Cosmic and superconducting strings

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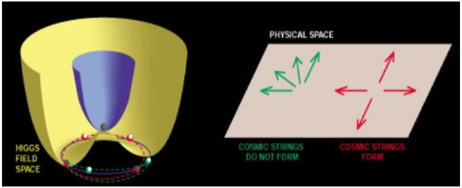
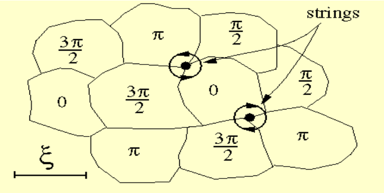
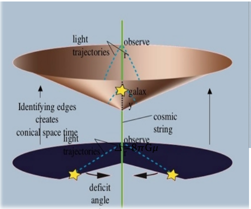
Outline

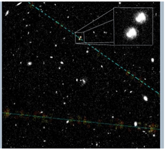
- Cosmic strings (as field theoretical solutions)...
  - form whenever a U(1) symmetry is spontaneously broken (Kibble mechanism)





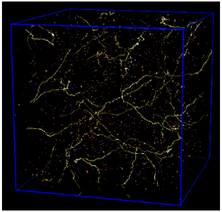
- **topological** solitons, i.e. possess a topological charge
- width inversely proportional to mass
- ruled out as (main) source of primordial density perturbations (by CMB data)
- have analogues in condensed matter: *flux tubes* in type-II superconductors, *vortices* in superfluid helium
- space-time has *deficit angle* $\delta = 8\pi GU$, U: energy per unit length $\Rightarrow$ **gravitational lensing**





Numerical simulation
for the Hubble deep field

- Cosmic string loops
  - form in the evolution of cosmic string networks
  - decay due to string tension under emission of gravitational radiation $\Rightarrow$ important energy loss mechanism $\Rightarrow$ contribution of strings to total energy density of the universe is kept constant (“scaling regime”)



  - BUT: loops could be stable if they carry additional internal structure (bosonic or fermionic currents)

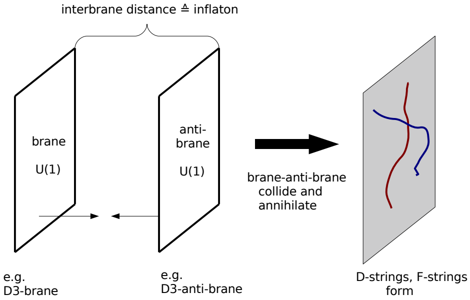
    $\Rightarrow$ **superconducting strings**

- Fundamental (F-) strings (of string theory) ...
  - have zero width
  - have tension close to the Planck scale
  - end on D-branes
  - D1-brane $=$ D-string

Connection between cosmic strings and fundamental strings ???

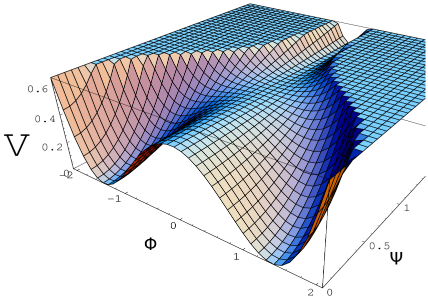
- **NO**: perturbative strings as cosmic strings ruled out (Witten (1985))

- **YES**: cosmic superstrings are formed in inflationary models originating from string theory
  - D-, F- and bound states of p F-strings and q D-strings (p-q-strings) are formed in **brane inflation**



Jones, Stoica, Tye (2002); Sarangi, Tye (2002)

- ... and also: Hybrid inflation (Linde (1994))



- two scalar fields
- inflation ends due to spontaneous symmetry breaking
- **cosmic strings form generically at the end of hybrid inflation in Supersymmetric Grand Unified Theories** (Jeannerot, Rocher, Sakellariadou (2003))

In agreement with PLANCK 2013?

- Inflationary models
  - Data favours "simple", single-field models
  - BUT: where do these models come from?
  - String inflationary models are **not** ruled out (Burgess, Cicoli, Quevedo, JCAP 11 (2013))
- Abelian-Higgs strings (Ade et al. [Planck Collaboration], arxiv: 1303.5085)

$$GU/c^2 \leq 3.2 \cdot 10^{-7} \quad , \quad f_{10} = 0.028$$

U: energy per unit length
f_{10}: fraction of contribution to power spectrum at $\ell = 10$

The $U(1) \times U(1)$ model

$U(1) \times U(1)$ gauge field model with Lagrangian density

$$\mathcal{L} = D_\mu \phi (D^\mu \phi)^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + D_\mu \xi (D^\mu \xi)^* - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - V(\phi, \xi)$$

with

$$D_\mu \phi = \nabla_\mu \phi - ie_1 A_\mu \phi \quad , \quad D_\mu \xi = \nabla_\mu \xi - ie_2 B_\mu \xi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \quad H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

where
ϕ, ξ: complex scalar fields
A_μ, B_μ: U(1) gauge fields
e_1, e_2: gauge couplings

The potential

$$V(\phi,\xi) = \frac{\lambda_1}{4}\left(\phi\phi^* - \eta_1^2\right)^2 + \frac{\lambda_2}{4}\left(\xi\xi^* - \eta_2^2\right)^2 + \lambda_3\left(\phi\phi^* - \eta_1^2\right)\left(\xi\xi^* - \eta_2^2\right)$$

$\eta_1 \neq 0$, $\eta_2 \neq 0$

$\lambda_1 > 0$, $\lambda_2 > 0$: self interaction couplings

$\lambda_3 > -\frac{1}{2}\sqrt{\lambda_1\lambda_2}$: interaction coupling

The potential

1. $4\lambda_3^2 < \lambda_1\lambda_2$
   - Minimum at $(\phi^2, \xi^2) = (\eta_1^2, \eta_2^2)$

     $\Rightarrow$ spontaneous symmetry breaking of both U(1)s
   - **describes two cosmic strings**
     - for $\lambda_3 = 0$: two non-interacting Nielsen-Olesen strings (Nielsen, Olesen (1973))
     - $\lambda_3 < 0$: toy model for *bound states* of *p* F-strings and *q* D-strings, i.e. *p*-*q*-strings (Saffin (2005))
2. $4\lambda_3^2 > \lambda_1\lambda_2$
   - Minimum at $(\phi^2, \xi^2) = (0, \eta_2^2 + \frac{2\lambda_3}{\lambda_2}\eta_1^2)$

     or $(\phi^2, \xi^2) = (\eta_1^2 + \frac{2\lambda_3}{\lambda_1}\eta_2^2, 0)$

     $\Rightarrow$ spontaneous symmetry breaking of only one of the U(1)s
   - **describes superconducting strings**

(Interacting) Cosmic strings

- Cylindrically symmetric Ansatz

$$\phi(\rho,\varphi) = \eta_1 h(\rho) e^{in\varphi} \ , \ \ \xi(\rho,\varphi) = \eta_2 f(\rho) e^{im\varphi}$$

$$A_\mu dx^\mu = \frac{1}{e_1}\left(n - P(\rho)\right) d\varphi \ , \ \ B_\mu dx^\mu = \frac{1}{e_2}\left(m - R(\rho)\right) d\varphi$$

  n, m: winding numbers

- magnetic fields $\vec{B}_i = B_{i,z}\vec{e}_z$, $i = 1, 2$ with

$$B_{1,z} = -\frac{1}{e_1}\frac{dP/d\rho}{\rho} \ , \ \ B_{2,z} = -\frac{1}{e_2}\frac{dR/d\rho}{\rho}$$

- quantized magnetic fluxes

$$\Phi_1 = -\frac{2\pi n}{e_1} \ , \ \ \Phi_2 = -\frac{2\pi m}{e_2}$$

$\lambda_3 = 0$: Nielsen-Olesen strings

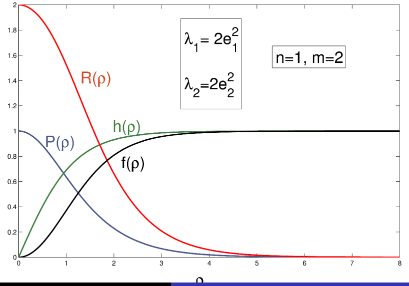
- scalar core width $\sim (\text{Higgs mass})^{-1} = M_{H,i}^{-1} = (\sqrt{2\lambda_i}\eta_i)^{-1}$, $i = 1, 2$
- width of flux tubes $\sim (\text{gauge boson mass})^{-1} = M_{W,i}^{-1} = (e_i\eta_i)^{-1}$, $i = 1, 2$
- $T_0^0 = T_z^z$: energy per unit length U = string tension T
- energy per unit length $U = U_1 + U_2$ where

$$U_1 = 2\pi\eta_1^2 n\, g_1(M_{H,1}^2/M_{W,1}^2) \quad , \quad U_2 = 2\pi\eta_2^2 m\, g_2(M_{H,2}^2/M_{W,2}^2)$$

$g_i = O(1)$ and weak dependence on $M_{H,i}^2/M_{W,i}^2$

$\lambda_3 = 0$: Nielsen-Olesen strings

- for $\lambda_1 = 2e_1^2$, $\lambda_2 = 2e_2^2 \Rightarrow$ **BPS limit**:
  - width of flux tube = width of scalar core
  - saturate energy bound $U_1 = 2\pi\eta_1^2 n$ and $U_2 = 2\pi\eta_2^2 m$
  - Equations of motion: 1.order
- Typical solution: BPS and $n = 1$, $m = 2$



$\lambda_3 < 0$: *p*-*q*-strings

(Saffin (2005))

- $p \to n$, $q \to m$
  $\Rightarrow$ interaction of *n*-string with *m*-string
- no longer BPS
- binding increases with
  - decreasing $-1 < \lambda_3 < 0$
  - increasing *n* and *m*
- formation of bound states $\Rightarrow$ effective energy loss mechanism to reach scaling regime for (p, q) networks

Gravitational effects

- Action

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} + \mathcal{L} \right)$$

- Ansatz for the metric

$$ds^2 = N^2(\rho)dt^2 - d\rho^2 - L^2(\rho)d\varphi^2 - N^2(\rho)dz^2$$

- weak gravitational fields

$$ds^2 = dt^2 - d\tilde{\rho}^2 - \tilde{\rho}^2 d\tilde{\varphi}^2 - dz^2$$

where $\tilde{\varphi} = (1 - 4GU)\varphi$ with $0 \leq \tilde{\varphi} < 2\pi(1 - 4GU)$

  - locally identical to flat space-time
  - globally: **deficit angle** $\delta = 8\pi GU$

Gravitational effects

- globally regular for $\delta < 2\pi$, i.e. $G \lesssim (8\pi\eta_1^2 n)^{-1}$
  - **String solutions**

$$N(\rho \to \infty) \to c_1 \quad , \quad L(\rho \to \infty) \to c_2\rho + c_3$$

    where c_1, c_2, c_3 constants and $\delta = 2\pi(1 - c_2)$
  - **Melvin solutions** (have no flat space-time counterpart):

$$N(\rho \to \infty) \to a_1\rho^{2/3} \quad , \quad L(\rho \to \infty) \to a_2\rho^{-1/3}$$

    where a_1, a_2 constants

Gravitational effects

- For $G > (8\pi\eta_1^2 n)^{-1}$: singular solutions
  - **supermassive string solutions** (Garfinkle, Laguna (1989); Ortiz (1991))

$$\delta > 2\pi \quad \text{with} \quad L(\rho = \rho_s) = 0 \quad , \quad N(\rho = \rho_s) \text{ finite}$$

  - **Kasner solutions** (Christensen, Larsen, Verbin (1999))

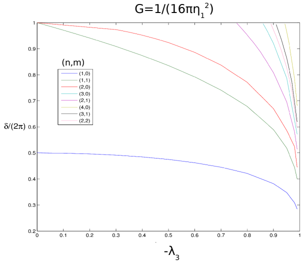
$$L(\rho = \rho_k) = \infty \quad , \quad N(\rho = \rho_k) = 0$$

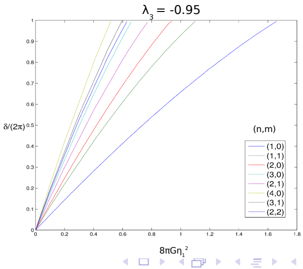
- for $\lambda_1 = 2e_1^2$, $\lambda_2 = 2e_2^2$, $\lambda_3 = 0$ **are still BPS** (Linet (1987))

Gravitating *p*-*q*-strings

(B.H. & J. Urrestilla (2008))

- G fixed
  - singular solutions can be made regular by decreasing λ_3
  - δ decreases for decreasing λ_3, i.e. increasing interaction
- λ_3 fixed
  - δ increases for G increasing





Cosmic strings in de Sitter

(B.H. & Y. Brihaye (2008))

- only one string: $R = f \equiv 0$, $m = 0$, $\lambda_2 = \lambda_3 = \eta_2 = 0$
- de Sitter background space-time

$$ds^2 = \left(1 - \frac{r^2}{l^2}\right) dt^2 - \left(1 - \frac{r^2}{l^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

de Sitter radius $l = \sqrt{3/\Lambda}$, Λ positive cosmological constant

- For $\Lambda > 24M_W^2$, $\Lambda > 2/3M_H^2$

$$\begin{aligned} P(\rho >> 1) &= P_0\rho^a \\ f(\rho >> 1) &= 1 - f_0\rho^b \end{aligned}$$

P_0, F_0, a, b : parameter dependent constants

Cosmic strings in de Sitter

(B.H. & Y. Brihaye (2008))

- For $\Lambda < 24 M_W^2$, $\Lambda < 2/3 M_H^2$

$$\begin{aligned} P(\rho >> 1) &= P_0 \rho^{-1/2} \sin\left(\sqrt{-R_1/2}\log\rho + \phi_1\right) \\ f(\rho >> 1) &= 1 - f_0 \rho^{-3/2} \sin\left(\sqrt{-R_2/2}\log\rho + \phi_2\right) \end{aligned}$$

P_0, f_0, R_1, R_2, ϕ_1, ϕ_2: parameter dependent constants

$\Rightarrow$ **oscillating**

- deficit angle

$$\delta \approx 8\pi G U \left(1 + \frac{16}{9}\frac{\Lambda}{M_H^2}\right)$$

increases for Λ increasing

Superconducting strings

- let $e_2 = 0$, only one U(1) is gauged
- Ansatz for “standard” string

$$\phi(\rho, \varphi) = \eta_1 h(\rho) e^{in\varphi} \quad , \quad A_\mu dx^\mu = \frac{1}{e_1} (n - P(\rho))\, d\varphi$$

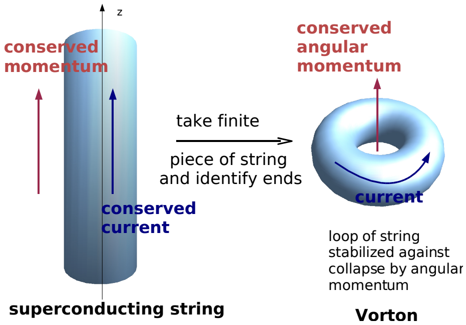
- “neutral” current carrier field

$$\xi(\rho, z, t) = \eta_1 f(\rho) e^{i(kz - \omega t)} \quad , \quad k \text{ integer}$$

$f(\rho \to \infty) \to 0 \Rightarrow$ unbroken U(1)
$f(\rho = 0) \neq 0 \Rightarrow$ “condensate” inside the string

Conserved quantities

- Conserved Noether current in z-direction $J_z = k \int dx^3 f^2$
- Conserved Noether charge $Q = \omega \int dx^3 f^2$
- Conserved momentum $P_z = \omega J_z = \omega k \int dx^3 f^2$



Parameter restrictions

- only one U(1) broken $\Rightarrow 4\lambda_3^2 > \lambda_1\lambda_2$
- minimum of potential at $(\phi^2, \xi^2) = (\eta_1^2 + \frac{2\lambda_3}{\lambda_1}\eta_2^2, 0)$
  $$\Rightarrow \lambda_1\eta_1^4 > \lambda_2\eta_2^4$$
- **New** restriction (B.H. & B. Carter (2008)):
  non-vanishing condensate inside string, i.e. $f(0) \neq 0$ only for
  $$c\lambda_3\eta_1^4 < \lambda_2\eta_2^4$$
  where $c = O(1)$ and $c \lesssim 2.8$

Stability of superconducting strings

Stability criterion (B. Carter (1989)):

speed of transversal (T) & longitudinal (L) perturbations real

$$c_{\mathrm{T}}^2 = \frac{T}{U} > 0 \quad , \qquad c_{\mathrm{L}}^2 = -\frac{dT}{dU} > 0$$

- energy per unit length

$$U = 2\pi \int \rho d\rho T^{00}$$

with $T^{00} = 2\omega^2 f^2 - \mathcal{L}$

- tension

$$T = -2\pi \int \rho d\rho T^{zz}$$

with $T^{zz} = 2k^2 f^2 + \mathcal{L}$

Stability of superconducting strings

Question: how does

$$L = 2\pi \int \rho d\rho \mathcal{L}$$

depend on ω and k?

- Suggestion (P. Peter (1992))

$$L = -m^2 - \frac{1}{2} m_*^2 \ln(1 + \delta_*^2 w) \quad , \quad w = k^2 - \omega^2$$

  - $\delta^* = 1/m_\xi$ with $m_\xi = 2\lambda_3\eta_1^2 - \lambda_2\eta_2^2$ mass of carrier field
  - $m^2 = -L_{w=0}$

A logarithmic equation of state

- Confirmation and evaluation of parameters (B.H. & B. Carter, PRD (2008))

$$m^2 = \pi \eta_1^2 \ln \left(\frac{M_H}{\sqrt{2} M_W} \right) \quad , \quad m_*^2 = \pi m_\xi^2 \frac{\lambda_2 \eta_2}{\lambda_1 \lambda_2 \eta_1}$$

valid for $M_W \ll M_H$

Semilocal strings

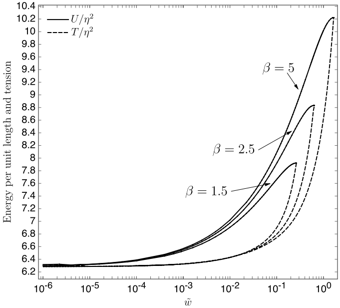
Cosmic strings in a $SU(2)_{global} \times U(1)_{local}$ model
(T. Vachaspati & A. Achucarro (1991))

$$\mathcal{L} = D_\mu \Phi (D^\mu \Phi)^\dagger - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} (\Phi^\dagger \Phi - \eta^2)^2$$

- $\Phi = (\phi, \xi)^{\mathrm{T}}$ complex doublet invariant under global SU(2) transformations
- $D_\mu \Phi = \partial_\mu \Phi - ieA_\mu \Phi$ with A_μ: U(1) gauge field

Stability of semilocal strings

(B.H. & P. Peter, PRD (2012))



$$c_{\mathrm{L}}^2 \equiv -\frac{dT}{dU} \leq 0$$

semilocal strings in $SU(2)_{global} \times U(1)_{local}$ are **always unstable**

Gravitational effects

- For **weak** gravitational fields

$$\begin{aligned} ds^2 &= (1+2\psi(\tilde{\rho}))\, dt^2 - (1-2\psi(\tilde{\rho}))\, dz^2 \\ &- d\tilde{\rho}^2 - (1-2G(U+T))^2\, \tilde{\rho}^2 d\varphi^2 \end{aligned}$$

where $\psi(\tilde{\rho}) = 4G(U-T)\ln(\tilde{\rho}/\tilde{\rho}_0)$

- **deficit angle** $\delta = 4\pi G(U+T)$
- **attractive force** towards string $\propto G(U-T)/\tilde{\rho}$

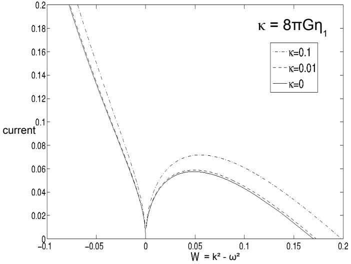
Gravitational effects

Full gravitating system (B.H. & F. Michel, PRD (2012))

- **No** boost symmetry along z-axis

$$ds^2 = N^2(\rho)dt^2 - d\rho^2 - L^2(\rho)d\varphi^2 - K^2(\rho)dz^2 \ , \ K(\rho) \neq N(\rho)$$

- bigger currents possible for gravitating superconducting strings



Summary

- Link between cosmic strings $\leftrightarrow$ fundamental strings
- to understand observational effects it is important to ...
  - ... understand gravitational effects (gravitational lensing,...)
  - ... know which type of strings form and how string networks evolve (CMB data, gravitational waves,...)
- in view of this ...
  - ... gauge and Higgs fields describing a cosmic string oscillate for all accepted values of the (positive) cosmological constant
  - ... gravitating *p*-*q*-strings can be made regular by increasing the interaction between the *p*- and the *q*-string
  - ... superconducting strings can be described by a logarithmic equation of state
  - ... semilocal strings are always unstable $\rightarrow$ cannot form stable vortons

And also....

Detection of cosmic strings?

- CMB data (power and polarization spectra)
  (e.g. P. Ade et al. [Planck Collaboration], arxiv: 1303.5085)
- motion of test particles in the space-time of cosmic strings
  (V. Diemer, B. H., E. Hackmann, C. Lämmerzahl, F. Michel, P. Sirimachan (2010-2013))

Selected References

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- ...

Beware: vortices are everywhere!

Muito obrigada pela sua atenção!