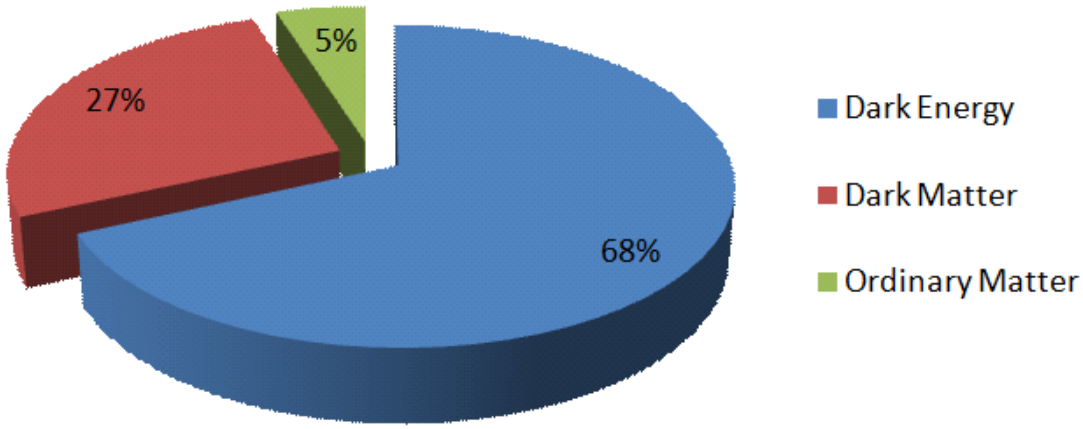
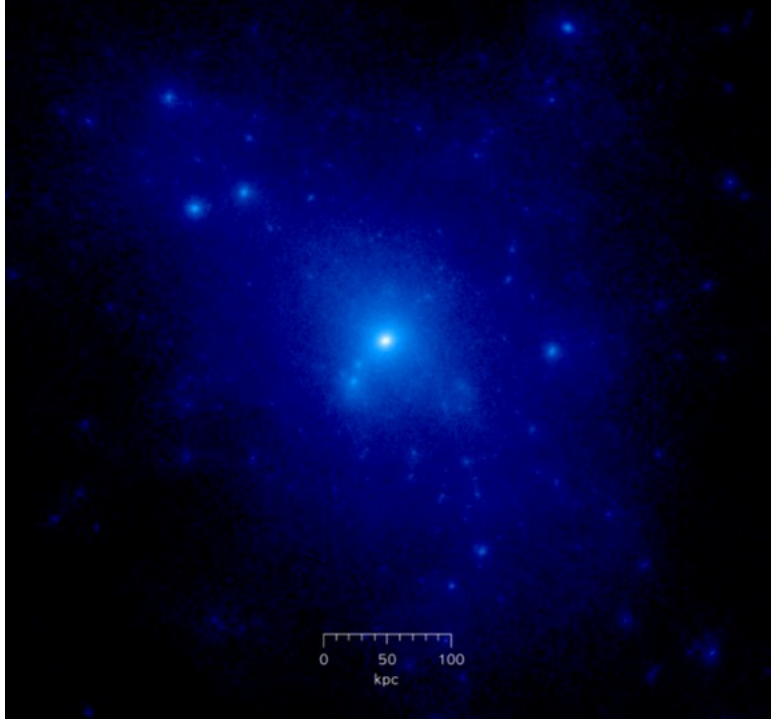
Scalar fields around compact objects: Scattering and absorption

Carolina L. Benone
Universidade Federal do Pará

Introduction



Planck collab. (2020)
[arXiv:1807.06205]

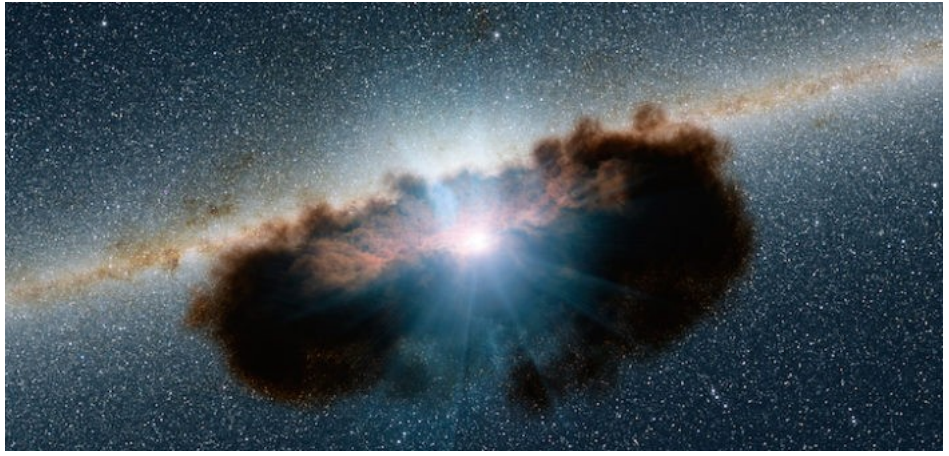


Sin (1994)
[arXiv:hep-ph/9205208]

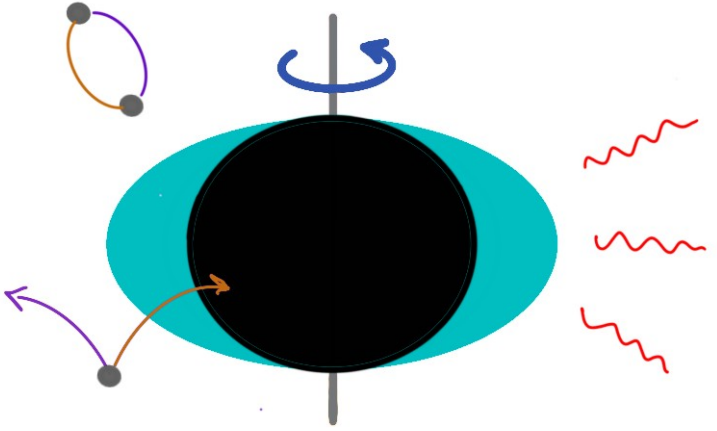
Lora+ (2012)
[arXiv:1110.2684]

$$m \sim 10^{-22}\mathrm{eV}$$

Introduction



NASA/JPL-Caltech



Absorption in a static spacetime

Simpson&Visser (2019)
[arXiv:1812.07114v3]

$$ds^2 = -\left(1 - \frac{2M}{\sqrt{r^2 + a^2}}\right) dt^2 + \left(1 - \frac{2M}{\sqrt{r^2 + a^2}}\right)^{-1} dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2\theta d\varphi^2)$$

a = 0 ⟶ Schwarzschild BH

0 < a < 2M ⟶ Regular BH

a = 2M ⟶ One-way traversable wormhole

a > 2M ⟶ Two-way traversable wormhole

Absorption in a static spacetime

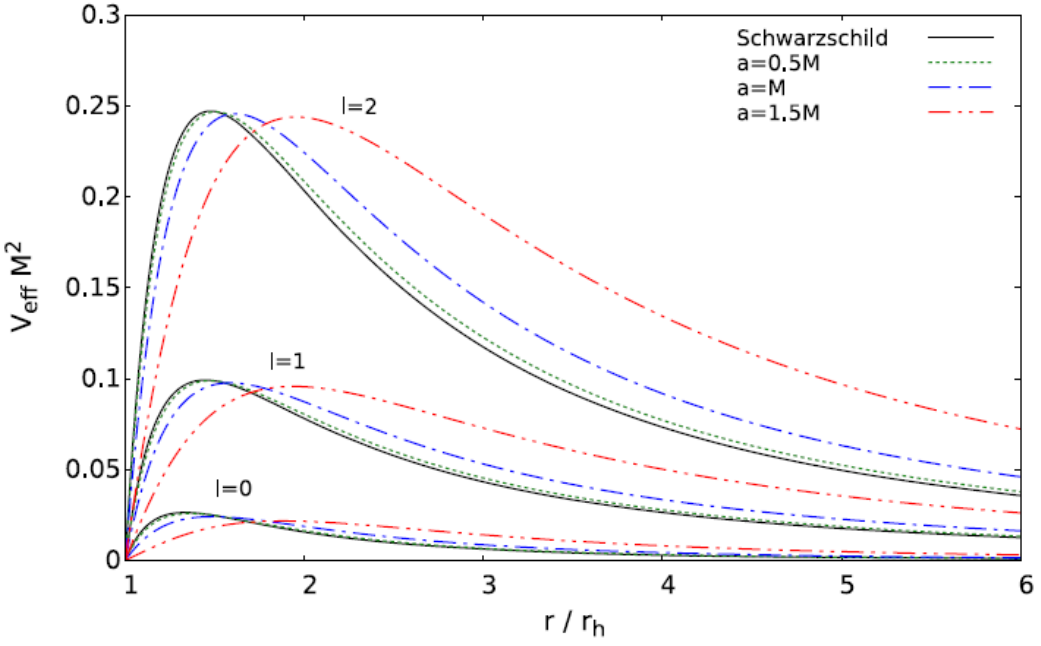
$$\nabla^{\mu}\nabla_{\mu}\Phi = 0$$

$$\Phi = \sum_{l,m} \frac{\phi(r)}{(r^2 + a^2)^{\frac{1}{2}}} Y_{lm}(\theta, \varphi) e^{-i\omega t}$$

$$f(r)\frac{d}{dr}\left(f(r)\frac{d\phi(r)}{dr}\right) + [\omega^2 - V_{\text{eff}}]\phi(r) = 0$$

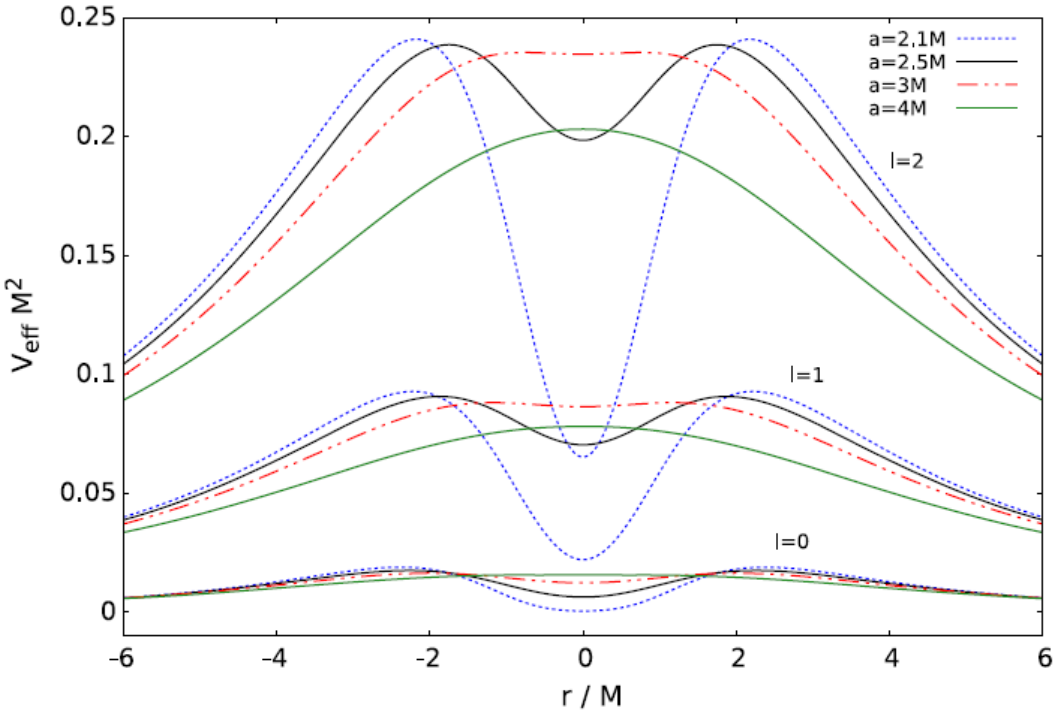
$$V_{\text{eff}} \equiv f(r)\left[\frac{f'(r)r}{(r^2 + a^2)} + \frac{a^2 f(r)}{(r^2 + a^2)^2} + \frac{l(l+1)}{(r^2 + a^2)}\right]$$

Absorption in a static spacetime



$$\phi(r) \approx \begin{cases} e^{-i\omega x} + e^{i\omega x} R_{\omega l}, & r \to +\infty (x \to +\infty), \\ T_{\omega l} e^{-i\omega x}, & r \to r_h (x \to -\infty). \end{cases}$$

Absorption in a static spacetime



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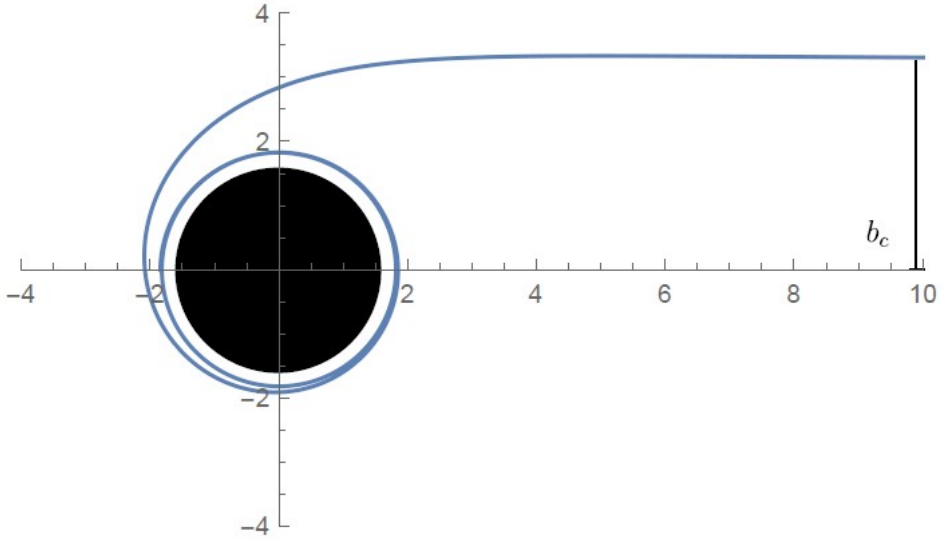
Absorption in a static spacetime



$$\sigma_{\text{geo}} = \pi b_c^2$$

$$r_{\text{ph}} = \begin{cases} \sqrt{9M^2 - a^2}, & \text{if } 0 \le a < 2M, \\ \pm\sqrt{9M^2 - a^2}, & \text{if } 2M \le a \le 3M, \\ 0, & \text{if } a > 3M. \end{cases}$$

$$\sigma_{\text{geo}} = \begin{cases} \pi b_1^2 = 27\pi M^2, & \text{if } 0 \le a \le 3M \\ \pi b_0^2 = \pi\left(\frac{a^3}{a-2M}\right), & \text{if } a > 3M. \end{cases}$$

Absorption in a static spacetime

Décanini+ (2011)
[arXiv:1101.0781v2]

$$\sigma_{\rm hf} \approx \sigma_{\rm geo} + \sigma_{\rm osc}$$

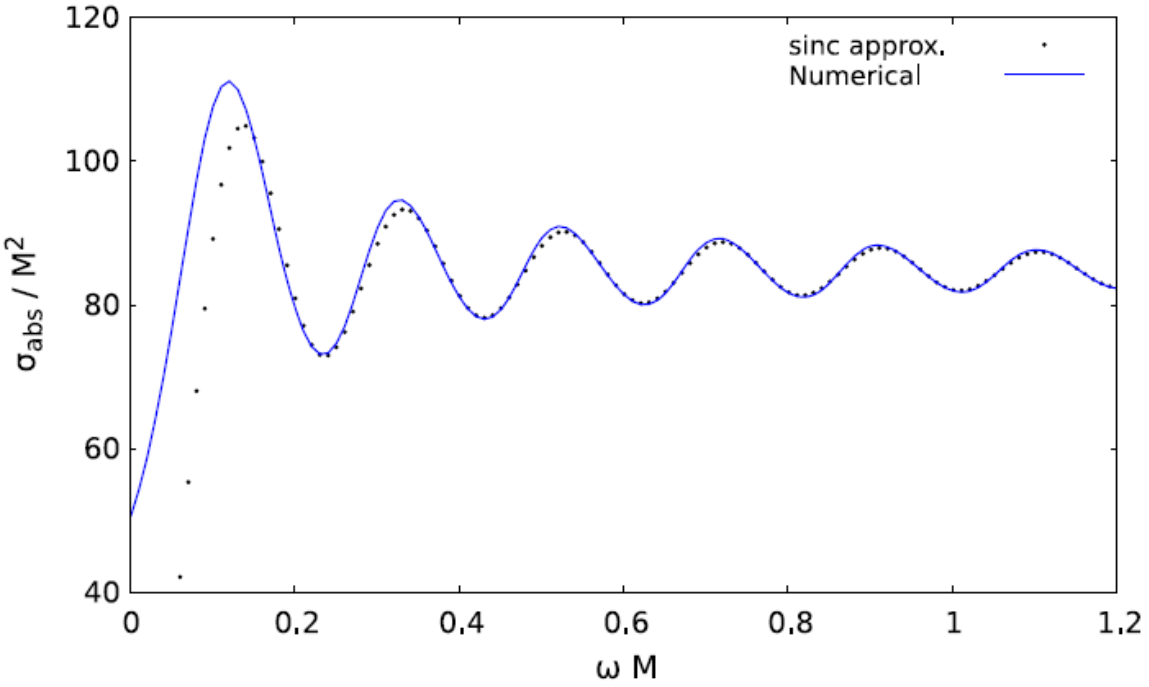
$$\sigma_{\rm osc} = -\frac{8\pi\lambda_l}{\Omega_l} e^{\frac{\pi\lambda_l}{\Omega_l}} \mathrm{sinc}\left(\frac{2\pi\omega}{\Omega_l}\right)\sigma_{\rm geo}$$

Das+ (1997)
[arXiv:hep-th/9609052]

$$\sigma_{lf} = A_h = 16\pi M^2$$

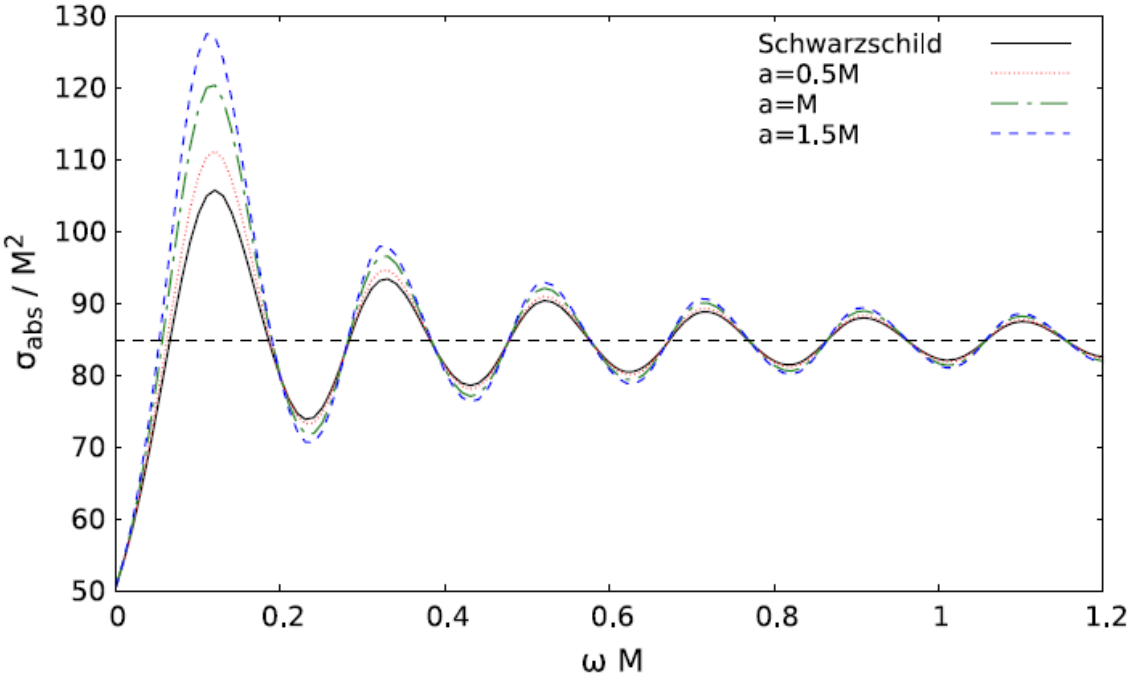
$$\sigma_{\rm abs} = \sum_{l=0}^{\infty} \sigma_l \qquad \sigma_l = \frac{\pi}{\omega^2}(2l+1)|T_{\omega l}|^2$$

Absorption in a static spacetime



Junior+ (2020)
[arXiv:2006.03967]

Absorption in a static spacetime



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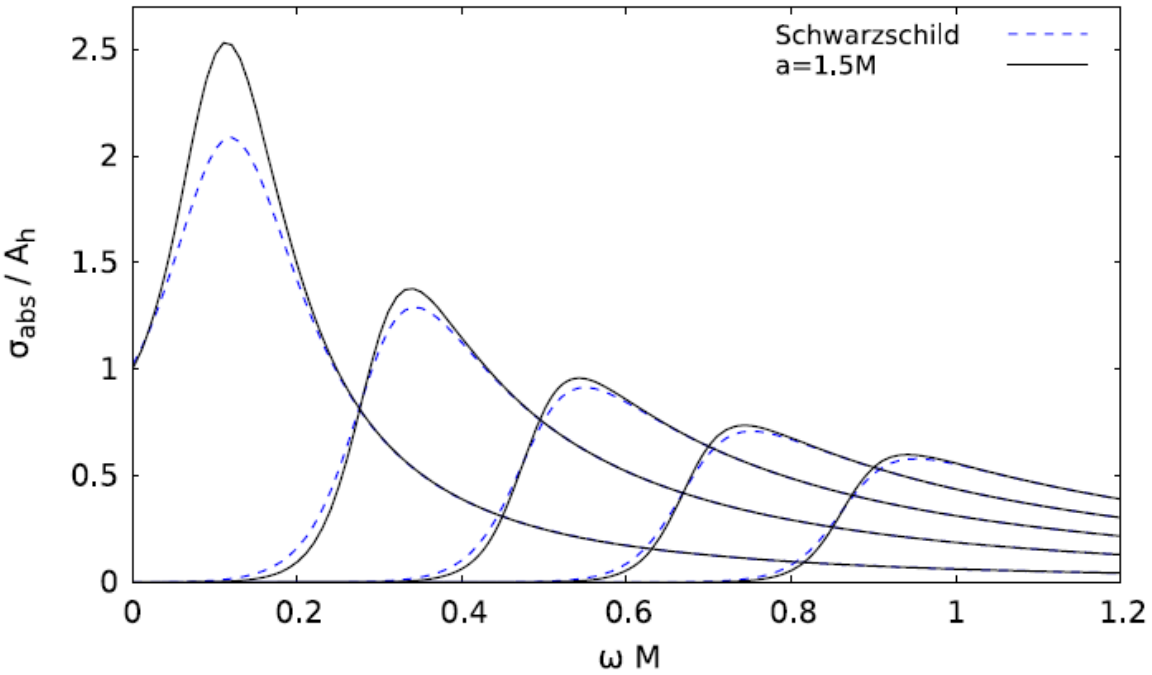
Absorption in a static spacetime



Junior+ (2020)
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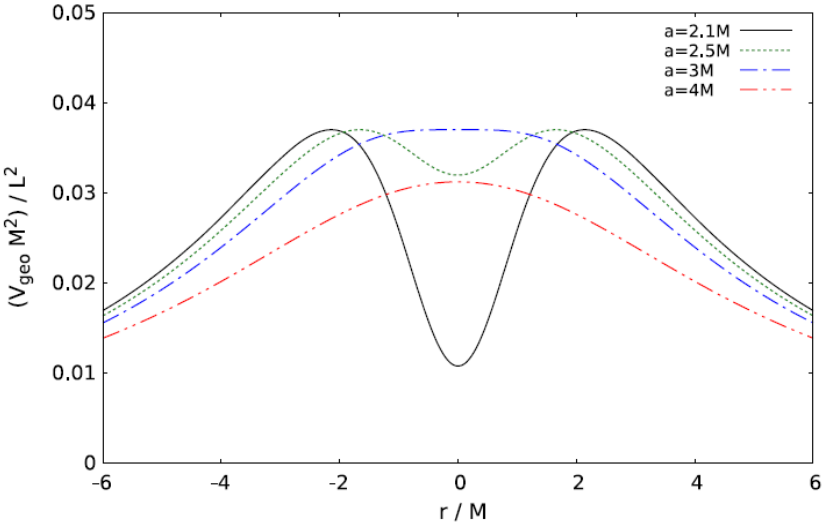
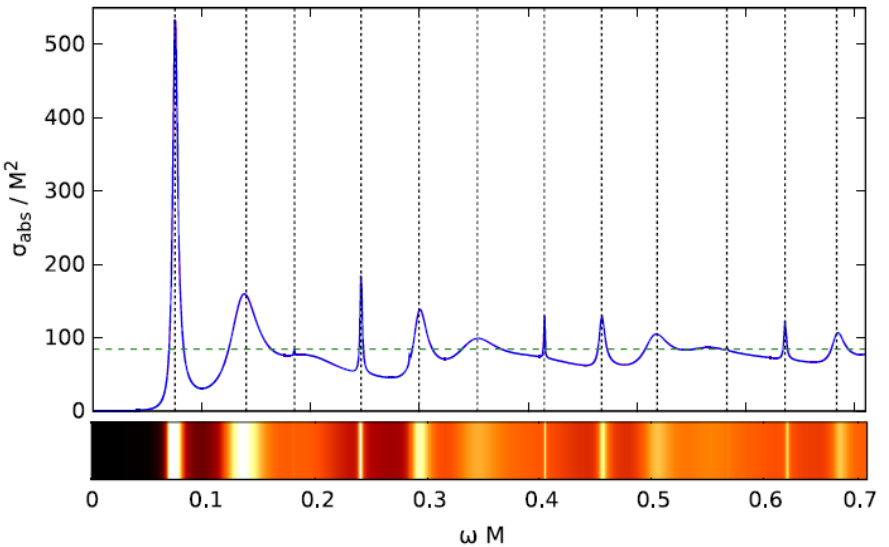
Absorption in a static spacetime

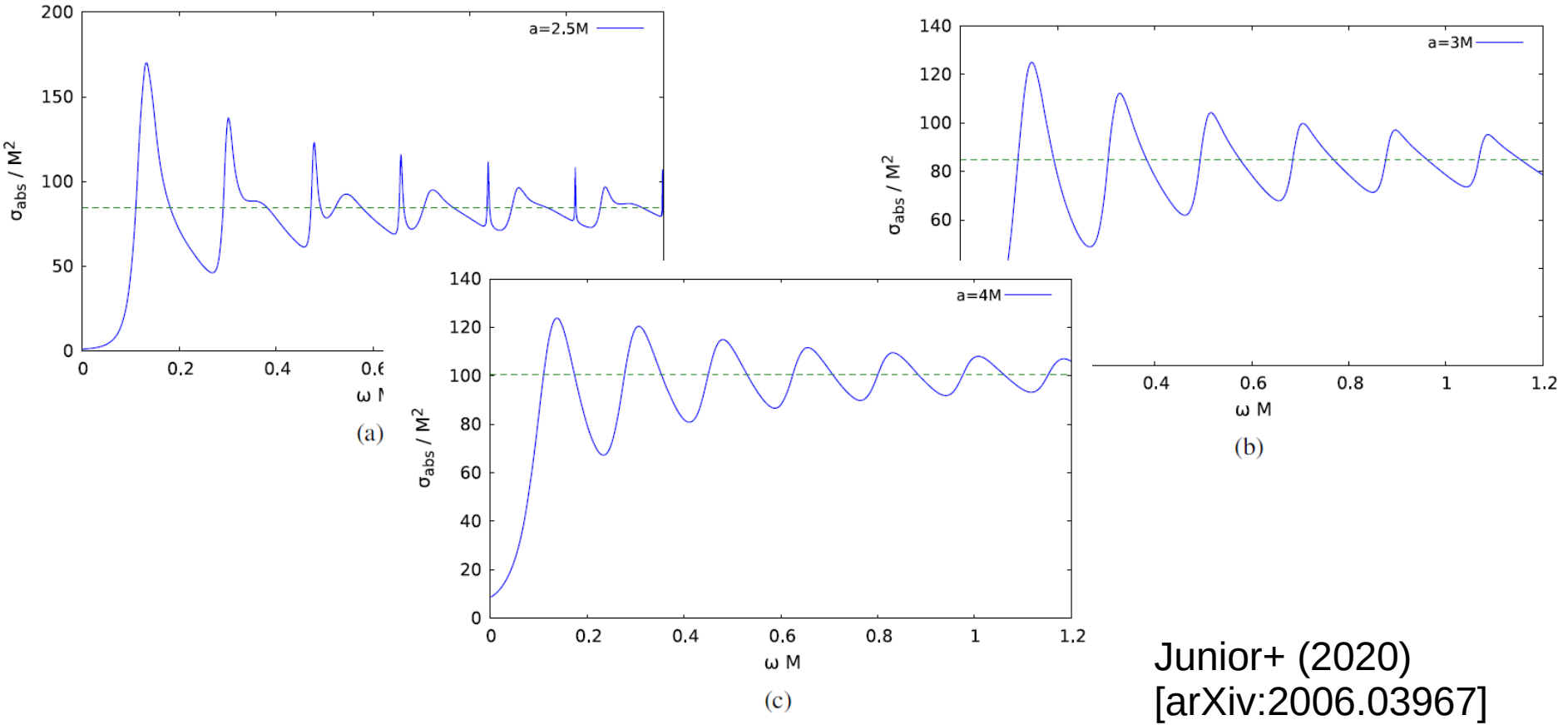


l	ω_R	$-i\omega_I$	l	ω_R	$-i\omega_I$
0	0.0753	2.673×10^{-3}	1	0.1852	4.249×10^{-5}
0	0.1408	8.304×10^{-3}	1	0.2464	1.011×10^{-3}
			1	0.2996	7.578×10^{-3}
			1	0.3532	9.417×10^{-3}
2	0.4147	3.432×10^{-4}	3	0.6360	1.222×10^{-3}
2	0.4674	3.154×10^{-3}	3	0.6834	6.861×10^{-3}
2	0.5184	9.281×10^{-3}	3	0.5825	1.096×10^{-4}



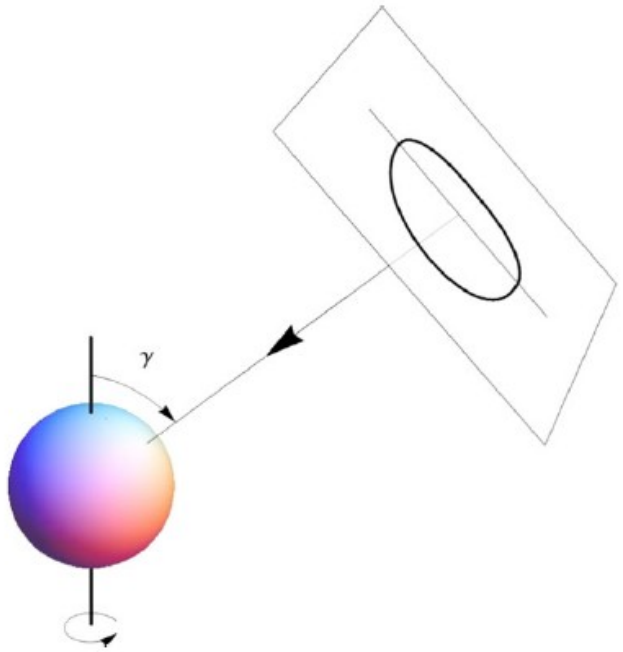
Junior+ (2020)
[arXiv:2006.03967]

Absorption in a static spacetime



Junior+ (2020)
[arXiv:2006.03967]

Absorption by a Kerr-Newman BH



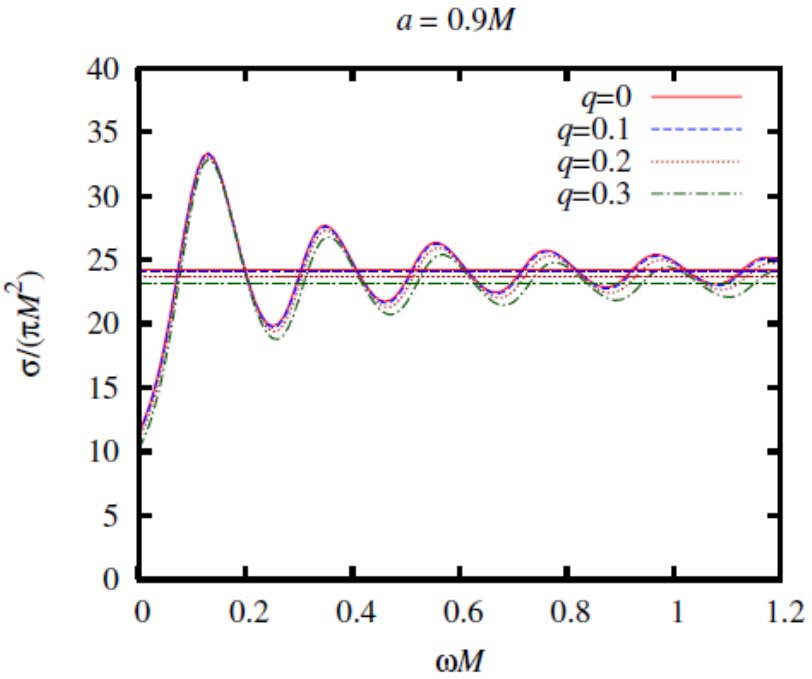
Macedo+ (2013)
[arXiv:1308.0018]

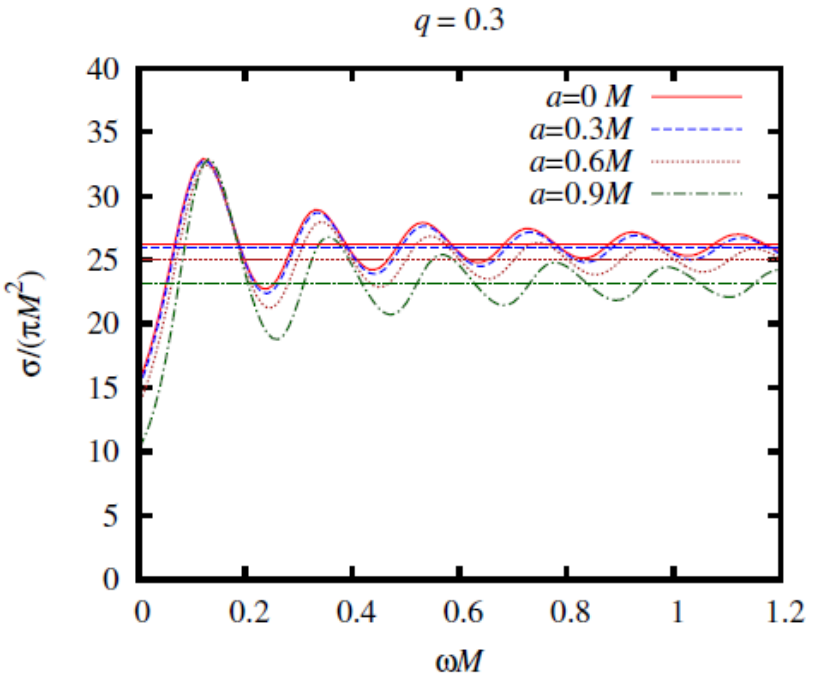
$$\sigma = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi^2}{\omega^2} |S_{\omega lm}(\gamma)|^2 \left(1 - \left|\frac{\mathcal{R}_{\omega lm}}{\mathcal{I}_{\omega lm}}\right|^2\right)$$

$$\sigma^{\mathrm{co}} = \sum_{l=1} \sum_{m=1}^{l} \sigma_{l,m}$$

$$\sigma^{\mathrm{counter}} = \sum_{l=0} \sum_{m=0}^{l} \sigma_{l,-m}$$

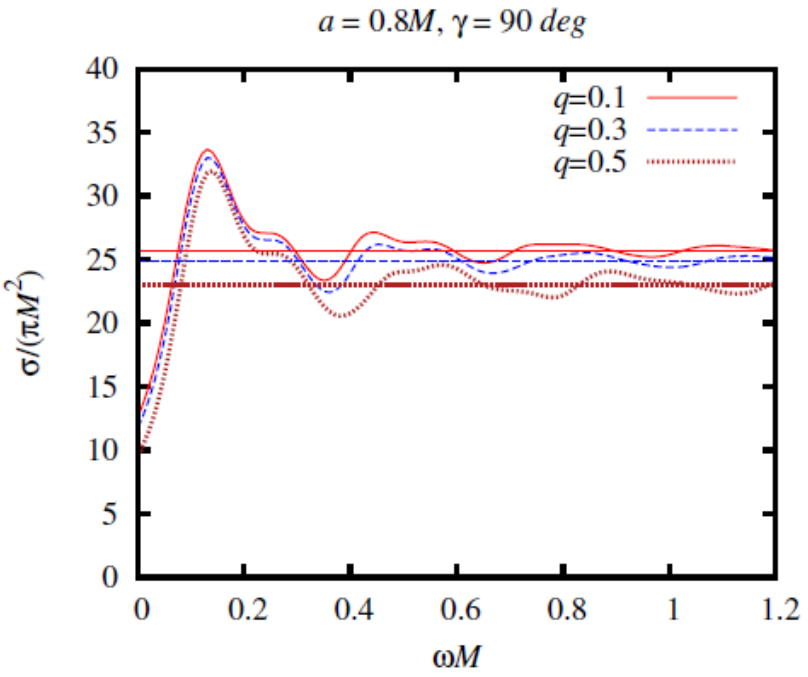
On-axis incidence

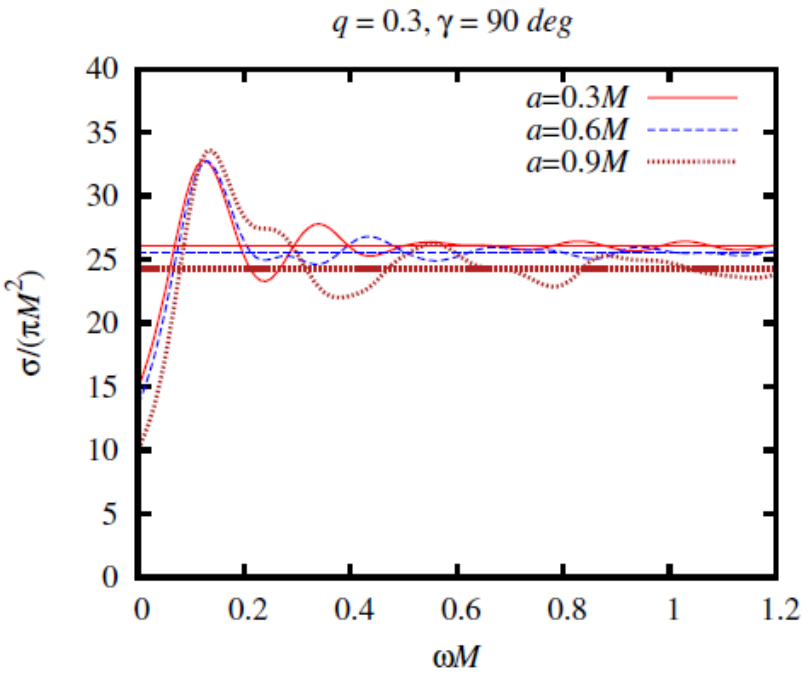




Leite+ (2017)
[arXiv:1708.03370]

Off-axis incidence





Leite+ (2017)
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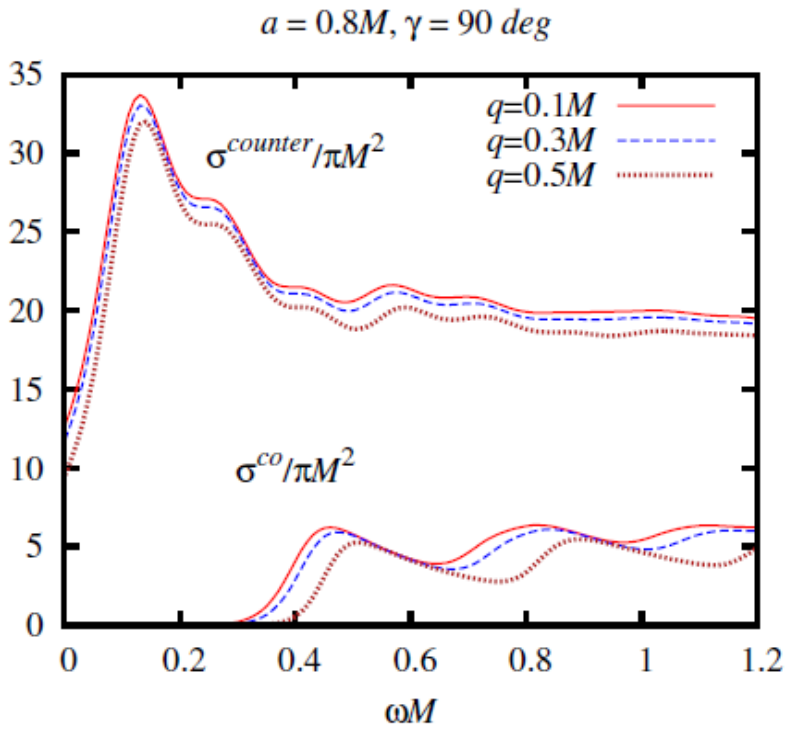
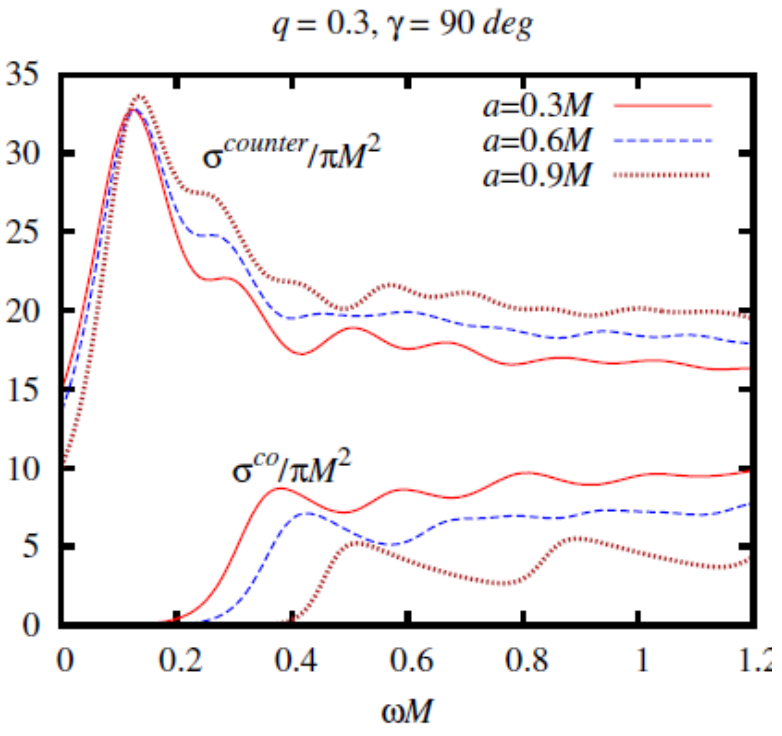
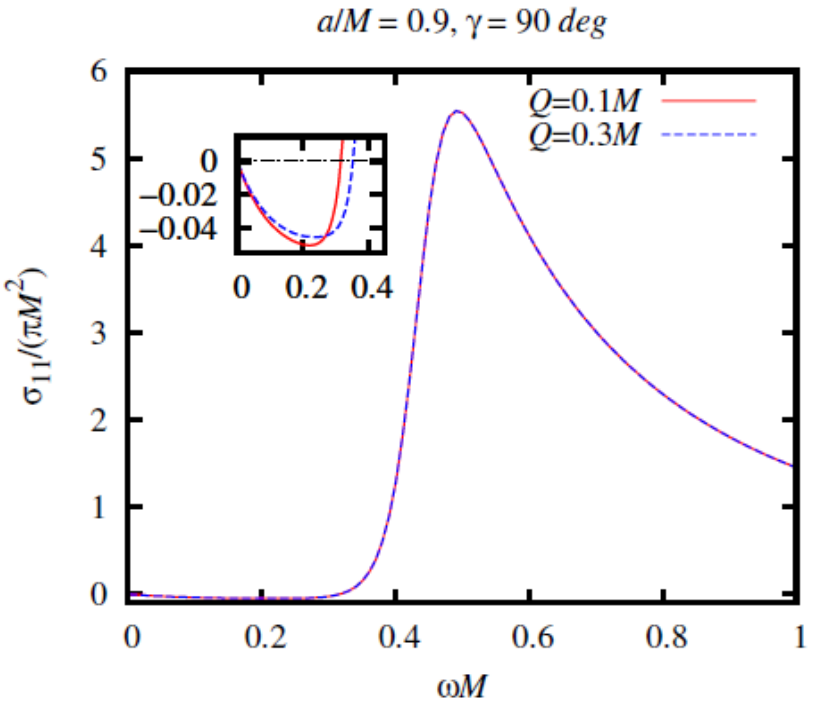
Off-axis incidence

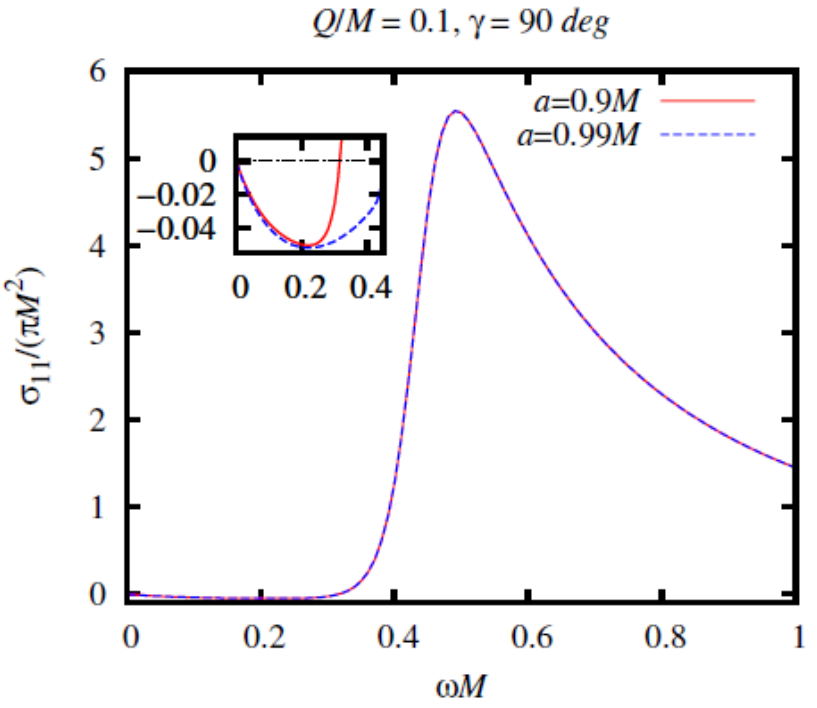




Leite+ (2017)
[arXiv:1708.03370]

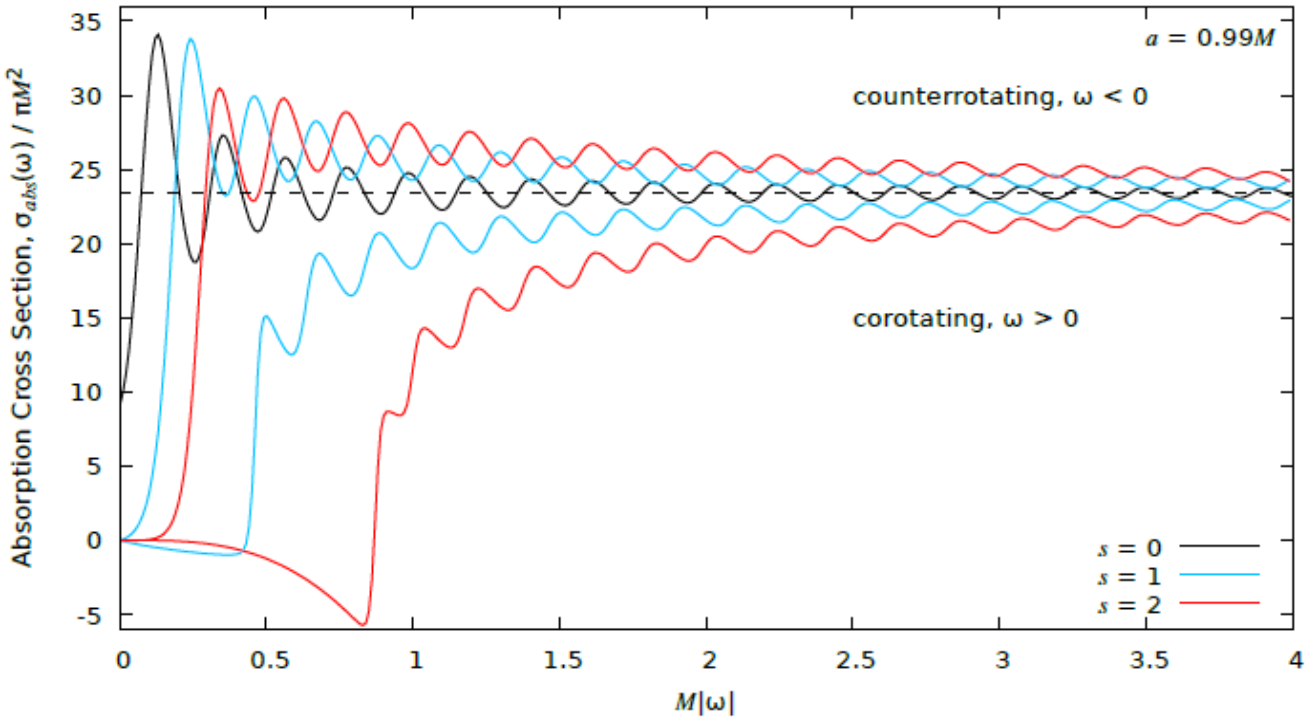
Superradiance





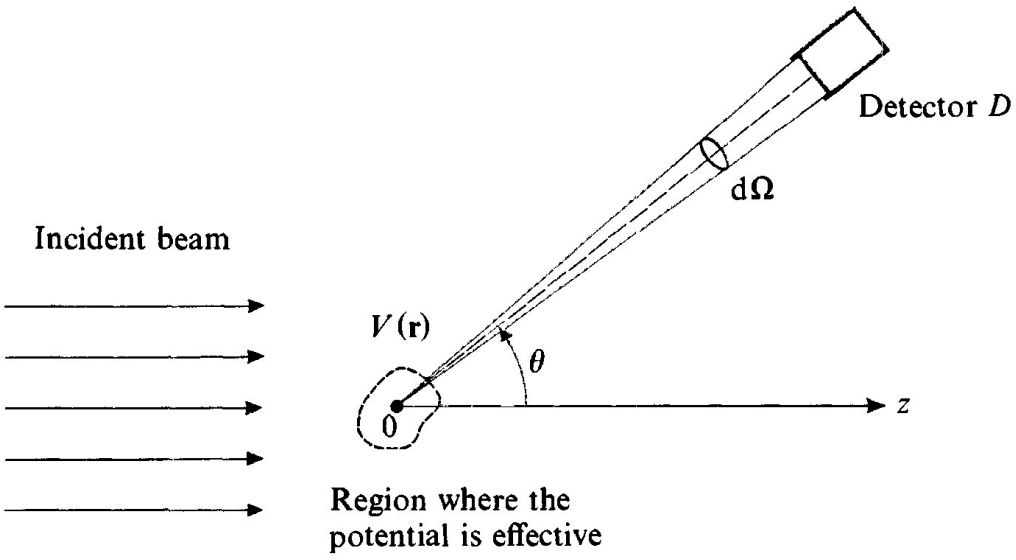
Leite+ (2017)
[arXiv:1708.03370]

Absorption by bosonic fields



Leite+ (2017)
[arXiv:1707.01144]

Scattering

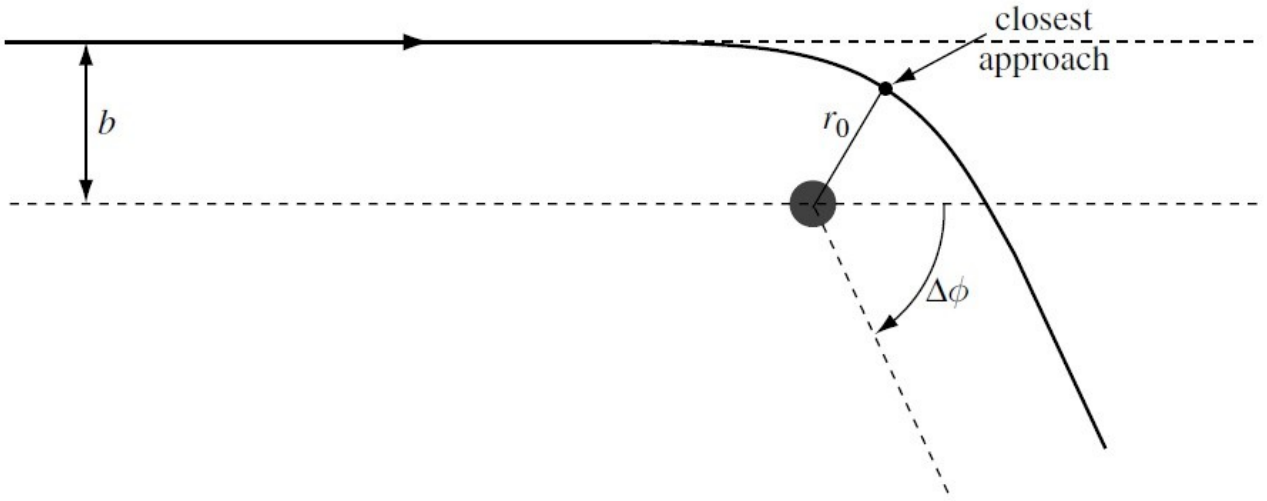


$$\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2$$

$$f(\theta) = \frac{2\pi}{i\omega} \sum_{l=0}^{\infty} S_{\omega l 0}(0) S_{\omega l 0}(\theta) \left[(-1)^{l+1} \mathcal{R}_{\omega l 0} - 1 \right]$$

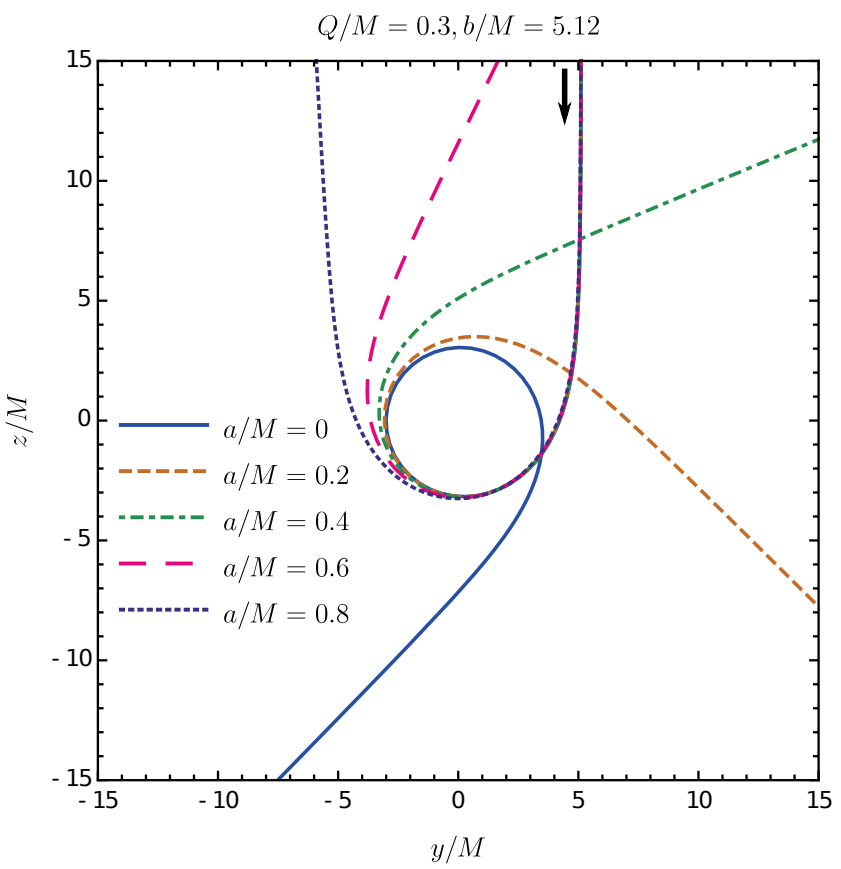
$$f(\theta) = \frac{1}{(1 - \cos\theta)^n} \sum_{j=0}^{+\infty} F_{\omega j}^{(n)} Y_{j0}(\theta)$$

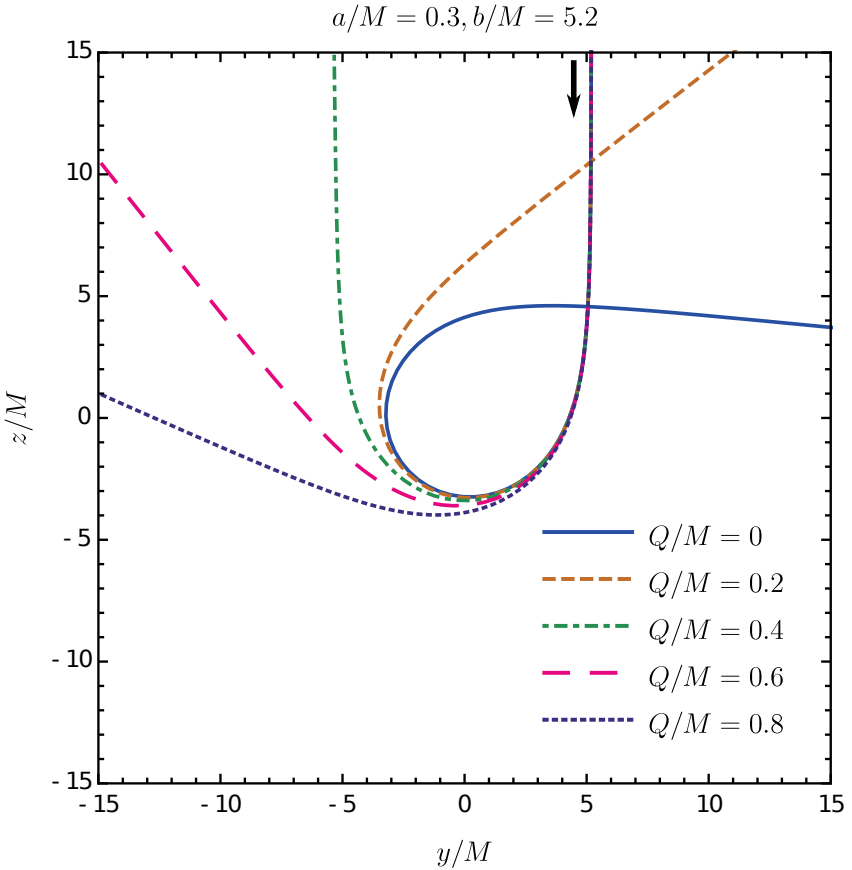
Scattering



$$\left(\frac{dr}{d\theta}\right)^2 = \frac{(a^2+r^2)^2 - b^2\Delta}{b^2 - a^2\sin\theta}$$

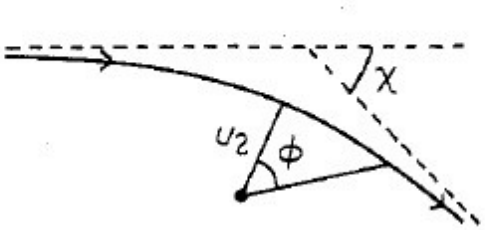
Scattering


$Q/M = 0.3, b/M = 5.12$
$a/M = 0$
$a/M = 0.2$
$a/M = 0.4$
$a/M = 0.6$
$a/M = 0.8$
z/M
y/M
15
10
5
0
- 5
- 10
- 15
- 15
- 10
- 5
0
5
10
15

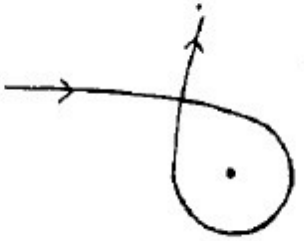

$a/M = 0.3, b/M = 5.2$
$Q/M = 0$
$Q/M = 0.2$
$Q/M = 0.4$
$Q/M = 0.6$
$Q/M = 0.8$
z/M
y/M
15
10
5
0
- 5
- 10
- 15
- 15
- 10
- 5
0
5
10
15

Scattering

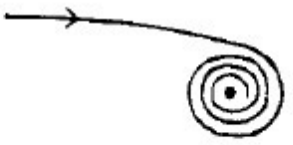
$$\frac{d\sigma}{d\Omega} = \frac{1}{\sin\theta} \sum_j b_j(\theta) \left| \frac{db_j}{d\theta} \right|$$



No spiral (a)

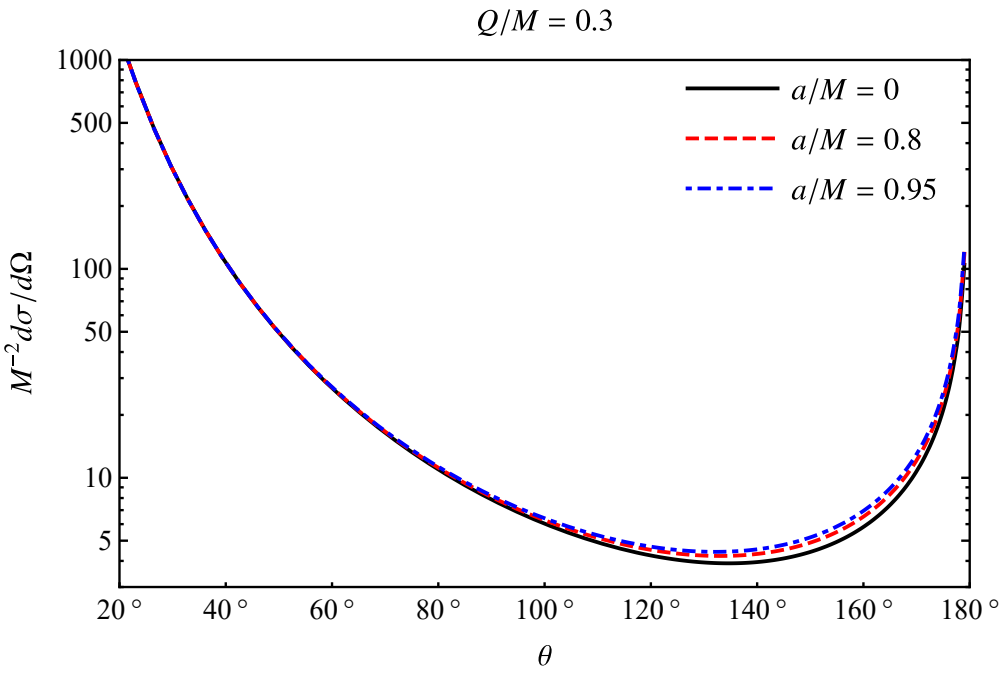


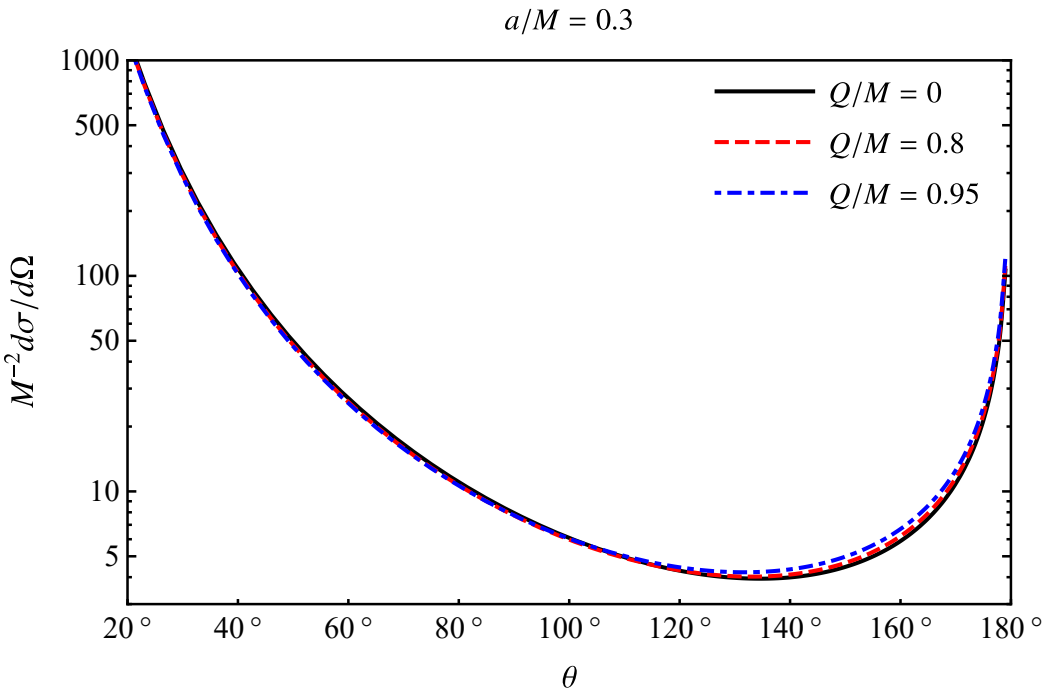
Spiralling (b)



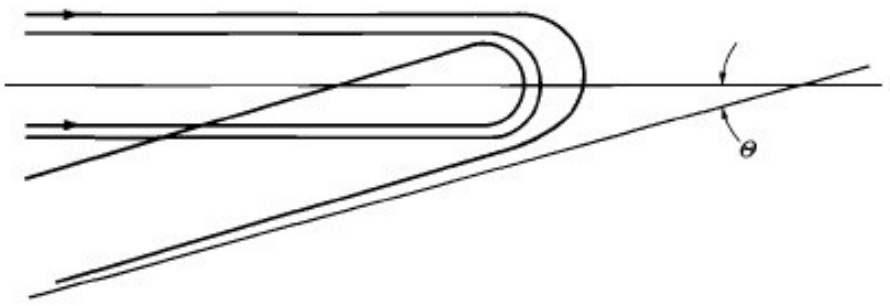
Capture (c)

Scattering


$Q/M = 0.3$
$a/M = 0$
$a/M = 0.8$
$a/M = 0.95$
$M^{-2}d\sigma/d\Omega$
1000
500
100
50
10
5
20°
40°
60°
80°
100°
120°
140°
160°
180°
θ


$a/M = 0.3$
$Q/M = 0$
$Q/M = 0.8$
$Q/M = 0.95$
$M^{-2}d\sigma/d\Omega$
1000
500
100
50
10
5
20°
40°
60°
80°
100°
120°
140°
160°
180°
θ

Scattering

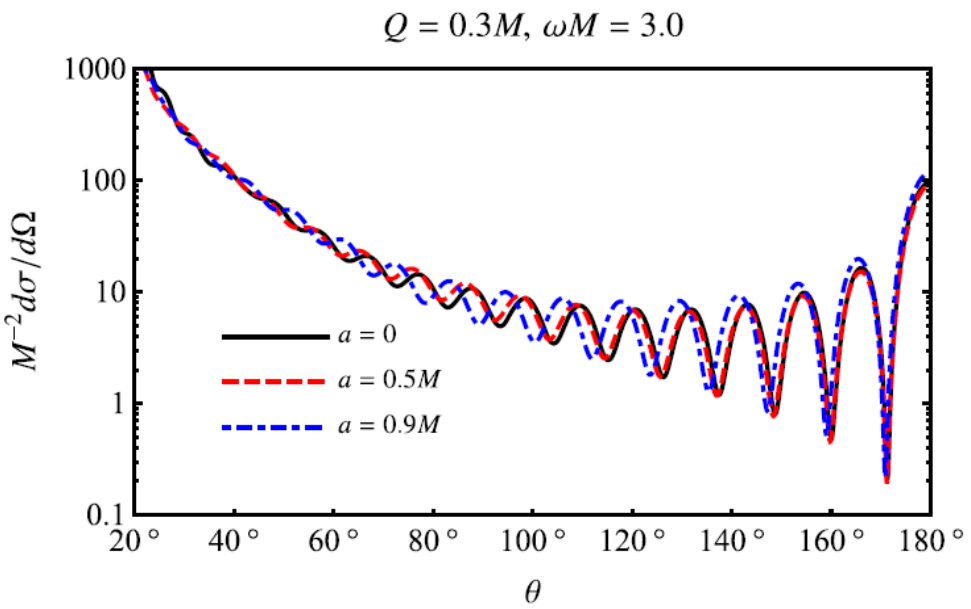


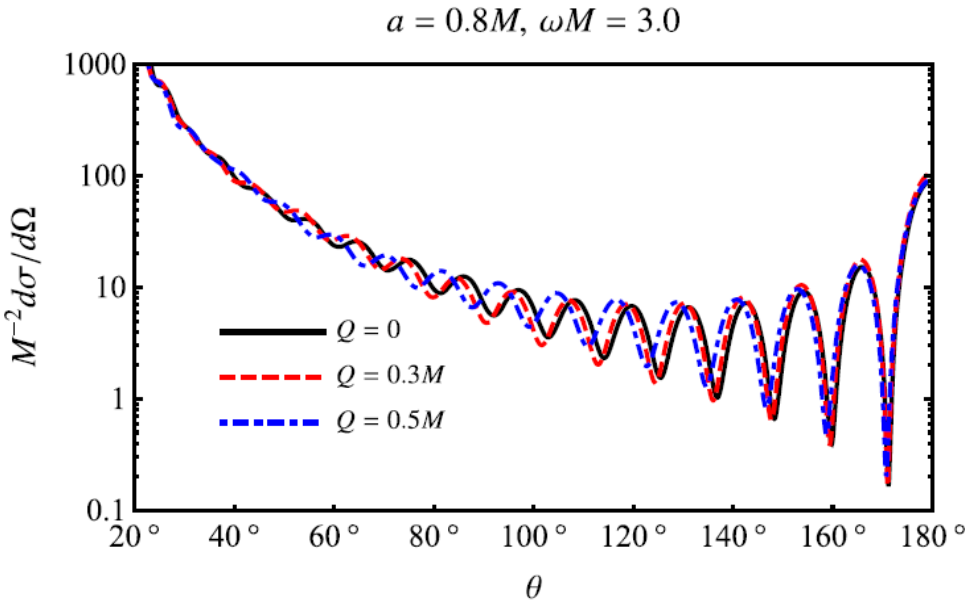
Ford&Wheeler (1959)
Nelson&DeWitt-Morette (1984)

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta\approx\pi} = \mathcal{A}\, J_0^2\left(\omega b_g \sin\theta\right)$$

$$\mathcal{A} \equiv 2\pi\,\omega b_g \left|\frac{db}{d\theta}\right|_{\theta=\pi}$$

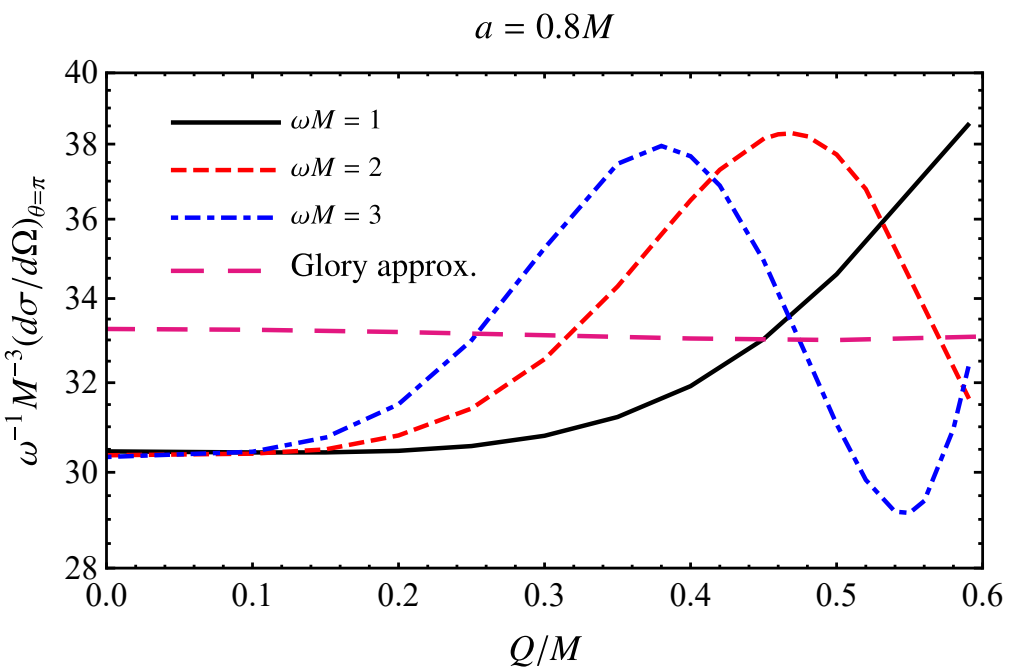
Scattering

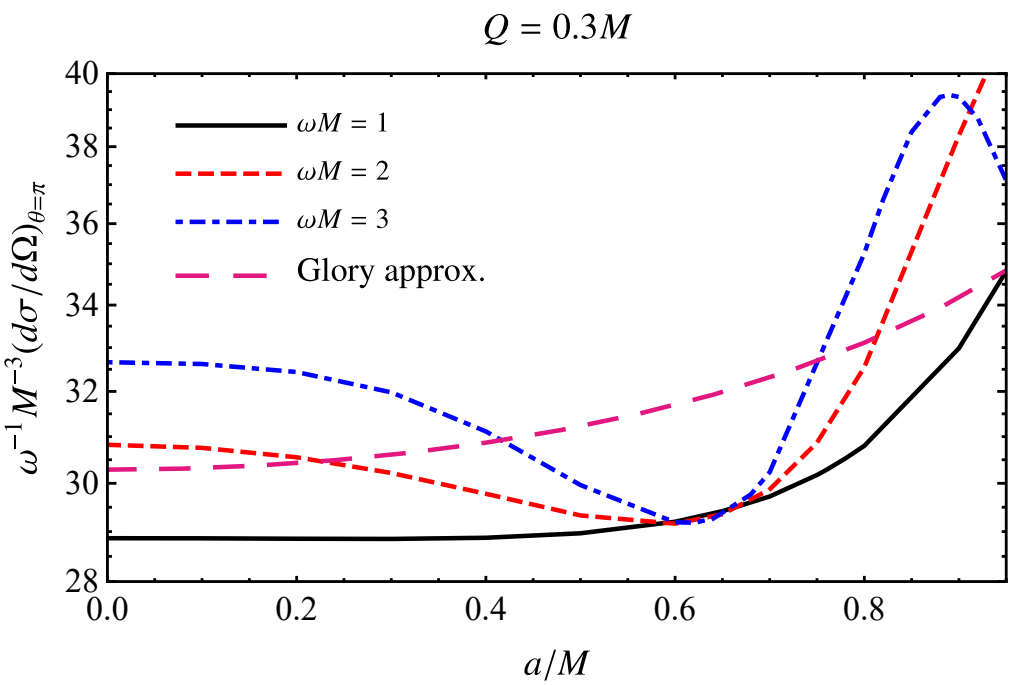




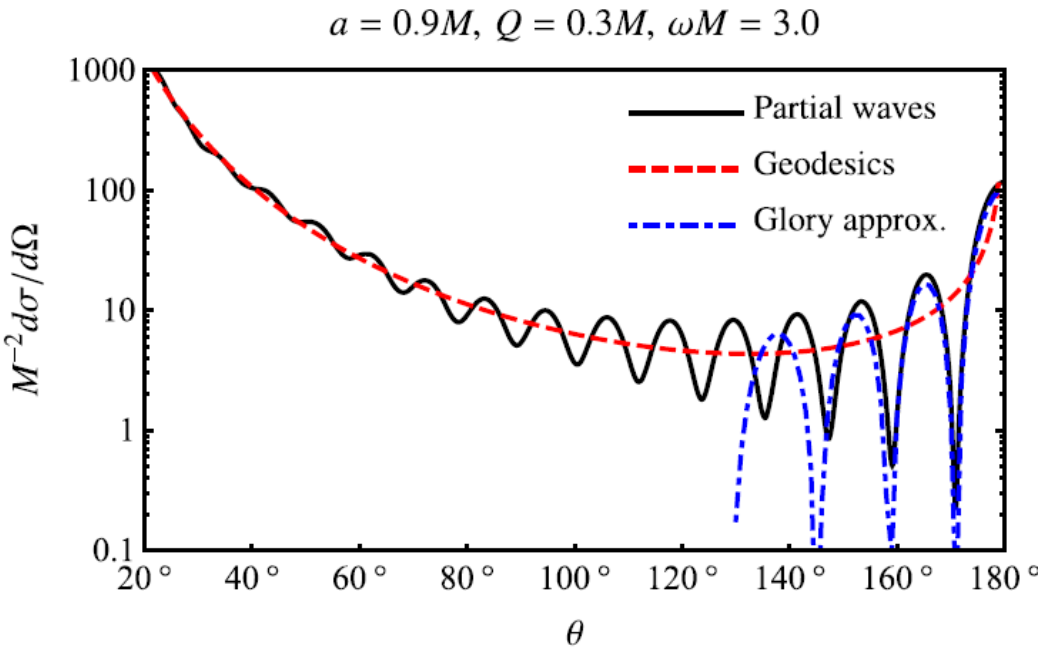
Leite+ (2019)
[arXiv:1907.04746]

Scattering


$a = 0.8M$
$\omega M = 1$
$\omega M = 2$
$\omega M = 3$
Glory approx.
$\omega^{-1} M^{-3} (d\sigma/d\Omega)_{\theta=\pi}$
Q/M
40
38
36
34
32
30
28
0.0
0.1
0.2
0.3
0.4
0.5
0.6

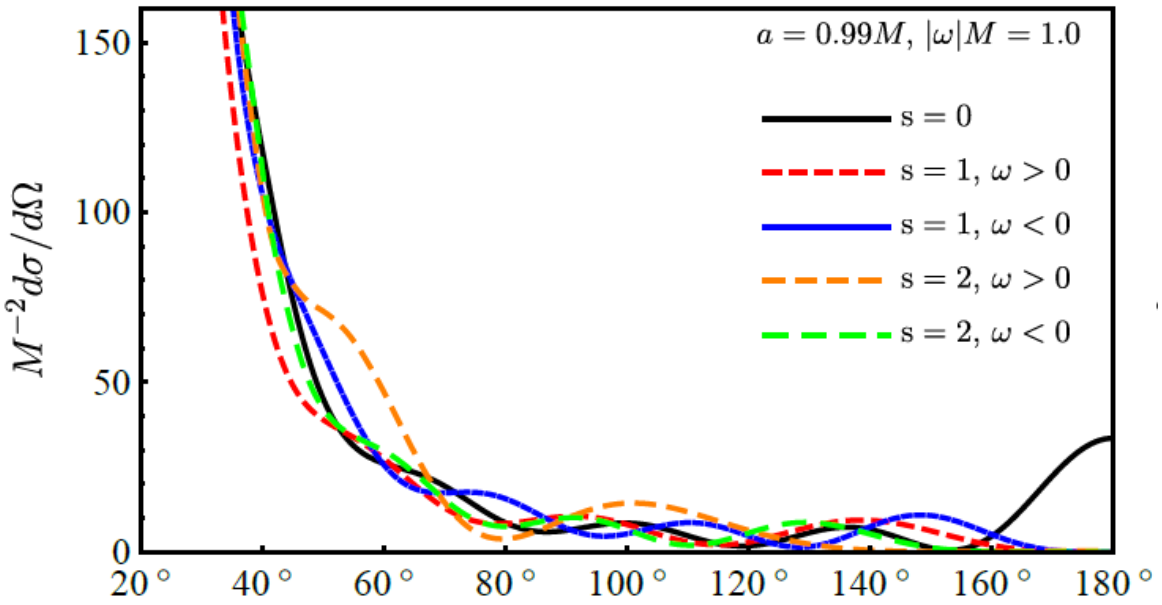

$Q = 0.3M$
$\omega M = 1$
$\omega M = 2$
$\omega M = 3$
Glory approx.
$\omega^{-1} M^{-3} (d\sigma/d\Omega)_{\theta=\pi}$
a/M
40
38
36
34
32
30
28
0.0
0.2
0.4
0.6
0.8

Scattering



Leite+ (2019)
[arXiv:1907.04746]

Scattering



Leite+ (2019)
[arXiv:1910.07666]

Final remarks

- The oscillatory pattern for the absorption cross section comes from contributions for different l.
- The resonance peaks in the absorption cross section are related to the quasibound states in the potential well.
- The differential scattering cross section presents an oscillatory pattern, which comes from contributions of different angular momentum.
- The angular widths of the spiral oscillations become larger as one increases the BHs rotation and/or the electric charge.
- The flux scattered in the backward direction is a non-monotonic function of both the rotation parameter and the electric charge of Kerr-Newman BHs.

Obrigada! Thank you!

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CAPES

MARIE CURIE ACTIONS

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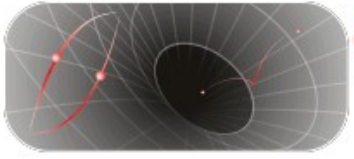

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