On the Massive Fierz-Pauli Lagrangian obtained from the Siegel-Zwiebach Action in String Theory Context Symmetries and Propagators

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April 13, 2023

Summary

1 - Massive Gravity

Lagrangian density

$$\mathcal{L}_{MFP} = \frac{1}{4}h_{\mu\nu}(\partial^2 - m^2)h^{\mu\nu} + \frac{1}{2}\partial^\nu h_{\mu\nu}\left(\partial^\rho h^\mu_{\ \rho} - \partial^\mu h\right) - \frac{1}{4}h(\partial^2 - m^2)h\,, \tag{1}$$

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$$h \equiv h^\mu_{\ \mu}$$

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M. Fierz, W. Pauli, Proc. R. Soc. Lond. A 173 (1939) 211.

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Hamiltonian density

$$\mathcal{H}_{MPF} = \pi_{ij}\pi_{ij} - \frac{1}{D-2}\left[\pi_{ii} + \frac{1}{2}\partial_i h^{0i}\right]^2 - \frac{1}{4}(\partial_i h^{0i})^2 + \frac{1}{4}\partial_i h^{\mu\nu}\partial^i h_{\mu\nu} - \frac{1}{2}\partial^i h_{\mu i}\partial^j h^\mu_j + \frac{1}{2}\partial^i h_{ij}\partial^j h - \frac{1}{4}\partial_i h\partial^i h + \frac{m^2}{4}(h_{\mu\nu}h^{\mu\nu} - h^2) \qquad (2)$$

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$$\phi_0 = \pi_{00} + \frac{1}{2}\partial_i h^{0i} \tag{3}$$

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$$\chi_0 = \frac{1}{2}\left(\partial_i\partial_i h^{jj} - \partial_i\partial_j h^{ij} - m^2 h^{ii}\right) \tag{5}$$

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$$\Psi = -\partial^i\partial_j\pi^{ij} - \partial_i\partial_i\partial_j h_{0j} - \frac{D-3}{D-2} m^2 \partial_i h_{0i} + \frac{m^2}{D-2} m^2 \pi^{ii} \tag{7}$$

$$\Lambda = \frac{m^2}{2}(3h_{00} - h_{ii}) \tag{8}$$

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2 - Siegel-Zwiebach Action

From BRST invariance in the bosonic string, by component fields expansion, Siegel and Zwiebach obtained the spin-two action

$$S_{SZ} = \int d^{26}x \, \frac{1}{2} \left\{ \frac{1}{2} h_{\mu\nu}(\partial^2 - m^2) h^{\mu\nu} + B_\mu(\partial^2 - m^2) B^\mu - \theta(\partial^2 - m^2)\theta \right.$$

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& \left.+(\partial^\nu h_{\mu\nu}+\partial_\mu\theta-mB_\mu)^2+\left(\frac{mh}{4}+\frac{3m\theta}{2}+\partial_\mu B^\mu\right)^2\right\} \qquad (13)
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The canonical quantization of this model has been studied recently in
H. Park and T. Lee, Nucl. Phys. B **954**, 115006 (2020).

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Gauge invariance

$$\delta h_{\mu\nu} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu - \frac{1}{2} m \eta_{\mu\nu} \epsilon \,, \tag{14}$$

$$\delta B_\mu = \partial_\mu \epsilon + m \epsilon_\mu \,, \quad \delta\theta = -\partial_\mu \epsilon^\mu + \frac{3}{2} m \epsilon \,, \tag{15}$$

$$\begin{aligned} S_{SZ} &= \int d^{26}x \, \frac{1}{2} \left\{ \frac{1}{2} h_{\mu\nu} (\partial^2 - m^2) h^{\mu\nu} + B_\mu (\partial^2 - m^2) B^\mu - \theta (\partial^2 - m^2) \theta \right. \\ & \left. + (\partial^\nu h_{\mu\nu} + \partial_\mu \theta - m B_\mu)^2 + \left(\frac{mh}{4} + \frac{3m\theta}{2} + \partial_\mu B^\mu \right)^2 \right\} \end{aligned}$$

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with

$$\begin{aligned} \delta\mathcal{L}_{MFP} &= (26-D)\left[\frac{m^3 h\epsilon}{32} - \frac{m^2}{8}B_\mu\partial^\mu\epsilon - \frac{3m^3}{16}\theta\epsilon\right] \\ & m^4\frac{(18-D)(26-D)}{128}\epsilon^2 \\ & + \frac{m^2(26-D)}{16}\epsilon\Box\epsilon \end{aligned} \tag{17}$$

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Reminiscent from anomaly cancelation and BRST closure in the bosonic string.

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This suggests (18) should be a suitable gauge-fixing condition for S_{SZ}.

In fact, given an arbitrary field configuration $(h^{\mu\nu}, B^{\mu}, \theta)$, by choosing the gauge parameters

$$
\begin{aligned}
\epsilon &= \frac{1}{5m}\left(\theta + \frac{h}{2}\right) , \\
\epsilon^{\mu} &= -\frac{B^{\mu}}{m} - \frac{1}{5m^2}\partial^{\mu}\left(\theta + \frac{h}{2}\right) ,
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it is always possible to pass to $(h'^{\mu\nu}, B'^\mu, \theta')$ and achieve (18), namely,

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B'^\mu = 0, \quad \theta' = -\frac{h'}{2} , \tag{22}
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However, in order to rigorously implement this result at quantum level, one needs to write down the complete Green's functions generating functional, by taking advantage of the BRST symmetries.

5 - BRST Symmetries

Introducing $(c, \bar{c})$, $(c^\mu, \bar{c}_\mu)$ and (b, b^μ), we obtain

$$\begin{gathered} sh_{\mu\nu} = \partial_\mu c_\nu + \partial_\nu c_\mu - \frac{m}{2}\eta_{\mu\nu} c \,, \\ sB_\mu = \partial_\mu c + mc_\mu \,, \quad s\theta = -\partial_\mu c^\mu + \frac{3}{2} mc \,, \\ sc_\mu = 0 \,, \quad s\bar{c}_\mu = b_\mu \,, \qquad sb_\mu = 0 \,, \\ sc = 0 \,, \quad s\bar{c} = b \,, \quad sb = 0 \,, \end{gathered} \tag{23}$$

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which can be checked to be nillpotent

$$s^2 = 0 \tag{24}$$

with ghost numbers

$$\text{gh } s = \text{gh } c = \text{gh } c_\mu = 1 \tag{25}$$

$$\text{gh } \bar{c} = \text{gh } \bar{c}_\mu = -1 \tag{26}$$

Now the proper inclusion of the additional terms

$$S_{gg} = \int d^D x \left[b_\mu B^\mu + b\left(\theta + \frac{h}{2}\right) - 5m\bar{c}c + \bar{c}_\mu \left(\partial^\mu c + mc^\mu\right)\right] , \quad (27)$$

allows for a consistent BRST functional quantization.

Adding S_{gg} to the classical action (13), we write the generating functional

$$Z = \int d\mu\, e^{-i\,\{S_{SZ} + S_{gg}\}} \tag{28}$$

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with integration measure

$$d\mu \equiv [dh_{\mu\nu}][dB_\mu][d\theta][dc][dc_\mu][d\bar{c}][d\bar{c}_\mu][db][db_\mu]\,, \tag{29}$$

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which can be explicitly checked to be BRST invariant. Now, from (28), we may calculate the massive graviton propagator as

$$\begin{aligned} G_{\alpha\beta,\sigma\lambda} \;=\; & \frac{i}{p^2+m^2}\left\{ \frac{2}{25}\left(\eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2}\right)\left(\eta_{\sigma\lambda} + \frac{p_\sigma p_\lambda}{m^2}\right)\right. \\ & - \left(\eta_{\alpha\sigma} + \frac{p_\alpha p_\sigma}{m^2}\right)\left(\eta_{\beta\lambda} + \frac{p_\beta p_\lambda}{m^2}\right) \\ & \left. - \left(\eta_{\alpha\lambda} + \frac{p_\alpha p_\lambda}{m^2}\right)\left(\eta_{\beta\sigma} + \frac{p_\beta p_\sigma}{m^2}\right)\right\} \end{aligned} \tag{30}$$

It is clear that the $m \to 0$ limit is not well-defined.

To circumvent this issue, a transverse-tracelessness gauge, similar to the Lorentz condition in QED, for $h_{\mu\nu}$ along with a corresponding trace nulity demand, namely,

$$\partial_\mu h^\mu_{\ \nu} = 0 \quad \text{and} \quad h = 0\,, \tag{31}$$

can be worked out.

For gauge accessibility, the transformation parameters should satisfy the simultaneous conditions

$$\partial_\mu \epsilon^\mu - \frac{13m}{2}\epsilon = -h/2 \tag{32}$$

and

$$\Box \epsilon_\mu + \partial_\mu \partial_\nu \epsilon^\nu - \frac{m}{2}\partial_\mu \epsilon = -\partial_\nu h^\nu_{\ \mu} \tag{33}$$

Given an arbitrary field configuration, by choosing gauge parameters as

$$\epsilon = \frac{2}{25m}\left[h - \Box^{-1}\partial^\mu\partial^\nu h_{\mu\nu}\right] \tag{34}$$

$$\epsilon^\mu = \frac{12}{25}\Box^{-1}\partial^\mu\partial^\rho\partial^\nu h_{\rho\nu} + \frac{1}{50}\Box^{-1}\partial^\mu h - \Box^{-1}\partial_\mu h^{\mu\nu} \tag{35}$$

it is always possible to perform a gauge transformation

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu - \frac{1}{2}m\eta_{\mu\nu}\epsilon\,, \tag{36}$$

such that

$$\partial_\mu h'^\mu_{\ \nu} = 0, \quad h' = 0 \tag{37}$$

This gauge can be realized in a functional quantization framework in a very similar fashion to the Landau gauge in QCD and its Abelian version in QED.

$$
\begin{aligned}
S \;=\; & \int d^{26}x\,\frac{1}{2}\left\{\frac{1}{2}h_{\mu\nu}(\partial^2-m^2)h^{\mu\nu}+B_\mu(\partial^2-m^2)B^\mu-\theta(\partial^2-m^2)\theta\right.\\
& +(\partial^\nu h_{\mu\nu}+\partial_\mu\theta-mB_\mu)^2+\left(\frac{mh}{4}+\frac{3m\theta}{2}+\partial_\mu B^\mu\right)^2\\
& +\frac{\zeta}{2}b^2+bh+\frac{\xi}{2}b^\mu b_\mu+b^\mu\partial_\nu h^\nu_\mu+\bar{c}^\mu\Box c_\mu+\bar{c}^\mu\partial_\mu\partial_\nu c\\
& \left.-\frac{m}{2}\bar{c}^\mu\partial_\mu c+2\bar{c}\partial_\mu c^\mu-13m\bar{c}c\right\} \qquad (38)
\end{aligned}
$$

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Z=\int d\mu\, e^{-i\,\{S_{SZ}+S_{gg}\}} \qquad (39)
$$

Functional integration in the Nakanishi-Lautrup fields b and b^μ leads to the familiar gauge-fixing terms

$$-\frac{h^2}{2\zeta} \quad \text{and} \quad -\frac{1}{2\xi}(\partial_\nu h^\nu_\mu)(\partial_\rho h^\rho_\mu)\,. \tag{40}$$

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Hence, we may finally obtain the small mass graviton propagator.

In the limit $\xi, \zeta \to \infty$ we have

$$\begin{aligned} G^{\alpha\beta,\sigma\lambda}(m) \;=\; & \frac{i}{p^2+m^2}\left\{\frac{2}{25}\left(\eta^{\alpha\beta}-\frac{p^\alpha p^\beta}{p^2}\right)\left(\eta^{\sigma\lambda}-\frac{p^\sigma p^\lambda}{p^2}\right)\right. \\ & -\left(\eta^{\alpha\sigma}-\frac{p^\alpha p^\sigma}{p^2}\right)\left(\eta^{\beta\lambda}-\frac{p^\beta p^\lambda}{p^2}\right) \\ & \left. -\left(\eta^{\alpha\lambda}-\frac{p^\alpha p^\lambda}{p^2}\right)\left(\eta^{\beta\sigma}-\frac{p^\beta p^\sigma}{p^2}\right)\right\} \end{aligned} \tag{41}$$

which has a pretty well defined massless limit and satisfies the further properties

$$p_\alpha G^{\alpha\beta,\sigma\lambda}(m) = p_\beta G^{\alpha\beta,\sigma\lambda}(m) = 0 \tag{42}$$

$$p_\sigma G^{\alpha\beta,\sigma\lambda}(m) = p_\lambda G^{\alpha\beta,\sigma\lambda}(m) = 0 \tag{43}$$

and

$$G^{\alpha}{}_{\alpha}{}^{,\sigma\lambda}(m) = G^{\alpha\beta,\sigma}{}_{\sigma}(m) = 0 \tag{44}$$

Conclusion and Final Remarks

- We have discussed a gauge-invariant massive gravity model originated in the bosonic string framework.
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- The massless limit is not well-defined in the unitary gauge.

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- A transverse traceless gauge condition leads to a consistent small mass graviton propagator.

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- We have performed a rigorous BRST symmetry analysis, working out two specific gauge-fixing conditions.
- The massless limit is not well-defined in the unitary gauge.
- A transverse traceless gauge condition leads to a consistent small mass graviton propagator.
- As a next step we are connecting the Green functions in the two discussed gauges by finite field BRST transformations (FFBRST) (joint work in progress with Vipul Pandey).

References

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