Robinson-Trautman solutions in (2+1) dimensions

Alberto Saa

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PPGCosmo - 06/11/2020

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http://vigo.ime.unicamp.br/RT3/

FAPESP SÃO PAULO RESEARCH FOUNDATION

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CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

Outline

1) Robinson-Trautman solutions in GR

2) Robinson-Trautman in arbitrary dimensions

3) Robinson-Trautman in $N=2+1$

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   The Calabi flow

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4) Final Remarks

Robinson-Trautman solutions in GR

PROCEEDINGS OF THE ROYAL SOCIETY A

MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

Some spherical gravitational waves in general relativity†

By I. Robinson

Department of Physics, Syracuse University, Syracuse, New York

and A. Trautman‡

Institute of Physics, Polish Academy of Science, Warsaw

(*Communicated by H. Bondi, F.R.S.—Received* 11 *July* 1961)

Einstein's equations for empty space are solved for the class of metrics which admit a family of hypersurface-orthogonal, non-shearing, diverging null curves. Some of these metrics may be considered as representing a simple kind of spherical, outgoing radiation. (Among them are solutions admitting no Killing field whatsoever.) Examples of solutions to the Maxwell–Einstein equations with a similar geometry are also given.

Robinson-Trautman solutions in GR

- Spacetimes admitting a an expanding congruence of null geodesic with vanishing shear and twist

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The Calabi flow

$Q(u,\theta,\phi) = Q_0$ Schwarzschild solution

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RT dynamics (Calabi flow)

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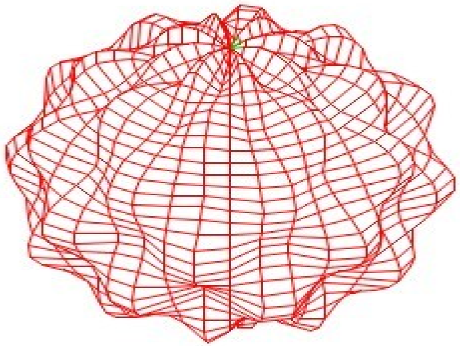

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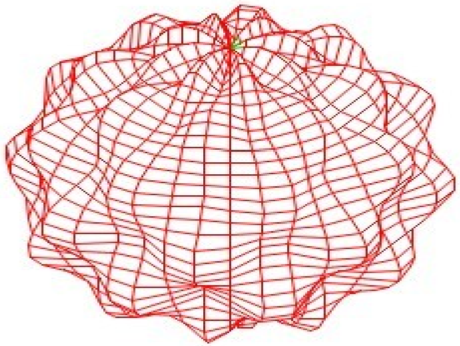

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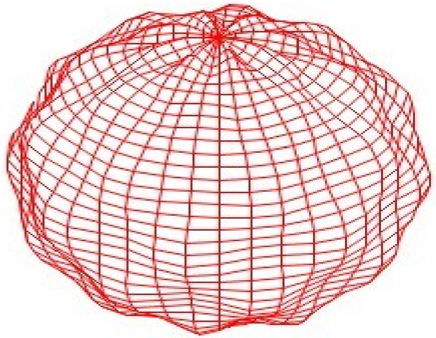

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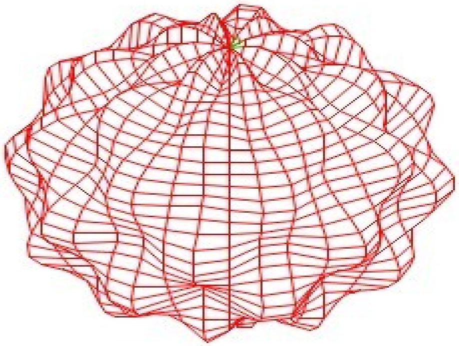

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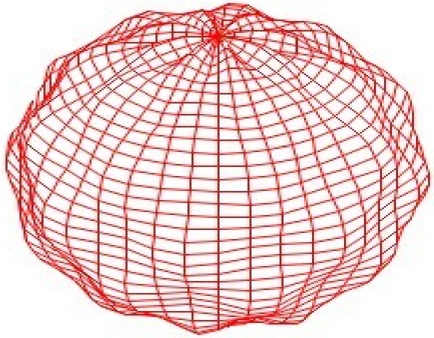

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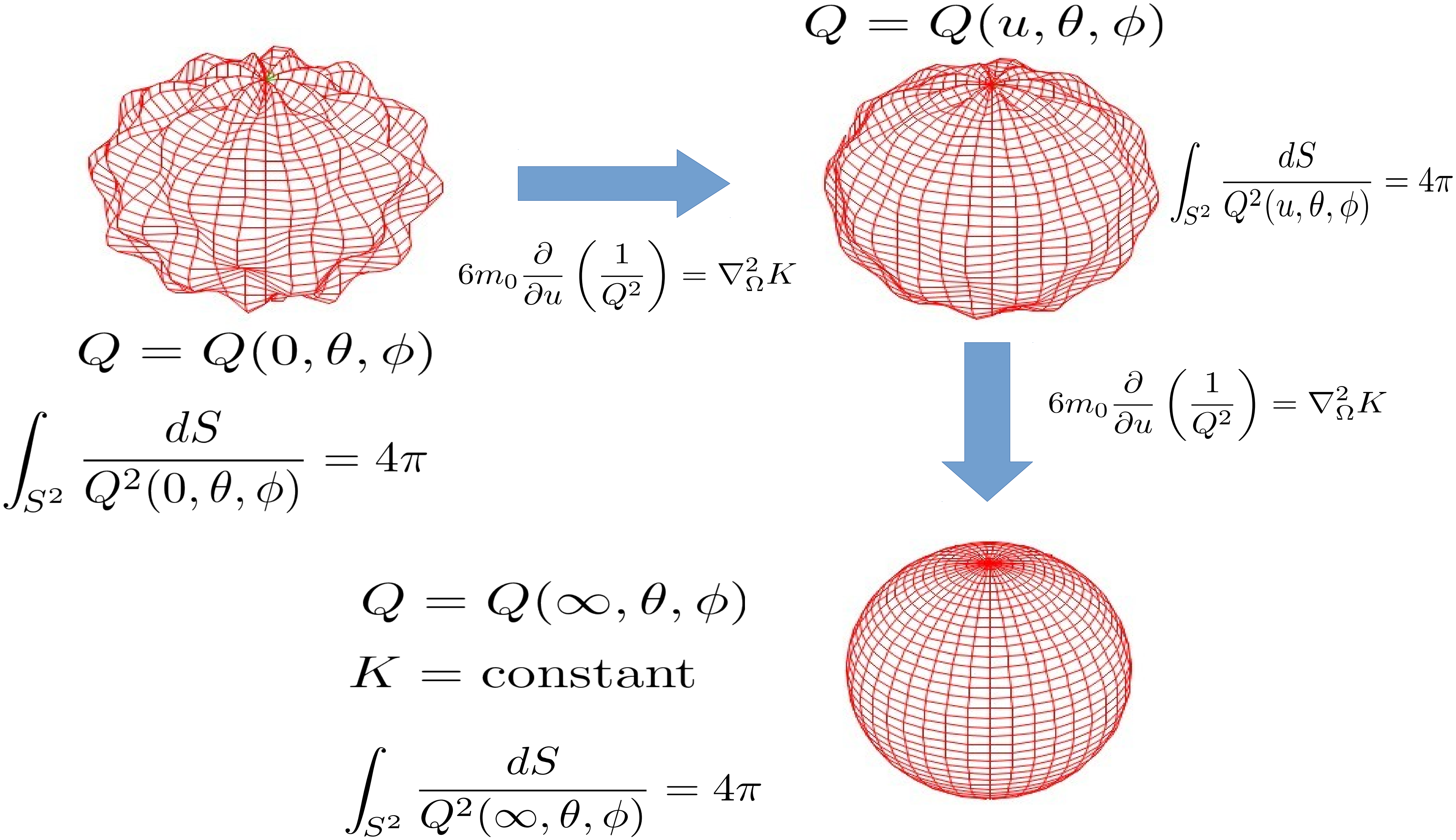
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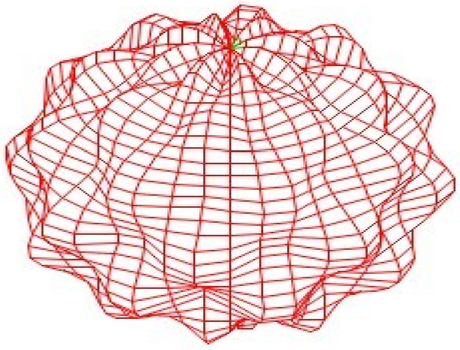

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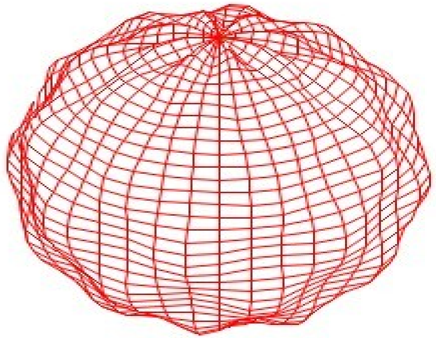

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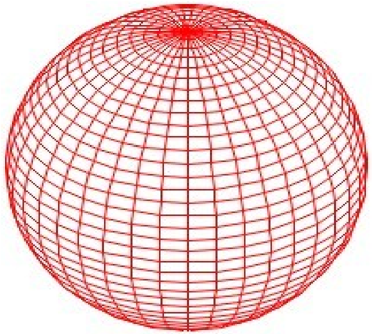

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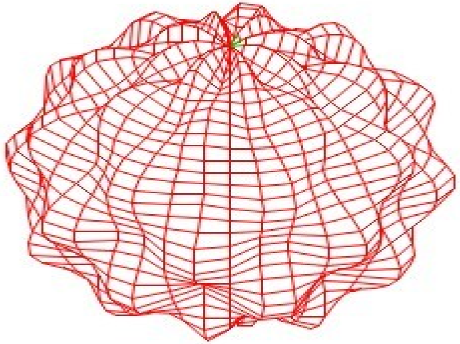

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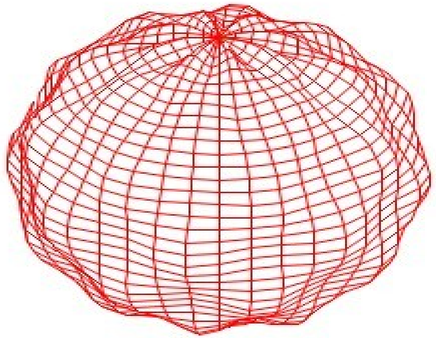

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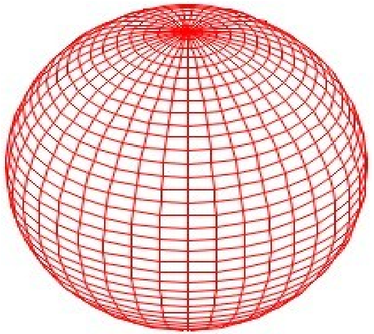

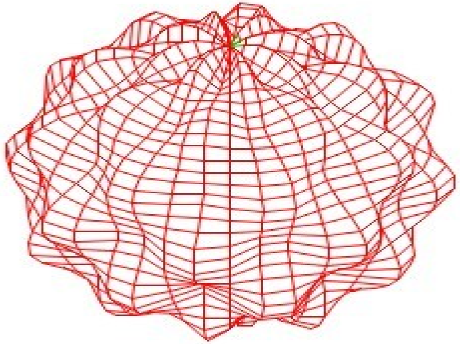

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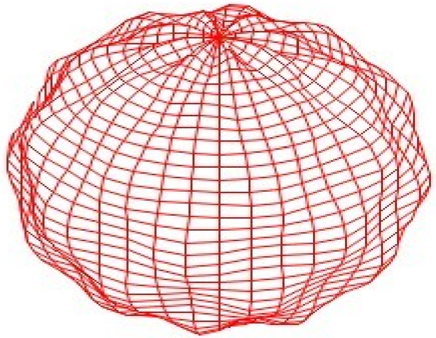

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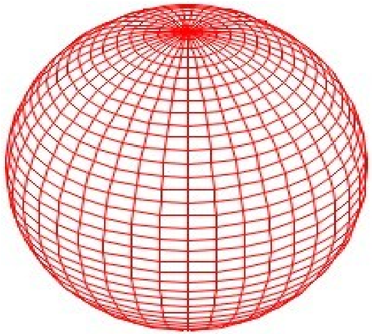

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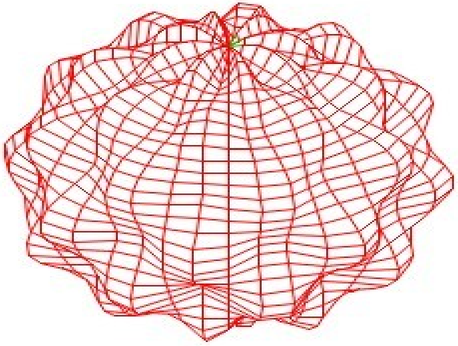

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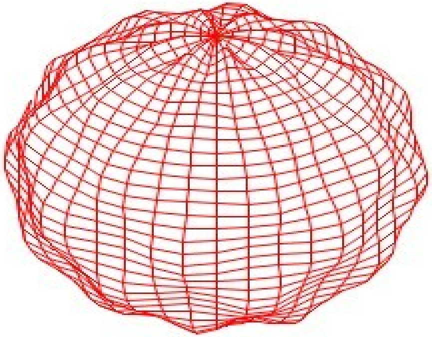

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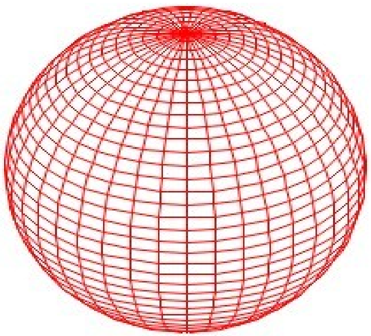

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Boosted Schwarzschild BH

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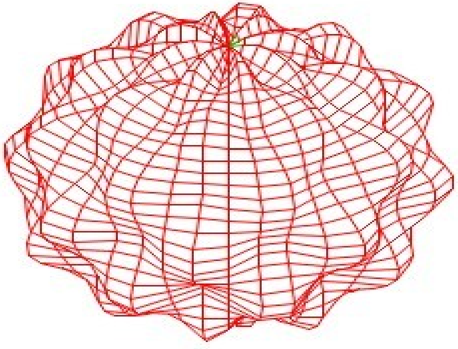

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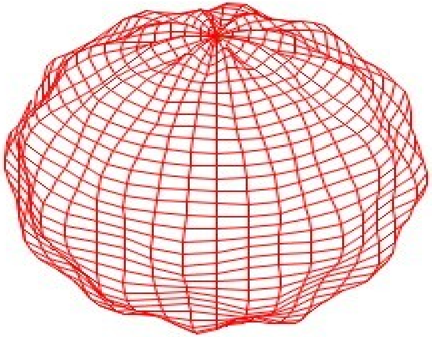

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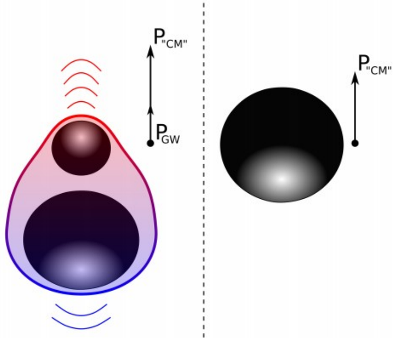

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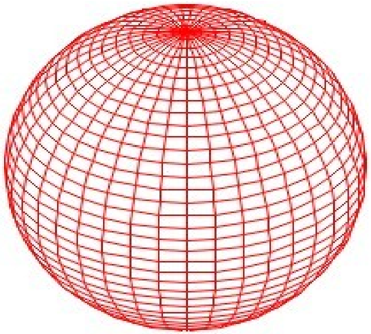

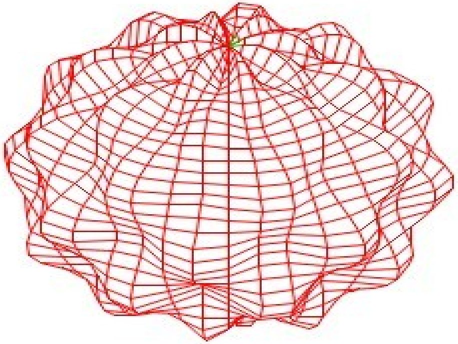

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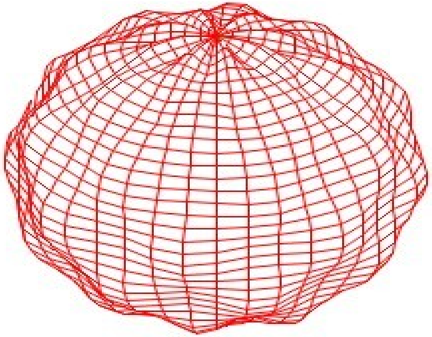

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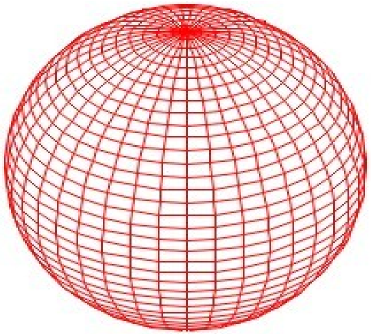

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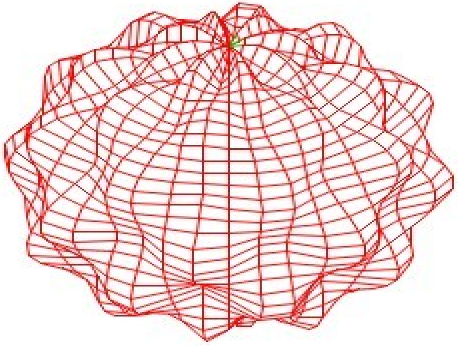

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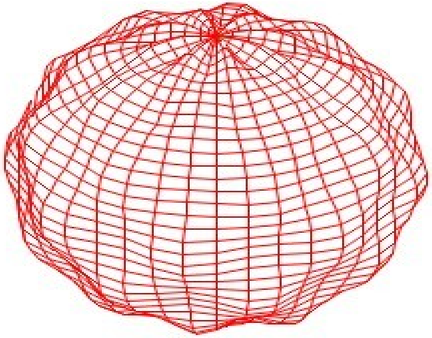

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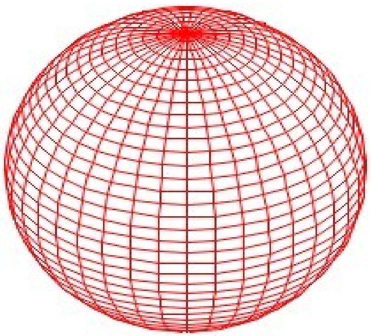

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Gravitational waves (Bondi news function)

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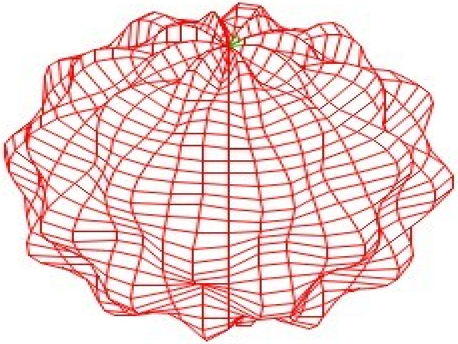

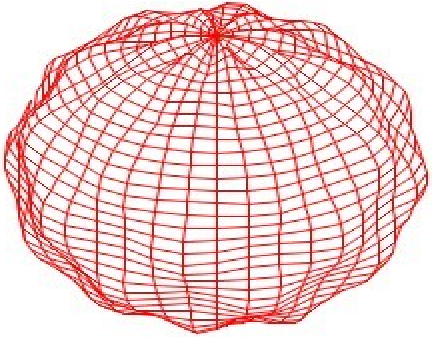

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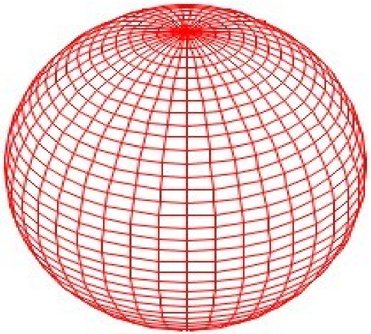

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Compact object surrounded by gravitational waves

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N-dimensional Schwarzschild

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$$P(0,\phi) \quad \Rightarrow \quad P(\infty,\phi) = \mathrm{constant}$$

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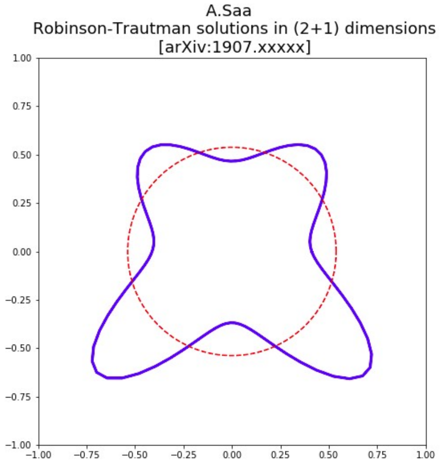

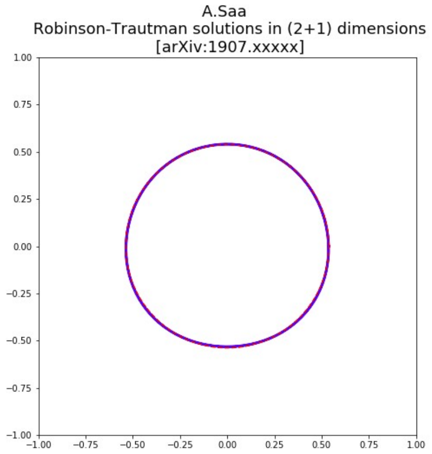

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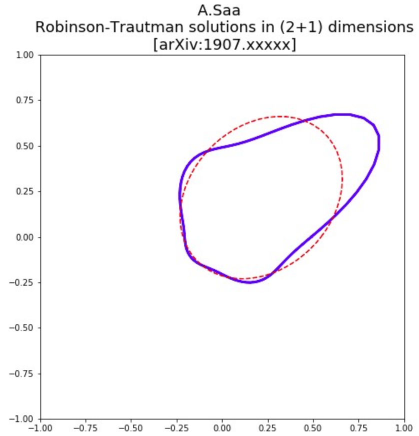

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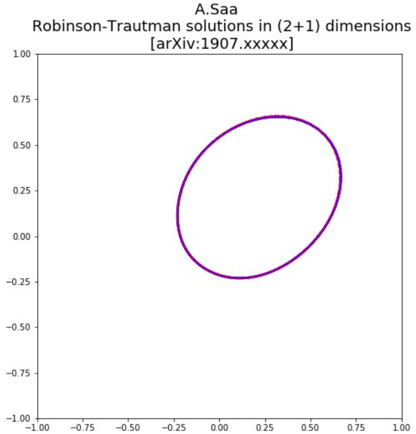

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Numerical instabilities

Galerkin methods

Conserved quantities

Thanks!

http://vigo.ime.unicamp.br/RT3/

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