# Gravitational Waves from Coalescing Binaries and the post-Newtonian Theory

Riccardo Sturani

Instituto de Física Teórica UNESP/ICTP-SAIFR São Paulo (Brazil)

Ubu - Anchieta, April $16^{\mathrm{th}}$ 2015

# Outline

1. Introduction to GW: sources and observations
2. GW detectors and sensitivities
3. Fundamental physics with GWs

# Outline

1. Introduction to GW: sources and observations
2. GW detectors and sensitivities
3. Fundamental physics with GWs

# Wave generation: localized sources

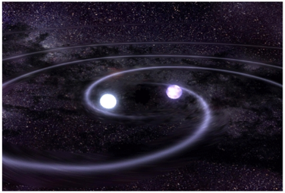
Einstein formula relates $h_{ij}$ to the source quadrupole moment $Q_{ij}$

$$
\begin{aligned}
Q_{ij} &= \int d^3x\rho\left(x_ix_j-\frac{1}{3}\delta_{ij}x^2\right) \qquad v^2\simeq G_N M/r \\
h_{ij} &= \frac{2G_N}{D}\frac{d^2Q_{ij}}{dt^2}\simeq\frac{2G_N\mu v^2}{D}\cos(2\phi(t))
\end{aligned}
$$

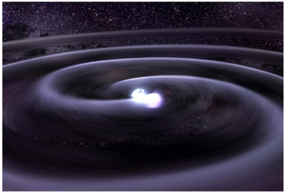
$$
\begin{aligned}
f &= 1\mathrm{kHz}\left(\frac{r}{14\mathrm{Km}}\right)^{-3/2}\left(\frac{M}{m_\odot}\right)^{1/2} \\
v &= 0.3\left(\frac{f}{1\mathrm{kHz}}\right)^{1/3}\left(\frac{m}{m_\odot}\right)^{1/3}<\frac{1}{\sqrt{6}}
\end{aligned}
$$

$$
\frac{dE}{dt}=\frac{D^2}{16\pi G_N}\int d\Omega\langle\dot{h}_+^2+\dot{h}_\times^2\rangle=\frac{32\nu^2}{5G_N}v^{10} \qquad \nu\equiv\mu/M=\frac{m_1m_2}{(m_1+m_2)^2}
$$

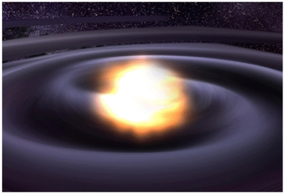
# Binary coalescence: a tale made of three stories



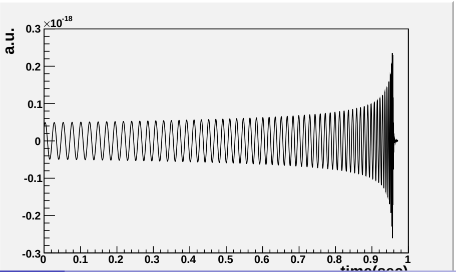
Inspiral phase
post-Newtonian
approximation: $v/c$



Merger: fully
non-perturbative



Ring-down:
Perturbed
Kerr Black Hole



# Data analysis technique: Matched filtering

An experimental apparatus output: time series

$$O(t) = h(t) + n(t) \qquad h(t) = D^{ij} h_{ij}(t)$$

Noise is conveniently characterized by its spectral function

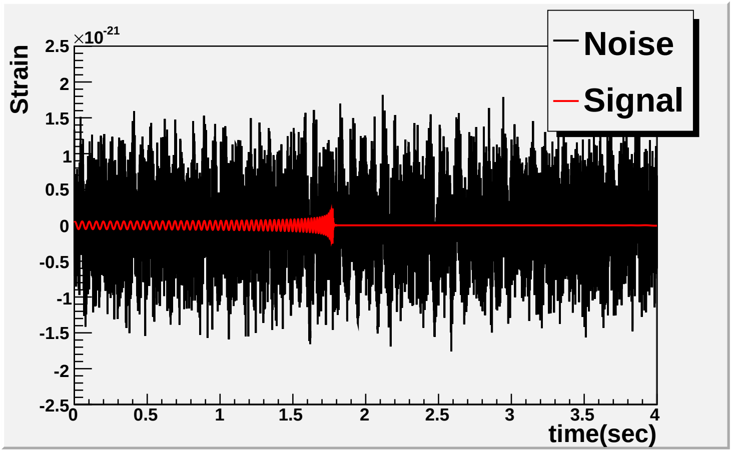
$$\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \delta(f - f') S_n(f) \qquad [Hz^{-1}]$$

Matched filter enhances the sensitivity

$$\frac{1}{T}\int_0^T O(t)h(t)\,dt = \frac{1}{T}\int_0^T h^2(t)\,dt + \frac{1}{T}\int_0^T n(t)h(t)\,dt \sim h_0^2 + \sqrt{\frac{\tau_0}{T}} n_0 h_0$$

# Hunting for tiny signals

Detector's output is flooded with noise:

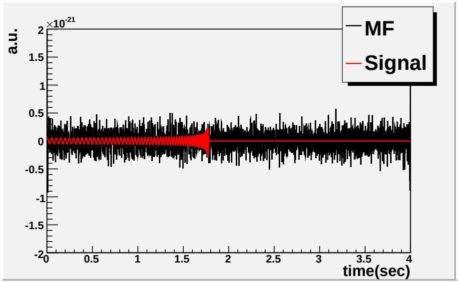



Noise + GW signal from 2+12 $M_\odot$ system at 50 Mpc distance

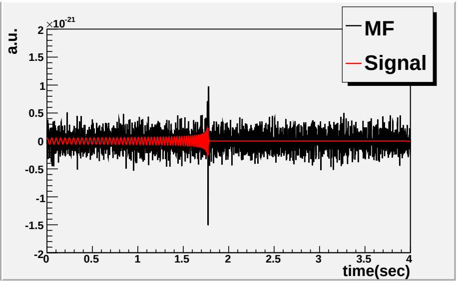
# Matched filtering

Matched filtering enhances sensitivity:

$$O(t) \to MF(t) \propto \int \frac{O(f)h^*(f)}{S_n(f)} e^{2\pi i f t} df$$




MF with $h' \neq h$




MF with $h$

but requires good model of the signal $h$

or a complete bank of $h's$

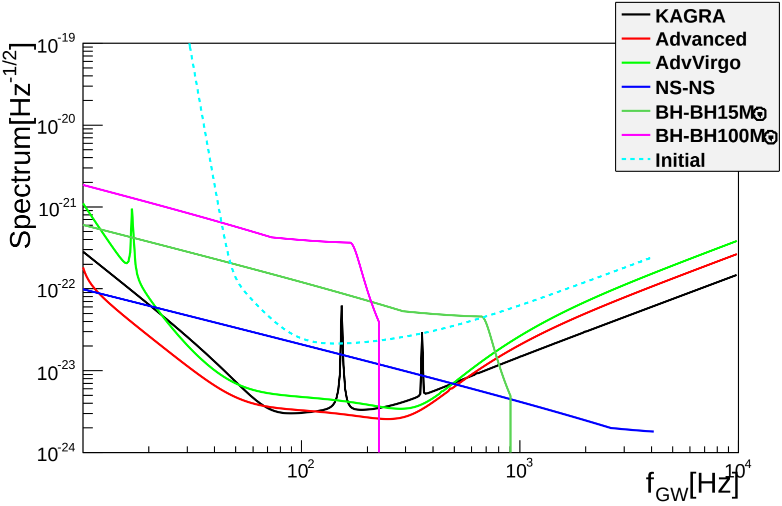
# Outline

# Detector locations

All are now being upgraded to their Advanced LIGOs (Virgo) due to to start data taking in Sept 2015 ( spring 2016)
2017+ for design sensitivity

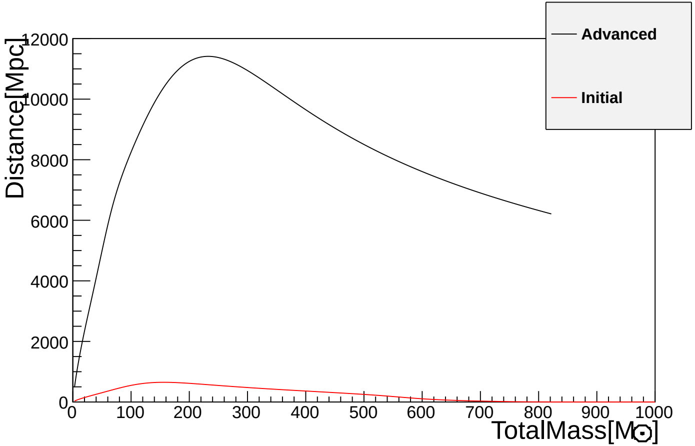
Old coincident runs terminated in October 2010

# Advanced detectors




Sensitivity vs. signals @ 200Mpc w. optimal orientation

# Distance reach for compact binary coalescence



# Advanced LIGO/Virgo goals

- Make first "direct" detection of GWs from neutron stars and/or black holes
- Probe intermediate mass black hole range: $M \sim 100 M_\odot$
- Measure rate of binary coalescences
- Measure pulsar parameters
- Possible probe of neutron star interior/nuclear matter at high density
- Verify association between short GRB's and GW's
- Combine EM and GW detection
- Make strong tests of GR
- Use coalescing binaries as standard sirens for cosmology

# Outline

# GW detection

Inspiral $h = A\cos(\phi(t))$ $\qquad \frac{\dot{A}}{A} \ll \dot{\phi}$

Virial relation:

$$v \equiv (G_N M \pi f_{GW})^{1/3} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\left.\begin{array}{rcl} E(v) & = & -\frac{1}{2}\nu M v^2 \left(1 + \#(\nu)v^2 + \#(\nu)v^4 + \ldots\right) \\ P(v) \equiv -\frac{dE}{dt} & = & \frac{32}{5G_N} v^{10}\left(1 + \#(\nu)v^2 + \#(\nu)v^3 + \ldots\right) \end{array}\right\} \text{PN corrections}$$

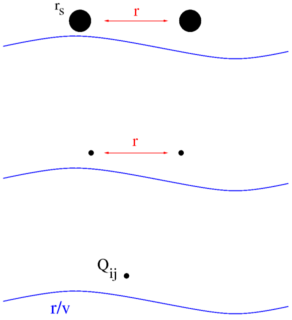
E(v)(P(v)) known up to 3(3.5)PN

$$\begin{array}{rcl} \frac{1}{2\pi}\phi(T) & = & \frac{1}{2\pi}\int^T \omega(t)dt = -\int^{v(T)} \frac{\omega(v)dE/dv}{P(v)} dv \\ & \sim & \int \left(1 + \#(\nu)v^2 + \ldots + \#(\nu)v^6 + \ldots\right)\frac{dv}{v^6} \end{array}$$

# The EFT point of view on the 2-body problem

Different scales in EFT:

- Very short distance $\lesssim r_s = G_N M$ negligible up to 5PN (effacement principle)
- Short distance: potential gravitons $k_\mu \sim (v/r, 1/r)$ with $r \sim r_s/v^2$
- Long distance: GW's $k_\mu \sim (v/r, v/r)$ coupled to point particles with moments




W. Goldberger and I. Rothstein, PRD '06

# EFT for compact binary systems

## Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}h(x) \exp\left[iS_{EH}(h + h_{GW}) + iS_{pp}(h + h_{GW}, x_a)\right]$$

$$\begin{aligned} S_{pp} &= -m \int d\tau \\ S_{EH} &= \frac{1}{32\pi G_N} \int d^4x \left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N} h (\partial h)^2 + G_N h^2 (\partial h)^2 \ldots\right] \end{aligned}$$

# EFT for compact binary systems

## Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}h(x) \exp\left[iS_{EH}(h+h_{GW}) + iS_{pp}(h+h_{GW}, x_a)\right]$$

$$\begin{aligned} S_{pp} &= -m \int dt d^3x\, \delta(\vec{x} - \vec{x}_a(t)) \left(h_{00}/2 + v_i h_{0i} + v^i v^j h_{ij}/2 + h_{00}^2 \ldots\right) \\ S_{EH} &= \frac{1}{32\pi G_N} \int d^4x \left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N} h (\partial h)^2 + G_N h^2 (\partial h)^2 \ldots\right] \end{aligned}$$

# EFT for compact binary systems

## Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

$$\exp\left[iS_{eff}(x_a, h_{GW})\right] = \int \mathcal{D}h(x) \exp\left[iS_{EH}(h+h_{GW}) + iS_{pp}(h+h_{GW}, x_a)\right]$$

$$\begin{aligned} S_{pp} &= -m\int dtd^3x\, J_{\mu\nu}(x)h^{\mu\nu}(x) \\ S_{EH} &= \frac{1}{32\pi G_N}\int d^4x\left[(\partial_i h)^2 - (\partial_t h)^2 + \sqrt{G_N}h(\partial h)^2 + G_N h^2(\partial h)^2 \ldots\right] \end{aligned}$$

# EFT for compact binary systems

## Fundamental

- Fundamental gravitational fields
- Fundamental coupling to particle world line

## Effective

- Generic $v$-dependent potential terms
- Instantaneous couplings between world-line particles

$$\exp\left[iS_{eff}(x_a,h_{GW})\right]=\int\mathcal{D}h(x)\exp\left[iS_{EH}(h+h_{GW})+iS_{pp}(h+h_{GW},x_a)\right]$$

$$\begin{aligned}S_{pp} &= -m\int dtd^3x\, J_{\mu\nu}(x)h^{\mu\nu}(x)\\ S_{EH} &= \frac{1}{32\pi G_N}\int d^4x\left[(\partial_i h)^2-(\partial_t h)^2+\sqrt{G_N}h(\partial h)^2+G_N h^2(\partial h)^2\ldots\right]\end{aligned}$$

$$S_{eff}=\int d^4x\, m_1\left[\frac{1}{2}v_1^2+\frac{G_N m_2}{2r}+\frac{1}{8}v_1^4+\frac{G_N^2 m_2}{2r^2}\left(\frac{G_N m_1}{2r}+v_1^2+v_{1r}v_{2r}\right)+1\leftrightarrow 2\right]$$

3PN: Jaranowski, Schäfer, Damour; Blanchet, Faye; Itoh Futamase; Foffa & RS

4PN: Jaranowski, Schäfer, Damour

# How to compute effective potential?

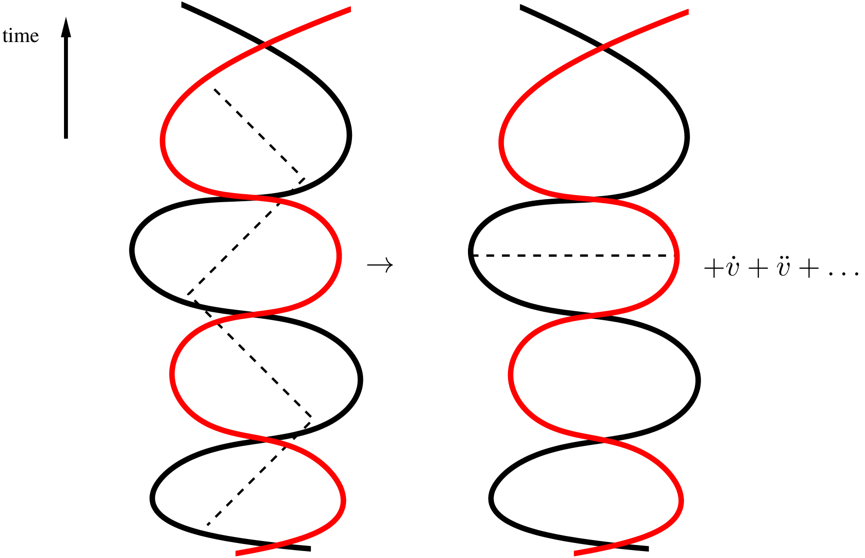
Iteratively solve Einstein equations:

$$
\begin{aligned}
-\int dt V_{1-2}(t,r) &\sim \int dt\, dt'\, d^3x\, d^3x'\, J(x) G(x-x') J(x') \\
&= 32\pi G_N m_1 m_2 \int dt\, dt' \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x_1(t_1)-x_2(t_2))}}{k^2-k_0^2} \\
&\simeq G_N \int dt\, dt' \delta(t-t') \frac{m_1 m_2}{|x_1(t)-x_2(t')|}
\end{aligned}
$$

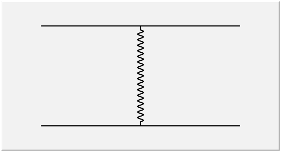
where $k_0$ has been neglected. Considering effects of $v$

$$
\begin{aligned}
V &\propto \int d^3k \frac{e^{ik(x_1-x_2)}}{k^2-k_0^2} = \int \frac{e^{ik(x_1-x_2)}}{k^2}\left(1+\frac{k_0^2}{k^2}+\ldots\right) \\
&= \int \frac{e^{ik(x_1-x_2)}}{k^2}\left(1-\frac{\partial_t^2}{k^2}+\ldots\right) = \int \frac{e^{ik(x_1-x_2)}}{k^2}\left(1-\frac{k\cdot v_1 k\cdot v_2}{k^2}+\ldots\right)
\end{aligned}
$$

Trading knowledge over the full trajectory with knowledge of all derivatives of the trajectory at equal time

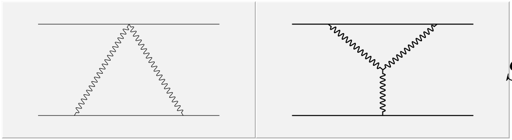


Newtonian potential:

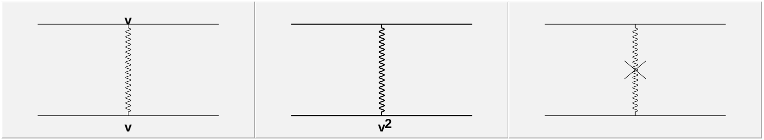


$$S_{eff} \sim \int dt\, \frac{G_N m_1 m_2}{r} \sim L$$

At 1PN:



$$S_{eff} = \int dt \frac{G_N^2 m_1^2 m_2}{r^2} \sim Lv^2$$

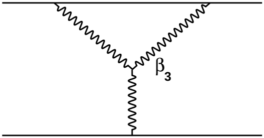



$$V_{1PN} = -\frac{Gm_1m_2}{2r}\left[1 - \frac{G_N m_1}{2r} + \frac{3}{2}(v_1^2) - \frac{7}{2}v_1v_2 - \frac{1}{2}v_1\hat{r}v_2\hat{r}\right] + 1 \leftrightarrow 2$$

# Graviton self-interaction vertices

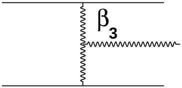
- Conservative dynamics

$$V \supset \beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2}$$




- Emission

$$L_{pp} \supset h_{ij} \beta_3 (\nu M x_i \ddot{x}_j)$$




Example of tagging fundamental physics effects

$\beta_3$ is not a viable modification of General Relativity

# Bound on self-interaction triple vertex from binary pulsars

At present the binary pulsars give best constraint on non-conservative effect from $\beta_3$

$$\dot{P}_{\beta_3} = \dot{P}_{GR}(1 + c\beta_3) \qquad c \simeq 3.21$$

Given that $\frac{\dot{P}_{obs}}{\dot{P}_{GR}} - 1 \simeq 0.1\% \implies \beta_3 = (4.0 \pm 6.4) \cdot 10^{-4}$

Conservative effect of $\beta_3$ already constrained by Lunar Laser Ranging, as @ 1PN

$$\beta_3 = \beta_{PPN} < 2 \cdot 10^{-4}$$

RS et al. '09

# Testing GR

$$\phi(t) = v^{-5} \sum_{n=0}^{7} \left( \phi_n + \phi_n^{(l)} \log(v) \right) v^n$$

Bayesian model selection between two hypothesis

- $\mathcal{H}_{GR}$
- $\mathcal{H}_{modGR}$: one or more $\phi_i$'s are not as predicted by GR

Odds ratio:

$$\mathcal{O}_{GR}^{modGR} \equiv \frac{P\left(\mathcal{H}_{modGR} | d, I\right)}{P\left(\mathcal{H}_{GR} | d, I\right)}$$

In absence of noise $\mathcal{O}_{GR}^{modGR} \gtrless 0$ favours $\begin{matrix}\mathbf{modGR} \\ GR\end{matrix}$

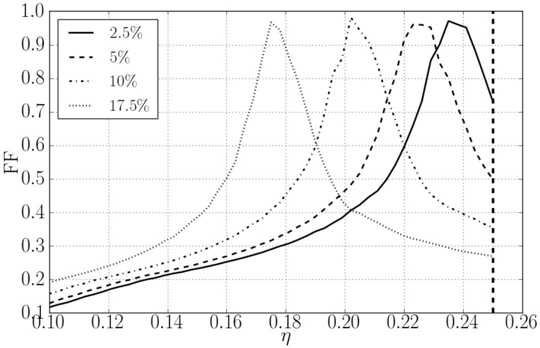
Li, RS et al. JPCS (2012), PRD (2012)

# Parameter estimation bias

Waveform match (fitting factor)

$$FF = \int df \frac{h_1^*(f)h_2(f) + h_1(f)h_2^*(f)}{S_n(f)}$$

for $\delta\chi_3$ injections varying $\eta$ (maximized over other params)



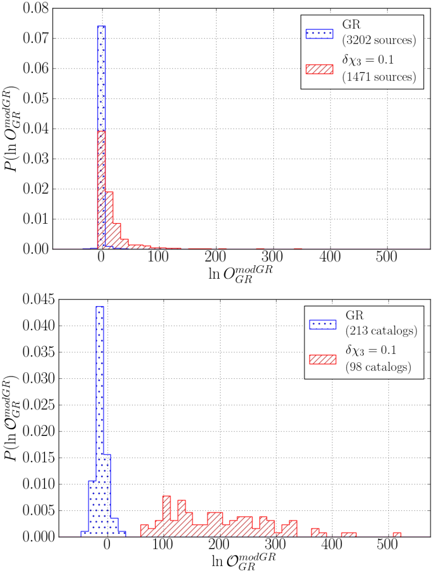

GR deviation do not prevent detection, but considerable bias!

# Simulated signals in noise

Constant shift in $\phi_3 \to \phi_3(1+\delta\chi_3)$ SNR's limited to 8-25

Single sources with noise:
Odds ratio overlaps

15-sources catalogs
can disentangle
fundamental effects



New era is about to start ($\sim$ 4 years)

- GW astronomy will open a new window onto the Universe, both astrophysically and cosmologically
- Observational test of strong gravity field dynamics not available so far: GW detection will give access to strong gravity phenomena

School on Gravitational Waves:
From data to theory and back
August 3-11 2015
www.ictp-saifr.org/gwaves

Lecturers: A. Buonanno, S. Foffa, S. Klimenko, E. Ramirer-Ruiz, C. Will, RS, A. Vecchio

Astrophysics workshop Astro-GR 2015
August 11-15 2015
www.ictp-saifr.org/astrogr