Asymptotic Freedom

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Verão Quântico 2025 - Iriri, Espírito Santo

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FAPERJ
Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro

Can quantum fluctuations distinguish standard gravity from its unimodular version?

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I don't think so...

(at least locally…)

Outline:

- General introduction and motivation
- Path integral of unimodular gravity
- Perturbative results
- Going beyond perturbation theory
- Perspectives and conclusions

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Part I

- Going beyond perturbation theory
- Perspectives and conclusions

Part II

1. General introduction and motivation

Classical reasons to *(not)* change GR

- **Classical gravitational dynamics: very well described by GR.**
- **Reasons to modify GR (classically) are typically related to the dark sector.**
- **Simply assuming that the cosmological constant accounts for the cosmic expansion, GR does a great job. "Maximally boring" Universe?. (What about DESI results? Tensions?)**

Quantizing GR

- **Standard perturbative techniques lead to a non-renormalizable QFT. Effective QFT.**
- **Alternative theories of classical gravitational dynamics will typically lead to different quantum theories.**
- **What about theories which are classically equivalent to GR? Can we safely state that the resulting quantum theory is strictly equivalent to quantum GR?**

- GR has a close relative: unimodular gravity.
- It is almost as old as GR and it was put forward by Einstein as well.
- It consists in fixing the determinant of the metric to a fixed scalar density.

[J. L. Anderson, D. Finkelstein 1971; J.J. van de Bij, H. van Dam, Y. J. Ng 1982; W. Buchmuller, N. Dragon 1988; 1989; W.G. Unruh 1989; M. Henneaux and C. Teitelboim 1990,...]

$$\begin{tikzcd} \boxed{\det g = \omega^2(x)} \arrow[r] & \boxed{\delta g_{\mu\nu} = g_{\nu\alpha}\nabla_\mu \epsilon^\alpha + g_{\mu\alpha}\nabla_\nu \epsilon^\alpha} \arrow[d] \\ \boxed{\nabla_\mu \epsilon^\mu = 0} & \boxed{\delta\sqrt{|\det g_{\mu\nu}|} = 0} \arrow[l] \end{tikzcd}$$

The group of diffeomorphisms (Diff) is constrained to volume-preserving diffeomorphisms (SDiff)

DO GRAVITATIONAL FIELDS PLAY AN ESSENTIAL PART IN THE STRUCTURE OF THE ELEMENTARY PARTICLES OF MATTER?

BY

A. EINSTEIN

Translated from "Spielen Gravitationsfelder im Aufber der materiellen Elementarteilchen eine wesentliche Rolle?" Sitzungsberichte der Preussischen Akad. d. Wissenschaften, 1919.

The difficulties set forth above are removed by setting in place of field equations (1) the field equations

$$G_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}G = -\kappa T_{\mu\nu} \qquad . \qquad . \qquad (1a)$$

where $T_{\mu\nu}$ denotes the energy-tensor of the electromagnetic field given by (3).

The unimodularity condition can be imposed in several different ways.

In particular, the classical equations of motion are obtained by a constrained variation of the Einstein-Hilbert action

$$\delta\sqrt{|\mathrm{det} g_{\mu\nu}|} = \frac{1}{2} g^{\mu\nu}\delta g_{\mu\nu} = 0 \quad \longleftrightarrow \quad S_{\mathrm{GR}} = -\frac{1}{16\pi G}\int \mathrm{d}^d x \sqrt{g}(R + 2\Lambda)$$

Including matter fields, the equations of motion are

$$\boxed{R^{\mu\nu} - \frac{1}{d} g^{\mu\nu} R = 8\pi G \left(T^{\mu\nu} - \frac{1}{d} g^{\mu\nu} T \right)}$$

<u>NOTE:</u> The tensor T, as derived, is <u>not</u> (in general) what we would call as the energy-momentum tensor.

An unconstrained variation leads to the standard Einstein field equations,

$$\boxed{R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + g^{\mu\nu}\Lambda = 8\pi G\, T_{\mathrm{GR}}^{\mu\nu}} \longrightarrow \boxed{T_{\mathrm{GR}}^{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta S_{\mathrm{matter}}}{\delta g_{\mu\nu}}}$$

Conservation of energy-momentum tensor is guaranteed in GR

$$\boxed{\nabla_\mu T_{\mathrm{GR}}^{\mu\nu} = 0}$$

Consider a scalar field coupled to gravity,

$$S_\phi = \frac{1}{2}\int \mathrm{d}^d x\sqrt{g}\, g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$$

$$\longrightarrow\; T_{\mathrm{GR}}^{\mu\nu} = \nabla^\mu\phi\nabla^\nu\phi - \frac{1}{2}g^{\mu\nu}\nabla^\alpha\phi\nabla_\alpha\phi$$

$$\longrightarrow\; T^{\mu\nu} = \nabla^\mu\phi\nabla^\nu\phi$$

Invariance under SDiff ensures that

$$\boxed{\nabla_\mu T^{\mu\nu} = \nabla^\nu \Omega \;\; \Rightarrow \;\; \nabla_\mu \left(T^{\mu\nu} - g^{\mu\nu}\Omega\right) = 0}$$

This implies that the tensor obtained from the constrained variation can be improved to the one extracted from standard GR.

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This implies that the tensor obtained from the constrained variation can be improved to the one extracted from standard GR. Energy-momentum is *conserved* in unimodular gravity.

> In many works, it is invoked a conservation of the tensor T obtained from the constrained variation. In particular, it is said that this is essential to recover Einstein's equations from the trace-free unimodular equations.

However, the unimodular eom is invariant under

$$T^{\mu\nu} \to T^{\mu\nu} - g^{\mu\nu}\Omega \qquad \text{with} \qquad \Omega \sim \mathcal{L}_{\text{matter}}$$

$$\boxed{R^{\mu\nu} - \frac{1}{d} g^{\mu\nu} R = 8\pi G \left(T^{\mu\nu}_{\text{GR}} - \frac{1}{d} g^{\mu\nu} T_{\text{GR}}\right)}$$

Taking the four-divergence of the eom, *making use* of energy-momentum conservation, and employing the Bianchi identities, one obtains,

$$\nabla^\nu \left(\frac{d-2}{2} R - 8\pi G T_{\rm GR} \right) = 0 \longrightarrow \frac{d-2}{2} R - 8\pi G T_{\rm GR} = \alpha$$

$$\boxed{R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + g^{\mu\nu} \tilde{\Lambda} = 8\pi G T^{\mu\nu}_{\rm GR}}$$

Cosmological constant regained as an integration constant!

Locally, unimodular gravity provides the same eom as GR. CC appears as an integration constant.

Globally, however, fixing the volume form imposes a physical restriction, i.e.,

$$\sqrt{g} = \omega(x) \longrightarrow \boxed{\int \mathrm{d}^d x \sqrt{g} = \int \mathrm{d}^d x\, \omega(x) = \text{“}V\text{”}}$$

In summary,

- **Unimodular gravity is equivalent to GR locally at the level of eoms;**
- **It is possible to make a clear definition of energy-momentum conservation in classical UG;**
- **There is a single global dof that plays a different role in GR and UG. This is associated to the spacetime total volume (and thus to the CC);**
- **In UG, the CC enters as an integration constant – it should be fixed by some boundary condition;**
- **What happens if one aims at constructing a quantum theory of unimodular metrics?**

2. Path integral of unimodular gravity

de Brito, Melichev, Percacci, *ADP* [2105.13886]

...a ghost/gauge story

Formal path integral of GR,

$$\boxed{Z_{\mathrm{GR}} = \int \frac{(\mathcal{D}g)}{\mathrm{V}_{\mathrm{Diff}}}\, \mathrm{e}^{iS_{\mathrm{EH}}[g]}} \longrightarrow \boxed{S_{\mathrm{EH}} = -\frac{1}{16\pi G}\int \mathrm{d}^d x \sqrt{g}(R + 2\Lambda)}$$

Formal path integral of UG,

$$\boxed{Z_{\mathrm{UG}} = \int \frac{(\mathcal{D}\tilde{g})}{\mathrm{V}_{\mathrm{TDiff}}}\, \mathrm{e}^{iS_{\mathrm{UG}}[\tilde{g}]}} \longrightarrow \boxed{S_{\mathrm{UG}} = -\frac{1}{16\pi G}\int \mathrm{d}^d x\, \omega\, R}$$

Path integrals seem to be very different. In (continuum) perturbative calculations, one introduces a gauge-fixing term. In the case of GR, one needs to fix the full Diff group.

Let us consider the full Diff invariant path integral. We employ a gauge fixing by means of the background field method,

$$g_{\mu\nu} = g_{\mu\nu}(\bar{g}; h)$$

Background fixed metric

Quantum fluctuations

Gauge fixing achieved by the Faddeev-Popov procedure

$$\epsilon^{\mu} = \epsilon^{\mu}_{\mathrm{T}} + \nabla^{\mu}\phi$$

Instead of gauge-fixing all Diff at once, we introduce a partial gauge fixing to the transformations generated by the "longitudinal Diffs",

$$1 = \Delta_{\mathcal{F}}(g) \int \mathcal{D}\phi\, \delta(\mathcal{F}(g^{\phi}))$$

Faddeev-Popov unity

$$\longrightarrow$$

$$Z_{\mathrm{GR}} = \int \frac{\mathcal{D}\phi \mathcal{D}h_{\mu\nu}}{V_{\mathrm{Diff}}} \Big(\Delta_{\mathcal{F}}(g)\, \delta(\mathcal{F}(g)) \Big) \mathrm{e}^{iS_{\mathrm{EH}}(\bar{g};h)}$$

Partial gauge-fixed path integral

$$\boxed{V_{\mathrm{Diff}} = \mathrm{Det}(-\nabla^2) \times V_{\mathrm{SDiff}} \times \int \mathcal{D}\phi}$$

de León Ardón, Ohta, Percacci [*Phys.Rev.D* 97 (2018) 2, 026007]
Percacci [*Found.Phys.* 48 (2018) 10, 1364-1379]

$$\boxed{Z_{\mathrm{Diff}} = \int \frac{\mathcal{D}h_{\mu\nu}}{V_{\mathrm{SDiff}}} \frac{1}{\mathrm{Det}(-\nabla^2)} \Delta_{\mathcal{F}}(g)\delta(\mathcal{F}(g))\mathrm{e}^{iS_{\mathrm{EH}}(\bar{g};h)}}$$

Next to that, let us choose an explicit form for the gauge fixing,

$$\mathcal{F}(g) = \mathrm{det} g_{\mu\nu} - \omega^2(x) \longrightarrow \Delta_{\mathcal{F}}(g) = \mathrm{Det}(\omega^2(x)(-\nabla^2))$$

$$\boxed{Z_{\mathrm{Diff}} = \int \frac{\mathcal{D}h_{\mu\nu}}{V_{\mathrm{SDiff}}} \delta(\mathrm{det} g_{\mu\nu} - \omega^2(x))\mathrm{e}^{iS_{\mathrm{EH}}(\bar{g};h)}}$$

Path integral of unimodular gravity

Thus, we have recovered the path integral of unimodular gravity by a partial gauge-fixing of the Diff-invariant path integral.

Remarks:

- **The partial gauge fixing involves the introduction of a delta function – in a general parameterization of the metric, this condition is highly complicated;**
- **The volume of the Diff group can be decomposed as the volume of SDiff group times the volume of the "longitudinal diffs" times the determinant of a differential operator;**
- **The chosen partial gauge fixing "decouples" the cosmological constant from the path integral;**
- **Using a perturbative scheme for the evaluation of expectation values seems to lead to the same results in full Diff- or SDiff- invariant theories;**

Practical implementation of the partial gauge fixing:

$$g_{\mu\nu} = \bar{g}_{\mu\alpha}\left(\mathrm{e}^{h}\right)^{\alpha}{}_{\nu} \longrightarrow \boxed{\det g = (\det \bar{g})\mathrm{e}^{h^{\mathrm{tr}}}}$$

Exponential parameterization

$$\boxed{\begin{aligned} \det g &= \omega^2(x) \\ \det \bar{g} &= \omega^2(x) \\ h^{\mathrm{tr}} &= 0 \end{aligned}}$$

The exponential parameterization imposes the unimodularity condition by setting to zero the trace of the quantum fluctuation h.

Eichhorn [***Class.Quant.Grav.* 30 (2013) 115016**]
Eichhorn [***JHEP 04 (2015) 096***]

Another possibility is the implementation of a densitizied parameterization of the metric.

$$\boxed{\begin{aligned} g_{\mu\nu} &= \gamma_{\mu\nu}(\det(\gamma_{\mu\nu}))^{m} \\ g_{\mu\nu} &= \gamma_{\mu\nu}(\det(\gamma_{\mu\nu}))^{-1/d} \end{aligned}}$$

Introduces extra Weyl invariance.

Ohta, Percacci, ***ADP*** **[*JHEP* 06 (2016) 115]**
Álvarez, González-Martin, Herrero-Valea, Martín **[*JHEP* 08 (2015) 078], and more...**

For a practical computation, we adopt the exponential parameterization of the metric, remove the trace mode, and gauge fix the SDiff invariance by a linear covariant gauge choice

$$F^{\mathrm{T}}_{\mu} = \mathcal{P}^{\mathrm{T}\nu}_{\mu}\bar{\nabla}_{\lambda}h^{\lambda}{}_{\nu} = \alpha b_{\mu} \qquad \textbf{with} \qquad \mathcal{P}^{\mathrm{T}\nu}_{\mu} = \delta^{\nu}_{\mu} - \bar{\nabla}_{\mu}(\bar{\nabla}^2)^{-1}\bar{\nabla}^{\nu}$$

This will be called "minimal unimodular" implementation.

SETUP :

$$h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu} + \bar{\nabla}_{\mu}\xi_{\nu} + \bar{\nabla}_{\nu}\xi_{\mu} + \bar{\nabla}_{\mu}\bar{\nabla}_{\nu}\sigma - \frac{1}{d}\bar{g}_{\mu\nu}\bar{\nabla}^2\sigma$$

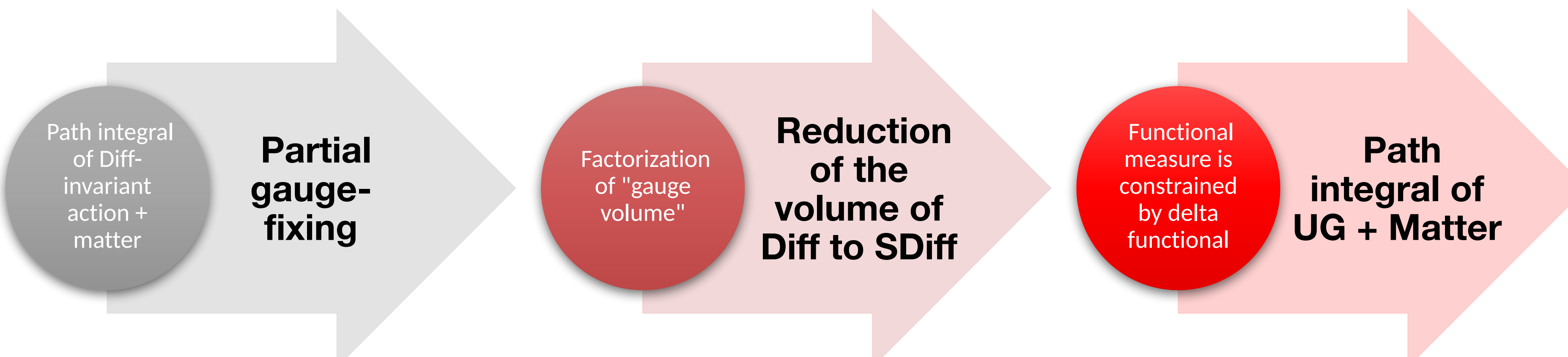
$$g_{\mu\nu} = \bar{g}_{\mu\alpha}\left(\mathrm{e}^{h}\right)^{\alpha}{}_{\nu}$$

$$S_{\mathrm{FP}} = \int \mathrm{d}^d x\,\omega\left(\bar{g}^{\mu\nu}b_{\mu}F^{\mathrm{T}}_{\nu} - \frac{\alpha}{2}\bar{g}^{\mu\nu}b_{\mu}b_{\nu} + \bar{C}_{\mu}\mathcal{M}^{\mu}{}_{\nu}C^{\nu}\right)$$

We will adopt the Euclidean version for explicit calculations.

The "proof" of equivalence between the path integral of Diff- and SDiff- invariant path integrals does not rely on the explicit form of the classical gravitational action and on the presence of matter. Essentially, it assumes that,

- **The fundamental gravitational field is the metric;**
- **It is possible to write a Diff-invariant functional measure;**
- **The partial gauge-fixing procedure is well-defined at least perturbatively.**



3. Perturbative results

Ohta, Percacci, ***ADP*** **[*Phys.Rev.D* 97 (2018) 10, 104039]**
de Brito, Melichev, Percacci, ***ADP*** **[2105.13886]**

In order to explicitly verify the claims regarding the "formal" proof of equivalence between the Diff-invariant path integral and its SDiff version, we provide the computation of one-loop beta functions in both settings. As a concrete example, we take a scalar-tensor theory defined by

$$\boxed{S[\phi, g] = \int \mathrm{d}^d x \sqrt{g}\Big(V(\phi) - F(\phi) R + \frac{1}{2}\nabla_\mu \phi \nabla^\mu \phi\Big)}$$

In the unimodular version, this is a fixed density. The "potential" does not gravitate.

Exponential parameterization

$$\sqrt{g} = \sqrt{\bar{g}}\, \mathrm{e}^{h^{\mathrm{tr}}}$$

In UG: h = 0

In Diff invariant theory, h = 0 as a gauge condition.

We can evaluate the one-loop effective action in three particular interesting ways for our purposes (all employing the exponential parameterization for simplicity):

1. **Consider the full-Diff invariant action and employ a two-parameter covariant gauge fixing**

$$\bar{\nabla}^{\nu} h_{\nu\mu} - \frac{1+\beta}{d}\bar{\nabla}_{\mu} h^{\text{tr}} = \alpha B_{\mu} \qquad \textbf{with} \qquad \alpha \to 0 \ , \ \ \beta \to -\infty$$

2. **Employ different gauge conditions to SDiff and "longitudinal diffs";**

$$F^{\text{T}}_{\mu} = \mathcal{P}^{\text{T}\nu}_{\mu} \bar{\nabla}_{\lambda} h^{\lambda}{}_{\nu} = \alpha b_{\mu} \qquad h^{\text{tr}} = \lambda b \qquad \textbf{with} \qquad \alpha = 0 \ , \ \ \lambda = 0$$

OBS1: In both cases, the gauge conditions impose the vanishing of the trace of the quantum fluctuation. However, this condition requires the introduction of suitable FP ghosts. Therefore, we call this "unimodular *gauge*"

3. We consider the unimodular theory, i.e., the trace of h is taken to zero as a definition of the configuration space. The residual SDiff invariance is fixed by

$$F^{\mathrm{T}}_{\mu} = \mathcal{P}^{\mathrm{T}\,\nu}_{\mu} \bar{\nabla}_{\lambda} h^{\lambda}{}_{\nu} = \alpha b_{\mu} \qquad \textbf{with} \qquad \alpha = 0$$

OBS2: In this case, there is no FP ghost associated with the condition tr h=0. This is genuinely what we have defined as "unimodular *gravity*".

From our general argument: Schemes 2 and 3 should give exactly the same result. Moreover, since Scheme 1 effectively corresponds to the same gauge fixing as in Scheme 2, we should conclude that the one-loop effective action is the same, i.e.,

Scheme 1 = Scheme 2 = Scheme 3

Scheme 1:

The one-loop effective action received contributions from spin-2 and spin-0 fluctuations in the gravitational sector + spin-0 fluctuations from the scalar fields + spin-0 and spin-1 contributions from the FP ghosts (d=4; maximally symmetric space)

$$\boxed{\Gamma = S + \frac{1}{2}\mathrm{Tr}\log\Delta_2 - \frac{1}{2}\mathrm{Tr}\log\Delta_1 + \frac{1}{2}\mathrm{Tr}\log\Delta_S}$$

$$\Gamma_{div} = -\frac{1}{2}\frac{1}{16\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)\int \mathrm{d}^4x\sqrt{\bar{g}}\,[b_4(\Delta_2) - b_4(\Delta_1) + b_4(\Delta_S)]$$

$$\Delta_2 = -\bar{\nabla}^2 + \frac{\bar{R}}{6} \qquad \Delta_1 = -\bar{\nabla}^2 - \frac{\bar{R}}{4} \qquad \Delta_S = -\bar{\nabla}^2 + E_S \qquad E_S = \frac{FV'' - (F'^2 + FF'')\bar{R}}{F + 3F'^2}$$

Scheme 2:

In this scheme, the gauge-fixing term is written as

$$\boxed{S_{\mathrm{gf}} = \frac{1}{2\zeta_1}\int \mathrm{d}^4x\sqrt{\bar{g}}\,\bar{g}^{\mu\nu}F^{\mathrm{T}}_{\mu}F^{\mathrm{T}}_{\nu} + \frac{1}{2\zeta_2}\int \mathrm{d}^4x\sqrt{\bar{g}}\,h^2}$$

Generates a transverse vector ghost contribution.

Generates a scalar ghost contribution.

$$\boxed{S^{\mathrm{T}}_{\mathrm{gh}}[\bar{C}, C] = -\sqrt{2}\int \mathrm{d}^4x\sqrt{\bar{g}}\,\bar{C}^{\mathrm{T}}_{\mu}\left(-\bar{\nabla}^2 - \frac{\bar{R}}{4}\right)C^{\mathrm{T}\mu}}$$

$$\boxed{S^{\mathrm{scalar}}_{\mathrm{gh}} = -\int \mathrm{d}^4x\sqrt{\bar{g}}\,\bar{c}\bar{\nabla}^2 c}$$

Proper factorization of the SDiff group volume:

$$\boxed{\int \mathcal{D}\epsilon^{\mathrm{T}}\delta(F^{\mathrm{T}}_{\mu})\Delta_{\mathrm{FP}} = 1}$$

with

$$\boxed{V_{\mathrm{SDiff}} = \int \mathcal{D}\epsilon^{\mathrm{T}}\det{}^{-1/2}(-\nabla^2)}$$

The volume of the SDiff group is generated by the integration of one of the scalar contributions arising from the gravitational action.

Scheme 3:

In this scheme Tr h = 0 as a condition over the configuration space and one just has to fix the SDiff invariance, i.e.,

$$S_{\text{gf}} = \frac{1}{2\alpha}\int \mathrm{d}^4x\,\omega\,\bar{g}^{\mu\nu}F^{\text{T}}_{\mu}F^{\text{T}}_{\nu}$$

Generates a transverse vector ghost contribution.

$$\sqrt{\det{}_1\left(-\bar{\nabla}^2-\frac{\bar{R}}{4}\right)}$$

Again, a proper factorization of the volume of the SDiff group produces a scalar determinant which accounts for the scalar contribution arising from the gravitational action.

Scheme 1 = Scheme 2 = Scheme 3

$$\beta_{\mathcal{V}}=\frac{m^4}{32\pi^2}$$

$$\beta_{m^2}=\frac{3\lambda}{2\pi^2}m^2-\frac{6Gm^4\xi^2}{\pi}$$

$$\beta_{\lambda}=\frac{9\lambda^2}{2\pi^2}-\frac{72Gm^2\lambda\xi^2}{\pi}$$

$$\beta_{G}=-\frac{G^2m^2(1+6\xi)}{6\pi}$$

$$\beta_{\xi}=\frac{\lambda(1+6\xi)}{4\pi^2}+\frac{Gm^2\xi^2(1-12\xi)}{\pi}$$

$$V(\phi)=\mathcal{V}+\frac{1}{2}m^2\phi^2+\lambda\phi^4\dots\,,\qquad \mathcal{V}=\frac{\Lambda}{8\pi G_N}$$

$$F(\phi)=Z_N+\frac{1}{2}\xi\phi^2+\dots\,,\qquad Z_N=\frac{1}{16\pi G_N}$$

Those are the one-loop results where we just collected the regulator-*independent* contributions, i.e., those arising from log-divergences.

Remarks:

- The gravitational contribution to the matter-coupling beta functions are still gauge dependent.

$$\boxed{\begin{aligned}\beta_{m^2} &= \frac{3m^2\lambda}{2\pi^2} + \frac{2Gm^4(4\alpha - 3(2+(3-\beta)\xi)^2)}{(3-\beta)^2\pi}\,,\\ \beta_\lambda &= \frac{9\lambda^2}{2\pi^2} - \frac{8Gm^2\lambda(12-4\alpha+24(3-\beta)\xi+9(3-\beta)^2\xi^2)}{(3-\beta)^2\pi}\,,\\ \beta_\xi &= \frac{\lambda(1+6\xi)}{4\pi^2} - \frac{Gm^2}{12\pi}\mathcal{F}(\alpha,\beta,\xi)\,.\end{aligned}}$$

- Define couplings by an appropriate gravitational dressing of the correlation functions. [Work in progress]

- Similar strategy adopted in Yang-Mills theories with the non-Abelian dressed gauge-invariant composite field,

$$
\boxed{\begin{aligned}
A^h_\mu &= A_\mu - \partial_\mu \frac{1}{\partial^2}\partial A + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] - ig\frac{1}{\partial^2}\partial_\mu\left[A_\alpha, \partial_\alpha\frac{1}{\partial^2}\partial A\right] + \frac{ig}{2}\frac{1}{\partial^2}\partial_\mu\left[\frac{1}{\partial^2}\partial A, \partial A\right] \\
&+ \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu\frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)\,.
\end{aligned}}
$$

$$
\boxed{\begin{aligned}
&\delta A^h_\mu = 0 \\
&\langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_n}(x_n)\rangle
\end{aligned}}
$$

- For practical perturbative calculations, working with the non-local form is sufficient;

- For non-perturbative calculations, this composite field can be localized in terms of a Stueckelberg-like field – the local action is non-polynomial very much like SYM theories using superfields. Capri, Fiorentini, Sorella, ***ADP*** **[*Phys.Rev.D* 96 (2017) 5, 054022]**

4. Going beyond perturbation theory

Neither GR nor its unimodular version are perturbatively renormalizable.

— Infinitely many counterterms with free coefficients (to be fixed by external renormalization conditions) are needed.

However, perturbation theory around a free (Gaussian) fixed point might be a too strong requirement. Perhaps, a predictive quantum field theory of (unimodular) metrics might exist as an *asymptotically safe theory*

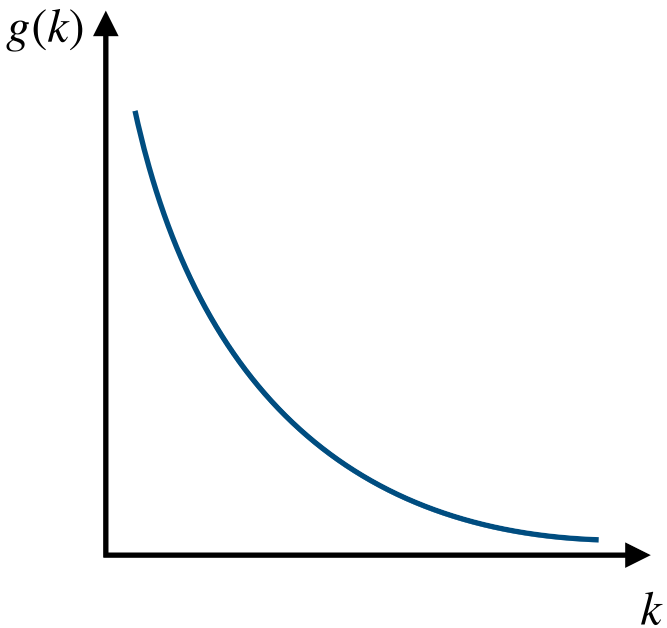
Weinberg **[*General Relativity, chapter 16, S.W. Hawking and W. Israel eds*];**
Smolin **[*Nucl.Phys.B* 208 (1982) 439-466];**
Kawai, Ninomiya **[*Nucl.Phys.B* 336 (1990) 115-145]**
Reuter **[*Phys. Rev. D57, 971 (1998)*]**
Souma **[*Prog. Theor. Phys. 102, 181 (1999)*]**
Eichhorn **[*Front.Astron.Space Sci.* 5 (2019) 47]**
Pawlowski & Reichert **[*Front. Phys., 24 February 2021*]**
Books by ***Percacci, and Reuter & Saueressig***
See also the 'recent' "Critique" by Donoghue **[*Front.in Phys.* 8 (2020) 56];** The "Critical Reflections" by several members of the AS community; Bonanno, Eichhorn et al. **[*Front.in Phys.* 8 (2020) 269]**, and the debate available in PIRSA between Donoghue and Percacci.

Asymptotic Safety in Unimodular QG:

Eichhorn **[*Class.Quant.Grav.* 30 (2013) 115016]**
Eichhorn **[*JHEP 04 (2015) 096*]**
Saltas **[*Phys.Rev.D* 90 (2014)]**
Benedetti **[*Gen.Rel.Grav.* 48 (2016) 5, 68]**
de Brito, Eichhorn, ***ADP*** **[*JHEP* 09 (2019)]**
de Brito, ***ADP*** **[*JHEP* 09 (2020) 196]**
de Brito, Vieira, ***ADP*** **[*Phys.Rev.D* 103 (2021) 10, 104023]**

Paradigmatic example: QCD

Coupling runs to zero at arbitrarily large energy scales: Asymptotic Freedom



The theory becomes free in the deep UV

Perturbation theory is a good toolbox to probe such a fixed point

Paradigmatic example: QCD

Coupling runs to zero at arbitrarily large energy scales: Asymptotic Freedom

Advanced Seminars:

1. **Quantum chromodynamics** - Gastão Krein (IFT, Brazil);
2. **Finslerian structure of black holes** - Alberto Saa (Unicamp, Brazil);
3. **Rotating black holes** - Oleg Zaslavskii (Kharkov, Ukraine);
4. **Analogue black holes** - Maurício Richartz (UFABC, Brazil);
5. **Black holes and wormholes** - Kirill Bronnikov (NITs PM/Rússia);
6. **Conformal symmetry and renormalization in QFT** - Ilya Shapiro (UFJF, Brazil);
7. **Asymptotic Freedom** - Antônio Duarte Pereira (UFF, Brazil).

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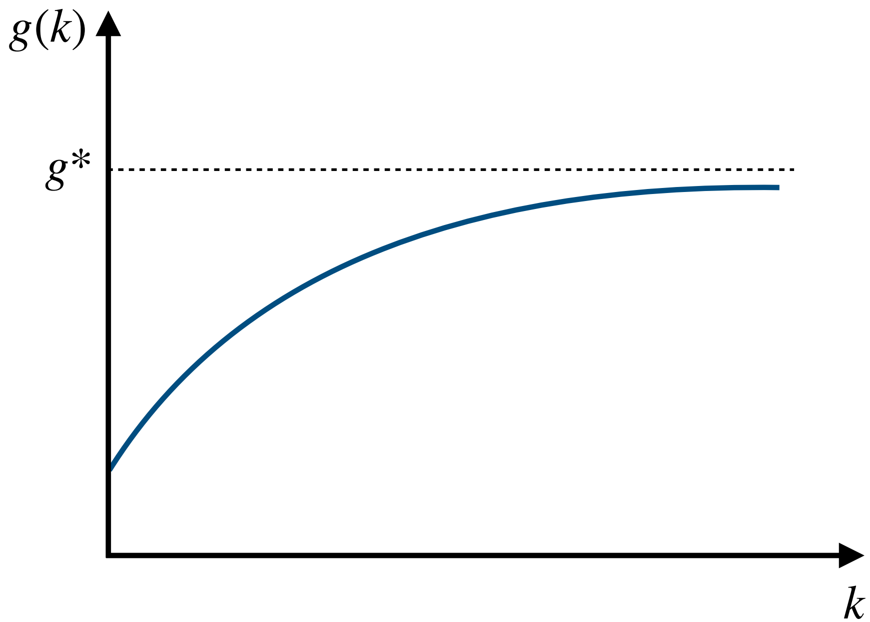
Another possibility…

Coupling runs to finite value at arbitrarily large energy scales: Asymptotic Safety



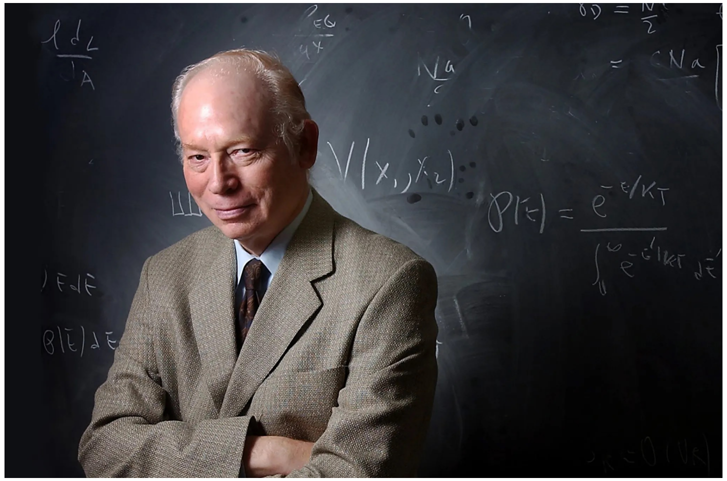
The theory remains interacting in the deep UV

Perturbation theory is NOT a good toolbox to probe such a fixed point

Could quantum gravity-matter system be asymptotically safe?

Weinberg

Go non-perturbative!



Steven Weinberg conjectured that Quantum Gravity could be an asymptotically safe quantum field theory

[“Ultraviolet divergences in quantum theories of gravitation” - S. Weinberg] - *An Einstein centenary survey* edited by Israel and Hawking, 1979

“[...] this paper will be chiefly concerned with another possibility, that a quantum field theory which incorporates gravitation may satisfy a generalized version of the condition of renormalizability known as asymptotic safety.”

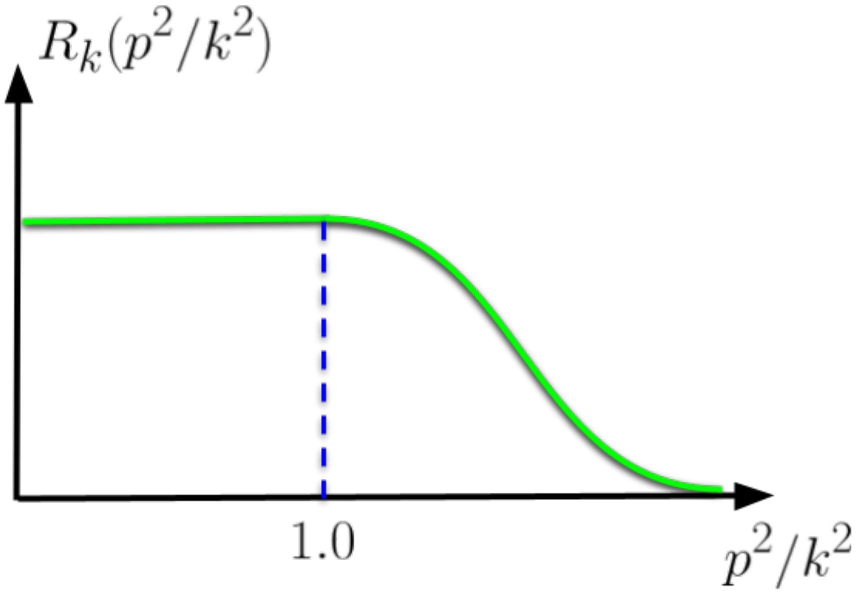
Asymptotic Safety Condition: the dimensionless counterparts of the *essential* couplings should reach a UV renormalization group fixed point

At the fixed point the couplings cease to run: quantum scale invariance

Most of the progress in the field is due to the functional renormalization group

Wetterich [*Phys.Lett.B* 301 (1993) 90-94]
Dupuis, Canet, Eichhorn, Metzner, Pawlowski, Tissier, and Wschebor [*Phys.Rept.* 910 (2021) 1-114]

$$\boxed{\int [\mathcal{D}\varphi]\, \mathrm{e}^{-S(\varphi) - \int_p \varphi(p) R_k(p^2) \varphi(-p)}}$$



$$\boxed{\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1} \right]}$$

Exact flow equation

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Wetterich [*Phys.Lett.B* 301 (1993) 90-94]
Dupuis, Canet, Eichhorn, Metzner, Pawlowski, Tissier, and Wschebor [*Phys.Rept.* 910 (2021) 1-114]

$$\boxed{\int [\mathcal{D}\varphi]\, \mathrm{e}^{-S(\varphi)-\int_p \varphi(p) R_k(p^2)\varphi(-p)}} \longleftrightarrow \boxed{\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left[\partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}\right]}$$

$$\boxed{\Gamma_k = \sum_i \bar{g}_i(k)\mathcal{O}^i(\phi)}$$

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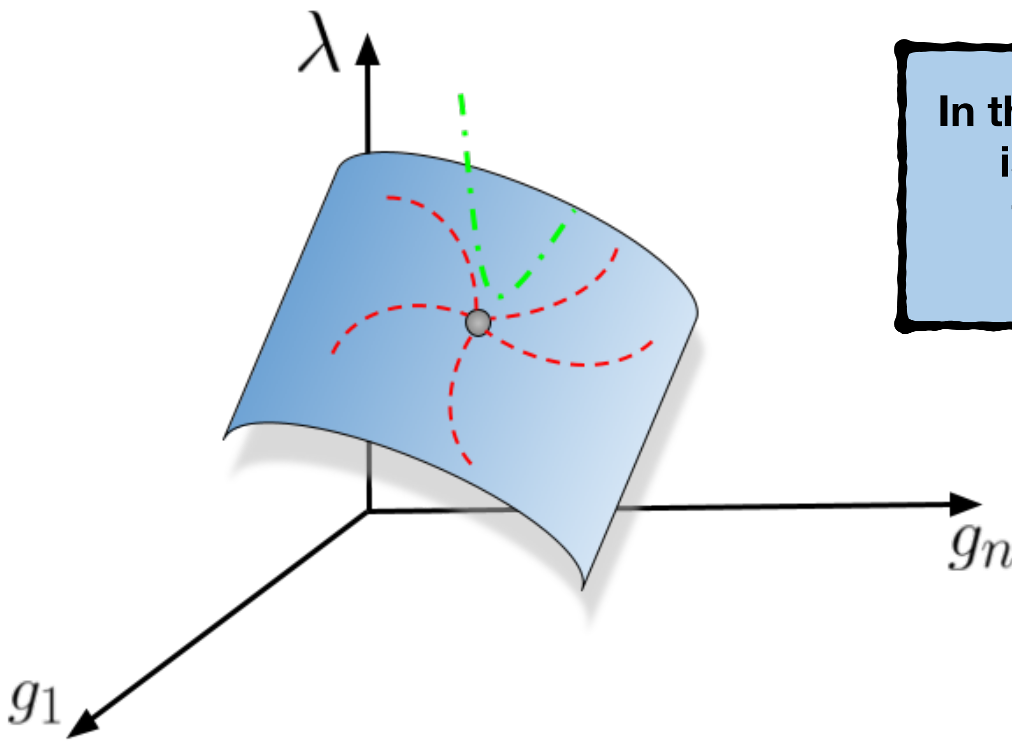
$$\Gamma_k = \sum_i \bar{g}_i(k)\, \boxed{\mathcal{O}^i(\phi)}$$

All operators compatible with the symmetries of the underlying theory, deformed by the regulator.

Suitable truncation schemes can be applied and the flow equation allows for the evaluation of the IR cutoff dependence of all couplings that span the theory space.

Thus, one can look for a scaling regime (non-trivial fixed point); UV behavior is controlled by *quantum scale invariance.*

For Diff-invariant theories, the theory space is defined by all operator compatible with the deformed BRST-invariant operators associated with the presence of gauge fixing + regulator. In particular, there is a direction associated to the cosmological constant.



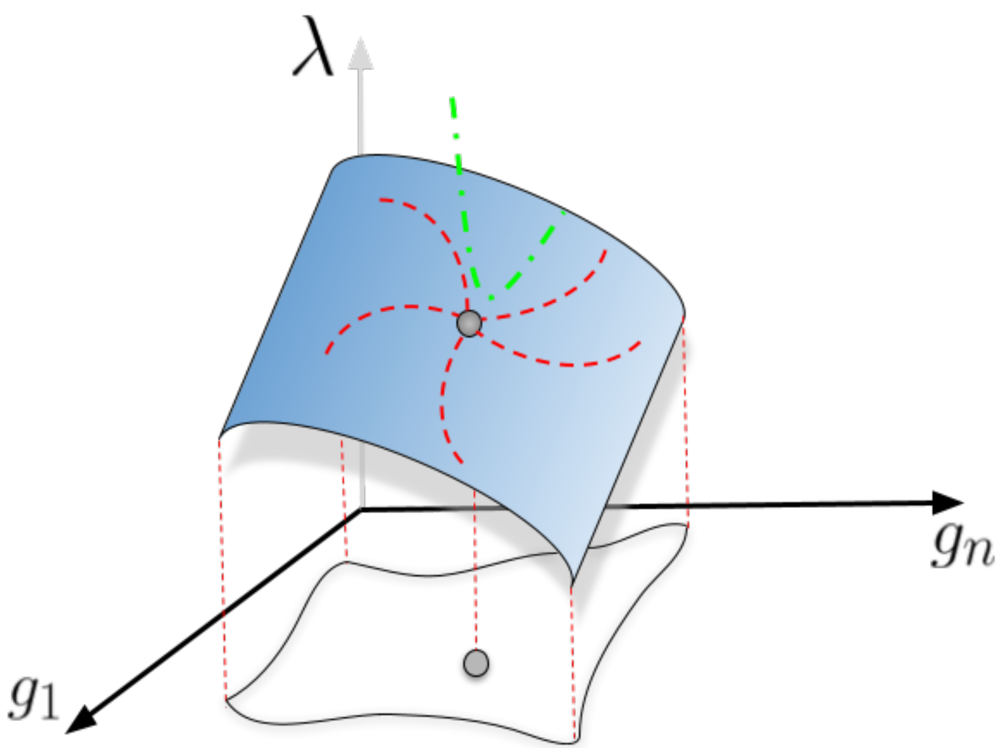
In the "standard" AS scenario, the cosmological constant is treated as an essential coupling. As such, it must feature a non-trivial fixed point – the cosmological constant runs up to the (quantum) scale regime.

Several investigations point to a cosmological constant as a *relevant* coupling, i.e., it must be fixed by external data.

Eichhorn [***Front.Astron.Space Sci.*** **5 (2019) 47]**
Pawlowski & Reichert [***Front. Phys., 24 February 2021***]
Books by ***Percacci, and Reuter & Saueressig***
Bonanno, Eichhorn et al. [***Front.in Phys.*** **8 (2020) 269]**
Dupuis, Canet, Eichhorn, Metzner, Pawlowski, Tissier, and Wschebor [***Phys.Rept.*** **910 (2021) 1-114]**

However, if one starts with a theory space defined by SDiff, there is no cosmological constant as a coupling constant that should feature a fixed point. From this perspective, unimodular quantum gravity is not equivalent to “standard quantum gravity".

But: path integral analysis implies equivalence between unimodular gauge and unimodular gravity. Possibility: "physical properties" of the fixed point live in the SDiff theory space;



Theory invariant under SDiff has one less coupling than theory invariant under Diff, very much like in SU(N) and U(N) gauge theories.

When dealing with the SDiff group, the derivation of the flow equation needs to account for the non-trivial factorization of the gauge group volume,

$$Z_{\mathrm{SDiff}} = \int \frac{\mathcal{D}h_{\mu\nu}}{V_{\mathrm{SDiff}}} \left(\int \mathcal{D}\epsilon^{\mathrm{T}} \Delta_{\mathrm{FP}} \delta(F^{\mathrm{T}}) \right) \mathrm{e}^{-S_{\mathrm{UG}}(\bar{g};h)}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right] - \frac{1}{2} \mathrm{Tr} \left(\frac{\partial_t R_k(-\bar{\nabla}^2)}{P_k(-\bar{\nabla}^2)} \right)$$

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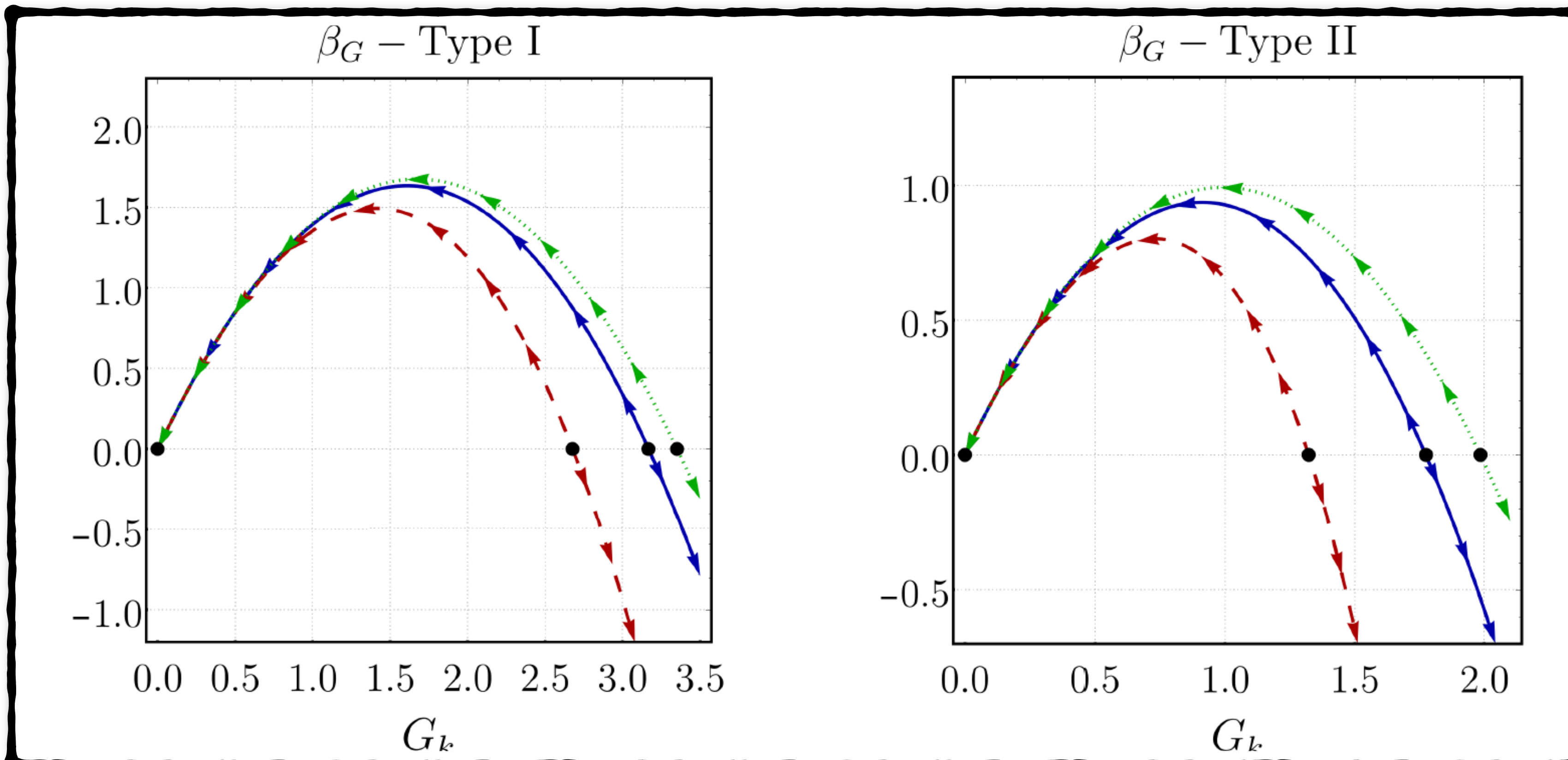
Contribution akin to the scalar ghost present in the Diff-invariant version.

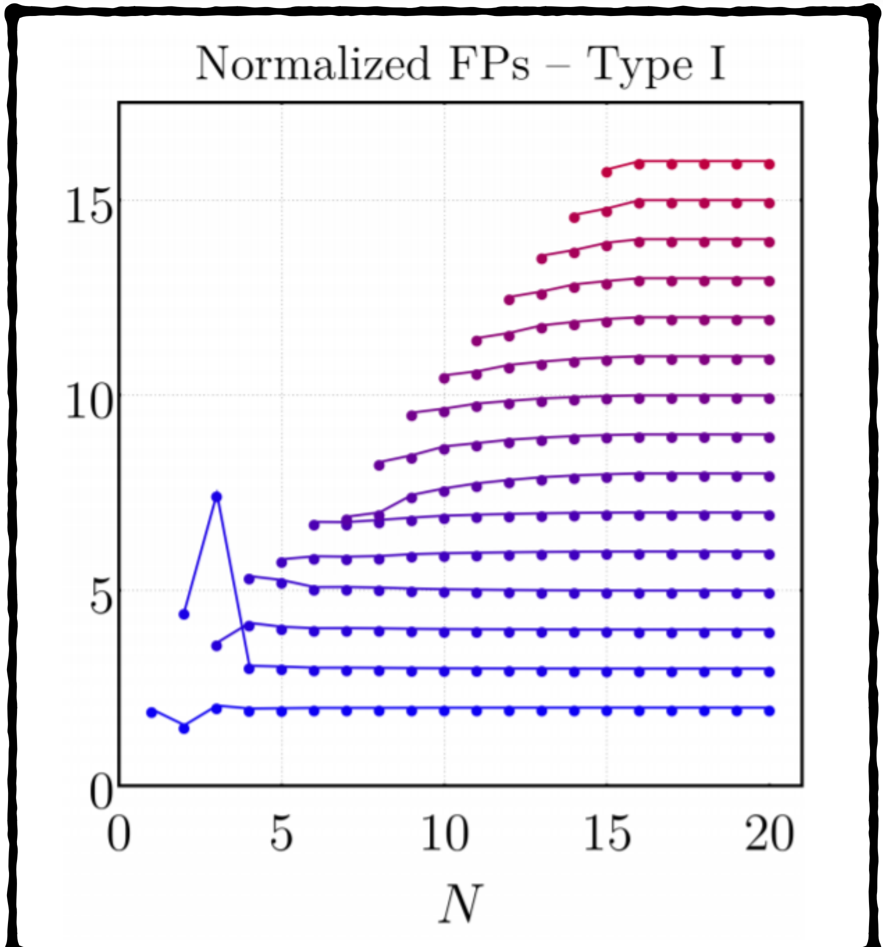
Search for a fixed point for Newton coupling in the simplest approximation for the flowing action;

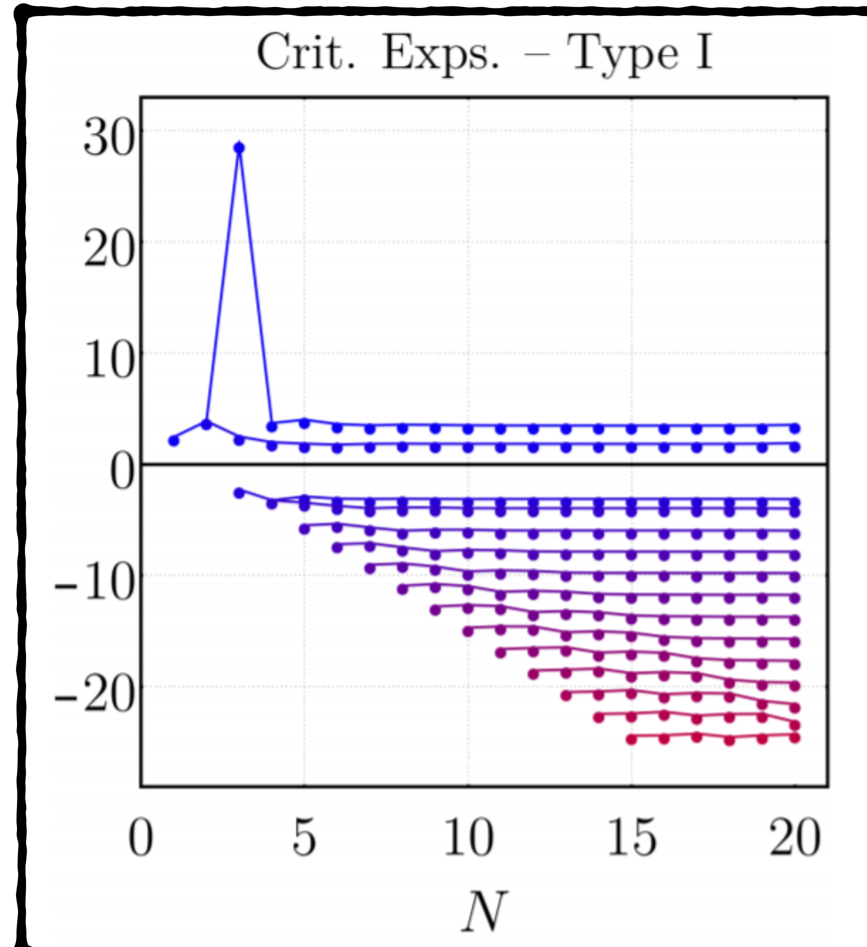
de Brito, *ADP* [*JHEP* 09 (2020) 196]



Enlarging the truncation, we have verified that the fixed point persists up to the considered order

de Brito, Vieira, ***ADP*** **[*Phys.Rev.D* 103 (2021) 10, 104023]**





f(R)-Polynomial truncations

Are "standard" quantum gravity and unimodular quantum gravity in different universality classes?

- **Most calculations in standard gravity indicate the existence of *three* relevant directions; In the unimodular gauge, this number reduces to two.**
- **As for unimodular *gravity* the number of relevant directions seems to be two**

Is the unimodular gauge not just a gauge choice but a definition of a different theory or is "standard" gravity being explored in a larger theory space than the “physical” one?

Moreover, the "unimodularity" condition can be imposed in several different ways. Are the underlying quantum theories all equivalent?

What different approaches to quantum gravity can tell about that? Loop quantum unimodular gravity Vs Loop quantum gravity; Canonical quantization; Lattice-like approaches,...

Baulieu, Bufalo, de León Ardón, Percacci, Ohta, Oksanen, Smolin, Tureanu, Yamashita,...

Adding matter degrees of freedom:

de Brito, Vieira, *ADP* [*Phys.Rev.D* 103 (2021) 10, 104023]

Stability of NGFP for some matter models							
Model	Matter content			Type I		Type II	
	N_ϕ	N_A	N_ψ	$f(R)$	$F(R_{\mu\nu}^2) + RZ(R_{\mu\nu}^2)$	$f(R)$	$F(R_{\mu\nu}^2) + RZ(R_{\mu\nu}^2)$
SM	4	12	45/2	✓(2)	✓(2)	✓(2)	✓(2)
SM + $3\nu_R$	4	12	24	✓(2)	✓(3)	✓(2)	✓(2)
MSSM	49	12	61/2	✖	✖	✖	✖
SU(5) GUT	124	24	24	✖	✖	✖	✖
SO(10) GUT	97	45	24	✓*(2)	✓*(3)	✓*(2)	✓*(2)

5. Perspectives and conclusions

Unimodular gravity provides the same local dynamics as GR, but treats the cosmological constant differently; This raises the natural question about the resulting quantum theory: is it equivalent to quantum GR or not?

For many practical purposes: they are the same!

If the unimodular gauge is a legitimate choice, then one can suppress trace fluctuations by a suitable gauge.

Should the cosmological constant "run" after all?

A particularly interesting question is whether the 'equivalence' between unimodular gauge and unimodular gravity is preserved when more gravitational structures are considered, such as torsion and non-metricity;

Thank you!