **D** field is completely defined in terms of the gradient of an auxilliary scalar potential ψ.
- We have demonstrated that the Bekenstein bound for a charged object obeying NLED is modified due to the regularity of the field.
- The full solution for the potential for BI electrodynamics in Schwarzschild allows for the study of the self-force problem within NLED.
- The full solution for the potential for BI electrodynamics in EBI spacetime allows for testing possible scenarios for the violation of WCCC.

Thank you!

For more information on our work see

[1] https://arxiv.org/abs/1712.02296

[2] https://arxiv.org/abs/1911.08467

[3] https://arxiv.org/abs/2103.14109