Fundamental Physics and Gravitational Waves

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The LIGO and Virgo observatories

- Observation run **O1** Sept '15 - Jan '16

$\sim$ 130 days, with 49.6 days of actual data, PRX (2016) 4, 041014, 2 detectors, 3BBH

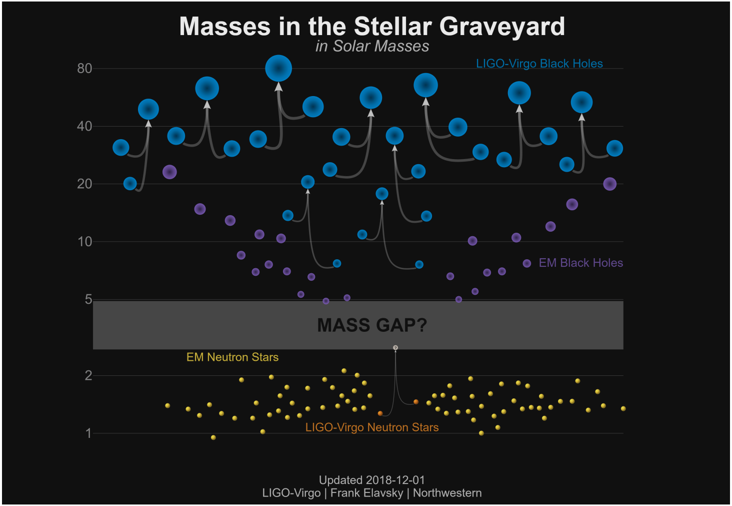
- **O2** Dec. '16 – Jul'17 2 det's + Aug '17 3 det's

3(+4) BBH + 1BNS in double (triple) coinc.

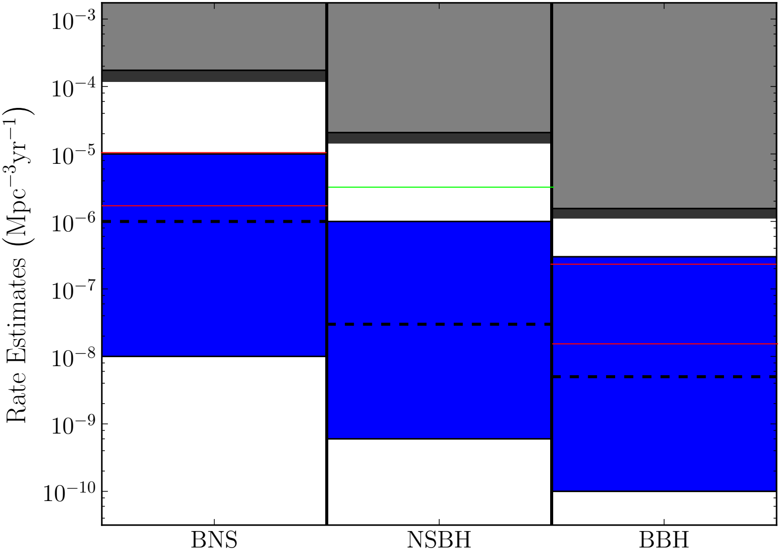
- **O3**, 3 detectors, started on April 1st 2019, will end in April 2020

Before the end of O3 Japanese KAGRA and 2025$^+$ Indian INDIGO will join the collaboration

Compact binary systems detected in O1+O2



1 confirmed detection in O3: GW190425, binary neutron star

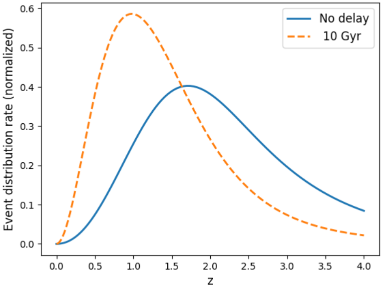


Astrophysical predictions, upper limit from S5/6/O1 and observation from O1/O2

LIGO/Virgo CQG ('10) 27 173001, PRX ('16), APJ (2016), PRL 119 ('17)

Ongoing O3: towards measuring merger distribution

At https://gracedb.ligo.org/superevents/public/O3
summary of candidate detections from O3a+b:
1 BNS+53(+1) candidate events, $\sim$ 10 of which possibly involving at least a NS



Expected merger distribution from Star Formation Rate (—) or SFR+Poissian distributed delay (– –)

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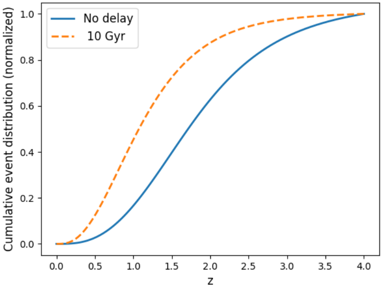
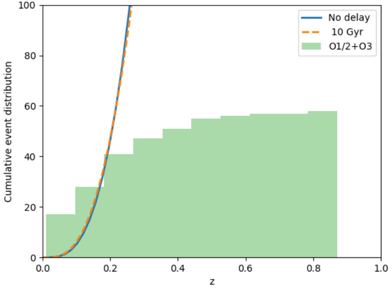




Expected merger distribution from Star Formation Rate (–) or SFR+Poissian distributed delay (– –)

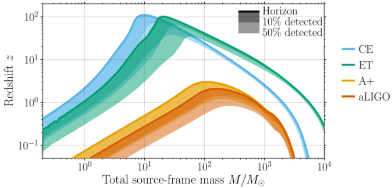
Future with ET and LISA looks very loud

Future 3rd generetaion detectors (Einstein Telescope, Cosmic Explore)/space telescope LISA will detect CBC signals with SNR $10 - 10^2$, with few golden events with SNR $\sim 10^3$.

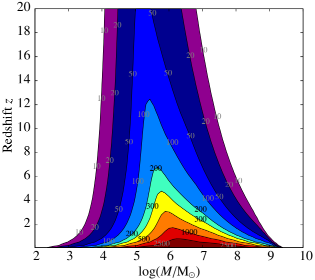
Templates few % accurate OK for characterising a source with SNR $O(10)$ (typical for LIGO/Virgo)

for SNR $\sim 10^3$ residual after extracting that source will have SNR $\sim O(10)$

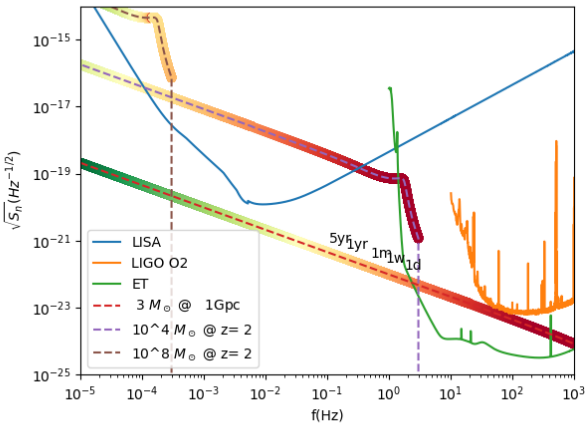
1. baising parameter estimation
2. contaminating the extraction of additional sources.



from arXiv:1902.09485



from arXiv:1201.3621



With ET event rate increased by 10^3, with LISA new type of sources
Currently used waveforms are hybridized with Numerical Relativity solutions describing the *merger* and *ring-down*

New lines of research

- Precision gravity
- General Relativity tests in strong gravity
- "Hairiness" of black holes
- Universe expansion history (standard sirens)
- Phase transitions in the Early Universe
- Black hole mass function
- How binary systems form and how frequent are they? (star formation rate)
- Fate of a massive star
- Probe the the interior of a neutron star
- Understand the life of a pulsar and its evolution
- Dark matter and Gravitational Waves (modification of compact object and/or their environment)
- Data analysis challenges for signals with $SNR \sim 100$
- Central Core of Galaxies, Massive Black Holes and their role in Galaxy Formation
- Multi-messenger astronomy

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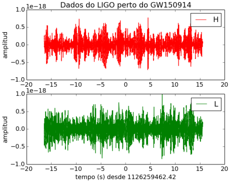
The importance of theoretical modeling



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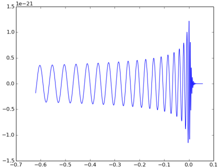

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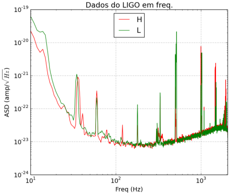

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The importance of theoretical modeling

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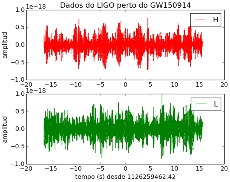

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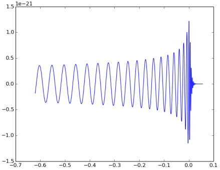

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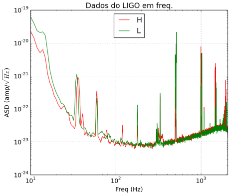

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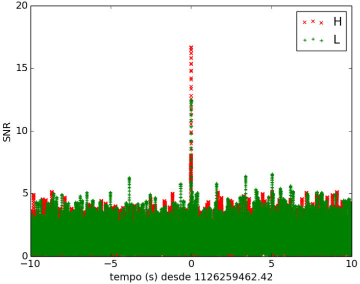

Data from https://losc.ligo.org/events/GW150914/

Fundamental GR: inspiral analytic model

Inspiral $h = A\cos(\phi(t))$ $\qquad \frac{\dot{A}}{A} \ll \dot{\phi}$
Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\begin{aligned} E(v) &= -\frac{1}{2}\eta M v^2 \left(1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^4 + \ldots\right) \\ P(v) \equiv -\frac{dE}{dt} &= \frac{32}{5G_N} v^{10} \left(1 + \#(\eta, S_i/m_i^2)v^2 + \#(\eta, S_i/m_i^2)v^3 + \ldots\right) \end{aligned}$$

E(v)(P(v)) known up to 3(3.5)PN

$$\begin{aligned} \frac{1}{2\pi}\phi(T) &= \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv \\ &\sim \int \left(1 + \#(\eta, S_i/m_i^2)v^2 + \ldots + \#(\eta, S_i/m_i^2)v^6 + \ldots\right)\frac{dv}{v^6} \end{aligned}$$

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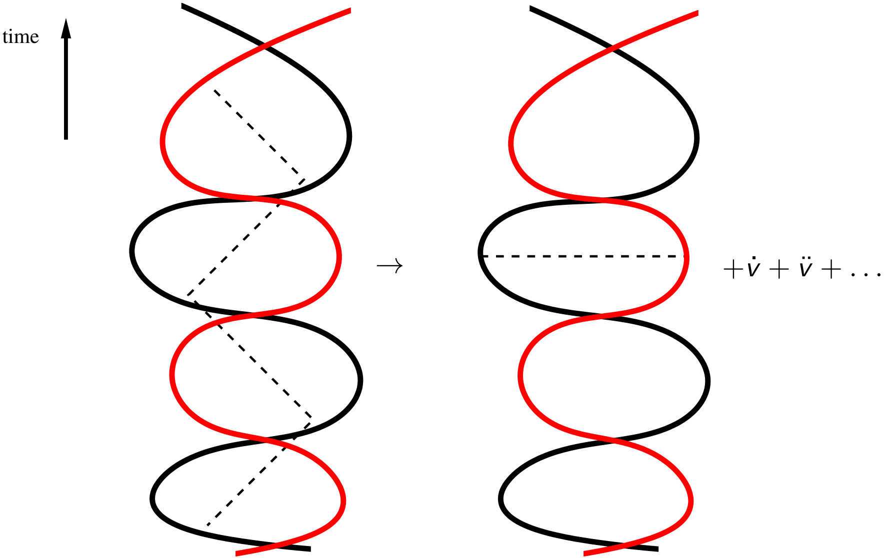
PN Coefficients (tidal $\sim v^{10}$)

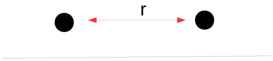
Trading knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time (PN approximation)



PN approximation to General Relativity

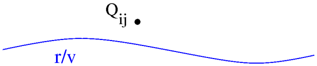
Small expansion parameter v, related to metric perturbation $v^2 \sim \frac{G_N M}{r}$

Near zone, $D \sim r$



Describe conservative dynamics

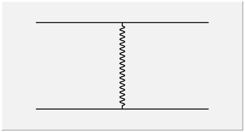
Far zone, $D \gtrsim \lambda = r/v$



conservative + dissipative

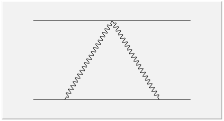
EFT framework pioneered by W. Goldberger and I. Rothstein, PRD '06

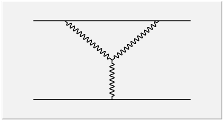
Newtonian potential:



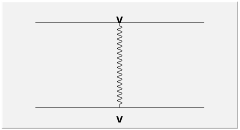
$$S_{eff} \sim \int dt\, \frac{G_N m_1 m_2}{r} \sim L$$

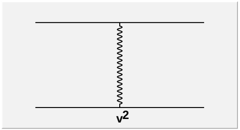
At 1PN:

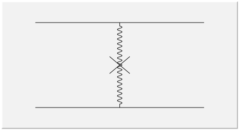




$$S_{eff} = \int dt\, \frac{G_N^2 m_1^2 m_2}{r^2} \sim L v^2$$







$$V_{1PN} = -\frac{G m_1 m_2}{2r}\left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2} v_1 v_2 - \frac{1}{2} v_1 \hat{r} v_2 \hat{r}\right] + 1 \leftrightarrow 2$$

Straightforward in this setup to consider constraints to UV gravity modifications $\sim (R^{\alpha}_{\mu\rho\sigma})^{\alpha}$, see arXiv:1704.01590

Effective potential from integration over regions

Internal graviton momentum can be expanded upon the following scaling:

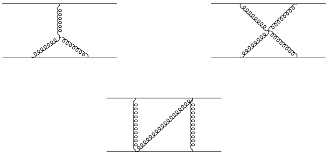
hard	(m, m)	quantum	×
soft	$(\lvert\vec{q}\rvert, \lvert\vec{q}\rvert)$	quantum	×
potential	$(v/r, 1/r)$	classical	✓
radiation	$(v/r, v/r)$	classical	✓

and then integrated over the full phase space

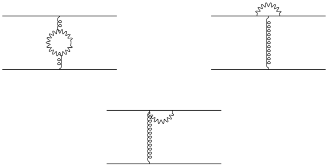
Only potential and radiation gravitons exchanged in classical processes: theory in terms of world lines selects diagrams that do not send source off-shell

Potential graviton $\to$ small change in energy wrt momentum, dominate classically

Ex. of classical connected diagrams



Ex. of quantum diagrams



Gravitational interactions do not create anti-BHs!

Graviton loop are negligible in astronomy!

From relativistic scattering amplitudes to 2-body potential

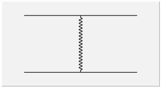
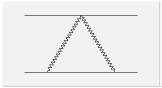
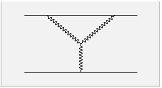
Equivalently, loops with massive particles contain a classical piece for $m \gg |\vec{q}|$:

$$V(\vec{r}) = \int \frac{d^3 q}{(2\pi)^3} A(\vec{q}, m_1, m_2, \vec{v}_1, \vec{v}_2, \ldots) e^{i\vec{q}\cdot\vec{r}}$$

$$\supset \frac{1}{m^2 - \hbar^2 \vec{q}^2} \rightarrow \underset{classical}{\frac{1}{m^2}} + \underset{quantum}{\frac{\hbar^2 \vec{q}^2}{m^4}} + \ldots$$

E.g.: $\frac{G_N m^2}{q^2}, \frac{G_N^2 m^3}{|q|}, G_N^3 m^4 \log |q| \ldots$ are classical contributions

The 4PN sector ✓

- G:  : 3 diagrams
- G^2:   21 diagrams, RS and S. Foffa, PRD '12
- G^3: 212 diagrams
- G^4: 317 diagrams RS and S. Foffa PRD'19
- G^5: 50 diagrams, RS et al. PRD '18

Also derived with other methods by Damour, Jaranowski, Schäfer, Blanchet, Bernard, Marsat, Faye, Marchand, Bohé

2 body dynamics expansions (spin-less)

Post-Minkowskian expansion parameter is $G_N M/r$, vs PN expansion

$$\mathcal{L} = -Mc^2 + \frac{\mu v^2}{2} + \frac{GM\mu}{r} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots]$$

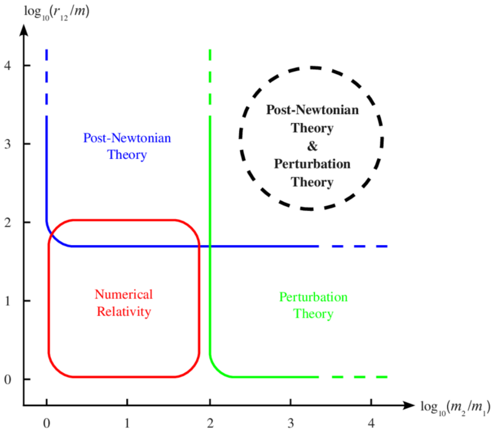
Terms known so far

		N	1PN	2PN	3PN	4PN	5PN	...
0PM	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM					$1/r^4$	v^2/r^4	v^4/r^4	...
5PM						$1/r^5$	v^2/r^5	...
...							$1/r^6$(!)	...

3PM recently computed by Z. Bern, C. Cheung et al. arXiv:1901.04424 PRL 122 ('19)

What's next? Amplitude program with modern methods: generalized unitarity, gravity as a double copy of gauge theory. . .

Different approximation methods



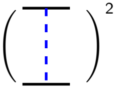
(Blanchet et al. arXiv:1007.2614)

Numerical relativity solution are expensive for large separation (large orbital scale) and large mass ratios (long dynamical evolution time)

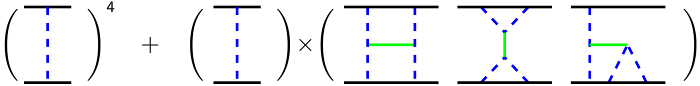
5PN static subsector ✓

Via factorization theorem the static 5PN sector could be computed gluing amplitudes
1PN

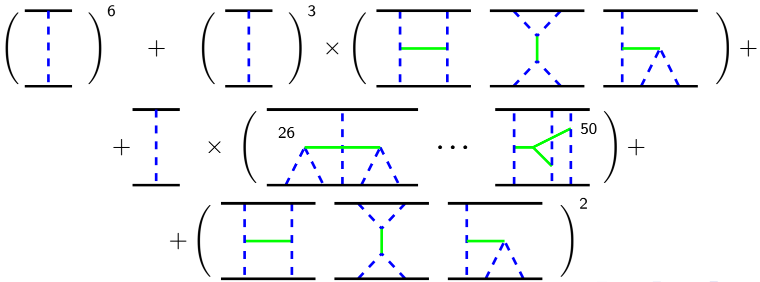
at lower order (RS et al. PRL '19)



3PN



5PN



Towards the full 5PN result

5PN sector is qualitatively different because it is the lowest order at which Black Holes can have tidal deformability

However classical results (Damour & Nagar PRD '09, Kol & Smolkin JHEP '12) showed that BH tidal deformation of BHs vanishes (spin-less, leading order)

A measure from observations of 5PN effects can test if GR is violated:

handle to *quantum effect* in GR!

The computation of the 5PN sector is on-going, $O(10^3)$ diagrams, but factorization + Double Copy techniques relating YM and GR *on-shell* 3-vertices can help:

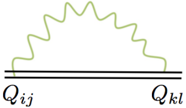
$$-gf^{abc}\left(\eta_{\mu\nu}(k_1-k_2)_\rho+\text{cyclic}\right)\to$$
$$\sqrt{G_N}\left(\eta_{\mu\nu}(k_1-k_2)\rho+\ldots\right)\left(\eta_{\alpha\beta}(k_1-k_2)\gamma+\ldots\right)$$

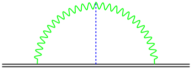
(and use Kawai-Lewellen-Tye relations for higher order vertices)

Conservative-dissipative interplay (Lamb-shift)

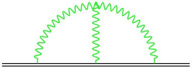
Theory at short distance (exchange of potential modes) + theory at large distance (GWs) $\rightarrow$ Self-energy diagrams of composite object

purely imaginary:





real and imaginary:



Foffa & RS PRD ('13), 1907.02869, Galley & al. PRD ('15),Damour & al. PRD ('13)

UV and IR divergences

Instantaneous interactions correspond to

$$\frac{1}{\vec{k}^2 - k_0^2} \simeq \frac{1}{\vec{k}^2}\left(1 + \frac{k_0^2}{\vec{k}^2} + \dots\right)$$

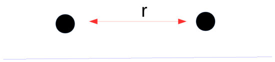
Hence the potential V:

$$\begin{aligned} V &\propto \int dk_0 d^3k \frac{e^{-ik_0 t_{12} + i\vec{k}\cdot(\vec{x}_1(t_1) - \vec{x}_2(t_2))}}{k^2 - k_0^2} = \int dk_0 d^3k \frac{e^{-ik_0 t_{12} + i\vec{k}\cdot\vec{x}_{12}}}{k^2}\left(1 + \frac{k_0^2}{k^2} + \dots\right) \\ &= \delta(t_1 - t_2) \int d^3k \frac{e^{ikx_{12}}}{k^2}\left(1 + \frac{\partial_{t_1}\partial_{t_2}}{k^2} + \dots\right) = \int d^3k \frac{e^{ikx_{12}}}{k^2}\left(1 - \frac{k\cdot v_1 k\cdot v_2}{k^2} + \dots\right) \end{aligned}$$

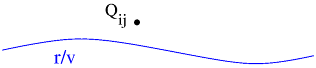
This must be complemented by the contribution of far-zone

UV and IR divergences II

Near zone, $D \sim r$



Far zone, $D \gtrsim \lambda = r/v$



UV-divs

IR- UV-divs

- Near UV: cancelled by local counterterms $R_{\mu\nu}u^\mu u^\nu$, $a^\mu \dot{v}^\mu$ which happen to vanish on-shell
  Caveat at 5PN: new counterterm $R^2_{\alpha\beta\gamma\delta} \neq 0$, fingerprint of finite size effect
- IR divergences mactched by far zone UV ones

$$M\dddot{Q}_{ij}^2\left(\frac{1}{\epsilon_{UV}} + \log\omega\mu + \dots\right) - M\dddot{Q}_{ij}^2\left(\frac{1}{\epsilon_{IR}} + \log\frac{\mu}{r} + \dots\right) \simeq M\dddot{Q}_{ij}^2\log(\omega r) + \dots$$

RS, Foffa, Porto, Rothstein PRD '19

Renormalization group

Theory at short and large distances have compensating spurious divergences:
the UV divergences in the long distance theory are of the type $\log(\omega/\mu)$ and combines with IR $\log(\mu r)$ from short distance

- ω: GW frequency
- μ: renormalization group scale
- r: orbital scale

enabled derivation of leading logarithmic terms in the Energy function at all orders (checked up to 21PN order from self-force computations)

$$E(v) = -\frac{1}{2}\mu v^2 \left[1 + \frac{16\eta v^2}{15\beta_I}\left(\left(1 + 24\beta_I v^6 \log v^2\right) v^{8\beta_I v^6} - 1\right)\right]$$

$\beta_I = -214/105$ RG coefficient of the quadrupole (Goldberger et al. PRD ('13))

$$\frac{dQ_{ij}(\omega,\mu)}{d\log\mu} = \beta_I (G_N M\omega)^2 Q_{ij}(\omega,\mu)$$

RS et al. arXiv:1912.12359, confirming earlier perturbative results by Kavanagh et al. PRD '15 (up to 21PN order)

Data analysis

LIGO searches made into 2 steps:

1. Matched-filtering for detection against simplified waveforms over all data to find triggers
2. Parameter estimation over triggers only, with more expensive waveforms

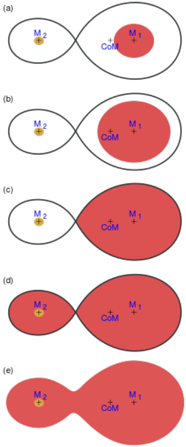
Future challenges in LIGO will include:

- inclusion of higher order perturbative computation in the LAL waveform codes
- matched-filtering is easily maximized (over time and phase) when dominant modes alone are relevant ($f_{GW} = 2f_{orb}$)
  For unequal masses and strong precession (large spins not aligned with angular momentum) subdominant modes: analyze their impact into detection loss and parameter bias

Astrophysical impact of GW Parameter Estimation

Astrophyisical source properties → formation models:

- field binaries
  - details of the common envelope phase, e.g. radius expansion, rotation, (non-)conservative mass transfer
  - minimum mass of the neutron star
  - metallicity
  - physics of the supernova explosion → the compact objects
  - natal kicks
  - cosmological star formation rate
- massive stars ($\gtrsim 300 M_\odot$) in dense populations
  - binary-single interactions accelerate merger (without common envelope)
  - globular clusters

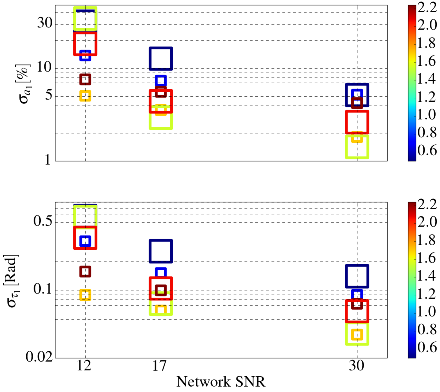


Izzard et al. Proc. IAU 2012

see e.g. Domink et al. APJ 2012

Binary parameter spin measure example: NS-BH $m_1/m_2 = 7$

Precession effects stronger for high mass ratio as spin $\propto m_i^2$ can be larger then L (but signal strength $\propto$ reduced mass), stronger with $\hat{J}\cdot\hat{n} = 0$ (tropical region)



- Color: angle between J and view direction
- size:
  1. small: both spin tilts 60^o
  2. large: tilt 45^o and 135^o

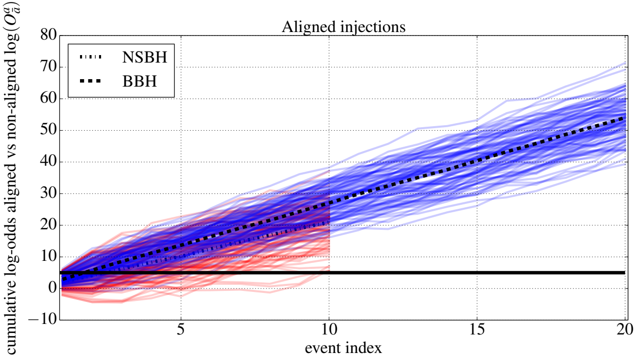
work with S. Vitale, R. Lynch, P. Graff PRL '14

Tilts poorly measured, turn to Bayesian model selection between twp hypotheses:

$$\bullet\mathcal{H}_{aligned-spins} \text{ vs. } \bullet\mathcal{H}_{isotropic-spins}$$

Odds ratio:

$$\mathcal{O}^{ali}_{iso} \equiv \frac{P\left(\mathcal{H}_{ali}|d,I\right)}{P\left(\mathcal{H}_{iso}|d,I\right)}$$



RS et al. PRD'17

Wave generation: localized sources and cosmology

Einstein formula relates h_{ij} to the source quadrupole moment Q_{ij}

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j - \frac{1}{3}\delta_{ij}x^2\right), \qquad v^2 \simeq G_N M/r\,, \quad \eta \equiv m_1m_2/M^2 \\
h_{ij} &\sim g(\theta_{LN})\frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2} \simeq \frac{2G_N\eta M v^2}{D}\cos(2\phi(t))
\end{aligned}
$$

$$
\begin{aligned}
f &= 2\mathrm{kHz}\left(\frac{r}{30\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{3M_\odot}\right)^{1/2} < f_{Max} \simeq 12kHz\left(\frac{M}{3M_\odot}\right)^{-1} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{M}{M_\odot}\right)^{1/3} < \frac{1}{\sqrt{6}}
\end{aligned}
$$

Geometric factor $g(\theta_{LN})$ takes account of transversality projection (angular momentum L of the binary, observation direction N)

$$
\begin{aligned}
h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\,\eta\frac{Mv^2}{D}\cos\phi(t_s/M,\eta,S_i^2/m_i^4,\ldots) \\
h_\times &\sim \cos(\theta_{LN})\,\eta\frac{Mv^2}{D}\sin\phi(t_s/M,\eta,\ldots)
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Amplitudes of 2 polarizations modulated by θ_{LN} ($h \nearrow$ for $\theta_{LN} \searrow_0$), never both vanishing unlike dipolar motion for the electromagnetic case

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h_+ &\sim \frac{1+\cos^2(\theta_{LN})}{2}\,\eta\frac{\mathcal{M}v^2}{d_L}\cos\phi(t_O/\mathcal{M},\eta,S_i^2/m_i^4,\ldots)\\
h_\times &\sim \cos(\theta_{LN})\,\eta\frac{\mathcal{M}v^2}{d_L}\sin\phi(t_O/\mathcal{M},\eta,\ldots)
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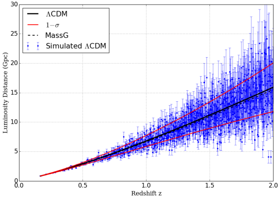
h sensitive to red-shifted masses $M\to M(1+z)\equiv\mathcal{M}$

GW and cosmology with ET

In GR the luminosity distance is related to red-shift via the matter content of the Universe

$$d_L = \frac{1+z}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z)^3+\Omega_\Lambda(1+z)^{3(1+w_\Lambda)}}}$$

where Ω_Λ is still misterious *dark energy* and $w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} \simeq -1$
GWs are *standard sirens*, with calibrated d_L



- Need for an EM counterpart to know z and/or *complete* galaxy catalog
- Use of average properties of galaxy populations
- Possibility of model independent measure of equation of state of the dark energy