Superradiance scattering and superradiant instability in the Einstein-Maxwell-dilaton theory

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Introduction

- The EMd gravity emerges as the bosonic sector in the $N = 4$ supergravity theory. It also appears in the low-energy limit of the heterotic string theory as an effective model.
- The Lagrangian for the EMd model and the external charged massive scalar field is given by

$$L = \Big(R - 2\partial_\mu\phi\partial^\mu\phi - e^{-2\phi}F_{\mu\nu}F^{\mu\nu}\Big) - [D_\mu\Psi^\dagger D^\mu\Psi + m^2\Psi\Psi^\dagger], \quad (1)$$

where $D_\mu = \nabla_\mu - ieA_\mu$. Here Ψ denotes an external (ultralight) field with $r_g/\lambda_c = Mm \lesssim 1$.
- The black hole solution reads as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 g(r)d\Omega^2, \;\; f(r) = 1 - \frac{2M}{r} \quad (2)$$

$$F = \frac{Q}{r^2}dr\wedge dt, \;\; e^{2\phi} = g(r), \;\; g(r) = \left(1 - \frac{Q^2}{Mr}\right). \quad (3)$$

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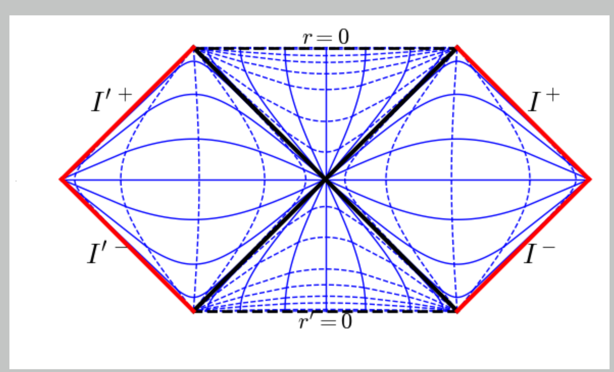
- For $Q < \sqrt{2}M$, the event horizon hides the singularity, and the Penrose diagram is similar to the Schwarzschild one.



Superradiant scattering

- If a charged massive scalar field is scattered off by a charged black hole, under certain conditions, the energy of the outgoing wave likely becomes greater than the energy of the incoming wave. As a result, the outgoing modes at spatial infinity are amplified.
- To explore the superradiant scattering (SS), the Klein-Gordon equation of the external field is written as a Schrödinger equation for the radial modes in the tortoise coordinate,

$$-\partial_{xx}\psi_{\omega\ell m} + \left(V - \Big[\omega - eA_t\Big]^2\right)\psi_{\omega\ell m} = 0, \quad (4)$$

where the potential reads

$$V(r) = f(r)\Big[\frac{rf'(r)g'(r) + f(r)\,(2g'(r) + rg''(r))}{2rg(r)} + \frac{f'(r)}{r} - \frac{f(r)g'(r)^2}{4g(r)^2} + \frac{\ell(\ell+1)}{r^2 g(r)} + m^2\Big]. \quad (5)$$

- The boundary condition at the horizon must be an ingoing plane wave, but at infinity, there is a superposition of the outgoing wave plus an ingoing wave:

$$\psi_{\omega\ell m} = \begin{cases} T_{\omega\ell}e^{-i\xi x}, & x \to -\infty \\ e^{-i\rho x} + R_{\omega\ell}e^{i\rho x}, & x \to \infty, \end{cases} \quad (6)$$

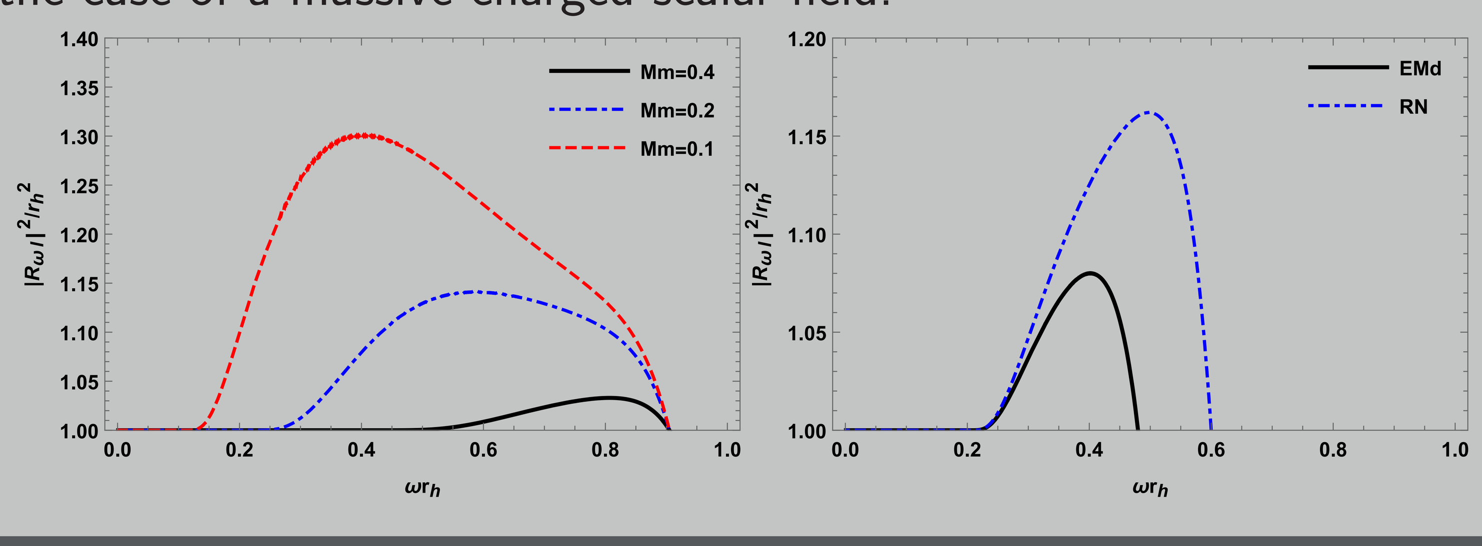
where $\xi = \omega - eA_t(r_h)$ and $\rho = \sqrt{\bar{\omega}^2 - m^2}$.
- From the conservation of the flux associated with the spatial 3-current, $J^T_x = J^I_x + J^R_x$, the amplitudes read

$$|R_{\omega l}|^2 + \frac{\xi}{\rho}|T_{\omega l}|^2 = 1, \quad (7)$$

where the extra factor in front of the transmission coefficient (7) is signaling a SS as long as $\xi/\rho < 0$.

Numerical Simulation

- Larger values of Me lead to bigger amplitudes in the $|R_{\omega\ell}|^2$ coefficient. But, the opposite effect by increasing the values of Mm.
- For a dilatonic black hole, the SS is less powerful than the RN case, at least for the case of a massive charged scalar field.



Superradiant Mechanism: Trapped modes.

- Using the first law of thermodynamics for charged black holes and the local conservation of energy for the composed system (BH+SF), one arrives at

$$\delta\mathcal{M} = \frac{\kappa_s}{8\pi}\Big(\frac{\omega}{\omega - q\Phi_H}\Big)\delta\mathcal{A}_H.$$

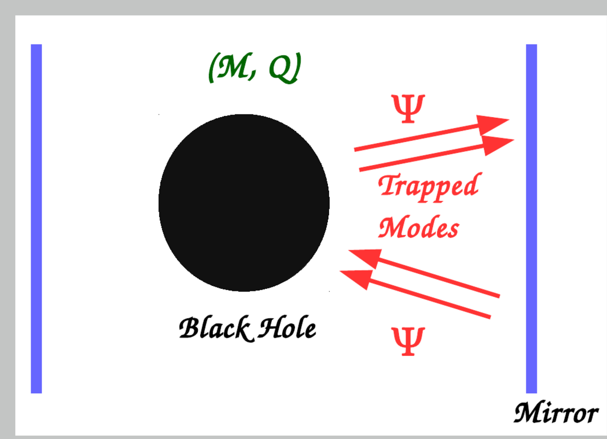
- The second law of black hole thermodynamics ($\delta\mathcal{A}_H \geq 0$) and the extraction of energy from the black hole ($\delta\mathcal{M} < 0$) imply the superradiance condition:

$$\omega < q\Phi_H = \omega_c.$$

- A massive charged scalar field has bound states ($\omega^2 < m^2$) and a superradiance effect ($\omega < eA_t(r_h)$) if and only if the frequency lies on the following interval:

$$0 < \omega < \mathrm{Min}\{eA_t(r_h), m\}. \quad (8)$$

- To achieve a superradiant instability (SRI) it is mandatory to place a reflecting mirror for $r > r_h$ at the location $r = r_m$ where $\psi_\omega(r = r_m) = 0$.



- This condition implies the following bound for the charged of the external field:

$$\frac{eq}{\sqrt{2}} > \sqrt{\frac{r_m}{(r_m - r_h)}\frac{\mathcal{H}(r_m)}{r_m^2}}, \quad (9)$$

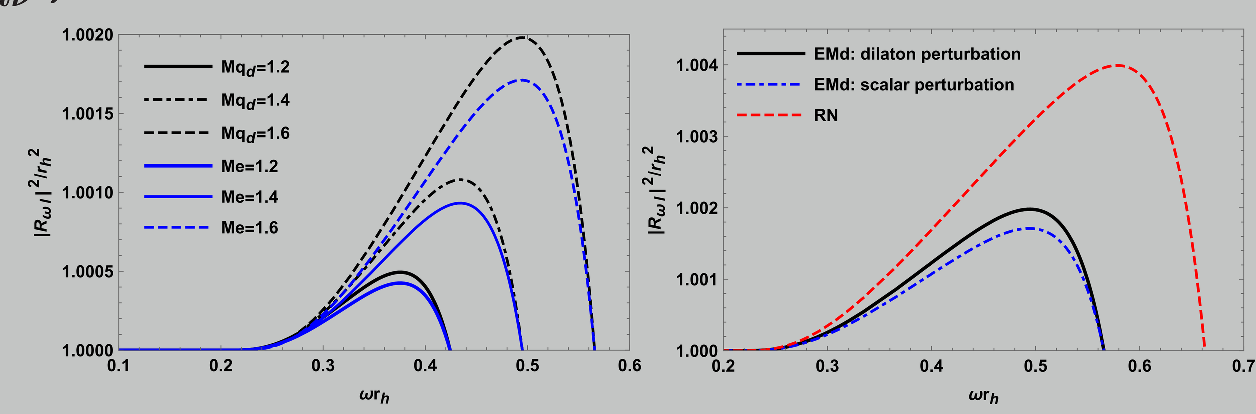
where $0 < s = r_h/r_m < 1$, $0 < q = Q/\sqrt{2}M < 1$, $\bar{m} = 2Mm > 0$ and $\mathcal{H}(\ell, s, q, \bar{m}) > 0$. One obtains that (9) satisfies the lower bound $e/m \gtrsim 1$.

Dilatonic perturbations

- Consider a quadratic perturbed Lagrangian for the dilaton field ($\phi = \phi_{bg} + \Pi$),

$$\Delta\mathcal{L}_d = -\mathcal{D}_\mu\Pi\left(\mathcal{D}^\mu\Pi\right)^\dagger - \mu_\Pi^2\Pi\Pi^\dagger - \lambda_g(\phi_{bg})\Pi\Pi^\dagger F^2_{bg}, \quad (10)$$

where $\mathcal{D}_\mu = \nabla_\mu - iq_dA^{bg}_\mu$ and $\lambda_g(\phi_{bg}) = \lambda_0 e^{-2\phi_{bg}}$, $0 < \lambda_0 \leq 1$.
- To perturb the dilaton field consistently while the metric and the gauge field are kept frozen requires that the mixed terms $\mathcal{O}(\partial\phi_{bg}\partial\Pi)$ and $\mathcal{O}(\Pi T^{em}_{\mu\nu})$ remain irrelevant at the linear level.



- The dilaton perturbation scheme enhances the reflection coefficient, but the amplitudes remain below the Reissner-Nordström case.
- Using the LHC bounds on the dilaton mass, $\mu_\Pi \in [10, 300]\mathrm{Gev}/c^2$, and the ultralight mass condition $(GM/\hbar c)\mu_\Pi = \mathcal{O}(1)$, one shows that the (primordial) BH mass window's is $M_{BH} \in [1.23, 4.45]10^{-22}M_\odot$.
- The SRI in the dilaton channel will happen as long as its typical time-scale remains several orders of magnitudes greater than the lifetime of the light dilaton, $\tau_{ins} \gg \mathcal{O}(10^{-27})s$ for $\mu_\Pi = 300\mathrm{Gev}/c^2$.

Reference

- **M.G. Richarte , E. L. Martins, and J.C. Fabris. PRD 105, 064043 (2022).**
- V. Cardoso, O.J.C. Dias, J. P.S. Lemos, and S. Yoshida. **PRD** 70 044039, 2004.S. Hod and O. Hod. **PRD (R)**81, 061502 (2010).
- W. H. Press and S. Teukolsky. **Nature** 238, 211 (1972). A.A. Starobinsky. **Sov. Phys. JETP**, 37(1),28-32 (1973).