Stability of black holes and solitons in Anti-de Sitter space-time

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Models of Gravity

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References

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Y. Brihaye and B. Hartmann, JHEP 1203 (2012) 050

Y. Brihaye and B. Hartmann, Phys. Rev. D 85 (2012) 124024

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Y. Brihaye, B. Hartmann and S. Tojiev, CQG 30 (2013)

Outline

Outline

Introduction

Solitons

localized, finite energy, stable, regular solutions of non-linear equations (**particle-like**)

Topological solitons

- carry topological charge (non-trivial vacuum manifold)
- In theories with spontaneous symmetry breaking
- Examples: (Cosmic) Strings, Monopoles, Domain walls ...

Non-topological solitons

- carry globally conserved Noether charge
- In theories with continuous symmetry
- Examples: Q-balls, boson stars ...

Introduction

Black holes

- exact solutions of Einstein's equations (and its generalizations)
- solutions of 4-dimensional Einstein-Maxwell equations **uniquely** characterized by mass M, angular momentum J and charge Q (**"No hair conjecture"**)
- **Black hole thermodynamics**

$$\text{temperature} \quad T = \frac{\hbar\kappa}{2\pi k_{\rm B} c} \quad , \quad \text{entropy} \quad S = \frac{k_{\rm B} c^3 A}{4G\hbar}$$

  κ: surface gravity, A: horizon area, G: Newton's constant
  c: speed of light, $k_{\rm B}$: Boltzmann's constant, $\hbar$: Planck's constant

- Information encoded in surface $\Rightarrow$ **Holographic principle**

Holographic principle (Gauge/gravity duality)

- d-dim String Theory ⇔ (d-1)-dim SU(N) gauge theory
  (Maldacena; Witten; Gubser, Klebanov & Polyakov (1998))
- Couplings

$$\lambda = (\ell/l_s)^4 = g^2 N \quad , \quad g_s \sim g^2$$

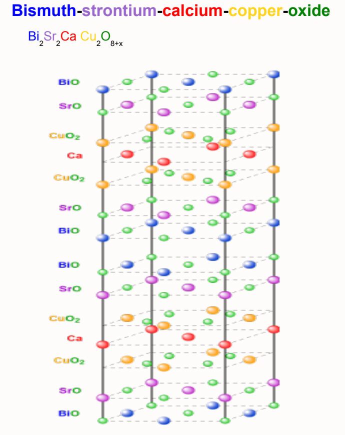
λ: 't Hooft coupling	N: rank of gauge group
ℓ: bulk scale/AdS radius	l_s: string scale
g: gauge coupling	g_s: string coupling

- classical gravity limit:

$$g_s \to 0 \qquad l_s/\ell \to 0$$

dual to **strongly coupled QFT with $N \to \infty$**

Application: high temperature superconductivity



Taken from wikipedia.org

- superconductivity associated to CuO_2-planes
- $T_c \approx 95$ K
- energy gap $\omega_g/T_c = 7.9 \pm 0.5$ (Gomes et al., 2007)

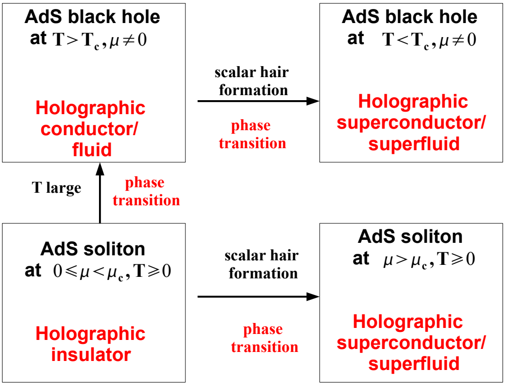
  $\Rightarrow$ compare to BCS value:

  $$\omega_g/T_c = 3.5$$

  $\Rightarrow$ **Strongly coupled** electrons in high-T_c superconductors?

Holographic phase transitions

T: temperature, μ: chemical potential



Introduction

- Instability of black holes in d-dimensional AdS_d (radius L) with respect to condensation of scalar field related to **Breitenlohner-Freedman bound** on mass m of scalar field (Breitenlohner & Freedman, 1982)

$$m^2 \geq m^2_{\mathrm{BF,d}} = -\frac{(d-1)^2}{4L^2} \Rightarrow \ AdS_d \ \ \textbf{stable}$$

Introduction

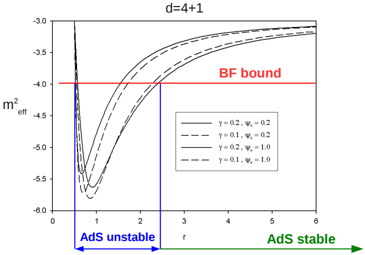
- Two type of instabilities
  - **uncharged** scalar field
    near-horizon geometry of **extremal** black holes given by $AdS_2 \times M_{d-2}$ (Robinson, 1959; Bertotti, 1959; Bardeen & Horowitz, 1999)
    if $m^2_{\mathrm{BF},2} > m^2 > m^2_{\mathrm{BF,d}} \Rightarrow$ asymptotic AdS_d stable, but (near-)extremal black hole forms scalar hair close to horizon
  - (Gubser, 2008) **scalar field charged** under $U(1)$, charge e

$$m^2_{\mathrm{eff}} = m^2 - e^2 |g^{tt}| A_t^2$$

    if $m^2 \geq m^2_{\mathrm{BF,d}}$: asymptotic AdS_d stable
    $e^2|g^{tt}|$ large close to horizon of black hole $\Rightarrow m^2_{eff} < m^2_{\mathrm{BF,d}}$ close to horizon $\Rightarrow$ black hole forms scalar hair

Introduction

- Example (Y. Brihaye & B.Hartmann, Phys.Rev. D81 (2010) 126008)



- Pioneering example: RNAdS black hole close to $T = 0$ unstable to form **scalar hair** close to horizon (Gubser, 2008)

Introduction

- **Anti-de Sitter/Conformal fiel theory (AdS/CFT) correspondence** applied: use AdS black hole and soliton solutions of classical gravity theory to describe phenomena appearing in strongly coupled Quantum Field Theories
- values of bulk field on AdS boundary $\Leftrightarrow$ sources for dual theory
- Applications
  - gravity $\Leftrightarrow$ condensed matter:
    holographic superconductors/superfluids
  - gravity $\Leftrightarrow$ Quantumchromodynamics (QCD)
    e.g. holographic description of quark-gluon plasma

Higher order curvature corrections

- *Coleman-Mermin-Wagner theorem*: **no** spontaneous symmetry breaking of continuous symmetry in (2+1) dim at finite temperature
- BUT: holographic superconductors have been constructed using (3+1)-dim black hole solutions of classical gravity (Hartnoll, Herzog & Horowitz (2008) and many more...)

$\Rightarrow$ *higher order curvature corrections* on gravity side ???

Outline

Action

Gauss-Bonnet gravity + scalar field ψ + U(1) gauge field A_μ

$$\begin{aligned} S &= \frac{1}{16\pi G}\int d^5x\sqrt{-g}\,\Big(16\pi G\mathcal{L}_{\rm matter} \\ &+ R - 2\Lambda + \frac{\alpha}{4}\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)\Big) \end{aligned}$$

with matter Lagrangian

$$\mathcal{L}_{\rm matter} = -\frac{1}{4}F_{MN}F^{MN} - (D_M\psi)^* D^M\psi - m^2\psi^*\psi \ , \ \ M,N = 0,1,2,3,4$$

$F_{MN} = \partial_M A_N - \partial_N A_M$ field strength tensor
$D_M\psi = \partial_M\psi - ieA_M\psi$ covariant derivative
$\Lambda = -6/L^2$: cosmological constant
G: Newton's constant , α: Gauss–Bonnet coupling
e: gauge coupling , m^2: mass of the scalar field ψ

Ansatz for static solutions

- Metric

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}d\Sigma^2_{k,3}$$

with

$$d\Sigma^2_{k,3} = \begin{cases} d\Xi^2_3 & \text{for} \quad k=-1 \quad \text{hyperbolic} \\ dx^2+dy^2+dz^2 & \text{for} \quad k=0 \quad \text{flat} \\ d\Omega^2_3 & \text{for} \quad k=1 \quad \text{spherical} \end{cases}$$

- Matter fields

$$A_M dx^M = \phi(r)dt \quad , \quad \psi = \psi(r)$$

Equations of motion

$$
\begin{aligned}
f' &= 2r\frac{k - f + 2r^2/L^2}{r^2 + 2\alpha(k-f)} \\
&- \gamma\frac{r^3}{2fa^2}\left(\frac{2e^2\phi^2\psi^2 + f(2m^2a^2\psi^2 + \phi'^2) + 2f^2a^2\psi'^2}{r^2 + 2\alpha(k-f))}\right) \\
a' &= \gamma\frac{r^3(e^2\phi^2\psi^2 + a^2f^2\psi'^2)}{af^2(r^2 + 2\alpha(k-f))} \\
\phi'' &= -\left(\frac{3}{r} - \frac{a'}{a}\right)\phi' + 2\frac{e^2\psi^2}{f}\phi \\
\psi'' &= -\left(\frac{3}{r} + \frac{f'}{f} + \frac{a'}{a}\right)\psi' - \left(\frac{e^2\phi^2}{f^2a^2} - \frac{m^2}{f}\right)\psi
\end{aligned}
$$

where $\gamma = 16\pi G$

Conditions on the horizon (Black holes)

- Regular horizon at $r = r_h > 0$

$$f(r_h) = 0 \quad , \quad a(r_h) \ \text{finite}$$

$$\phi(r_h) = 0 \ , \ \psi'(r_h) = \left.\frac{m^2\psi\left(r^2 + 2\alpha k\right)}{2rk + 4r/L^2 - \gamma r^3\left(m^2\psi^2 + \phi'^2/(2a^2)\right)}\right|_{r=r_h}$$

Behaviour on the AdS boundary

- Matter field

$$\phi(r \gg 1) = \mu - \frac{Q}{r^2} \ , \ \psi(r \gg 1) = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}$$

where

$$\lambda_\pm = 2 \pm \sqrt{4 + m^2 L_{\rm eff}^2}$$

with effective AdS radius

$$L_{\rm eff}^2 \equiv \frac{2\alpha}{1 - \sqrt{1 - 4\alpha/L^2}} \sim L^2 \left(1 - \alpha/L^2 + O(\alpha^2)\right)$$

Q: charge ($k = 1$); charge density ($k = -1$, $k = 0$)

Behaviour on the AdS boundary

Asymptotically Anti-de Sitter space-time

- Metric functions

$$
\begin{aligned}
f(r \gg 1) &= k + \frac{r^2}{L_{\mathrm{eff}}^2} + \frac{f_2}{r^2} + O(r^{-4}) \\
a(r \gg 1) &= 1 + \frac{a_4}{r^4} + O(r^{-6})
\end{aligned}
$$

f_2, a_4 constants (have to be determined numerically)

Properties of black holes

- **specific heat**

$$C = T_{\mathrm{H}} \left(\partial S / \partial T_{\mathrm{H}}\right)$$

in canonical ensemble: $C > 0 \Rightarrow$ black hole stable
$C < 0 \Rightarrow$ black hole unstable

- **Energy** E

$$\frac{16\pi G E}{V_3} = \sqrt{1 - \frac{\alpha}{L^2}} \left(-3 f_2 - 8 \frac{a_4}{L_{\mathrm{eff}}^2}\right)$$

V_3: volume of the 3-dimensional space

- **Free energy** $F = E - T_{\mathrm{H}} S - \mu Q$ with **Hawking temperature** T_{H} and **entropy** S

$$T_{\mathrm{H}} = \frac{f'(r_h) a(r_h)}{4\pi} \quad , \quad \frac{S}{V_3} = \frac{r_h^3}{4G} \left(1 - \frac{6\alpha}{r_h^2}\right)$$

Black holes without scalar hair

(Boulware & Deser, 1982; Cai, 2003)

For $m^2 > m^2_{\mathrm{BF},2} \Rightarrow \psi(r) \equiv 0$ and

$$\phi(r) = \frac{Q}{r_h^2} - \frac{Q}{r^2}$$

$$f(r) = k + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 - \frac{4\alpha}{L^2} + \frac{4\alpha M}{r^4} - \frac{4\alpha\gamma Q^2}{r^6}}\right) \quad , \quad a(r) \equiv 1$$

M: mass parameter , Q: charge (density)
event horizon at $f(r_h) = 0$

Outline

Uncharged black holes for $k = -1$

- Uncharged black holes $Q = 0$; uncharged scalar field $e = 0$
- for $k = -1$ extremal solution with $f(r_h) = 0$, $f'(r)|_{r=r_h} = 0$ exists
- extremal solution has $T_{\rm H} = 0$, $r_h^{\rm (ex)} = L/\sqrt{2}$
- close to **extremality** horizon topology is $AdS_2 \times H^3$
  (Astefanesei, Banerjee & Dutta, 2008)
- hyperbolic Gauss-Bonnet black holes in $d = 5$ have AdS_2 radius

  $$R = \sqrt{L^2/4 - \alpha}$$

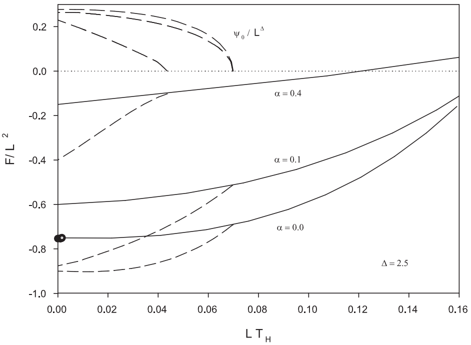
  (Y. Brihaye & B.H., PRD 84, 2011)
- asymptotic AdS_5 stable, near-horizon AdS_2 unstable for

  $$m^2_{\rm BF,5} = -\frac{4}{L^2_{\rm eff}} \le m^2 \le -\frac{1}{4R^2} = m^2_{\rm BF,2}$$

Black holes with scalar hair, $\gamma \neq 0$, $\alpha \neq 0$

(Y. Brihaye & B.H., PRD 84, 2011)

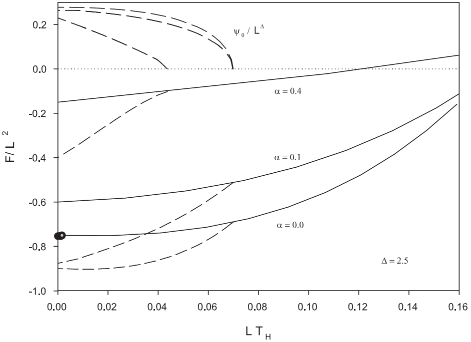
- black holes with scalar hair **thermodynamically preferred**



Black holes with scalar hair, $\gamma \neq 0$, $\alpha \neq 0$

(Y. Brihaye & B.H., PRD 84, 2011)

- the larger α the lower $T_{\rm H}$ at which instability appears



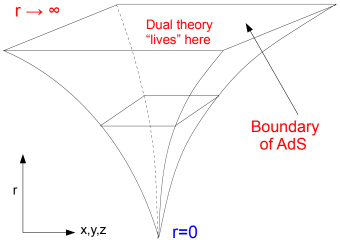
Outline

1. Introduction
2. The model
3. Black holes in hyperbolic AdS ($k = -1$)
4. Black holes & solitons in planar AdS ($k = 0$)
5. Black holes & solitons in global AdS ($k = 1$)
6. Conclusions & Outlook

Planar Anti-de Sitter (AdS) space-time

- For $\alpha = 0$, $\phi \equiv 0$, $\psi \equiv 0$

$$ds^2 = -\frac{r^2}{L^2}dt^2 + \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}\left(dx^2 + dy^2 + dz^2\right)$$



Taken from arxiv: 0808.1115

- Dilatation symmetry $\Rightarrow$ dual theory **scale-invariant**

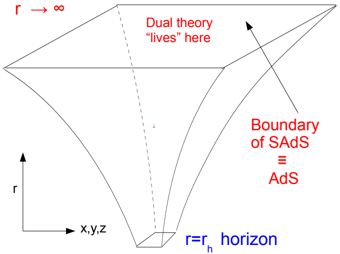
$$(t, x, y, z) \to \lambda(t, x, y, z) \ , \ \ r \to \frac{r}{\lambda}$$

- r: scale in dual theory
- r large $\Leftrightarrow$ dual theory strongly coupled (UV regime)

Planar Schwarzschild-Anti-de Sitter (SAdS)

- For $\alpha = 0$, $\phi \equiv 0$, $\psi \equiv 0$

$$ds^2 = -\left(\frac{r^2}{L^2} - \frac{{r_h}^4}{r^2 L^2}\right) dt^2 + \left(\frac{r^2}{L^2} - \frac{{r_h}^4}{r^2 L^2}\right)^{-1} dr^2 + \frac{r^2}{L^2}\left(dx^2 + dy^2 + dz^2\right)$$

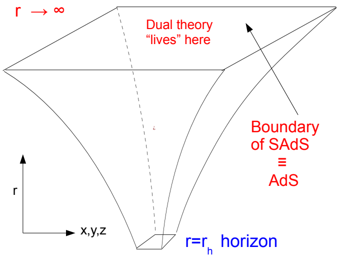


- Black hole with planar horizon at $r = r_h$
- **temperature** $T = r_h/(\pi L^2)$
- asymptotically AdS_{4+1}

Planar Schwarzschild-Anti-de Sitter (SAdS)

- For $\alpha = 0$, $\phi \equiv 0$, $\psi \equiv 0$

$$ds^2 = -\left(\frac{r^2}{L^2} - \frac{{r_h}^4}{r^2 L^2}\right) dt^2 + \left(\frac{r^2}{L^2} - \frac{{r_h}^4}{r^2 L^2}\right)^{-1} dr^2 + \frac{r^2}{L^2}\left(dx^2 + dy^2 + dz^2\right)$$



Taken from arxiv: 0808.1115

- new (dimensionless) scale $r_h/L \Rightarrow$ scale-invariance broken at $r < \infty$
- BUT: **scale-invariant** dual theory at **finite** temperature ($r \to \infty$)
- all $T \neq 0$ equivalent

Planar Reissner-Nordström-Anti-de Sitter (RNAdS)

- For $\alpha=0$, $\psi\equiv 0$

$$ds^2=-f(r)a^2(r)dt^2+\frac{1}{f(r)}dr^2+\frac{r^2}{L^2}\left(dx^2+dy^2+dz^2\right)$$

with

$$f(r)=\frac{r^2}{L^2}-\frac{2M}{r^2}+\frac{\gamma Q^2}{r^4}\quad,\quad a(r)\equiv 1$$

planar (outer) horizon at $r=r_h$, asymptotically AdS_{4+1}

- U(1) gauge field in AdS

$$A_M dx^M=A_t(r)dt=\frac{Q}{r_h^2}\left(1-\frac{r_h^2}{r^2}\right)=\mu\left(1-\frac{r_h^2}{r^2}\right)$$

Planar Reissner-Nordström-Anti-de Sitter (RNAdS)

- **temperature**

$$T = 3r_h/(\pi L^2) - \gamma\mu^2/(2\pi r_h)$$

  can be varied continuously (in particular $T = 0$ possible)

- $r \to \infty$:
  - scale-invariant dual theory at finite T
  - global U(1) with μ chemical potential
    - *In principle*: fluid/superfluid interpretation since U(1) **not** dynamical
    - *Mostly*: conductor/superconductor interpretation (photons neglected in many condensed matter processes)
  - only (dimensionless) scale: T/μ

RNAdS is gravity dual of a conductor/fluid

Planar Anti-de Sitter soliton

- **double Wick rotation** ($t \to i\eta$, $z \to it$) of SAdS
- metric

$$ds^2 = -\frac{r^2}{L^2}dt^2 + \left(\frac{r^2}{L^2} - \frac{r_h{}^4}{r^2L^2}\right)^{-1} dr^2 + \left(\frac{r^2}{L^2} - \frac{r_h{}^4}{r^2L^2}\right) d\eta^2 + \frac{r^2}{L^2}\left(dx^2 + dy^2\right)$$

- to avoid conical singularity $\to$ η **periodic** with period

$$\tau_\eta = \frac{\pi L^2}{r_0} \quad \text{where } r_0 > 0$$

- space-time unchanged for $A_M dx^M = \mu dt$
- can be considered at **any temperature** T

Planar Anti-de Sitter soliton

- only (dimensionless) scale T/μ
- energy spectrum discrete and strictly positive **(mass gap)**
- **unstable** to decay to planar SAdS for large T
  (Surya, Schleich & Witt, 2001)

AdS soliton is gravity dual of an insulator

Onset of superconductivity

- RNAdS black hole forms scalar “hair” for $T < T_c$, $\mu \neq 0$ (Gubser, 2008)

  $\Rightarrow$ conductor/superconductor phase transition
  $\Rightarrow$ fluid/superfluid phase transition

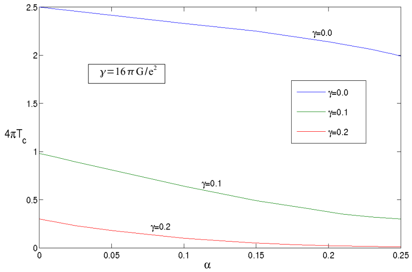
- AdS soliton forms scalar “hair” for $\mu > \mu_c$ (even at $T = 0$) (Nishioka, Ryu, Takayanagi, 2010)

  $\Rightarrow$ insulator/superconductor phase transition
  $\Rightarrow$ insulator/superfluid phase transition

Including Gauss-Bonnet corrections

(Brihaye & B. Hartmann, Phys. Rev. D 81, 2010)

- Gauss-Bonnet coupling $0 \leq \alpha \leq L^2/4$



$\Longrightarrow$ **condensation gets harder for $\alpha > 0$, but not suppressed**

Outline

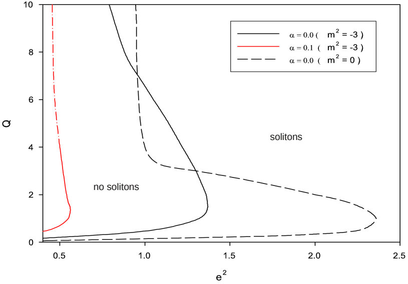
Spherically symmetric AdS solitons

- For $k = 1$ solitons regular on $r \in [0 : \infty[$ exist
  (Basu, Mukherjee & Shieh, 2009;
  Dias, Figueras, Minwalla, Mitra, Monteiro & Santos, 2011)
- Boundary conditions at $r = 0$

$$f(0) = 1 \ , \ \phi'(0) = 0 \ , \ \psi'(0) = 0 \ , \ a'(0) = 0$$

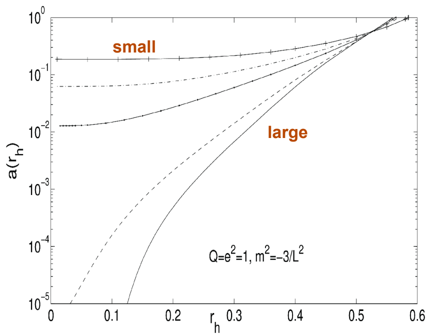
Charged solitons with scalar hair

- exist only in limited domain of Q-e^2-plane
- at $e = e_c(Q, \alpha)$: numerical results suggest $a(0) \to 0$
- for $\alpha \neq 0$ range of allowed Q and e values enlarged



Charged black hole with scalar hair, $k = 1$

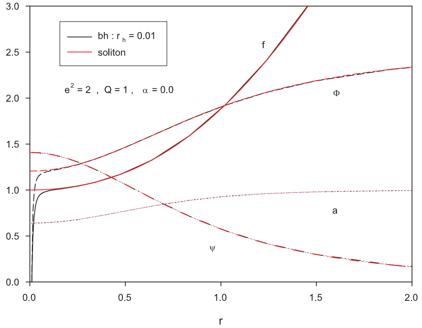
(Brihaye & B. Hartmann, PRD 85 (2012) 124024)



- For small α: solution exists down to $r_h = 0$
  $\rightarrow$ **soliton?**
- For large α: solution has $a(r_h) \rightarrow 0$ for $r_h \rightarrow r_h^{(cr)} > 0$
  $\rightarrow$ **extremal black hole?**

Charged black hole with scalar hair, $k = 1$

(Brihaye & B. Hartmann, PRD 85 (2012) 124024)

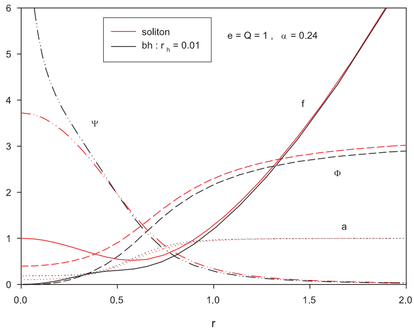


For $\alpha = 0$:

- Black hole tend to soliton solutions in the limit $r_h \to 0$

Charged black hole with scalar hair, $k = 1$

(Brihaye & B. Hartmann, PRD 85 (2012) 124024)



$\alpha \neq 0$:

- Gauss-Bonnet solitons with scalar hair exist
- black holes with scalar hair do **not** tend to corresponding solitons for $r_h \to 0$

Charged black hole with scalar hair, $k = 1$

(Brihaye & B. Hartmann, PRD 85 (2012) 124024

There exist no extremal Gauss-Bonnet black holes with scalar hair.

Charged black hole with scalar hair, $k = 1$

Proof:

- assume near-horizon geometry to be $AdS_2 \times S^3$:

$$ds^2 = v_1 \left(-\rho^2 d\tau^2 + \frac{1}{\rho^2} d\rho^2\right) + v_2 \left(d\psi^2 + \sin^2\psi \left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right)$$

  v_1, v_2: positive constants

- Combination of equations of motion yields

$$0 = 16\pi G \left(\frac{\rho^2}{v_1}\psi'^2 + \frac{e^2\phi^2\psi^2}{\rho^2 v_1}\right)$$

  This leads to: $\psi' = 0$ and $\phi^2\psi^2 = 0$ in near horizon geometry

- $\phi^2 = 0$ ruled out $\rightarrow \psi \equiv 0$ **in near horizon geometry** q.e.d.

The planar limit

($\alpha = 0$: Gentle, Pangamani & Withers, (2011))
($\alpha \neq 0$: Brihaye & B. Hartmann, PRD 85 (2012) 124024)

- apply rescaling $r \to \lambda r$, $t \to \lambda^{-1} t$, $M \to \lambda^4 M$, $Q \to \lambda^3 Q$ to Boulware-Deser-Cai solution with

$$f(r) = r^2 \left(\frac{1}{r^2} + \frac{1}{2\alpha} \left(1 - \sqrt{1 - \frac{4\alpha}{L^2} + \frac{4\alpha M}{r^4} - \frac{4\alpha\gamma Q^2}{r^6}} \right) \right)$$

- For $\lambda \to \infty$: $\lambda^2 d\Omega_3^2 \to dx^2 + dy^2 + dz^2$
- For $Q \to \infty$ the $k = 1$ **solution becomes** $k = 0$ **solution**, i.e. becomes comparable in size to AdS radius L
- electromagnetic repulsion balanced by gravitational attraction present in AdS

Conclusions & Outlook

Conclusions

- Two mechanisms can make AdS black holes unstable to scalar condensation
  - **uncharged scalar field**: black holes with AdS_2 factor in near-horizon geometry (near-extremal black holes)
    Example in this talk: uncharged, static black holes with hyperbolic horizon ($k = -1$)
  - **charged scalar field**: coupling to gauge field lowers effective mass of scalar field
    Examples in this talk: charged, static black holes with flat or spherical horizon ($k = 0$, $k = 1$)
- Black holes with scalar hair thermodynamically preferred

Outlook

- Instabilities of other black holes and solitons in AdS_d
  - charged and rotating Einstein black holes:
    Y. Brihaye & B.H., JHEP 1203 (2012) 050
  - charged and rotating Gauss-Bonnet black holes:
    Y. Brihaye, B.H. & S. Tojiev, Phys. Rev. D 87(2013)
  - charged boson stars:
    Y. Brihaye, B.H. & S. Tojiev, CQG 30 (2013)
  - solitons in conformal theories:
    Y. Brihaye, B.H. & S. Tojiev, Phys. Rev. D 88 (2013)
- Implications on CFT side?
- Implications for stability of String Theory/Supergravity black holes?

Still sceptical about Holography?

Muito orbigada pela sua atenção !