Cosmological perturbations in closed universes and the CMB

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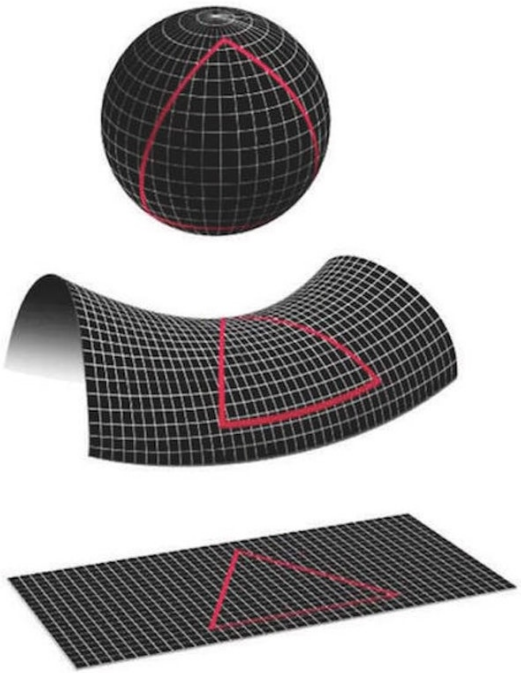
Summary

1. Introduction
2. Inflationary background dynamics in a closed universe
   [JCAP 10 (2016), with B. Bonga e B. Gupt]
   - Curvature effects in the background dynamics
   - Initial conditions
3. Scalar and tensor perturbations [JCAP 10 (2016) and JCAP 05 (2017), with B. Bonga e B. Gupt]
   - Hamiltonian formalism for the cosmological perturbations
   - Dynamics for gauge-invariant perturbations
   - Initial conditions
4. Implications for the CMB

Introduction



Closed: $\Omega_k < 0$

Flat: $\Omega_k = 0$

Open: $\Omega_k > 0$

Constraints on curvature from Planck:

The combination of the *Planck* temperature and polarization power spectra give

$$\Omega_K = -0.056^{+0.028}_{-0.018} \quad (68\%, \textit{Planck}\ \text{TT+lowE}), \tag{46a}$$

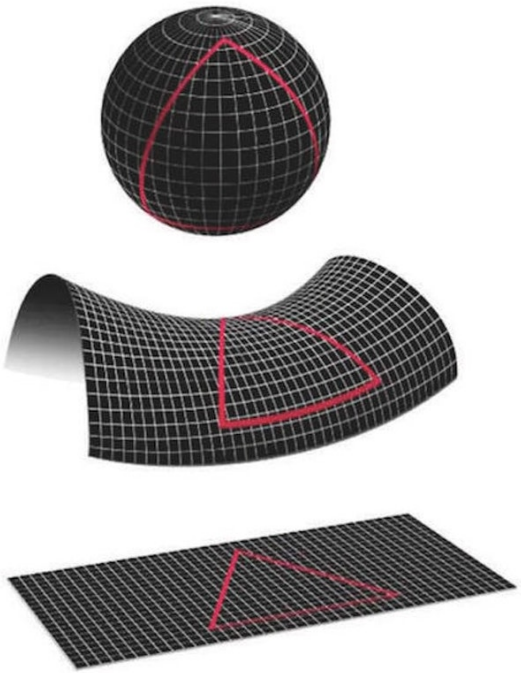
$$\Omega_K = -0.044^{+0.018}_{-0.015} \quad (68\%, \textit{Planck}\ \text{TT,TE,EE+lowE}), \tag{46b}$$

an apparent detection of curvature at well over 2σ. The 99% probability region for the TT,TE,EE+lowE result is $-0.095 < \Omega_K < -0.007$, with only about 1/10 000 samples at $\Omega_K \geq 0$.

The constraint can be further sharpened by combining the *Planck* data with BAO data; this convincingly breaks the geometric degeneracy to give

$$\Omega_K = 0.0007 \pm 0.0019 \quad (68\%, \text{TT,TE,EE+lowE+lensing+BAO}). \tag{47b}$$

[Planck 2018 VI. Cosmological parameters]



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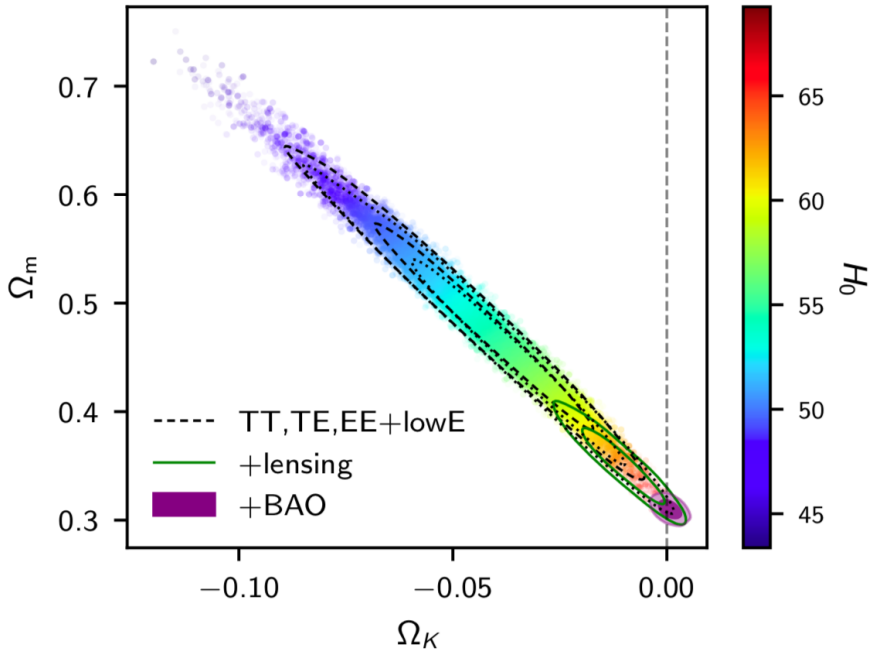
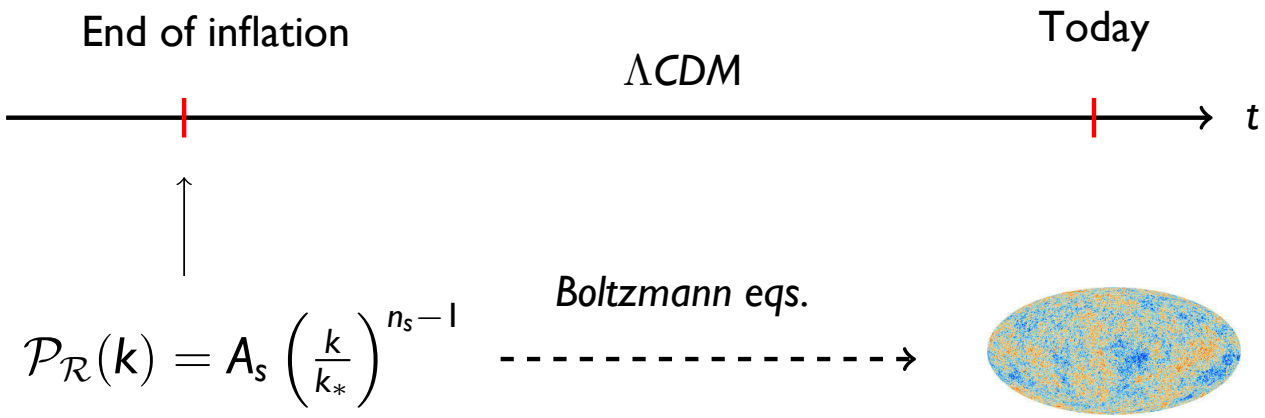


[Planck 2018 I. Overview and the cosmological legacy of Planck]

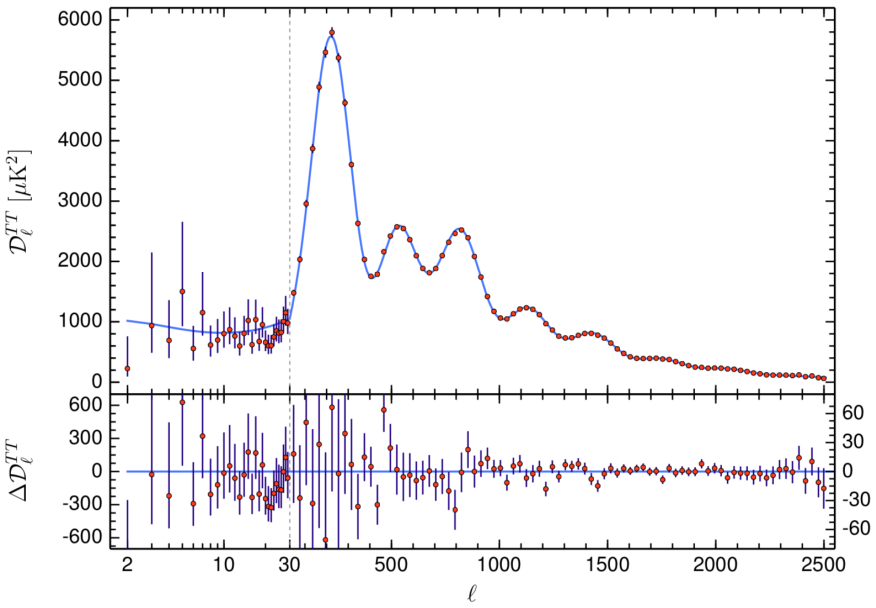
Primordial curvature perturbations produced during inflation described by a power law spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1},$$

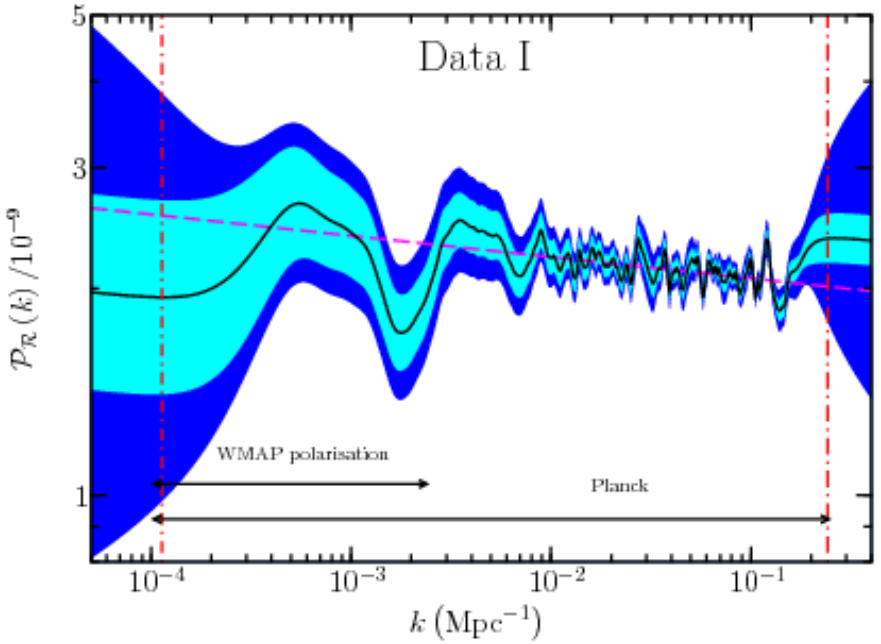
setting initial conditions for the evolution of cosmological perturbations in the ΛCDM model and the estimation of cosmological parameters.



Spatial curvature is then bounded by : $|\Omega_k| \lesssim 0.005$.



Power spectrum of temperature fluctuations [Planck 2018]



Reconstruction of power spectrum of primordial perturbations [Hunt and Sarkar, JCAP 12 (2015) 052]

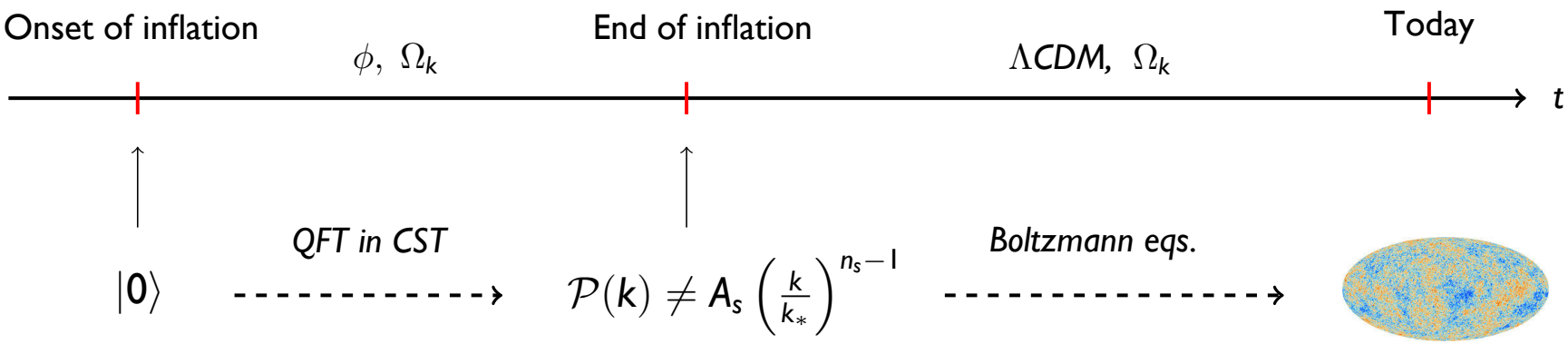
- Predictions fit observations for small angular scales.
- Anomalies observed for large angular scales.

Physical mechanisms leading to large scale anomalies? Consequences for polarization spectra?

1. How does the presence of spatial curvature in a closed universe affect the primordial power spectrum produced during inflation?
2. What are the consequences for the CMB? Is it consistent to use a power-law spectrum to estimate Ω_k?

In a closed universe, spatial curvature sets an upper bound on the duration of slow-roll inflation [Uzan, Kirchner and Ellis '2003]. Modes with small k do not cross the horizon during slow-roll ["Just enough inflation", Cicoli et al '2014].

Large scale cutoff at the curvature scale, leading to power supression [Efstathiou '2003]. Dynamics of perturbations also affected by spatial curvature.



Analysis in [Bonga, Gupt and NY, JCAP 1610 (2016) 031 and JCAP 1705 (2017) 021].

Tensions and spatial curvature

Renewed interest in closed universes:

- [di Valentino et al '2019] Positive curvature explains observed enhanced lensing in CMB power spectra ($A_L > 1$ in flat universe) and removes internal tension for estimation of cosmological parameters at distinct angular scales.
- [Handley '2021] Planck CMB and BAO data are inconsistent datasets and should not be combined. Planck CMB data alone better described by a closed universe (odds of 50:1 against flat).

Claims countered, however, for instance, in:

- [Efstathiou '2020] Questions choice of prior for Ω_k. Milder pull towards closed universes in Planck data if larger fraction of CMB sky is used for the estimations of cosmological parameters.

Background dynamics

Background dynamics

Closed FLRW universe:

$$ds^2 = -dt^2 + \tilde{a}(t)^2 r_0^2 \left[d\chi^2 + \sin^2\chi \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]$$

Radius of the universe today:

$$r_0 = \frac{1}{\sqrt{H_0^2 |\Omega_k|}} \qquad (\simeq 4.5\, r_{LS} \text{ for } \Omega_k = -0.005)$$

- Presence of spatial curvature modifies the inflationary dynamics.

$$H^2 = \frac{8\pi G}{3}\rho - \frac{1}{a^2}$$

Curvature term becomes relevant in the far past even for small Ω_k (large radius today).

Background dynamics

- Universe filled by a scalar field:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V = 0$$

Two potentials were considered:

$$V(\phi) = \begin{cases} \frac{1}{2}m^2\phi^2 & \text{(Quadratic)} \\ \frac{3m^2}{32\pi G}\left(1 - e^{-\sqrt{\frac{16\pi G}{3}}\phi}\right)^2 & \text{(Starobinski)} \end{cases}$$

Background dynamics

Initial conditions: $a_*, \phi_*, \dot{\phi}_*$

Parameter to be fixed: m

Requirement: inflation takes place and is long enough so that observable modes are within the Hubble horizon at the onset of inflation.

Initial time t_* when mode $k_* = 0.05\ \mathrm{Mpc}^{-1}$ exits horizon during inflation:

$$\tilde{a}_* H_* = k_*$$

Assume:

- Spatial curvature negligible at t_*
- Slow-roll inflation at t_*

Background dynamics (quadratic potential)

Then the background equations at t_* give:

$$3H_*\dot{\phi}_* + m^2\phi_* \simeq 0$$

$$H_*^2 \simeq \frac{8\pi G}{3}\left(\frac{\dot{\phi}_*^2}{2} + \frac{m^2\phi_*^2}{2}\right)$$

First slow-roll parameter satisfies:

$$(\epsilon - 3)\dot{\phi}^2 + \epsilon m^2\phi_*^2 = 0$$

These equations can be solved to give $\phi_*, \dot{\phi}_*, m$ in terms of H_* and ϵ.

Background dynamics

Primordial power spectrum near k_* described by flat space inflation:

$$n_s = 1 - 4\epsilon\,, \qquad A_s = \frac{GH_*^2}{\pi\epsilon}$$

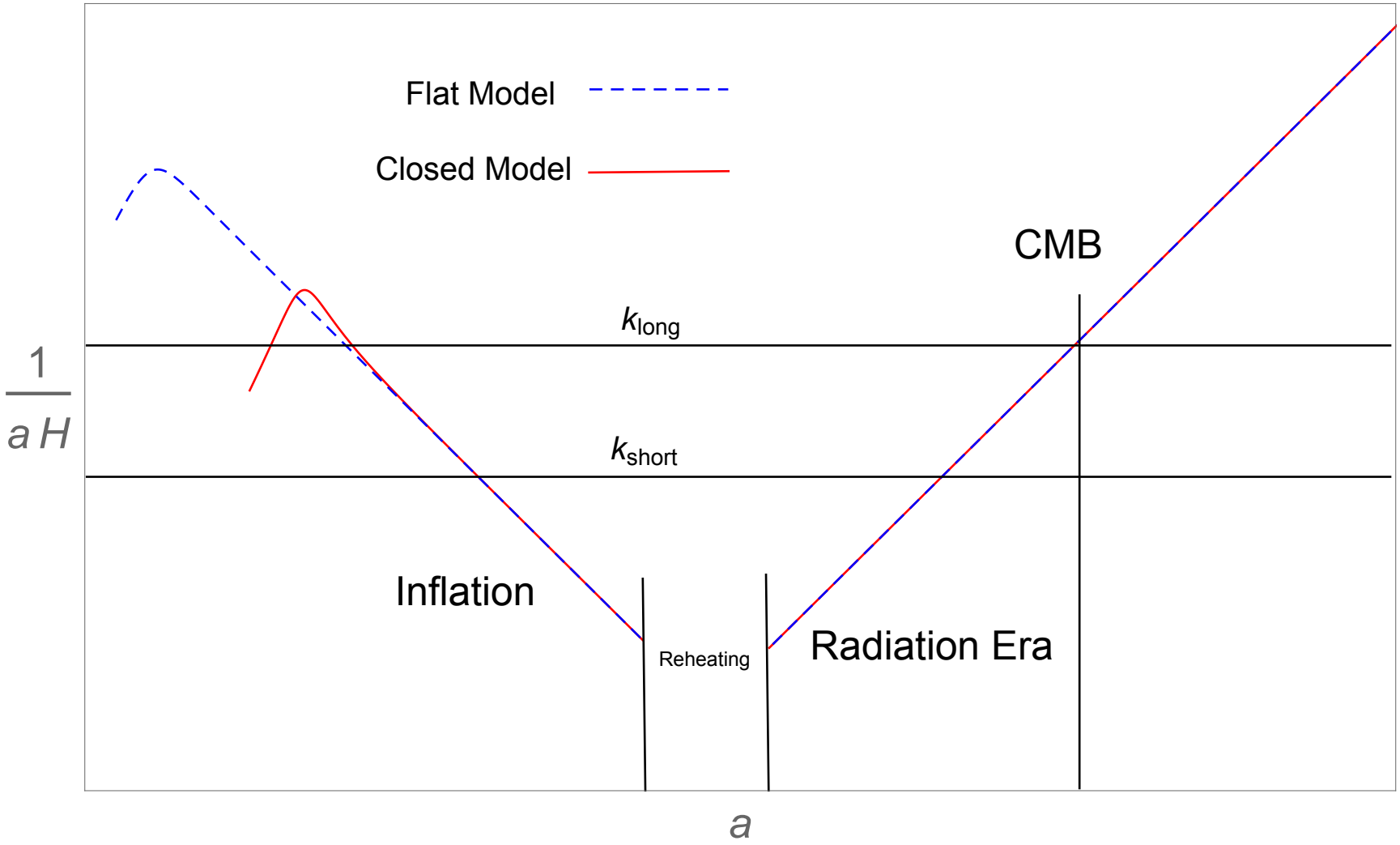
H_* and ϵ can then expressed in terms of the observable quantities A_s, n_s.

Observational input:

$$A_s = (2.2065 \pm 0.0035) \times 10^{-9}$$

$$n_s = 0.9645 \pm 0.0049$$

Evolution of background geometry ($\Omega_k = -0.005$)



Cosmological perturbations

Linear perturbations:

$$\gamma_{ij} = \bar{\gamma}_{ij} + \delta\gamma_{ij}\,, \qquad \phi = \bar{\phi} + \delta\phi$$

- Expansion of scalar field perturbations in normal modes:

$$\delta\phi = \sum_{nlm} \delta\phi^{nlm} Q_{nlm}\,,$$

Hyperspherical harmonics Q_{nlm} analogous to Fourier modes.

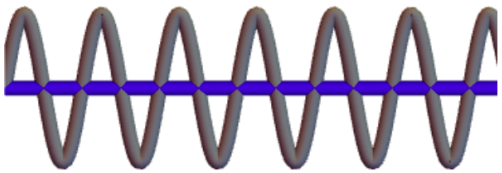
$$\nabla^2 Q_{nlm} = -(n^2-1)Q_{nlm}\,, \qquad Q_{nlm}(\chi,\theta,\varphi) = \Phi^l_n(\chi)Y_{lm}(\theta,\varphi)$$

Perturbations on a 3-sphere

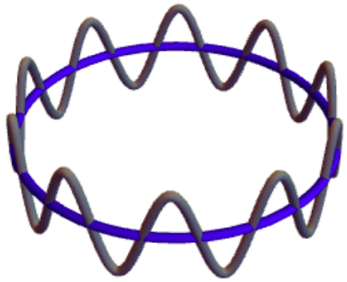
Flat limit: large n and $\chi \ll 1$.

Perturbations in flat space recovered, in spherical coordinates.

$$\frac{n^2-1}{r_0^2} \to k^2$$



Flat: plane waves labelled by *k*



Closed: spherical harmonics labelled by *n*

- Expansion of scalar modes of the metric:

$$\delta\gamma_{ij}^{(s)} = \sum_{nlm}\left[\gamma_{nlm}^{(1)}\mathcal{S}_{ij}^{nlm} + \gamma_{nlm}^{(2)}\mathcal{T}_{ij}^{nlm}\right]$$

Normal modes:

$$\mathcal{S}_{ij}^{nlm} = \frac{1}{3}\Omega_{ij}Q_{nlm}\,, \qquad \nabla^2 \to -(n^2-1)$$

$$\mathcal{T}_{ij}^{nlm} = \left(\frac{1}{n^2-1}\nabla_i\nabla_j + \frac{1}{3}\Omega_{ij}\right)Q_{nlm}\,, \qquad \nabla^2 \to -(n^2-7)$$

Ω_{ij} is the metric of the unit 3-sphere and ∇ the covariant derivative compatible with this metric. (Mode $n = 1$ is homogeneous.)

[Halliwell and Hawking '1985]

- Expansion of tensor modes of the metric:

$$\delta\gamma_{ij}^{(t)} = a^2 h_{ij}^{TT}$$

$$h_{ij}^{TT} = \sum_s \sum_{nlm} h_{nlm,s}\, \mathcal{Q}_{ij}^{nlm,s}(\chi,\theta,\varphi)$$

Properties of normal modes:

$$\Omega^{ij}\,\mathcal{Q}_{ij}^{nlm,s}(\chi,\theta,\varphi) = 0\,, \qquad \nabla^i \mathcal{Q}_{ij}^{nlm,s}(\chi,\theta,\varphi) = 0$$

$$\nabla^2 \mathcal{Q}_{ij}^{nlm,s}(\chi,\theta,\varphi) = -(n^2-3)\mathcal{Q}_{ij}^{nlm,s}(\chi,\theta,\varphi)$$

Explicit form in [Gerlach and Sengupta '1978].

Dynamics of scalar modes

- ADM formalism for the gravitational field coupled to the scalar field.
- Hamiltonian expressed in terms of background quantities and linear perturbations, together with their conjugate momenta, keeping terms up to second order in the perturbations.
- Canonical transformation leads to quadratic Hamiltonian in terms of gauge invariant scalar perturbations. Technique developed in [Goldberg, Newman and Rovelli '1991] and applied to cosmological perturbations in flat and closed universes in [Langlois '1994].

Dynamics of scalar modes

Gauge invariant perturbation:

$$q = \frac{\dot{\phi}}{H}\zeta$$
$$= \sum q_{nlm} Q_{nlm}$$

Expression in terms of field and metric perturbations:

$$q_{nlm} = \delta\phi_{nlm} - \sqrt{V_0}\frac{\dot{\phi}}{H}\left(\gamma^{(1)}_{nlm} + \gamma^{(2)}_{nlm}\right)$$

Equation of motion

$$\ddot{q}_{nlm} + b(n,t)\dot{q}_{nlm} + c(n,t)q_{nlm} = 0$$

Coefficients:

$$b(n,t) = 3H + \frac{32\pi G a^3 \dot{a}\dot{\phi}V'(\phi) + 48\pi G a^2 \dot{a}^2\dot{\phi}^2 - 8\pi G a^2\dot{\phi}^2\left(8\pi G a^2\left(\dot{\phi}^2 - 2V\right) + 2\right)}{2a\dot{a}\left(2(n^2-4)\dot{a}^2 + 8\pi G a^2\dot{\phi}^2\right)}$$

$$c(n,t) = \frac{8\pi G}{a^2\dot{a}^2\left(2\left(n^2-4\right)\dot{a}^2 + 8\pi G a^2\dot{\phi}^2\right)}$$

$$\left[\frac{\dot{a}^4(n^2-4)\left(n^2-1+a^2V''\right)}{4\pi G} + \left(4n^2-7\right)a^3\dot{a}^3\dot{\phi}V' - \pi G\frac{n^2-1}{n^2-4}a^4\dot{\phi}^4\left[8\pi G a^2\left(\dot{\phi}^2+2V\right) - 6\right]\right.$$

$$+ (n^2-1)a^2\dot{a}^2\left(-6\pi G\frac{n^2-5}{n^2-4}a^2\dot{\phi}^4 + 4\pi G a^2\dot{\phi}^2V + \frac{3}{2}\dot{\phi}^2 + \frac{9}{2}\dot{a}^2\dot{\phi}^2\right)$$

$$\left.+ a^3\dot{a}\left[a\dot{a}\dot{\phi}^2V'' + 2a\dot{a}V'^2 + 4\pi G a^2\dot{\phi}V'\left(\dot{\phi}^2+2V\right) - \dot{\phi}V'\right]\right]$$

Comments:

- For large radius r_0 and n, equations for flat universe are recovered.
- For modes within the horizon, coefficients b, c vary slowly, and the eq. of motion is well approximated by:

$$\ddot{q}_{nlm} + 3H\,\dot{q}_{nlm} + \left[\frac{n^2-1}{a^2} + m^2\left(1 + \frac{6}{n^2-4}\right)\right] q_{nlm} = 0$$

In conformal time, with $v = qa$,

$$v''_{nlm} + \left[m^2 a^2\left(1 + \frac{6}{n^2-4}\right) + n^2 - 1\right] v_{nlm} = 0$$

Initial conditions for the scalar perturbations

Initial conditions set at the onset of inflation, $\ddot{a}(t_0) = 0$. All observable modes are within the horizon at t_0.

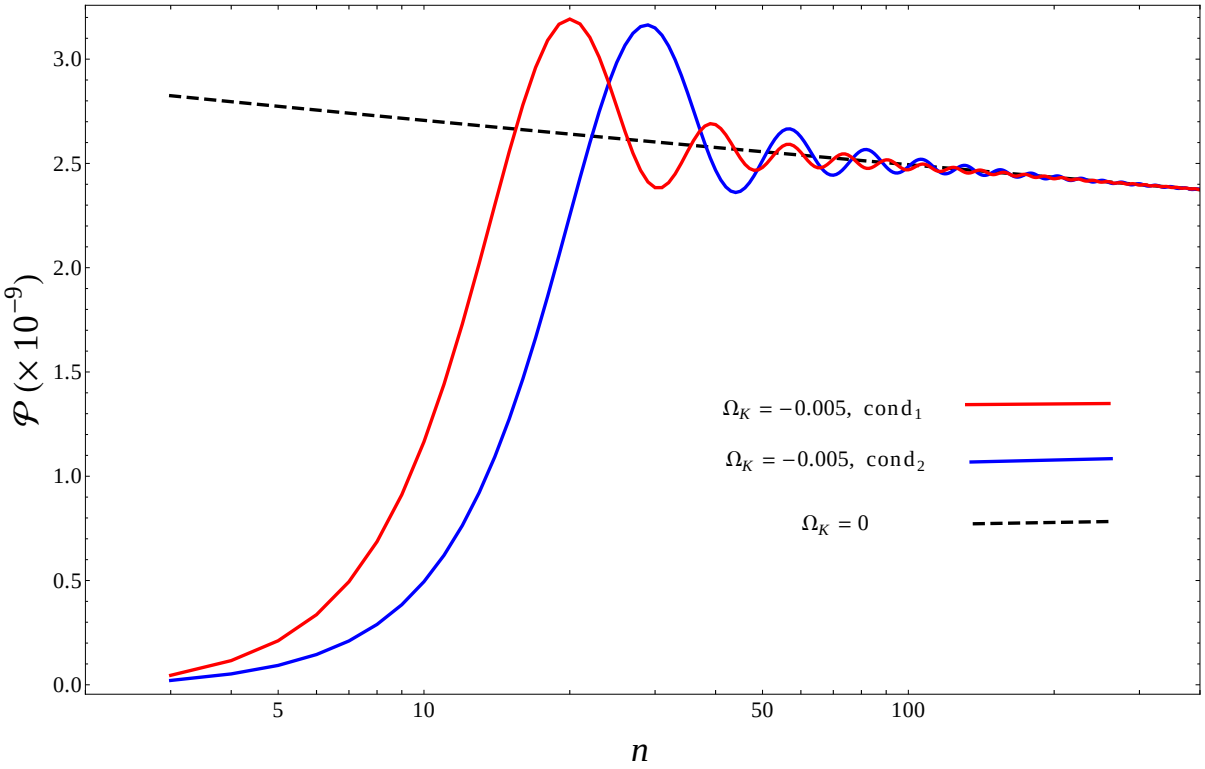
Quantized perturbations at ground state of the instantaneous Hamiltonian $H(t_0)$. Positive-frequency modes determined by the initial conditions:

$$v_{nlm}(\eta_0) = \frac{1}{\sqrt{2\omega_n}}\,, \qquad v'_{nlm}(\eta_0) = -i\frac{\omega_n}{2}\,,$$

with

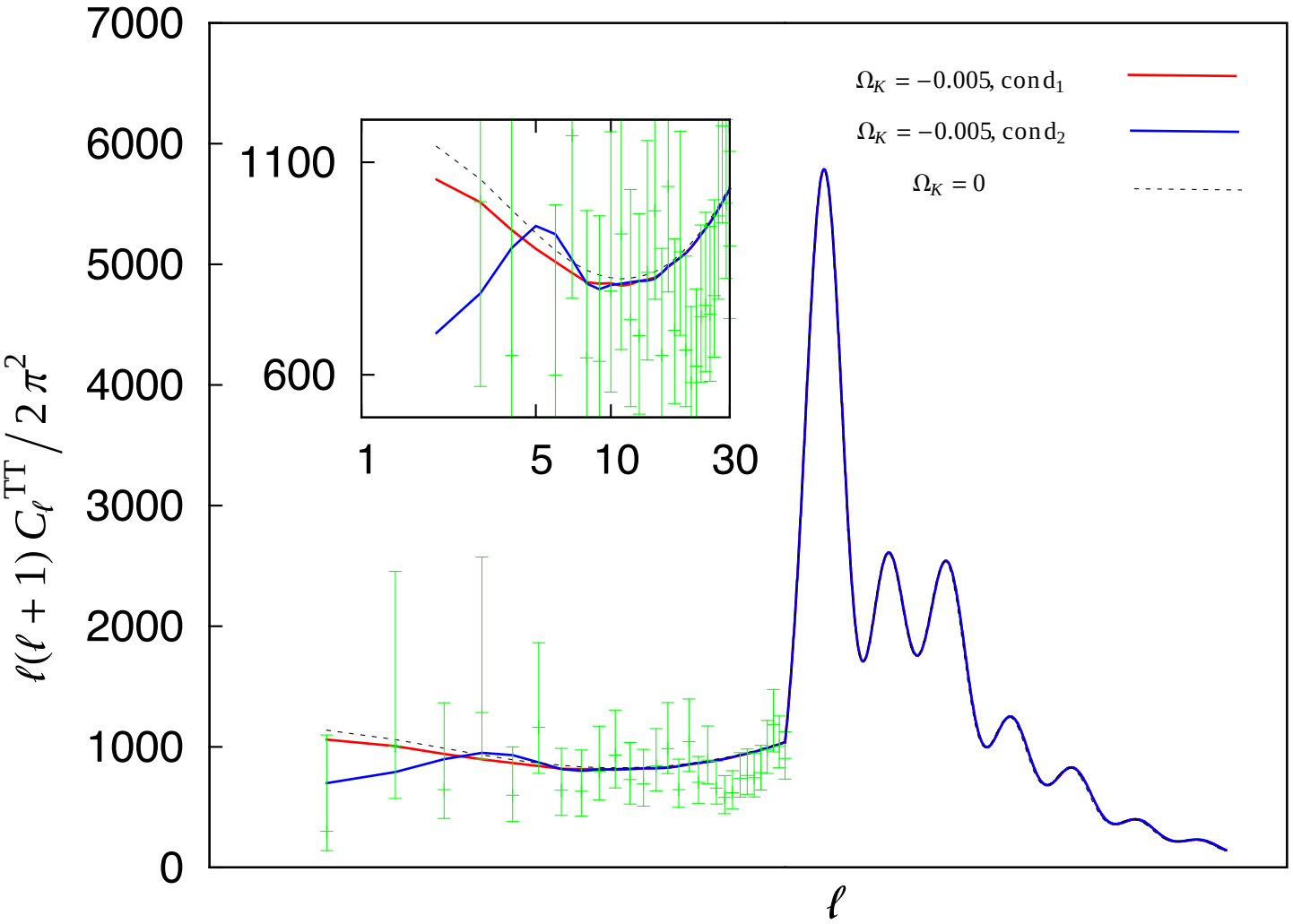
$$\omega_n = \sqrt{m^2a^2\left(1 + \frac{6}{n^2-4}\right) + n^2 - 1}$$

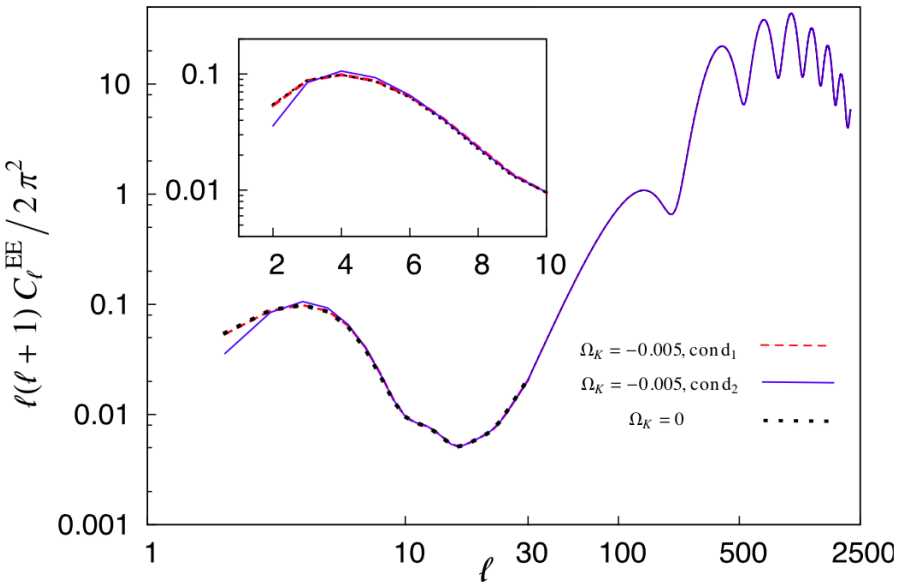
Primordial scalar perturbations ($\Omega_k = -0.005$)



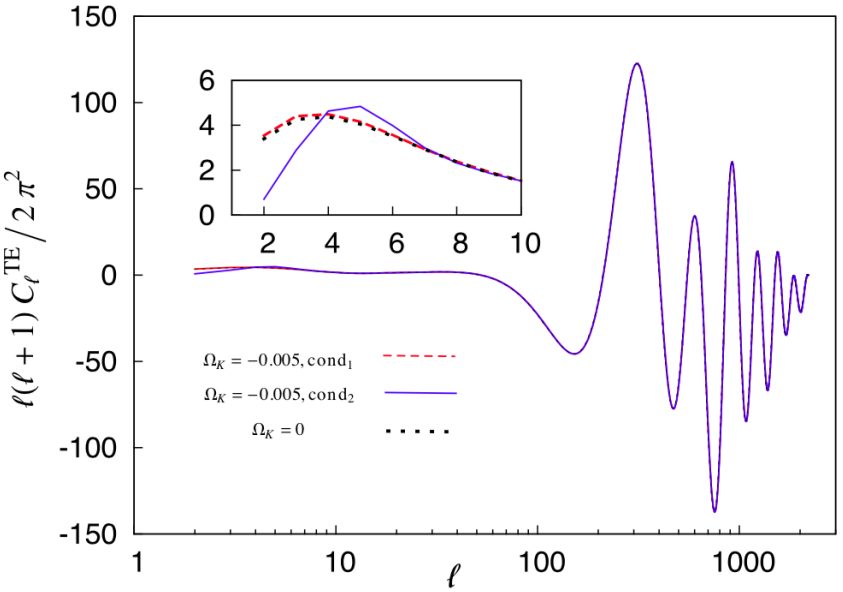
- Spectrum displays oscillations at large scales
- Oscillations accompanied by power suppression

Temperature fluctuations in the CMB ($\Omega_k = -0.005$)





E-mode polarization



TE cross correlation

Equation of motion

$$h''_{nlm,s} + 2\frac{\tilde{a}'}{\tilde{a}} h'_{nlm,s} + \left(\frac{n^2-1}{r_0^2}\right) h_{nlm,s} = 0$$

In terms of $\mu = ah$,

$$\mu''_{nlm,s} + \left(\frac{n^2-1}{r_0^2} - \frac{\tilde{a}''}{\tilde{a}}\right) \mu_{nlm,s} = 0$$

Obs: In the analysis of tensor modes in [JCAP 05 (2017)], the normal modes are expanded in spherical harmonics over a sphere of radius r_0.

Initial conditions for the tensor perturbations

Quantized perturbations at ground state of the instantaneous Hamiltonian $H(t_0)$. Positive-frequency modes determined by the initial conditions:

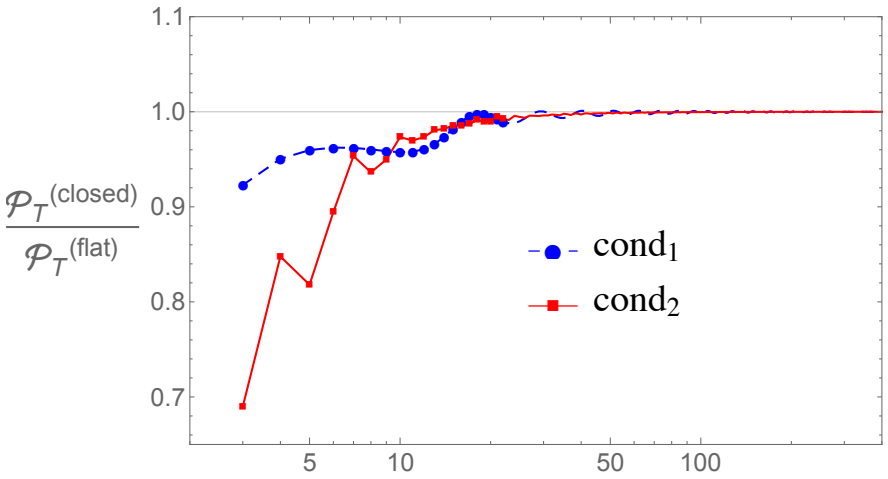
$$\mu_{nlm}(\eta_0) = \frac{1}{\sqrt{2\omega_n r_0}}\,, \qquad \mu'_{nlm}(\eta_0) = -i\sqrt{\frac{\omega_n}{2r_0}}\,,$$

with

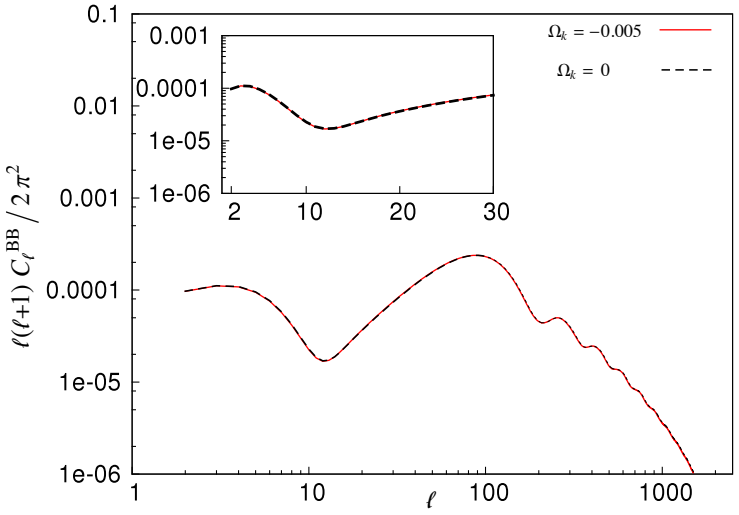
$$\omega_n = \sqrt{\frac{n^2-1}{r_0^2} - \frac{\tilde{a}''}{\tilde{a}}}$$

The term $\tilde{a}''/\tilde{a}$ is negligible for modes within the horizon (verified numerically).

Tensor modes and polarization



Primordial tensor perturbations

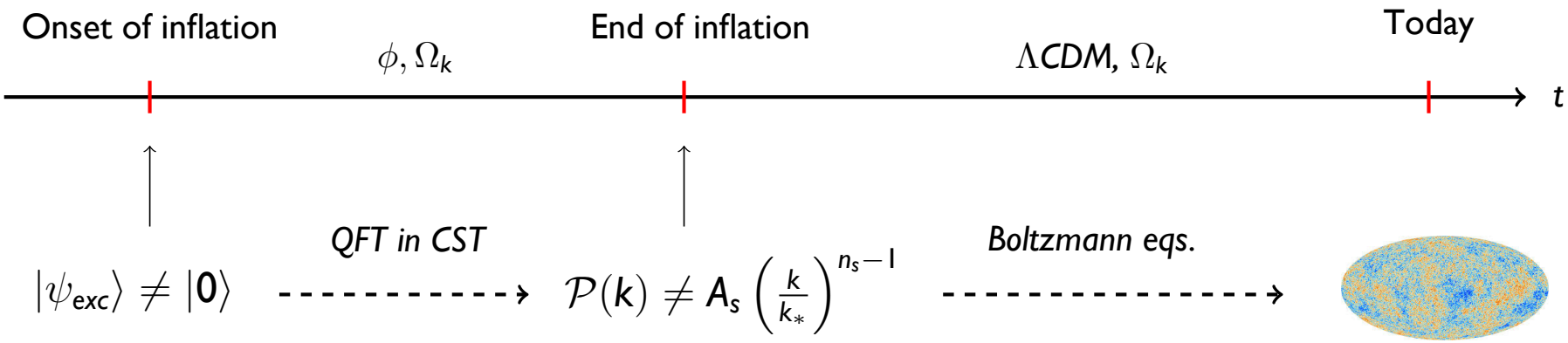


B-mode polarization

- Power suppression at large scales, but less pronounced than for scalar modes.
- Negligible modification in the power spectrum of the B-mode of polarization.

Pre-inflationary dynamics

- Quantum state of cosmological perturbations must include excitations produced in the pre-inflationary era.



In loop quantum cosmology, quantum corrections for the background dynamics near the bounce lead to power suppression at large scales for the scalar modes [A. Ashtekar, I. Agullo, P. Singh, B. Gupt].

Conclusion

1. Determination of modifications due to spatial curvature of inflationary predictions for the spectra of primordial scalar and tensor modes. Analysis of consequences for the power spectra of temperature and polarization fluctuations in the CMB.

2. Primordial scalar perturbations display power suppression and oscillations at large scales. Temperature fluctuations suppressed at large angular scales ($\ell \lesssim 20$), where anomalies are observed.

3. Milder suppression for primordial tensor modes at large scales. Negligible effects on polarization B-mode.