The Unruh effect from the de Broglie-Bohm perspective

Matheus Paixão[1], Olesya Galkina[1] and Nelson Pinto-Neto[1]

Centro Brasileiro de Pesquisas Físicas[1]

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Outline

Table of Contents

de Broglie-Bohm interpretation of Quantum Mechanics

For the dBB interpretation (or pilot wave interpretation) the trajectory of quantum objects has reality independent of observation

velocity field determined by a pilot wave Ψ through

$$\frac{dx_a(t)}{dt} = \frac{j_a\left(x_1, ..., x_N, t\right)}{|\Psi(x_1, ..., x_N, t)|^2},$$

with $j_a\left(x_1, ..., x_N, t\right)$ the quantum current associated to $\Psi(x_1, ..., x_N, t)$ which is given by Schrödinger equation

$$i\frac{\partial\Psi(x_1, ..., x_N, t)}{\partial t} = \hat{H}\Psi(x_1, ..., x_N, t) \quad \text{with} \quad \Psi = Re^{iS}$$

Hamiltonians are usually of the type $\hat{H} = \sum_a \frac{\hat{p}_a^2}{2m_a} + \hat{V}(x_1, ..., x_N, t)$. The guidance equations may also be written as

$$\frac{dx_a(t)}{dt} = \frac{1}{m_a}\frac{\partial S(x_1, ..., x_N, t)}{\partial x_a},$$

de Broglie-Bohm interpretation of Quantum Mechanics

For 1 particle, when we substitute $\Psi = Re^{iS}$ in the Schrödinger equation we have

$$\frac{\partial S}{\partial t} + \frac{1}{2}\left(\frac{\partial S}{\partial x}\right)^2 + V + Q = 0,$$

$$\frac{\partial R^2}{\partial t} + \frac{\partial}{\partial x}\left(R^2 \frac{\partial S}{\partial x}\right) = 0$$

with $Q = -\frac{1}{2R}\frac{\partial^2 R}{\partial x^2}$

Harmonic Oscillator: $H = \frac{p^2}{2} + \frac{x^2}{2} - f(t)x,$ with $f(t < t_0) = 0$

$$f(t > t_f) = 0$$

de Broglie-Bohm interpretation of Quantum Mechanics

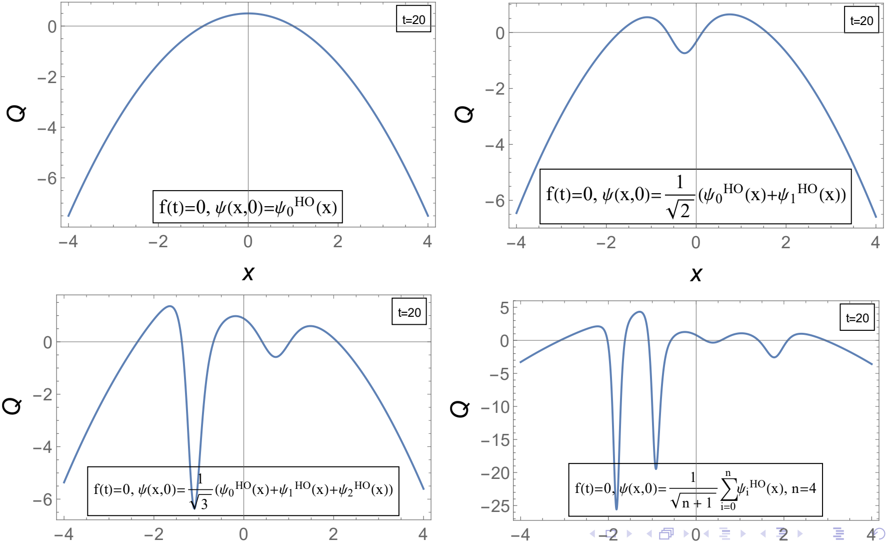
M.Paixão, H.Lima, W.Lima, F.Lustosa, N.Pinto-Neto *in preparation*

Outline:

- Write the Schrödinger equation in dimensionless form for different types of force
- Solve it numerically for different initial wave functions
- Compute the quantum potential
- Solve the guidance equations for a set of initial conditions

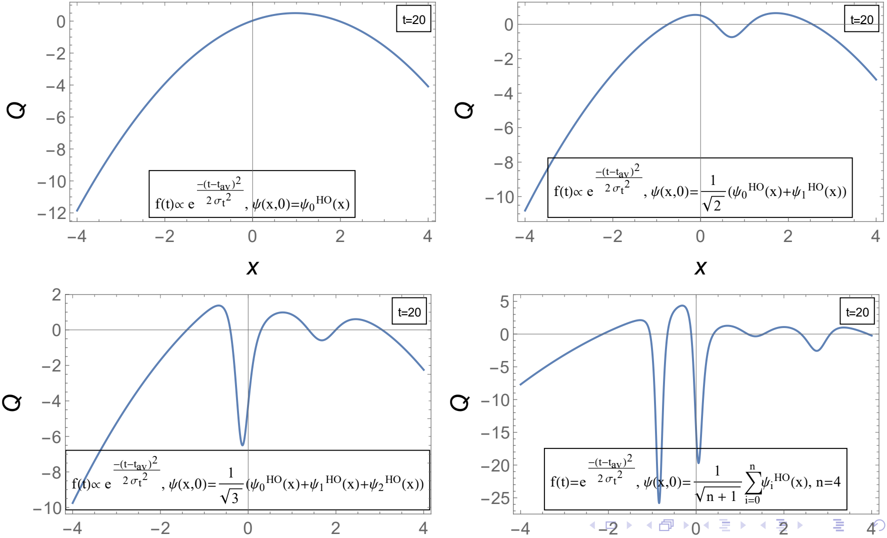
de Broglie-Bohm interpretation of Quantum Mechanics

$f(t) = 0$



de Broglie-Bohm interpretation of Quantum Mechanics

$$f(t) = e^{\frac{-(t-t_{\mu})^2}{2\sigma^2}}$$



de Broglie-Bohm interpretation of Quantum Mechanics

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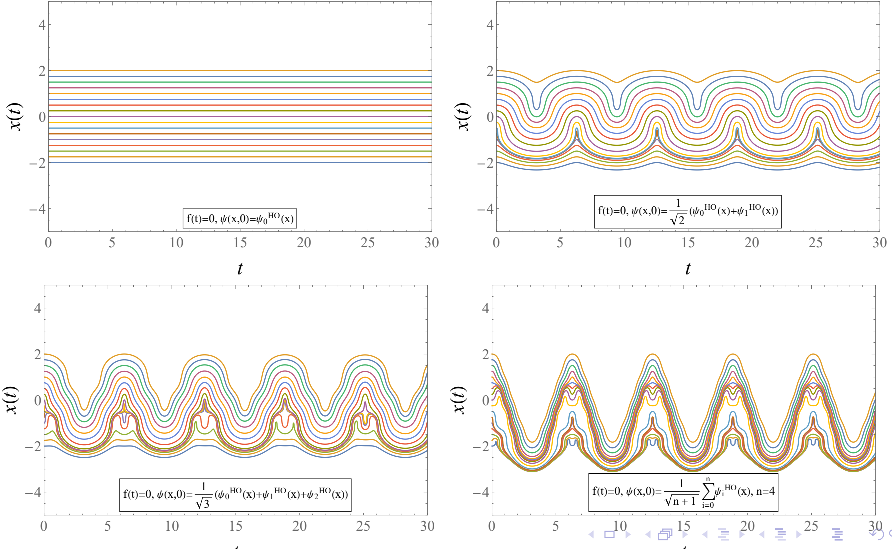

de Broglie-Bohm interpretation of Quantum Mechanics

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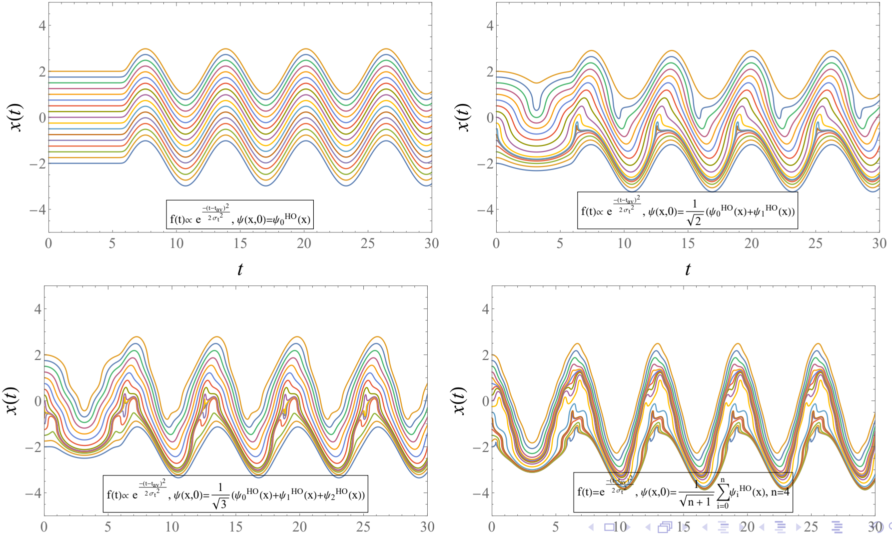


de Broglie-Bohm interpretation of Quantum Mechanics

Why use de Broglie-Bohm interpretation of quantum mechanics?

Very suitable to cosmology: N. Pinto-Neto, Universe 2021, 7(5), 134

- Solves the measurement problem
- Allows us to understand the quantum-to-classical transitions of cosmological perturbations
- the classical limit of quantum systems can be obtained in terms of the quantum potential Q
- dBB interpretation allows us to study quantum singularities in more detail
- identify non-singular quantum models through the Bohmian solutions such as the bouncing models

Table of Contents

The Unruh Effect

$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$, $\quad \mu, \nu = 0, 1, ..., n-1$

Consider a massive scalar field $\phi(x)$

$$S = \frac{1}{2} \int \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 - \xi R \phi^2 \right) d^n x$$

Equations of motion:

$$\delta S = 0 \implies \left(\Box + m^2 + \xi R \right) \phi(x) = 0$$

Inner product:

$$(\phi_1, \phi_2) := -i \int_\Sigma \sqrt{-g} \left(\phi_1(x) \overleftrightarrow{\partial_\mu} \phi_2^*(x) \right) d\Sigma^\mu,$$

with $d\Sigma^\mu = n^\mu d\Sigma$ a vector orthogonal to the Cauchy hypersurface indicating the direction of time.

The Unruh Effect

Expand $\phi(x)$ in terms of mode solutions $\{u_k, u_k^*\}$:

$$\hat{\phi}(x) = \int \left(\hat{a}_k u_k(x) + \hat{a}_k^\dagger u_k^*(x) \right) dk,$$

with $\hat{a}_k$ and $\hat{a}_k^\dagger$ obeying

$$[\hat{a}_k, \hat{a}_{k'}] = 0, \quad [\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger] = 0 \quad [\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta(k - k'),$$

- we can define a vacuum state $|0\rangle$ such that $\hat{a}_k |0\rangle = 0$, $\forall k$
- Successive applications of $\hat{a}_k^\dagger$:

$$\begin{aligned} |1_k\rangle &= \hat{a}_k^\dagger |0\rangle \\ |2_k\rangle &= \hat{a}_k^\dagger |1_k\rangle \\ |3_k\rangle &= \hat{a}_k^\dagger |2_k\rangle \\ &\vdots \end{aligned}$$

The Unruh Effect

Assuming another basis $\{\bar{u}_k, \bar{u}_k^*\}$, the expansion of the field is

$$\phi(x) = \int \left(\hat{b}_k \bar{u}_k(x) + \hat{b}_k^\dagger \bar{u}_k^*(x)\right) dk,$$

with analog commutation relations. The $\hat{b}_k$ coefficient defines a new vacuum $|\bar{0}\rangle$ such that $\hat{b}_k |\bar{0}\rangle = 0$

.

The relation between the mode solutions are give by the Bogoliubov transformations:

$$\bar{u}_k = \int (\alpha_{kk'} u_{k'} + \beta_{kk'} u_{k'}^*) dk'$$
$$u_{k'} = \int ({\alpha_{kk'}}^* \bar{u}_k - \beta_{kk'} \bar{u}_k^*) dk,$$

where $\alpha_{kk'} = (\bar{u}_k, u_{k'})$ and $\beta_{kk'} = -(\bar{u}_k, u_{k'}^*)$.

The Unruh Effect

What is the expectation value of the number operator $\hat{N}_k = \hat{a}_k^\dagger a_k$ of u_k basis in the $|\bar{0}\rangle$ vacuum?

$$\langle\bar{0}|\,\hat{N}_k\,|\bar{0}\rangle = \int |\beta_{k'k}|^2 dk'$$

This means that the $|\bar{0}\rangle$ vacuum (of the $\bar{u}_{k'}$ modes) contains $\int |\beta_{k'k}|^2 dk'$ particles in the u_k mode.

- Particle concept is observer dependent

Minkowski: (2D and $m = 0$)

$$S = \frac{1}{2}\int dt dx \left\{\left(\frac{\partial\phi}{\partial t}\right)^2 - \left(\frac{\partial\phi}{\partial x}\right)^2\right\} \quad\Longrightarrow\quad \left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}\right)\phi = 0$$

The Unruh Effect

Mode solutions:

$$\text{p.f.m} : u_k(t,x) = \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k t - kx)}, \quad \text{n.f.m} : u_k^*(t,x) = \frac{1}{\sqrt{4\pi\omega_k}} e^{i(\omega_k t - kx)}$$

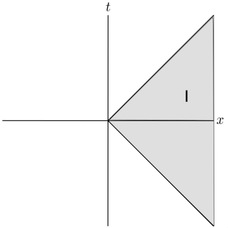
defines a vacuum $|0\rangle_M$, such that $\hat{a}_k |0\rangle_M = 0$, which is equivalent for all *inertial observers*.

And if we consider observers with *uniform acceleration a* in relation to some Minkowski reference frame?

Rindler Coordinates:

$$t = a^{-1} e^{a\xi} \sinh(a\tau)$$
$$x = a^{-1} e^{a\xi} \cosh(a\tau)$$

valid for $0 < x < \infty$ and $-t < x < t$



The Unruh Effect

$$ds^2 = -dt^2 + dx^2 = e^{2a\xi}(-d\tau^2 + d\xi^2)$$

$$\text{p.f.m} : \bar{u}_k(\tau,\xi) = \frac{1}{\sqrt{4\pi\omega_k}} e^{-i(\omega_k \tau - k\xi)}, \quad \text{n.f.m} : \bar{u}_k^*(\tau,\xi) = \frac{1}{\sqrt{4\pi\omega_k}} e^{i(\omega_k \tau - k\xi)}$$

$$\langle \hat{N}_k^R \rangle_M = \frac{1}{e^{\frac{2\pi}{a}|k|} - 1},$$

Bose-Einstein distribution with temperature $T = \dfrac{a}{2\pi}$

Table of Contents

The dBB perspective

Functional approach: *K. Freese et. al. Nucl.Phys.B 255 (1985)*

Hamiltonian:

$$H = \frac{1}{2}\int_0^{\infty}\left\{\left(\frac{\partial\phi}{\partial t}\right)^2 + \left(\frac{\partial\phi}{\partial x}\right)^2\right\}dx$$

Schrödinger equation:

$$\hat{H}\Psi[\phi,t] = i\frac{\partial\Psi[\phi,t]}{\partial t}, \quad \text{with} \quad \Psi[\phi,t] = \langle\phi|\Psi\rangle$$

The dBB perspective

Expand the field in half Fourier transform

$$\phi(t,x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \sin(kx)\phi_k^M(t)dk$$

and assuming the decomposition $\Psi[\phi,t] = \prod_{k>0} \Psi_k[\phi_k^M,t]$, each Ψ_k satisfies a Schrödinger equation:

$$i\frac{\partial \Psi_k(\phi_k,t)}{\partial t} = \frac{1}{2}\left(-\frac{\partial^2}{(\partial \phi_k^M)^2} + k^2|\phi_k|^2\right)\Psi_k^M(\phi_k,t),$$

Ground State: $\Psi_k^0[\phi_k^M, t=0] = \exp\left(-\frac{1}{2}k\left(\phi_k^M\right)^2\right).$

The dBB perspective

In terms of Rindler modes

$$\phi(\tau,\xi) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{ik'\xi} \phi^R_{k'}(\tau) dk'.$$

The relation between Minkowski and Rindler modes is given by

$$\phi^M_k = \int_{-\infty}^{\infty} A(k,k') \phi^R_{k'} dk',$$

with

$$A(k,k') = \frac{1}{\pi} \int_0^{\infty} \sin(kx) e^{ik'\xi(x)} dx = \frac{1}{\pi a} \Gamma\left(1 + \frac{ik}{a}\right) \cosh\left(\frac{\pi k}{2a}\right) \left|\frac{k'}{a}\right|^{-1-ik/a}$$

Therefore,

$$\Psi^0_{k'}(\phi, \tau = 0) = \exp\left[-k' \coth\left(\frac{\pi k'}{2a}\right) \phi^R_{k'} \phi^{R*}_{k'}\right].$$

The dBB perspective

So, computing

$$\langle \hat{N}_k^R \rangle_M = \frac{1}{e^{\frac{2\pi}{a}|k|} - 1},$$

that is a Bose-Einstein distribution with temperature $T = \frac{a}{2\pi}$

Temporal evolution of Minkowski ground state in Rindler space-time:

$$\Psi^0_{k'}(\phi, \tau) = \exp\left[-k' f_{k'}(\tau)\phi^R_{k'}\phi^{R*}_{k'} + \Omega(\tau)\right]$$

with $f_{k'}(0) = \coth(\pi k'/2a)$.

Schrödinger equation:

$$i\frac{\partial \Psi_k[\phi, \tau]}{\partial \tau} = \left(-\frac{\partial^2}{\partial \phi_k^{R*}\partial \phi_k} + k^2|\phi_k^R|^2\right)\Psi_k^R[\phi, \tau]$$

The dBB perspective

Solution:

$$f_k(\tau) = \coth\left(\frac{\pi k}{2a} + ik\tau\right), \qquad \Omega_k(\tau) = -\ln\left[\sinh\left(\frac{\pi k}{2a} + ik\tau\right)\right]$$

In dBB interpretation, it is usual to write $\Psi_k = R_k e^{iS_k}$, where

$$R_k(\phi_k, \phi_k^*, \tau) = \exp\left(-k\Re[f_k(\tau)]|\phi_k|^2 + \Re[\Omega_k(\tau)]\right),$$
$$S_k(\phi_k, \phi_k^*, \tau) = -k\Im[f_k(\tau)]|\phi_k|^2 + \Im[\Omega_k(\tau)].$$

Using the polar form, the Schrödinger equation gives us two equations:

$$\frac{\partial S_k}{\partial \tau} + \frac{\partial S_k}{\partial \phi_k^*}\frac{\partial S_k}{\partial \phi_k} + k^2\left|\phi_k\right|^2 + Q_k = 0$$
$$\frac{\partial R_k^2}{\partial \tau} + \frac{\partial}{\partial \phi_k}\left(R_k^2 \frac{\partial S_k}{\partial \phi_k^*}\right) + \frac{\partial}{\partial \phi_k^*}\left(R_k^2 \frac{\partial S_k}{\partial \phi_k}\right) = 0$$

The dBB perspective

Q_k is the quantum potential and is given by

$$\begin{aligned} Q_k &= -\frac{1}{R_k}\frac{\partial^2 R_k}{\partial\phi_k^*\partial\phi_k} \\ &= k\Re[f_k(\tau)] - k^2\Re^2[f_k(\tau)]\,|\phi_k|^2 \end{aligned}$$

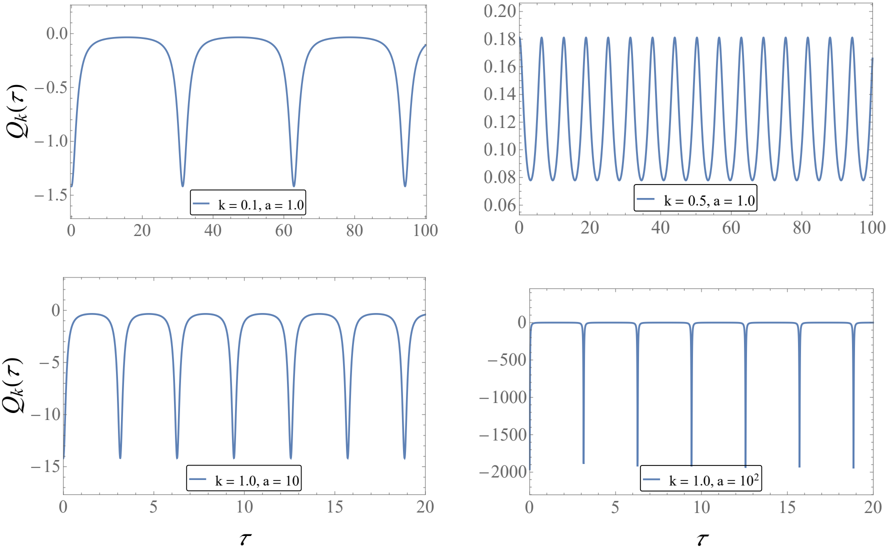
Guidance equations:

$$\frac{\partial\phi_k}{\partial\tau} = \frac{\partial S_k}{\partial\phi_k^*}, \qquad \frac{\partial\phi_k^*}{\partial\tau} = \frac{\partial S_k}{\partial\phi_k}$$

Setting the constant of integration equal to $\frac{1}{\sqrt{2k}}\frac{1}{\left(e^{\pi k/a}-1\right)}$,

$$\phi_k(\tau) = \frac{1}{\sqrt{2k}}\frac{\left(e^{2\pi k/a} - 2e^{\pi k/a}\cos(2k\tau) + 1\right)^{1/2}}{\left(e^{\pi k/a} - 1\right)}$$

The dBB perspective



The dBB perspective

Energy of each mode:

$$\begin{aligned} E_k &= -\frac{\partial S_k}{\partial \tau} \\ &= k\frac{\partial \Im[f_k(\tau)]}{\partial \tau}|\phi_k|^2 - \frac{\partial \Im[\Omega_k(\tau)]}{\partial \tau} \end{aligned}$$

Limit when $a \to 0$:

- $\phi_k(\tau) \to \frac{1}{\sqrt{2k}}$
- $Q_k = k - k^2|\phi_k|^2 = k/2$
- $E_k \to k$

Conclusion: When $a \to 0$, we reproduce Bohmian mechanics for Klein-Gordon field in Minkowski space.

The dBB perspective

Power Spectrum:

N. Pinto-Neto, G. Santos and W. Struyve, Phys.Rev.D 85 (2012)

$$\langle \phi(\tau,\xi)\phi(\tau,\xi+\rho)\rangle_{dBB} = \int \mathcal{D}\phi |\Psi(\phi,\tau)|^2 \phi(\xi)\phi(\xi+\rho)$$

After many, many calculations using QFT we have that

$$\langle \phi(\xi)\phi(0)\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \frac{1}{2|k|\Re[f_k(\tau)]} e^{-ik\xi}$$

Therefore,

$$\begin{aligned} P_k &= \int_{-\infty}^{\infty} e^{-ik\xi} \langle \phi(\xi)\phi(0)\rangle \, d\xi \\ &= \frac{1}{2|k|\Re[f_k(\tau)]} \end{aligned}$$

The dBB perspective

In the limit when $T >> 1$, then

$$P_k(\tau) = \frac{2T}{k^2} \sin^2(k\tau).$$

Table of Contents

Summary

- We obtained the wave functional of Minkowski vacuum in Rindler coordinates
- The Unruh effect is reproduced by the functional approach
- We solve the guidance equations and find a expression for the Bohmian field
- We find the quantum potential and the energy of each mode
- Bohmian mechanics for Klein-Gordon field in Minkowski space is reproduced in the limit when $a \to 0$
- We compute the power spectrum and its expansion in high temperature limit

Perspectives:

- We need to understand better the relation between the power spectrum and the quantum potential
- We can extend our analysis to the left wedge of Minkowski space-time
- Study Hawking radiation from the de Broglie-Bohm perspective

Thank you!

E-mail: matheus.mapaixao@gmail.com