Unimodular JT gravity and de Sitter quantum cosmology

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Joint work with **Altay Etkin** and **Bruno Alexandre** [arXiv:2501.17213]

What I want to convey in this talk

1. The emergence of "physical" time in Henneaux and Teitelboim Unimodular (HT) Gravity.
2. Naturalness of HT Unimodular gravity
   (emerges naturally in JT 1+1 gravity context).
3. A table top computation in de Sitter Quantum cosmology in JT gravity, using the new tools.

This is a step towards holographic interpretation of Unimodular Gravity.

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Original unimodular gravity (4D)

- We ask ourselves if **the diffeomorphism invariance in a theory of gravitation should be restricted to volume-preserving transformations** [Einstein, 1917]?

$$\sqrt{-g} = 1,$$

- The eoms are the trace-free Einstein equations (without matter part):

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 0.$$

- The unimodular gravity action is

$$S_{\mathrm{U}}[g, \Lambda] = \frac{1}{2} \int_{\mathcal{M}_4} d^4x \left[\sqrt{-g} R - 2\Lambda \left(\sqrt{-g} - 1 \right) \right].$$

$\Lambda(x)$ behaves as a Lagrange multiplier, enforcing the unimodular condition $\sqrt{-g} = 1$ on-shell. Conservation laws then imply that $\partial_\mu \Lambda = 0$.

Henneaux-Teitelboim Unimodular gravity (4D)

One can construct a **fully diffeomorphism invariant theory of unimodular gravity** by introducing an auxiliary vector density $\mathcal{T}^{\mu}$ [Henneaux and Teitelboim, 1989]:

$$\sqrt{-g} = 1 \quad \Longrightarrow \quad \sqrt{-g} = \partial_{\mu}\mathcal{T}^{\mu}. \tag{1}$$

The last equation is called the **unimodular condition**. The action for such a theory is described by the HT action

$$S_{\mathrm{HT}}[g, \Lambda, \mathcal{T}] = \frac{1}{2}\int_{\mathcal{M}_4} d^4x \left[\sqrt{-g}R - 2\Lambda\left(\sqrt{-g} - \partial_{\mu}\mathcal{T}^{\mu}\right)\right]. \tag{2}$$

Here $\Lambda(x)$ is an off-shell variable and becomes an on-shell constant:

$$\partial_{\mu}\Lambda = 0. \tag{3}$$

This theory is equivalent to standard GR.

Unimodular condition and Unimodular time

Assuming that our manifold $\mathcal{M} \simeq \Sigma \times \mathbb{R}$ upon integrating both size of the unimodular condition between two spacial hypersurfaces:

$$\int_{\Sigma_{t_0}}^{\Sigma_t} d^4x\sqrt{-g} = \int_{\Sigma_{t_0}}^{\Sigma_t} d^4x\, \partial_\mu \mathcal{T}^\mu$$

$$\mathrm{Vol}_4(\Sigma_t \to \Sigma_{t_0}) = T_\Lambda(\Sigma_t) - T_\Lambda(\Sigma_0) \tag{4}$$

Here, T_Λ behaves as a "flux of time": this is a good observable to define a flow of time.

- We have introduced the so-called **unimodular time**

$$T_\Lambda(\Sigma) := \int_\Sigma d\Sigma\, \mathcal{T}^0, \tag{5}$$

  Λ and T_Λ are promoted to phase-space variables, acting as canonically conjugate pairs

  $\Rightarrow$ They satisfy the Poisson bracket $\{T_\Lambda, \Lambda\}_{\mathrm{PB}} = 1$.

What are the consequence of introducing this unimodular time?

The problem of time

Different notions of "time":

- Quantum mechanics treats it as an external, absolute parameter, used to measure the evolution of a quantum system.
- General relativity treats is as part of the spacetime fabric.

  The Hamiltonian (and momentum) constraints vanishes in GR:

  $$\widehat{\mathcal{C}}_{\mathrm{GR}}\Psi = 0 \quad \Longrightarrow \quad e^{-it\widehat{\mathcal{C}}_{\mathrm{GR}}}\Psi = \Psi.$$

  Quantum states are therefore "frozen in time".

Henneaux and Teitelboim gravity one has physical relational times T_Λ as canonical conjugates to Λ.

- Unitary evolution can then be recast in term of these time observables: the Wheeler-DeWitt equation becomes a Schrödinger-type equation

$$\widehat{\mathcal{C}}_{\mathrm{HT}}\Psi = 0 \quad \Longrightarrow \quad i\partial_{T_\Lambda}\Psi(T_\Lambda) = \widehat{\mathcal{C}}_{\mathrm{GR}}\Psi(T_\Lambda) \quad \Longrightarrow \quad \Psi(T_\Lambda) = e^{iT_\Lambda\widehat{\mathcal{C}}_{\mathrm{GR}}}\Psi(0).$$

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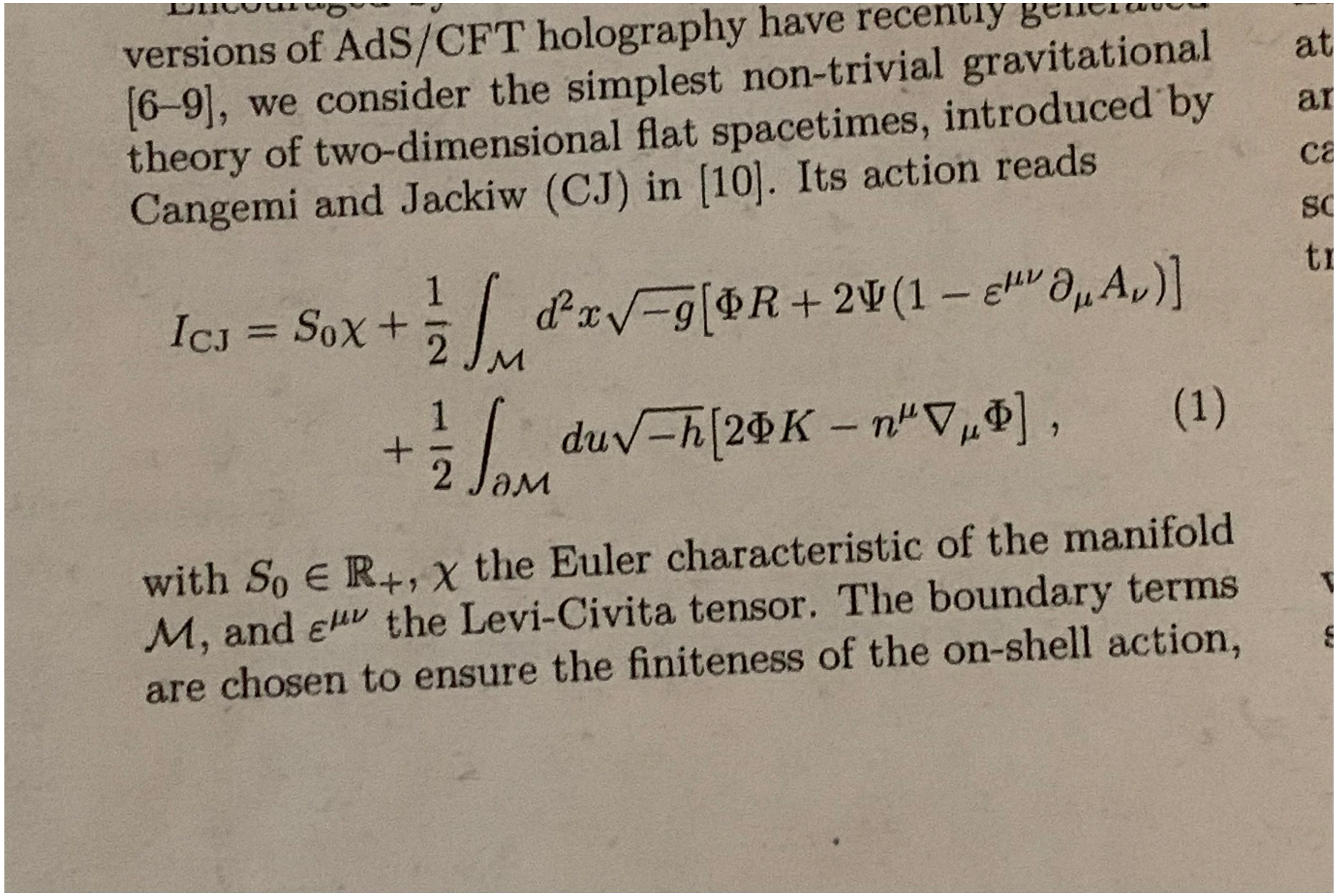

versions of AdS/CFT holography have recently generated [6–9], we consider the simplest non-trivial gravitational theory of two-dimensional flat spacetimes, introduced by Cangemi and Jackiw (CJ) in [10]. Its action reads

$$I_{\mathrm{CJ}} = S_0\chi + \frac{1}{2}\int_{\mathcal{M}} d^2x\sqrt{-g}\left[\Phi R + 2\Psi(1-\varepsilon^{\mu\nu}\partial_\mu A_\nu)\right] + \frac{1}{2}\int_{\partial\mathcal{M}} du\sqrt{-h}\left[2\Phi K - n^\mu\nabla_\mu\Phi\right] , \tag{1}$$

with $S_0 \in \mathbb{R}_+$, χ the Euler characteristic of the manifold $\mathcal{M}$, and $\varepsilon^{\mu\nu}$ the Levi-Civita tensor. The boundary terms are chosen to ensure the finiteness of the on-shell action,

FIGURE: Paper by Afshar, González, Grumiller, and Vassilevich on 2d flat holography and the complex SYK [1911.05739].

This structures are found and used in JT gravity unrecognized by the authors!
How do they found this formulation?

What is JT gravity?

One could naively write down the Einstein-Hilbert action in 2d but this is topological:

$$\frac{1}{4\pi}\int_{\mathcal{M}_2} d^2x\sqrt{-g}R = \chi(\mathcal{M}_2) \tag{6}$$

Instead Jackiw-Teitelboim (JT) proposed a non trivial action:

$$S_{\text{JT}}[\phi, g] = \frac{1}{2}\int_{\mathcal{M}_2} d^2x\,\sqrt{-g}\phi\big(R-2\lambda\big), \tag{7}$$

where λ is the "cosmological constant" and ϕ is the "dilaton" field introduced as a Lagrange multiplier.

- The field equations are

$$R = 2\lambda, \qquad \big(\nabla_\mu\partial_\nu - g_{\mu\nu}\Box\big)\phi = \lambda g_{\mu\nu}\phi. \tag{8}$$

This is the same action that you get near-extremal Reissner-Nordström black holes ($AdS_2 \times S^2$) and more recently near-extremal Bañados-Teitelboim-Zanelli black holes ($AdS_2 \times S^1$).

WHAT CANGEMI AND JACKIW DISCOVERED

JT Gravity admits a BF formulation, this is true for (A)dS$_2$ under $\mathfrak{sl}(2,\mathbb{R})$:

$$\frac{1}{2}\int_{\mathcal{M}_2} d^2x\,\sqrt{-g}\phi\left(R-2\lambda\right) \xrightarrow{\mathfrak{sl}(2,\mathbb{R})} \int_{\mathcal{M}_2} tr(\mathbf{BF})\ \checkmark \tag{9}$$

Cangemi and Jackiw where interested to find a BF theory for the flat case with the addition of the vacuum cosmological constant Λ:

$$\frac{1}{2}\int_{\mathcal{M}_2} d^2x\sqrt{-g}\,(\phi R - 2\Lambda) \xrightarrow{\mathfrak{iso}(1,1)} \int_{\mathcal{M}_2} tr(\mathbf{BF})\ \times \tag{10}$$

$$\xrightarrow{\mathfrak{iso}(1,1)\oplus\mathfrak{u}(1)} \int_{\mathcal{M}_2} tr(\mathbf{BF})\ \checkmark \tag{11}$$

This action in 2$^{\text{nd}}$-order formalism translates to the $\widehat{\text{CGHS}}$ gravity model:

$$\int_{\mathcal{M}_2} tr(\mathbf{BF}) \xrightarrow{2^{\text{nd}}\text{-order}} \frac{1}{2}\int_{\mathcal{M}_2} d^2x\,\sqrt{-g}\Big[\phi R - 2\Lambda\big(1-\epsilon^{\mu\nu}\partial_\mu A_\nu\big)\Big]. \tag{12}$$

Can we do the same for general non-flat?

BF-theory for JT gravity

A first-order formulation of JT gravity goes through a gauge-theoretic construction. This is achieved by taking the isometry group of (A)dS$_2$ spacetime and defining a *BF*-theory.

- JT gravity is based on the Lie algebra $\mathfrak{sl}(2,\mathbb{R})$:

$$[P_a, P_b] = 2\lambda J_{ab} \equiv -\lambda\epsilon_{ab}\widetilde{J}, \qquad [P_a, \widetilde{J}] = -\epsilon_a{}^b P_b. \tag{13}$$

We further define a gauge connection and its associated curvature two-form:

$$\mathbf{A} = e^a P_a + \widetilde{\omega}\widetilde{J} \qquad \text{and} \qquad \mathbf{F} = T^a P_a + \left(\widetilde{R} - \lambda\widetilde{\Sigma}\right)\widetilde{J}. \tag{14}$$

- Dynamics is obtained by considering a BF-like action with Lagrange multipliers $\mathbf{B} = (B^a, \phi)$:

$$S = \int_{\mathcal{M}_2} tr(\mathbf{BF}) = \int_{\mathcal{M}_2} B_a T^a + \phi\big(\widetilde{R} - \lambda\widetilde{\Sigma}\big) \quad \xRightarrow{T^a=0} \quad S_{\rm JT} = \int_{\mathcal{M}_2} \phi\big(\widetilde{R} - \lambda\widetilde{\Sigma}\big).$$

We have on-shell

$$\widetilde{R} = \lambda\widetilde{\Sigma} \qquad \text{and} \qquad D\mathbf{B} \equiv d\mathbf{B} + \left[\mathbf{A}, \mathbf{B}\right] = 0. \tag{15}$$

Extended de Sitter algebra and HT$_2$

We consider a central extension of the de Sitter group by $U(1)$. The corresponding algebra $\widehat{\mathfrak{sl}}(2,\mathbb{R}) \cong \mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{u}(1)$ is:

$$\left[P_a, P_b\right] = -\lambda \epsilon_{ab} \widetilde{J} + \epsilon_{ab} I, \qquad \left[P_a, \widetilde{J}\right] = -\epsilon_a{}^b P_b, \qquad \left[\widetilde{J}, \widetilde{J}\right] = 0, \tag{16}$$

- We can then consider a connection and curvature valued in the extended de Sitter algebra:

  $$\mathbf{A} = e^a P_a + \widetilde{\omega} \widetilde{J} + A I \quad \text{and} \quad \mathbf{F} = T^a P_a + \left(\widetilde{R} - \lambda \widetilde{\Sigma}\right) \widetilde{J} + \left(F + \widetilde{\Sigma}\right) I. \tag{17}$$

  Here, we used the abelian connection A with associated curvature $F = dA$.

- Dynamics is obtained via Lagrange multiplier fields (B^a, ϕ, Λ) to give a BF-like action:

  $$S = \int_{\mathcal{M}_2} B_a T^a + \phi(\widetilde{R} - \Lambda(F + \widetilde{\Sigma}) \xRightarrow{T^a=0} S_{\mathrm{HT}_2} = \int_{\mathcal{M}_2} \phi(\widetilde{R} - \lambda \widetilde{\Sigma}) - \Lambda(F + \widetilde{\Sigma}).$$

  Using Einstein-Cartan variables and going to 2$^{\text{nd}}$-order formalism gives the HT$_2$ action:

  $$S_{\mathrm{HT}_2} = \frac{1}{2} \int_{\mathcal{M}_2} d^2x \, \sqrt{-g} \Big[\phi(R - 2\lambda) - 2\Lambda\big(1 - \epsilon^{\mu\nu} \partial_\mu A_\nu\big)\Big]. \tag{18}$$

HT_2 GRAVITY AND UNIMODULAR TIME

Is this theory really a two-dimensional HT gravity?

- We re-write the HT_2 action in a suitable form:

$$S_{\mathrm{HT}_2}[g,\phi,\Lambda,\mathcal{A}] = \frac{1}{2}\int_{\mathcal{M}_2} d^2x \left[\sqrt{-g}\,\phi(R-2\lambda) - 2\Lambda\left(\sqrt{-g} - \partial_\mu \mathcal{A}^\mu\right)\right], \tag{19}$$

  where $\mathcal{A}^\mu = \varepsilon^{\mu\nu} A_\nu$ denotes the dual of the gauge field A_μ.

  $\Rightarrow$ Identify $\mathcal{T}^\mu \equiv \mathcal{A}^\mu = \varepsilon^{\mu\nu} A_\nu$.

- On-shell dynamics are identical to JT plus Λ, supplemented with the HT gravity equations:

$$\partial_\mu \Lambda = 0 \qquad \text{and} \qquad \sqrt{-g} = \partial_\mu \mathcal{T}^\mu. \tag{20}$$

- We find that the unimodular time is related to the holonomy of the gauge field:

$$T_\Lambda(\Sigma) := \int_\Sigma d\Sigma\; \mathcal{A}^0 = -\int_\Sigma d\Sigma\; A_1. \tag{21}$$

  On-shell we get the difference of unimodular time as the 2-volume:

$$\Delta T_\Lambda = \mathrm{Vol}_2(\Sigma_t \to \Sigma_{t_0}). \tag{22}$$

HT_2 GRAVITY

The HT_2 gravity action is re-written as:

$$S_{\mathrm{HT}_2}[g,\phi,\Lambda,\mathcal{A}] = \frac{1}{2}\int_{\mathcal{M}_2} d^2x \left[\sqrt{-g}\phi(R-2\lambda) - 2\Lambda\big(\sqrt{-g} - \partial_\mu \mathcal{T}^\mu\big)\right]. \tag{23}$$

- $\mathcal{T}^\mu = \varepsilon^{\mu\nu} A_\nu$.

  The action is diffeomorphism invariant and gauge invariant. The HT term is also topological.

- On-shell dynamics are identical to JT plus Λ, supplemented with the HT gravity equations:

$$\partial_\mu \Lambda = 0 \qquad \text{and} \qquad \sqrt{-g} = \partial_\mu \mathcal{T}^\mu. \tag{24}$$

Unimodular time in HT_2

- Unimodular time is the integral over Σ of the gauge field A (the holonomy of the gauge field):

$$T_\Lambda(\Sigma) := \int_\Sigma d\Sigma\, \mathcal{T}^0 = -\int_\Sigma d\Sigma\, A_1. \tag{25}$$

$\Rightarrow$ Time once again cares about topology!

On-shell we get the difference of unimodular time as the 2-volume:

$$\Delta T_\Lambda = \mathrm{Vol}_2(\Sigma_t \to \Sigma_{t_0}). \tag{26}$$

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Closed universes in de Sitter HT_2

We consider the mini-superspace approximation (MSS), where all the fields depend on global time coordinate t. The metric reads:

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\theta^2.$$

The HT_2 action in mini-superspace becomes

$$S_{HT_2} = -\int dt \left(\frac{\dot{\phi}\dot{a}}{N} + Na(\phi + \Lambda) - \Lambda \dot{T}_\Lambda \right) \equiv S_{JT} + \int dt\, \Lambda \dot{T}_\Lambda \tag{27}$$

$$T_\Lambda = \int_{S^1} A\Big|_{S^1} = -A_\theta \tag{28}$$

For global de Sitter we have this metric:

$$ds^2 = -dt^2 + \cosh^2(t)d\theta^2.$$

Geometrically, this describes a contracting closed universe at early times and an expanding closed universe at later times, with a minimal length $a(0) = 1$ reached at $t = 0$.

Unimodular time in HT$_2$ MSS

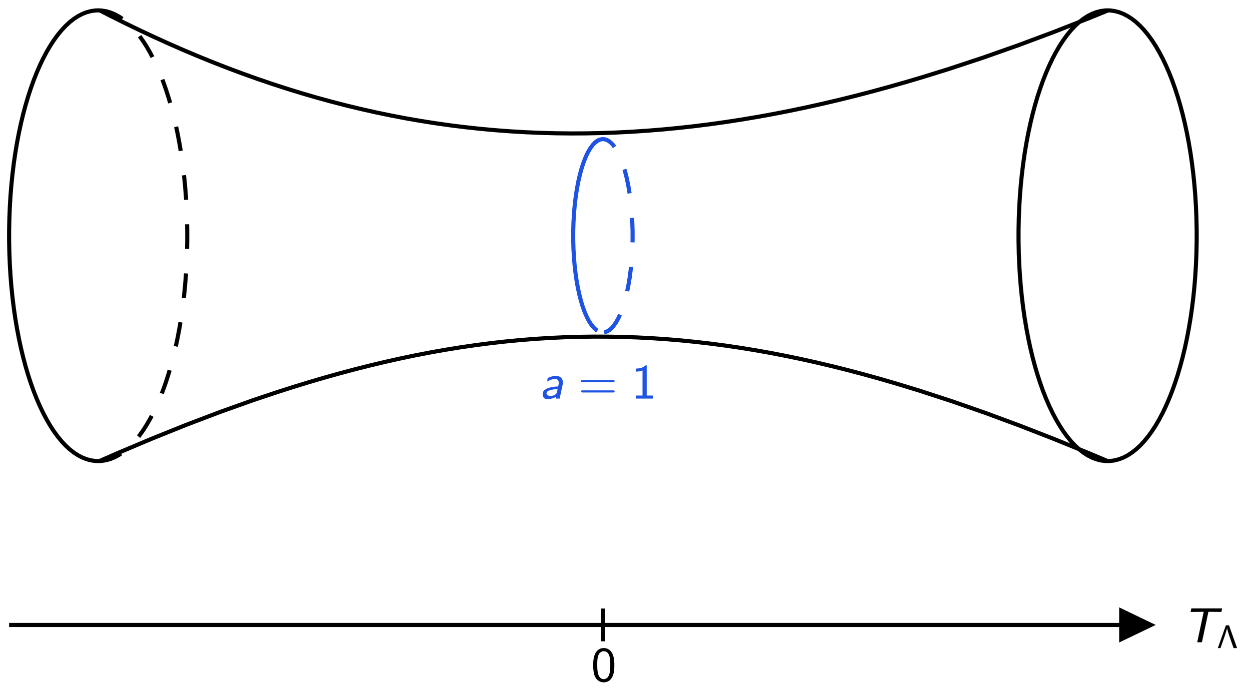
We can see that on-shell, we have:

$$\sqrt{-g} = \dot{\mathcal{T}}^0 \implies a(t) = \dot{T}_\Lambda, \tag{29}$$

That is unimodular time simply is

$$T_\Lambda(t) = \int_{\Sigma_t} dt \ a(t) = \sinh(t), \qquad \forall t \in \mathbb{R}. \tag{30}$$

Here a schematic drawing for the on-shell unitary time-evolution for global de Sitter geometry in unimodular time:



Hamiltonian for canonical quantisation

The mini-superspace variables being a, ϕ and T_Λ have associated conjugate momenta:

$$p_\phi = -\frac{\dot{a}}{N}, \qquad p_a = -\frac{\dot{\phi}}{N} \qquad p_{T_\Lambda} = \Lambda.$$

We can find the Hamiltonian for mini-superspace HT_2:

$$\begin{aligned} H &= \sum_i p_i \dot{q}_i - L && (31)\\ &= Na(-p_a(a^{-1})p_\phi + \lambda\phi + p_{T_\Lambda}) && (32)\\ &= Na(C_{\mathrm{JT}} + p_{T_\Lambda}) \equiv Na\mathcal{C}_{\mathrm{HT}_2} && (33)\end{aligned}$$

the Wheeler-DeWitt equation emerges as following:

$$\frac{\delta S_{\mathrm{HT}_2}}{\delta N} \Rightarrow \ \mathcal{C}_{\mathrm{HT}_2} \approx 0 \ \xrightarrow{\text{quantisation}} \ \widehat{\mathcal{C}}_{\mathrm{HT}_2}\Psi = 0 \qquad (34)$$

We promote the phase space variables to quantum operators

$$p_\phi \to \widehat{p}_\phi = -i\frac{\partial}{\partial\phi}, \qquad p_a \to \widehat{p}_a = -i\frac{\partial}{\partial a} \qquad p_{T_\Lambda} \to \widehat{p}_{T_\Lambda} = -i\frac{\partial}{\partial T_\Lambda},$$

The **Wheeler-DeWitt equation transforms into a Schrödinger-like equation**, that is

$$\left[\partial_a\left(\frac{1}{a}\partial_\phi\right) + \phi - i\frac{\partial}{\partial T_\Lambda}\right]\Psi(a,\phi;T_\Lambda) = 0 \tag{35}$$

$$\left[\widehat{\mathcal{C}}_{\mathrm{JT}} - i\frac{\partial}{\partial T_\Lambda}\right]\Psi(a,\phi;T_\Lambda) = 0 \tag{36}$$

$$i\frac{\partial}{\partial T_\Lambda}\Psi(a,\phi;T_\Lambda) = \widehat{\mathcal{C}}_{\mathrm{JT}}\Psi(a,\phi;T_\Lambda). \tag{37}$$

Next we want to find the wavefunction that governs this model universe.

We can base our general wavefunction Ψ solution on the timeless standard JT solution for ψ

$$\widehat{\mathcal{C}}_{\mathsf{JT}}\psi = 0 \tag{38}$$

We can write more explicit this Hamiltonian to find a WKB type of solution:

$$\left[\partial_a\left(\frac{1}{a}\partial_\phi\right) + \frac{1}{2}U'(\phi)\right]\psi\big(a, \phi, \Lambda\big) = 0, \qquad U(\phi) = \phi^2 + 2\phi\Lambda. \tag{39}$$

The resulting Hartle-Hawking wavefunction solution is time-independent:

$$\psi_\pm\big(a, \phi, \Lambda\big) = e^{\pm ia\sqrt{U(\phi)}}, \tag{40}$$

where $\pm$ stands for contracting and expanding universes, hence the general solution takes this form:

$$\psi\big(a, \phi, \Lambda\big) = \alpha\psi_-\big(a, \phi, \Lambda\big) + \beta\psi_+\big(a, \phi, \Lambda\big). \tag{41}$$

Schrödinger equation and time-evolution in HT_2

- The time-dependent equation is

$$i\frac{\partial}{\partial T_\Lambda}\Psi(a,\phi;T_\Lambda)=\widehat{\mathcal{C}}_{\mathrm{JT}}\Psi(a,\phi;T_\Lambda). \tag{42}$$

  This unfreezing of quantum states allows time-evolution with unimodular time:

$$\Psi(T_\Lambda)=\mathrm{e}^{-iT_\Lambda\mathcal{C}_{\mathrm{JT}}}\Psi(0). \tag{43}$$

- The time-independent wavefunction $\Psi(0)$ is obtained from the time-independent Schrödinger equation: this is obtained from the Wheeler-DeWitt equation by taking $\widehat{p}_{T_\Lambda}=\Lambda$:

$$\widehat{\mathcal{C}}_{\mathrm{JT}}\Psi(a,\phi,\Lambda)=\left[\partial_a\left(\frac{1}{a}\partial_\phi\right)+\phi\right]\Psi(a,\phi,\Lambda)=-\Lambda\Psi(a,\phi,\Lambda). \tag{44}$$

The time-dependent solution therefore is a wave-packet:

$$\Psi(a,\phi;T_\Lambda)=\int\frac{d\Lambda}{\sqrt{4\pi}}\,\mathrm{e}^{i\Lambda T_\Lambda}\mathcal{A}(\Lambda)\Big(\psi_-(a,\phi,\Lambda)+i\psi_+(a,\phi,\Lambda)\Big), \tag{45}$$

with Gaussian amplitudes

$$\mathcal{A}(\Lambda)=\frac{1}{(2\pi\sigma^2_{T_\Lambda})^{\frac{1}{4}}}\,\exp\left(-\frac{(\Lambda-\Lambda_0)^2}{4\sigma^2_{T_\Lambda}}\right). \tag{46}$$

We define the physical inner-product to be

$$\langle \Psi_1 \,|\, \Psi_2 \rangle := \int d\Lambda \; \mathcal{A}_1^*(\Lambda)\mathcal{A}_2(\Lambda). \tag{47}$$

We can define an inversion formula for $\mathcal{A}(\Lambda)$ by

$$\mathcal{A}(\Lambda) = \int \frac{da}{\sqrt{4\pi}} \; e^{-i\Lambda T_\Lambda}\Psi(a,\phi;T_\Lambda)\Big(\psi_+ - i\psi_-\Big), \tag{48}$$

To define the norm of the wavefunction:

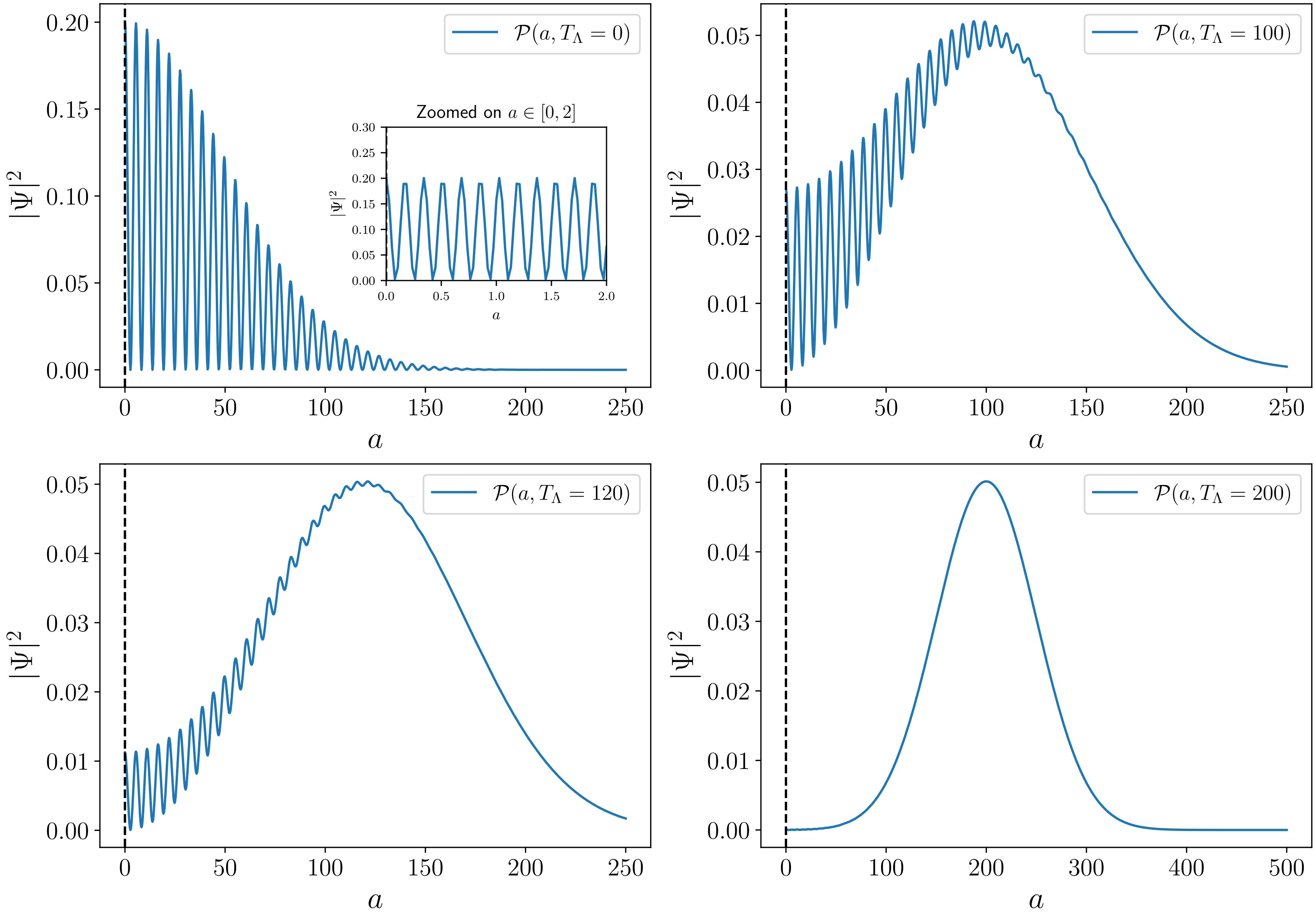
$$\langle \Psi \,|\, \Psi \rangle = \int d\Lambda \; \Big|\mathcal{A}(\Lambda)\Big|^2 \equiv \int da \; \Big|\Psi(a,\phi;T_\Lambda)\Big|^2. \tag{49}$$

- This product is automatically conserved with respect to unimodular time, i.e. unitarity is satisfied, since it is defined in terms of time-independent amplitudes. Given that we have unitarity, it is reasonable to define physical states as any state derived from a normalized Gaussian amplitude, which satisfies

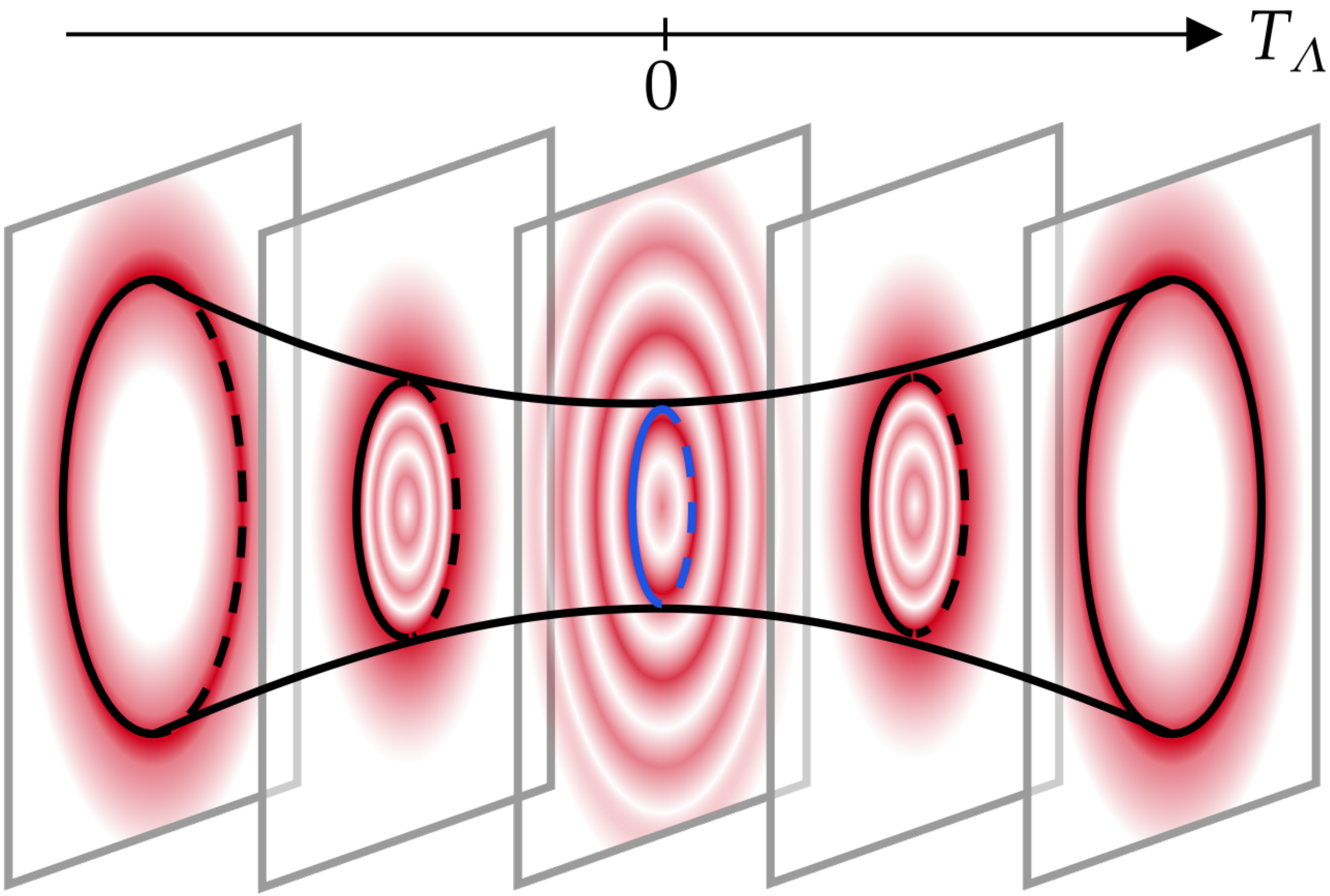
$$\langle \Psi \,|\, \Psi \rangle = 1. \tag{50}$$

Quantum interference

Here we illustrate $\left|\Psi(a, \phi; T_\Lambda)\right|^2$, plotted against a, the size of the universe:



Quantum interference



Towards topology change in HT_2

- The universe may evolve towards the singular point $a = 0$, enabling a transition from a circular spatial topology to a point and back again.
- Interpret this as a crunching universe, followed by the creation of an expanding universe at $a = 0$ with a bang: the universe undergoes a topology change process.
- We extend JT gravity theory by allowing the vacuum cosmological constant Λ to vary off-shell through the introduction of unimodular time.
- This modification allows the wavefunction to be in different energy eigenstates of Λ within the quantum theory.
- Consequently, quantum deformations of the global de Sitter geometry at early unimodular times can be viewed as the universe occupying various Λ-dependent energy levels.

Summary and Outlook

- We have shown that a Henneaux-Teitelboim gravity formulation of JT gravity is possible, and appears from a gauge-theoretic formulation.
- We have also seen that it can describe a unitary quantum cosmology, where the wavefunction of the universe is now unimodular time-dependent.
- In this perspective, the possibility of reaching the singular point $a = 0$ signals that topology change could occur quantum mechanically.

Whats next?:
What is the holographic perspective of Unimodular Time?
Can we derive HT gravity in the same manner for higher dimensions?
Does this formalism plays a role in extremal near horizon black holes?
...

Thank you!