Gravitational Wave Observations

Riccardo Sturani

International Institute of Physics - UFRN - Natal (Brazil)

Inverno Astrofísico, August 8th 2019

GW basics in 1 slide

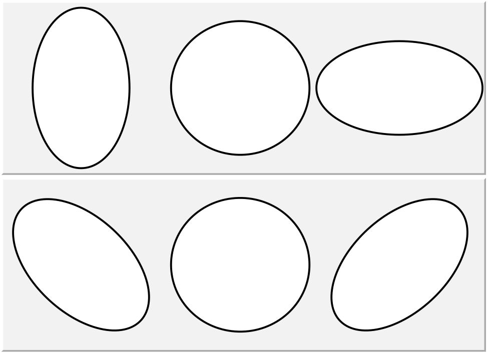
Gauged fixed metric perturbations $h_{\mu\nu}$ after discarding $h_{0\mu}$ components, which are not radiative $\rightarrow$ transverse waves

$$h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

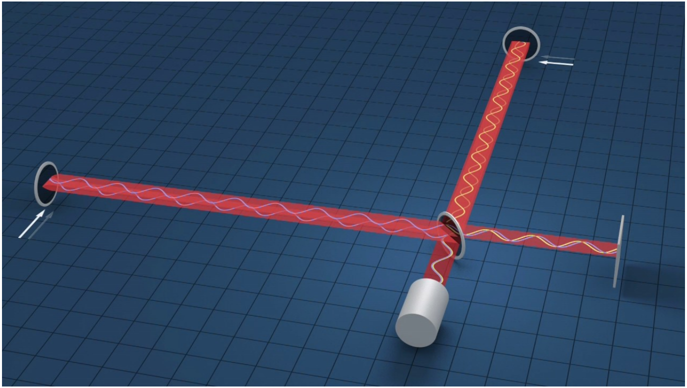
2 pol. state like any mass-less particle

h_+

$h_\times$



LIGO and Virgo: very precise rulers



Robert Hurt (Caltech)

Light intensity $\propto$ light travel difference in perpendicular arms
Effective optical path increased by factor $N \sim 500$ via Fabry-Perot cavities
Phase shift $\Delta\phi \sim 10^{-8}$ can be measured $\sim 2\pi N\Delta L/\lambda \rightarrow \Delta L \sim 10^{-15}/N$ m

Almost omnidirectional detectors

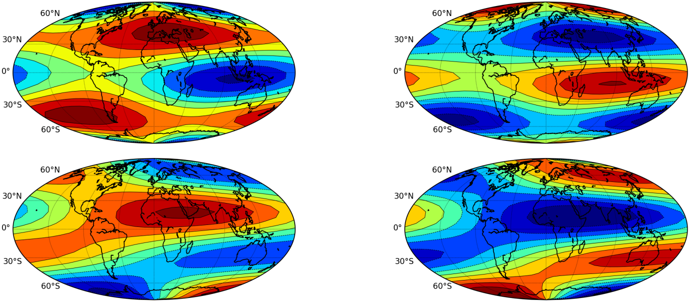
Detectors measure h_{det}: linear combination $F_+ h_+ + F_\times h_\times$

H L

-1 0 1

F_+

$F_\times$



$h_{+,\times}$ depend on source

pattern functions $F_{+,\times}$ depend on orientation source/detector

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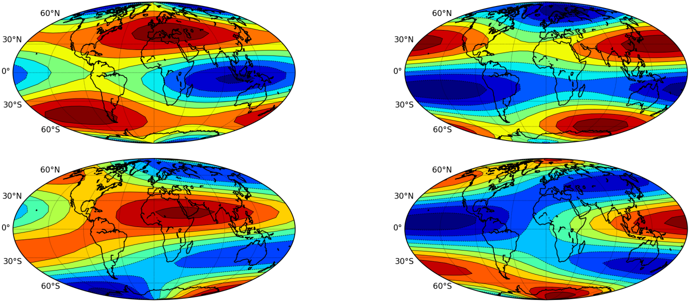

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The LIGO and Virgo observatories

- Observation run **O1** Sept '15 - Jan '16 $\sim$ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, 2 detectors, 3BBH
- **O2** Dec. '16 – Jul'17 2 det's + Aug '17 3 det's 3(+4) BBH + 1BNS in double (triple) coinc.
- **O3**, 3 detectors, started on April 1st 2019, last $\sim$ 1yr

In $\sim$ end of 2019 Japanese KAGRA and 2020+ Indian INDIGO will join the collaboration

Wave generation: localized sources

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j - \frac{1}{3}\delta_{ij}x^2\right)\,, \qquad v^2 \simeq G_N M/r\,, \quad \eta \equiv m_1 m_2/M^2 \\
h_{ij} &\sim g(\theta_{LN})\frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\eta M v^2}{D}\cos(2\phi(t))
\end{aligned}
$$

$$
\begin{aligned}
f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 12kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}}
\end{aligned}
$$

Geometric factor $g(\theta_{LN})$ takes account of transversality projection (angular momentum L of the binary, observation direction N)

$$
\begin{aligned}
h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\eta\frac{Mv^2}{D}\cos\phi(t_s/M,\eta,S_i^2/m_i^4,\ldots) \\
h_\times &\sim \cos(\theta_{LN})\,\eta\frac{Mv^2}{D}\sin\phi(t_s/M,\eta,\ldots)
\end{aligned}
$$

Amplitudes of 2 polarizations modulated by θ_{LN} ($h \nearrow$ for $\theta_{LN} \searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

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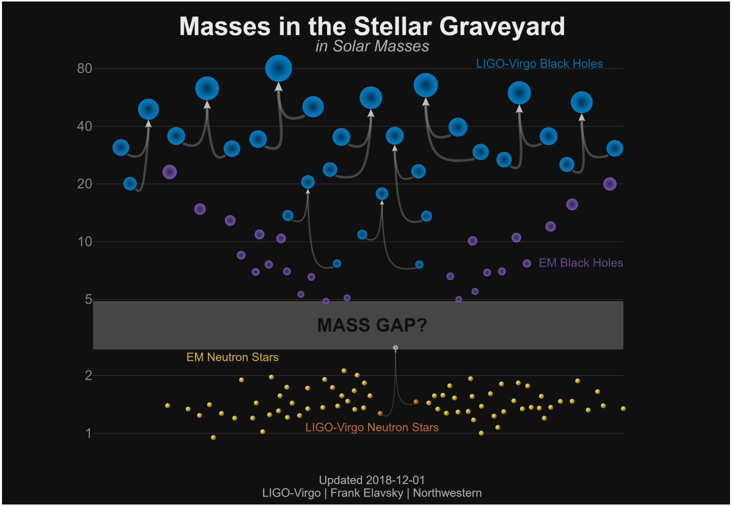
Amplitudes of 2 polarizations modulated by θ_{LN} ($h\nearrow$ for $\theta_{LN}\searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

h sensitive to red-shifted masses $M\rightarrow M(1+z)\equiv\mathcal{M}$

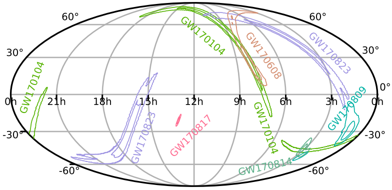
Binary systems and compact detected so far

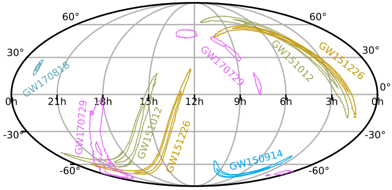


Event	Primary mass (M_sun)	Secondary mass (M_sun)	Effective inspiral spin	chirp mass (M_sun)	Final spin	Final mass (M_sun)	Luminosity distance (Mpc)	Sky localization (deg^2)
GW150914	$\mathbf{35.6}^{+4.8}_{-3.0}$	$\mathbf{30.6}^{+3.0}_{-4.4}$	$\mathbf{-0.01}^{+0.12}_{-0.13}$	$\mathbf{28.6}^{+1.6}_{-1.5}$	$\mathbf{0.69}^{+0.05}_{-0.04}$	$\mathbf{63.1}^{+3.3}_{-3.0}$	$\mathbf{430}^{+150}_{-170}$	**179**
GW151012	$\mathbf{23.3}^{+14.0}_{-5.5}$	$\mathbf{13.6}^{+4.1}_{-4.8}$	$\mathbf{0.04}^{+0.28}_{-0.19}$	$\mathbf{15.2}^{+2.0}_{-1.1}$	$\mathbf{0.67}^{+0.13}_{-0.11}$	$\mathbf{35.7}^{+9.9}_{-3.8}$	$\mathbf{1060}^{+540}_{-480}$	**1555**
GW151226	$\mathbf{13.7}^{+8.8}_{-3.2}$	$\mathbf{7.7}^{+2.2}_{-2.6}$	$\mathbf{0.18}^{+0.20}_{-0.12}$	$\mathbf{8.9}^{+0.3}_{-0.3}$	$\mathbf{0.74}^{+0.07}_{-0.05}$	$\mathbf{20.5}^{+6.4}_{-1.5}$	$\mathbf{440}^{+180}_{-190}$	**1033**
GW170104	$\mathbf{31.0}^{+7.2}_{-5.6}$	$\mathbf{20.1}^{+4.9}_{-4.5}$	$\mathbf{-0.04}^{+0.17}_{-0.20}$	$\mathbf{21.5}^{+2.1}_{-1.7}$	$\mathbf{0.66}^{+0.08}_{-0.10}$	$\mathbf{49.1}^{+5.2}_{-3.9}$	$\mathbf{960}^{+430}_{-410}$	**924**
GW170608	$\mathbf{10.9}^{+5.3}_{-1.7}$	$\mathbf{7.6}^{+1.3}_{-2.1}$	$\mathbf{0.03}^{+0.19}_{-0.07}$	$\mathbf{7.9}^{+0.2}_{-0.2}$	$\mathbf{0.69}^{+0.04}_{-0.04}$	$\mathbf{17.8}^{+3.2}_{-0.7}$	$\mathbf{320}^{+120}_{-110}$	**396**
GW170729	$\mathbf{50.6}^{+16.6}_{-10.2}$	$\mathbf{34.3}^{+9.1}_{-10.1}$	$\mathbf{0.36}^{+0.21}_{-0.25}$	$\mathbf{35.7}^{+6.5}_{-4.7}$	$\mathbf{0.81}^{+0.07}_{-0.13}$	$\mathbf{80.3}^{+14.6}_{-10.2}$	$\mathbf{2750}^{+1350}_{-1320}$	**1033**
GW170809	$\mathbf{35.2}^{+8.3}_{-6.0}$	$\mathbf{23.8}^{+5.2}_{-5.1}$	$\mathbf{0.07}^{+0.16}_{-0.16}$	$\mathbf{25.0}^{+2.1}_{-1.6}$	$\mathbf{0.70}^{+0.08}_{-0.09}$	$\mathbf{56.4}^{+5.2}_{-3.7}$	$\mathbf{990}^{+320}_{-380}$	**340**
GW170814	$\mathbf{30.7}^{+5.7}_{-3.0}$	$\mathbf{25.3}^{+2.9}_{-4.1}$	$\mathbf{0.07}^{+0.12}_{-0.11}$	$\mathbf{24.2}^{+1.4}_{-1.1}$	$\mathbf{0.72}^{+0.07}_{-0.05}$	$\mathbf{53.4}^{+3.2}_{-2.4}$	$\mathbf{580}^{+160}_{-210}$	**87**
GW170817	$\mathbf{1.46}^{+0.12}_{-0.10}$	$\mathbf{1.27}^{+0.09}_{-0.09}$	$\mathbf{0.00}^{+0.02}_{-0.01}$	$\mathbf{1.186}^{+0.001}_{-0.001}$	**≤ 0.89**	**≤ 2.8**	$\mathbf{40}^{+10}_{-10}$	**16**
GW170818	$\mathbf{35.5}^{+7.5}_{-4.7}$	$\mathbf{26.8}^{+4.3}_{-5.2}$	$\mathbf{-0.09}^{+0.18}_{-0.21}$	$\mathbf{26.7}^{+2.1}_{-1.7}$	$\mathbf{0.67}^{+0.07}_{-0.08}$	$\mathbf{59.8}^{+4.8}_{-3.8}$	$\mathbf{1020}^{+430}_{-360}$	**39**
GW170823	$\mathbf{39.6}^{+10.0}_{-6.6}$	$\mathbf{29.4}^{+6.3}_{-7.1}$	$\mathbf{0.08}^{+0.20}_{-0.22}$	$\mathbf{29.3}^{+4.2}_{-3.2}$	$\mathbf{0.71}^{+0.08}_{-0.10}$	$\mathbf{65.6}^{+9.4}_{-6.6}$	$\mathbf{1850}^{+840}_{-840}$	**1651**

https://www.gw-openscience.org/catalog/

Sky localization of GW binary systems detected so far





localization regions shrinks when Virgo in 3rd detector matters!

Astro sources emitting transient GWs/EM/ν radiation

- Coalescing binary systems of compact objects: $\eta \equiv m_1 m_2/M^2$

$$\Delta E_{GW} \sim 4\times 10^{-2} M_\odot \eta \left(\frac{M}{3M_\odot}\right)^{5/3}\left(\frac{f_{max}}{1\,\mathrm{kHz}}\right)^{2/3}$$

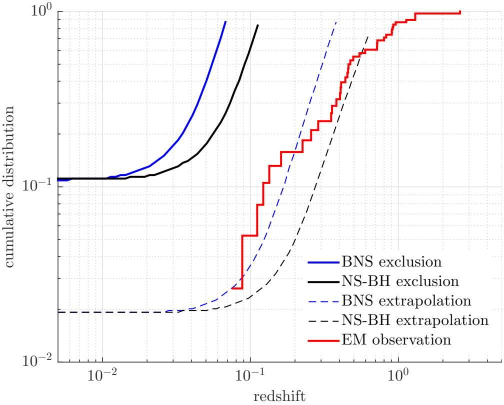
  A good proxy of the f_{max} is the Innermost Stable Circular Orbit frequency

$$f_{ISCO} = \frac{1}{6\sqrt{6}}\frac{1}{M} \simeq 2\mathrm{kHz}\left(\frac{M}{M_\odot}\right)^{-1}$$

  For NS-NS/BH ejected material supposed to produce short (<2 sec) GRBs $E_\gamma \sim$keV-MeV, ejected sub-relativistic material may produce radio signals

  ν: baryon loaded jets may emit νs: TeV-PeV νs @ γ emission; PeV-EeV ν @ optical afterglow
- NS-NS/BH $\rightarrow$ energetic outflows at different timescale/wavelength
- Relativistic jet $\rightarrow$ prompt short γ-ray burst (GRB) duration $\lesssim 1$ sec followed by X, optical, radio afterglows (hrs $-$ days)
- Rapid neutron capture in the sub-relativistic ejecta can produce a kilonova or macronova, with optical and near-infrared signal (hours - weeks)
- Eventually radio blast from sub-relativistic outflow (months to years) Several seconds prior to or tens of minutes after merger, coherent radio burst lasting milliseconds may be emitted

Association with 20 short GRB during O1 (before GW170817)

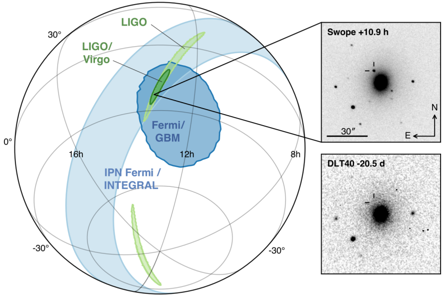


GW searched in a [-5,+1) sec window with GRB
GW constraints far from EM exclusion distance, but extrapolation to design Advanced LIGO sensitivity (2 years of data) will lead to detection/non-trivial constraints

LIGO/Virgo Astrophys.J. 841 (2017) no.2, 89

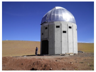
GW170817

- GW trigger on Aug 17th, 2017, ended at 12h 41' 04.4" UTC, first in in LIGO Hanford, then confirmed as a triple coincidence $\rightarrow$ localized in an area of $\sim$ 28 deg^2
- GRB trigger from Fermi-GBM 1.7" after
- first optical image 10.87 hr afterwards by One-Meter Two Hemisphere team with Swope telescope at Las Campanas Observatory in Chile
- X obtained by the X-Ray Telescope on Swift after 14.9 h (NuSTAR 16.8 h)
- radio ($\sim 3,6$ GHz) by VLA 16 days after GW event

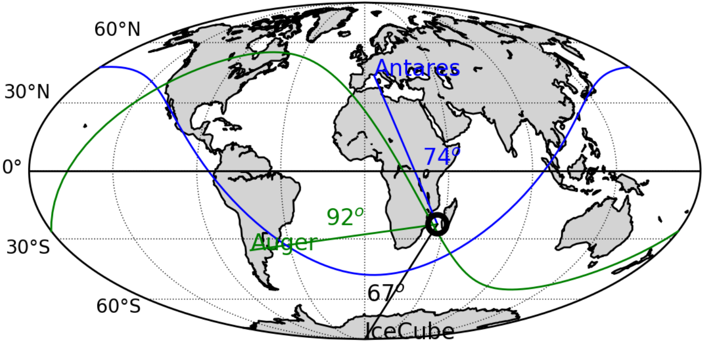


LIGO/Virgo & Partner Astronomy groups, Astrophys.J. 848 (2017) no.2, L12

An example of the optical image

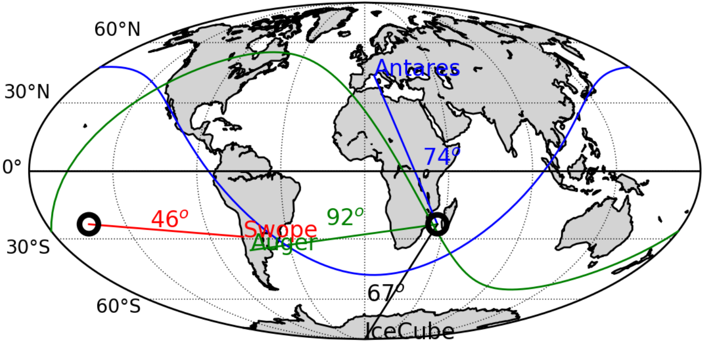
Transient Optical Robotic Observatory of the South 

GW170817 and ν detectors



Potential νs down-going for Antares and IceCube, grazing for Auger

GW170817 and ν detectors



Potential νs down-going for Antares and IceCube, grazing for Auger

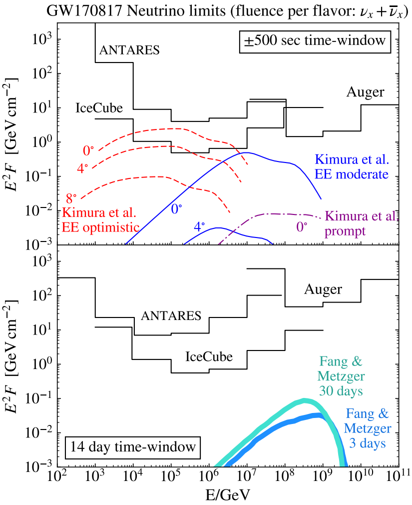
Source location at optical detection

ANTARES and ICECube look for ν

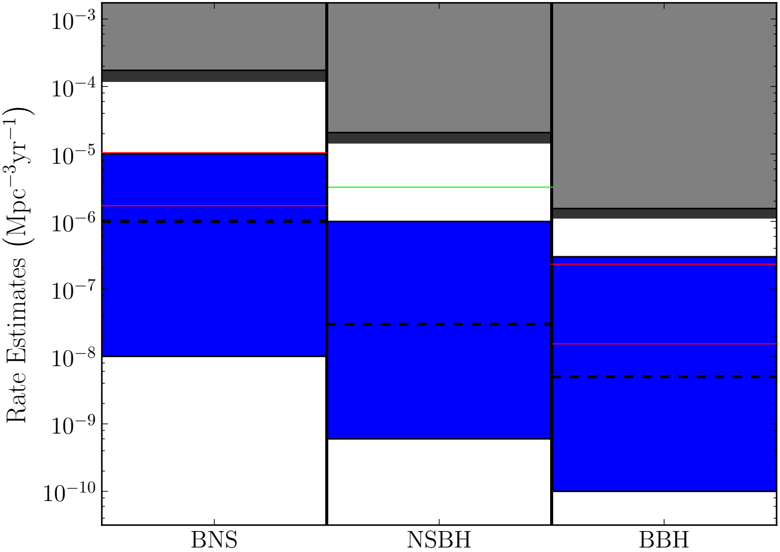
IceCube and ANTARES looked for $\mu - \nu$ at $\pm 500 sec$, observation for 2 weeks

no detection

Pierre Auger for $> 10^{17}$ eV, grazing good for τ production at 10^{18} eV



LIGO/Virgo, ANTARES, ICECUBE and Pierre Auger, Astrophys.J. 850 '17 2, L35



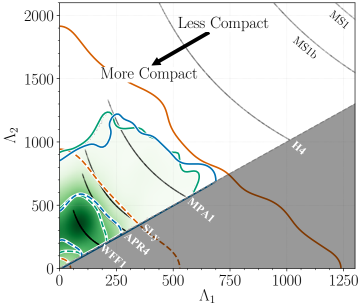
Astrophysical predictions, upper limit from S5/6/O1 and observation from O1/O2

LIGO/Virgo CQG ('10) 27 173001, PRX ('16), APJ (2016), PRL 119 ('17)

Fundamental physics of Neutron stars

Tidal deformability $\Lambda \simeq R^{-5} Q_{ab} / \left(\partial_a \partial_b U_{tidal}\right)$ depends on Equation of State

Determinaton of $\Lambda_{1,2}$
(assuming same EoS for 1,2)



Shading Common EoS for 2 bodies and constrain for $m_{max} > 1.97 M_\odot$
Lines 50%, 90% countours, No mimimum m_{max}, independent EoSs

LIGO/Virgo collaboration, arXiv:1805.11581

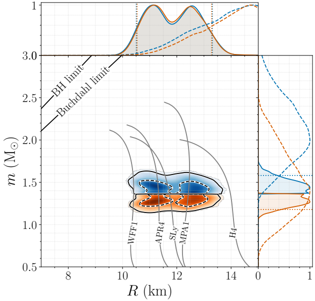
Fundamental physics of Neutron stars II

Parametrizing EoS with $\Gamma \equiv \frac{\epsilon+p}{p}\frac{dp}{d\epsilon}$, $\Gamma(x) = \exp\left(\sum_k \gamma_k x^k\right)$, $\qquad x \equiv \log\frac{p}{p_0}$

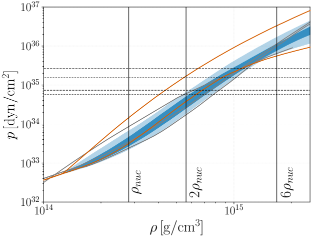
$$\gamma_0 \in [0.2, 2], \gamma_1 \in [1.6, 1.7], \gamma_2 \in [0.6, 0.6] \gamma_3 \in [0.02, 0.02], \Gamma(p) \in [0.6, 4.5]$$

L. Lindblom PRD 82, 103011 (2010)

Constraints on Mass vs- radius and p vs. rho



lighter, heavier



90% prior, posterior

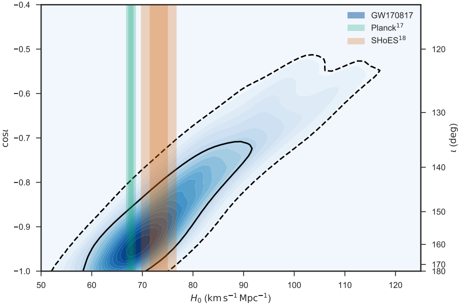
LIGO/Virgo collaboration, arXiv:1805.11581

Cosmology

- From GWs only: luminosity distance $d_L \sim 40^{+8}_{-14}$Mpc, viewing angle $\theta_{LN} > 125^o$
- Fixing d_L from info from EM location + Hubble relation $d_L H_0 = z$
  1. Hubble constant from SuperNovae (SHoES) $148^o < \theta_{LN} < 166^o$

     Riess et al. ApJ 826, 56 '16
  2. from Planck $157 < \theta_{LN} < 177^o$

     Planck A&A, 594, A13 '16
- GW almost coincident with EM $\Longrightarrow \Delta v_{GW}/c \sim 10^{-15}$ LIGO/Virgo, Fermi Gamma-Ray Burst Monitor, INTEGRAL Ap.J. 848 (2017) 2, L13



LIGO/Virgo Nature 551 '17 7678, 85

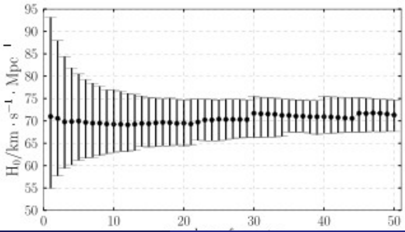
Determining H_0 without EM counterpart

Hubble law: $z = H_0 d_L$

D_L can be measured, z degenerate with M, however if

- the source in the sky has been localised (α, δ)
- GW sources are in the galaxy catalogue with known red-shift

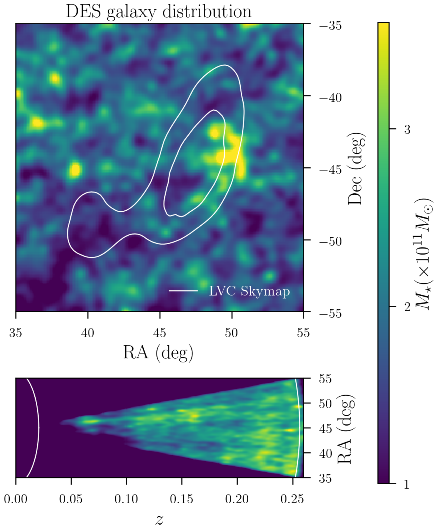
$$P(z, D_L|c_i) = \int d\mathcal{M}\, d\vec{\theta}\, d\alpha\, d\delta\, P(D_L \mathcal{M}, \vec{\theta}, \alpha, \delta|c_i)\pi(z, |\alpha, \delta)$$



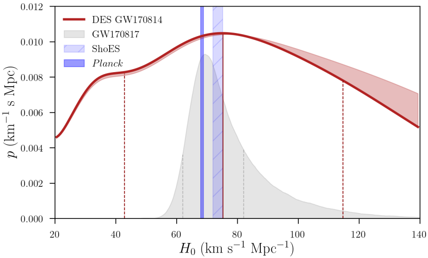
Schutz, Nature (1986) 323 310

W. Del Pozzo, Phys.Rev.D86 (2012) 043011

DES and H_0



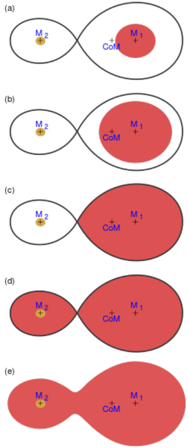
sky localization for GW170814



P.D.F for H_0

DES and LIGO/Virgo, arXiv:1901.01540

How binary systems progenitors formed? I



Izzard et al. Proc. IAU 2012

A binary system of massive stars ($M_{binary} < 100 M_{odot}$) can go through a common envelope phase that shrinks considerably the orbit and align the spins $\rightarrow$ collapse to compact objects

BNS requires complex evolution from massive binaries + mass transfer and 2 core-collapse SN explosions through which the binary system survives

Asymmetries in collapse imparts natal kick to the (galactic pulsar proper motion)

LIGO/Virgo Ap.J. 850 (2017) L40

vs.

For BBH another mechanism is possible

Globular Cluster: medium with high density of black holes/stars, when 3 black holes meet one is ejected and the binary shrinks

LIGO/VirgoAstrophys.J. 818 (2016) no.2, L22

Remnants of the first stars, produced at $z \sim 6$ can give only a small contribution to the total rate

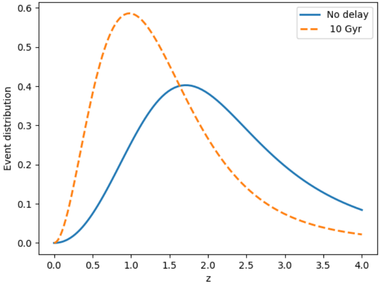
T. Hartwig et al. Mon.Not.Roy.Astron.Soc. 460 (2016) L74

Primordial black-holes? viable if $10^{-2} \lesssim M_{BH}/M_{\odot} \lesssim 1$ (constraints from evaporation, microlensing and CMB) but could grow later to 20-100 $M_{\odot}$ by merger
If present in globular cluster may also explain their cosmological abundance as dark matter

Sasaki M. et al. PRL 2016, Bird S. et al. PRL 2016, Clesse S. and García-Bellido Phys. Dark Univ. 2017

Ongoing O3

At https://gracedb.ligo.org/superevents/public/O3
Summary of candidate detections from O3:
25 alert sent out (3 retracted), 3 of 22 possibly involving at least a NS



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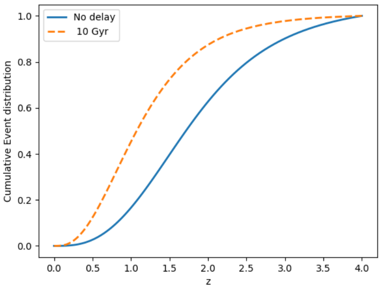
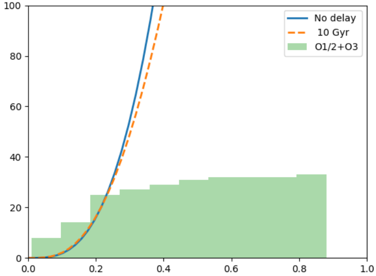
Summary of candidate detections from O3:
25 alert sent out (3 retracted), 3 of 22 possibly involving at least a NS





Fundamental GR: inspiral analytic model

Inspiral $h = A\cos(\phi(t))$ $\qquad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 \left(1 + \#(\eta)v^2 + \#(\eta)v^4 + \dots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} \left(1 + \#(\eta)v^2 + \#(\eta)v^3 + \dots\right) \end{aligned}$$

E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv \\ &\sim \int \left(1 + \#(\eta)v^2 + \dots + \#(\eta)v^6 + \dots\right) \frac{dv}{v^6} \end{aligned}$$

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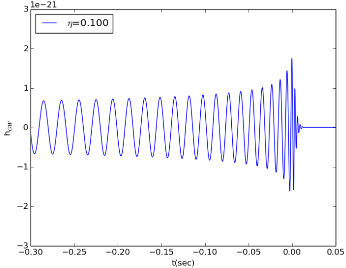
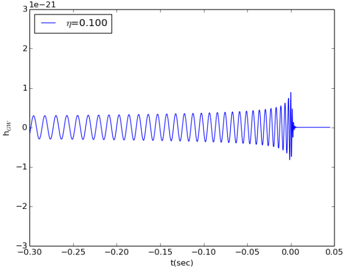
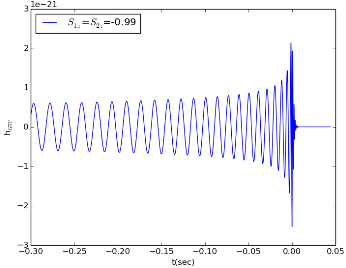
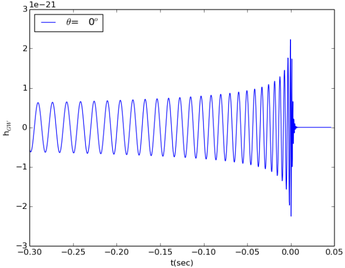
PN Coefficients (tidal $\sim v^{10}$)

Parameter estimation from $\phi(\mathbf{t})$









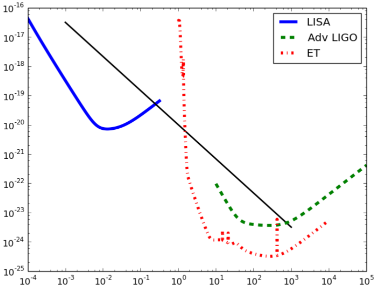
Precision gravity

Future 3rd generetaion detectors (Einstein Telescope, Cosmic Explore) / space telescope LISA will detect BBH binary signals with SNR $10 - 10^2$, with few golden events with SNR $\sim 10^3$.
Templates few % accurate OK for characterising a source with SNR $O(10)$ (typical for LIGO/Virgo)
for SNR $\sim 10^3$ residual after extracting that source will have SNR $\sim O(10)$

1. baising parameter estimation
2. contaminating the extraction of additional sources.

$$h(f) \propto f^{-7/6} M_c^{5/6}$$

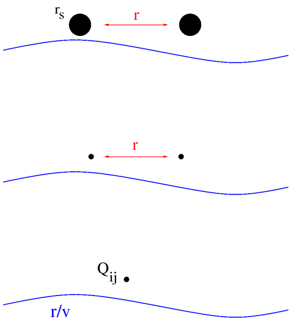
$$\Delta t_{i\to m} \propto M_c^{-5/3}(f_i^{-8/3} - f_m^{-8/3})$$



The EFT point of view on the 2-body problem

Different scales in EFT (borrowing ideas from NRQCD):

- Very short distance $\lesssim r_s = G_N M$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $H_{\mu\nu}$, $k_\mu \sim (v/r, 1/r)$ with $r \sim r_s/v^2$
- Long distance: GW's $k_\mu \sim (v/r, v/r)$ GWs $h^{GW}_{\mu\nu}$ coupled to point particles with moments



M. Beneke and V. A. Smirnov, NPB '98; I. Stewart and W. Manohar, PRD '07; W. Goldberger and I. Rothstein, PRD '06

EFT for compact binary systems

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x) \exp\left[iS_{EH}(H + h_{GW}) + iS_{pp}(H + h_{GW})\right]$$

$$\begin{aligned} S_{pp} &= -m \int d\tau \\ S_{EH} &= \frac{1}{32\pi G_N} \int d^4x \left[(\partial h)^2 + h(\partial h)^2 + h^2(\partial h)^2 \ldots\right] \end{aligned}$$

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$$\begin{aligned} S_{pp} &= -m\int dt\, d^3x\,\delta(\vec{x}-\vec{x}_a(t))\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + h_{00}^2\ldots\right) \\ S_{EH} &= \frac{1}{32\pi G_N}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + h(\partial h)^2 + h^2(\partial h)^2\ldots\right] \end{aligned}$$

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$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x)\exp\left[iS_{EH}(H+h_{GW}) + iS_{pp}(H+h_{GW})\right]$$

$$\begin{aligned}
S_{pp} &= -m\int dt\, d^3x\, \delta(\vec{x}-\vec{x}_a(t))\left(\sqrt{G_N}\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2\right) + G_N h_{00}^2 \ldots\right)\\
S_{EH} &= \frac{1}{2}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N}h(\partial h)^2 + G_N h^2(\partial h)^2 \ldots\right]
\end{aligned}$$

EFT for compact binary systems

Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

Effective

- Generic v-dependent potential terms
- Instantaneous couplings between world-line particles

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}H(x)\exp\left[iS_{EH}(H+h_{GW}) + iS_{pp}(H+h_{GW})\right]$$

$$\begin{aligned} S_{pp} &= -m\int dt\, d^3x\, \delta(\vec{x}-\vec{x}_a(t))\left(\sqrt{G_N}\left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2\right) + G_N h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{2}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N} h(\partial h)^2 + G_N h^2(\partial h)^2 \ldots\right] \end{aligned}$$

$$S_{eff} = m_1\int dt\, d^3x\left[\frac{1}{2}v_1^2 + \frac{G_N m_2}{2r} + \frac{1}{8}v_1^4 + \frac{G_N^2 m_2}{2r^2}\left(\frac{G_N m_1}{2r} + v_1^2 + v_{1r}v_{2r}\right) + 1 \leftrightarrow 2\right]$$

3PN: Jaranowski, Schäfer, Damour; Blanchet, Faye; Itoh Futamase; Foffa & RS
4PN: Jaranowski, Schäfer, Damour; Blanchet et al, RS Foffa; RS Foffa, Mastrolia, Sturm

How to compute effective potential?

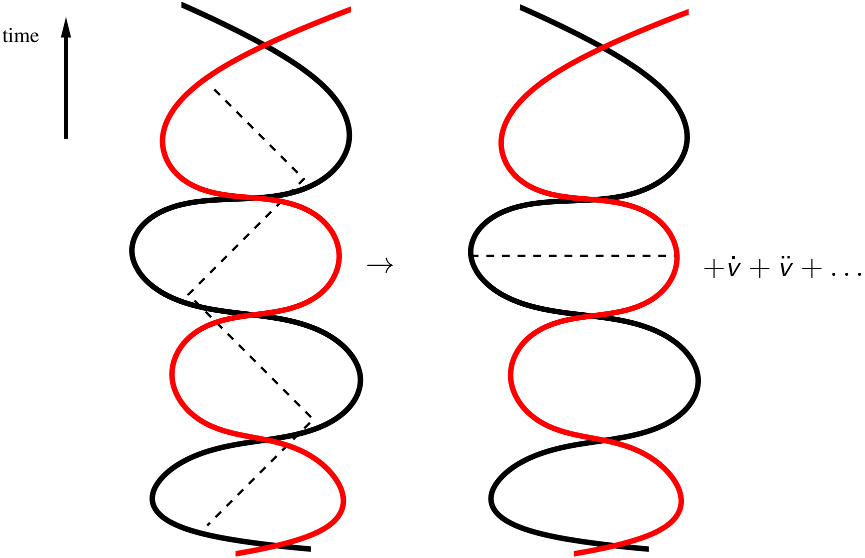
Iteratively solve Einstein equations for point particles ($J(x) \sim \delta^3(x - \bar{x})$):

$$
\begin{aligned}
-\int dt V_{1-2}(t,r) &\sim \int dt\, dt'\, d^3x\, d^3x' J(x) G(x-x') J(x') \\
&= 32\pi G_N m_1 m_2 \int dt\, dt' \frac{d^4k}{(2\pi)^4} \frac{e^{ik^\mu (x_1(t_1) - x_2(t_2))_\mu}}{k^2 - \omega^2} \\
&\simeq G_N \int dt\, dt' \delta(t-t') \frac{m_1 m_2}{|x_1(t) - x_2(t')|}
\end{aligned}
$$

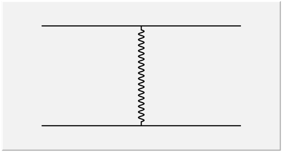
where k_0 has been neglected. Considering effects of v

$$
\begin{aligned}
V &\propto \int d\omega d^3k \frac{e^{-i\omega t_{12} + ikx_{12}}}{k^2 - \omega^2} = \int d\omega d^3k \frac{e^{-i\omega t_{12} + ikx_{12}}}{k^2} \left(1 + \frac{\omega^2}{k^2} + \ldots\right) \\
&= \delta(t_{12}) \int d^3k \frac{e^{ikx_{12}}}{k^2} \left(1 + \frac{\partial_{t_1}\partial_{t_2}}{k^2} + \ldots\right) = \int d^3k \frac{e^{ikx_{12}}}{k^2} \left(1 - \frac{k \cdot v_1 k \cdot v_2}{k^2} + \ldots\right)
\end{aligned}
$$

Trading knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time

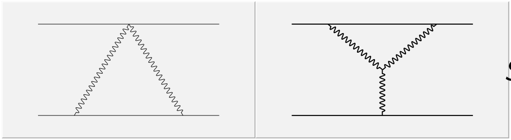


Newtonian potential:

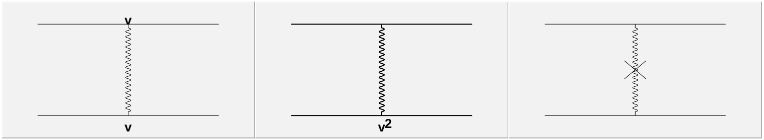


$$S_{eff} \sim \int dt \, \frac{G_N m_1 m_2}{r} \sim L$$

At 1PN:

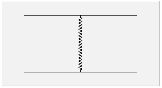
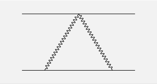
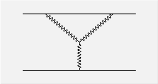


$$S_{eff} = \int dt \, \frac{G_N^2 m_1^2 m_2}{r^2} \sim L v^2$$

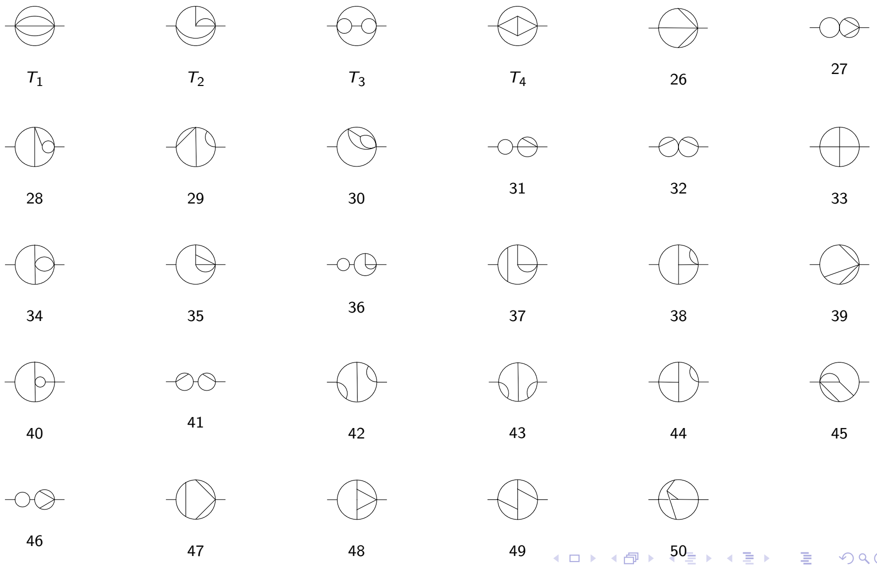


$$V_{1PN} = -\frac{G m_1 m_2}{2r}\left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2} v_1 v_2 - \frac{1}{2} v_1 \hat{r} v_2 \hat{r}\right] +$$
$$1 \leftrightarrow 2$$

The 4PN sector

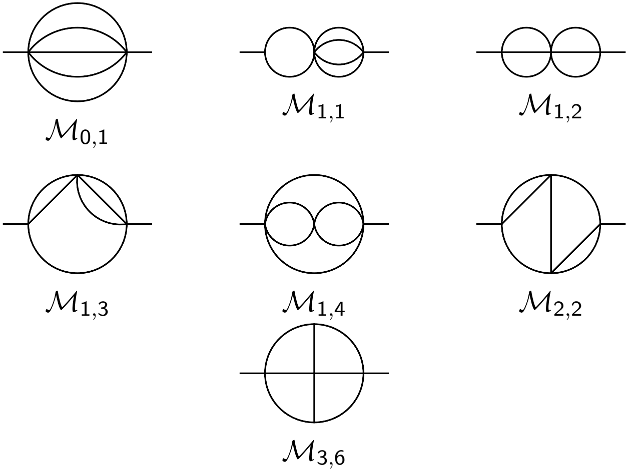
- G: : 3 diagrams
- G^2: 21 diagrams, RS and S. Foffa, PRD '12
- G^3: 212 diagrams
- G^4: 317 diagrams RS and S. Foffa ArXiv:1902.xxxx
- G^5: 50 diagrams, RS et al. PRD '18

4PN static G_N^5 topologies



G_N^5 master integrals

expressed via integration by parts reduction in terms of 7 master integrals:



Mapping the problem into 4-loop 3-D self-enrgy diagrams

RS, S. Foffa, P. Mastrolia, C. Sturm PRD (2017) 104009

2 body dynamics expansions (spin-less)

Post-Minkowskian expansion parameter is $G_N M/r$, vs PN expansion

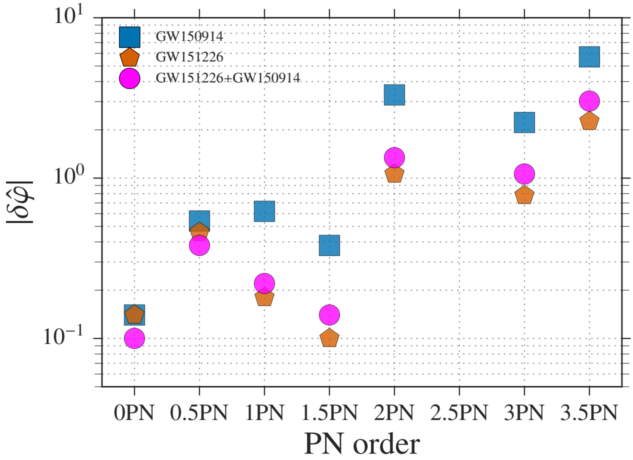
$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots]$$

Terms known so far

		N	1PN	2PN	3PN	4PN	5PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	...
5PM						$1/r^5$	v^2/r^5	...
...							...	...

3PM recently computed (!) by Z. Bern, C. Cheung et al. arXiv:1901.04424
What's next? Amplitude program with modern methods: generalized unitarity, gravity as a double copy of gauge theory...

Testing the PN series with GW150914 and GW151226



Precision in measuring the PN coefficients $\sim 100\%$: one cannot both measure the astro parameters and test GR with 1 detection! Still better precision than with binary pulsars

LIGO/Virgo Phys.Rev. X6 (2016) 4, 041015

Conclusions (Actually just the beginning!)

- After the beginning of GW astronomy, we have witnessed the start of multi-messenger astronomy
- New era for Cosmology: H_0 from GWs! (and EM)
- New input for fundamental physics: test of gravity in the strong field at short scale, radiative gravity sector tested at large distances and soon precision gravity is going to be needed