

Probing thermal fluctuations through scalar test particles

Alexsandre L. Ferreira Junior

PPGCosmo - UFES

PPGF - UNIFEI

alexsandrej@gmail.com

October 5, 2021

G. Camargo, V. A. De Lorenci, A.L. Ferreira Junior, C. H. Ribeiro
Eur. Phys. J. C 81, 424 (2021) [arXiv:2010.07146]

Stochastic motion on a charged particle due to the change in the background field

- Stochastic motion induced by a change in the background field, originally proposed by Ford & Yu (2004) [arXiv:0406122].
- Negative dispersion in the velocity $\rightarrow$ Subvacuum effects.
- General case of a massive scalar field in D spatial dimensions [Camargo et al. (2019) arXiv:1906.08322].
- In this paper we investigated temperature effects, and showed that increasing temperature facilitates the detection of subvacuum phenomena.

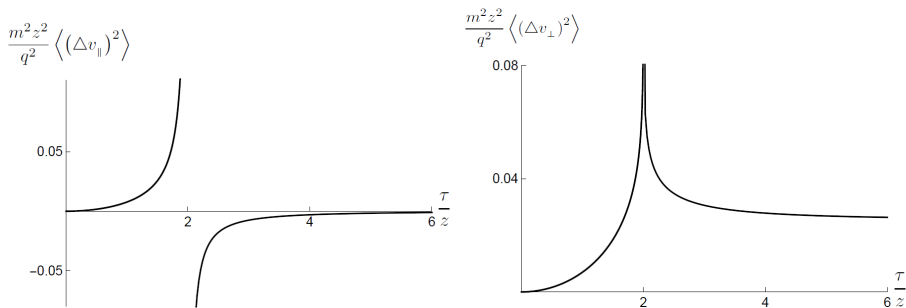


Figure: Figures taken from De Lorenci et al. (2014) arXiv:1404.3115

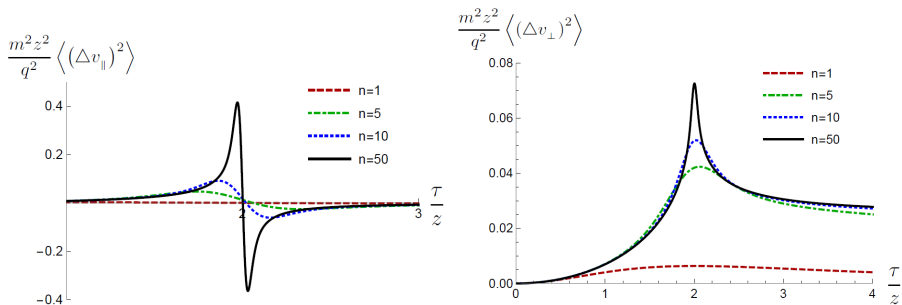


Figure: From De Lorenci et al. (2014) arXiv:1404.3115

Some properties of the Green functions

- Vacuum divergence due to high energy modes.
- Non-Huygesian character of the massive fields, or when D is even: Signals propagate with an arbitrary low velocity.

$$v_g = k / \sqrt{k^2 + m^2}$$

- Infrared divergence for the massless field when $D = 1$, and, when temperature is present, also for $D = 2$.

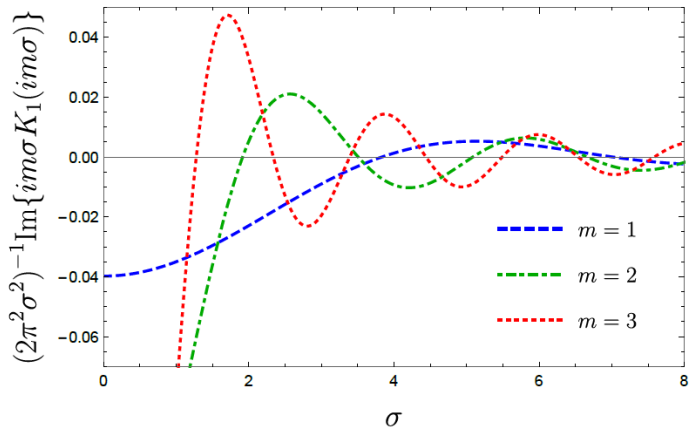


Figure: From Camargo et al. (2019) arXiv:1906.08322

Fluctuations in the presence of a perfectly reflective boundary

In the presence of an infinite Dirichlet's wall at $x_1 = 0$, where $\phi(t, x_1 = 0, x_2, \dots, x_D) = 0$

$$G_{\beta, \text{Ren}}^{(1)}(t, \vec{x}; t', \vec{x}') = -\frac{1}{\pi} \text{Re} \left[\left(\frac{m}{2\pi i \sigma_0^+} \right)^{\frac{D-1}{2}} K_{\frac{D-1}{2}}(im\sigma_0^+) \right] \\ + \frac{2}{\pi} \text{Re} \sum_{l=1}^{\infty} \left[\left(\frac{m}{2\pi i \sigma_l} \right)^{\frac{D-1}{2}} K_{\frac{D-1}{2}}(im\sigma_l) - (\sigma_l \leftrightarrow \sigma_l^+) \right].$$

Here $\sigma_l^+ = [(\Delta t - i\beta l)^2 - (\hat{\Delta}\vec{x})^2]^{1/2}$, $\hat{\Delta}\vec{x}$ being $\Delta\vec{x}$ when $x'_1 \rightarrow -x'_1$.

The interacting model

The interaction of a non-relativistic scalar charged test particle with the background field is given through

$$S[\phi, \partial_\mu \phi; \tau, \vec{x}] \simeq S_F + \int d\tau \left[\frac{M}{2} v^2 - e \phi(\tau, \vec{x}) \right] + \mathcal{O}(v^2/c^2). \quad (1)$$

Which gives

$$\frac{dv_i}{d\tau} = -g \frac{\partial \phi(\tau, \vec{x})}{\partial x_i}. \quad (2)$$

For the vacuum and thermal equilibrium $\langle v_i \rangle = 0$. So:

$$\langle (\Delta v_i)^2 \rangle_D = \frac{g^2}{2} \lim_{x \rightarrow x'} \left[\frac{\partial}{\partial x_i} \frac{\partial}{\partial x'_i} \int_0^\tau dt \int_0^\tau dt' G_{\beta, Ren}^{(1)}(t, \vec{x}; t', \vec{x}') \right]. \quad (3)$$

Henceforth, in order to describe a more realistic system, we introduce a switching function $F(t)$, such that

$$\int_{-\infty}^{\infty} dt F(t) = \tau. \quad (4)$$

The fluctuations are now given by

$$\langle (\Delta v_i)^2 \rangle_D = \frac{g^2}{2} \lim_{x \rightarrow x'} \left[\frac{\partial}{\partial x_i} \frac{\partial}{\partial x'_i} \int_{-\infty}^{\infty} dt F(t) \int_{-\infty}^{\infty} dt' F(t') G_{\beta, \text{Ren}}^{(1)}(t, \vec{x}; t', \vec{x}') \right]. \quad (5)$$

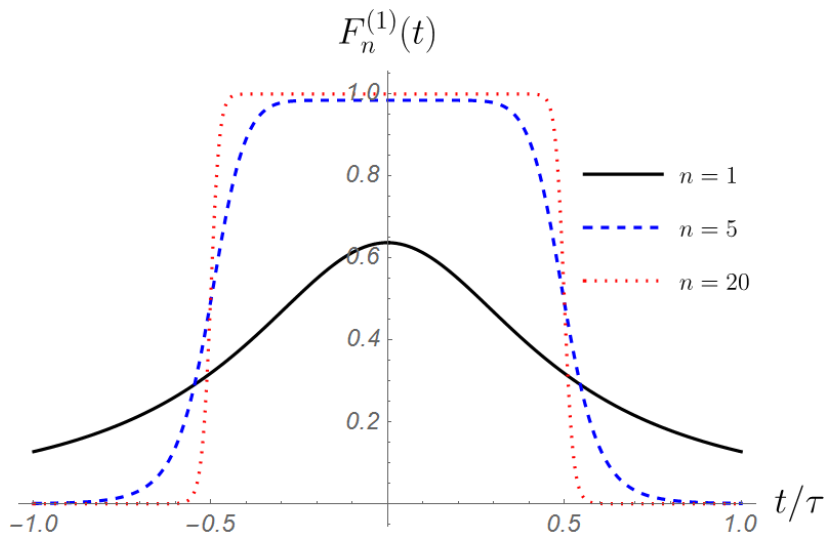
Switching Function

A suitable choice is the generalized Lorentzian

$$F_n^{(1)}(t) = \frac{c_n}{\left[1 + \left(\frac{2t}{\tau}\right)^{2n}\right]}, \quad (6)$$

with $c_n = (2n/\pi)\sin(\pi/2n)$. Which defines the transition time

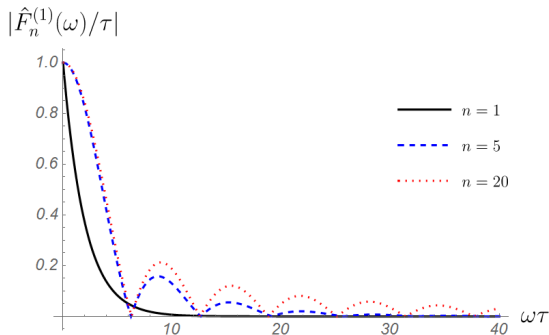
$$\tau_s = \frac{\tau}{2} \left(\frac{2n-1}{n+1}\right)^{\frac{1}{2n}} \left[\left(1 + \sqrt{1 - \frac{(n+1)(n-1)}{(2n+1)(2n-1)}}\right)^{\frac{1}{2n}} - \left(1 - \sqrt{1 - \frac{(n+1)(n-1)}{(2n+1)(2n-1)}}\right)^{\frac{1}{2n}} \right], \quad (7)$$



The Fourier transform of $F_n^{(1)}(t)$ is given by

$$\hat{F}_n^{(1)}(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} F_n^{(1)}(t) = \frac{i\tau\pi c_n}{2n} \sum_{q=n}^{2n-1} \psi_{n,q} e^{-i\omega\tau\psi_{n,q}/2}, \quad (8)$$

with $\psi_{n,p} = \exp[i(\pi/2n)(1 + 2p)]$.



Another choice of switching function is

$$F_{\tau_s}^{(2)}(t) = \frac{1}{\pi} \left[\arctan \left(\frac{t}{\tau_s} \right) + \arctan \left(\frac{\tau - t}{\tau_s} \right) \right]. \quad (9)$$

with fourier transform given

$$\hat{F}^{(2)}(\omega) = \frac{1}{i\omega} (1 - e^{-i\omega\tau}) e^{-\tau_s|\omega|}. \quad (10)$$

This switching function is useful because the transition time is a parameter and does not change with the interaction time.

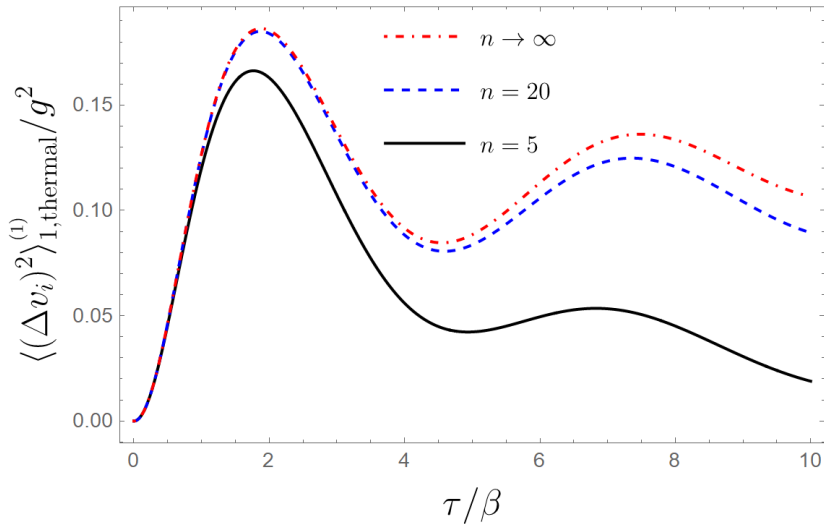
Dispersions due to a pure thermal bath

In order to obtain a closed expression we first choose the switching function $F_n^{(1)}(t)$. Which gives:

$$\begin{aligned} \langle (\Delta v_i)^2 \rangle_{D,\text{thermal}}^{(1)} &= \frac{2g^2}{\beta^{D-1}} \left[\frac{(\tau/\beta)\pi c_n}{2n} \right]^2 \sum_{p,q=n}^{2n-1} \sum_{l=1}^{\infty} \psi_{n,p} \psi_{n,q}^* \\ &\times \left(\frac{m\beta}{2\pi\sqrt{-a_l^2}} \right)^{\frac{D+1}{2}} K_{\frac{D+1}{2}} \left(m\beta\sqrt{-a_l^2} \right). \quad (11) \end{aligned}$$

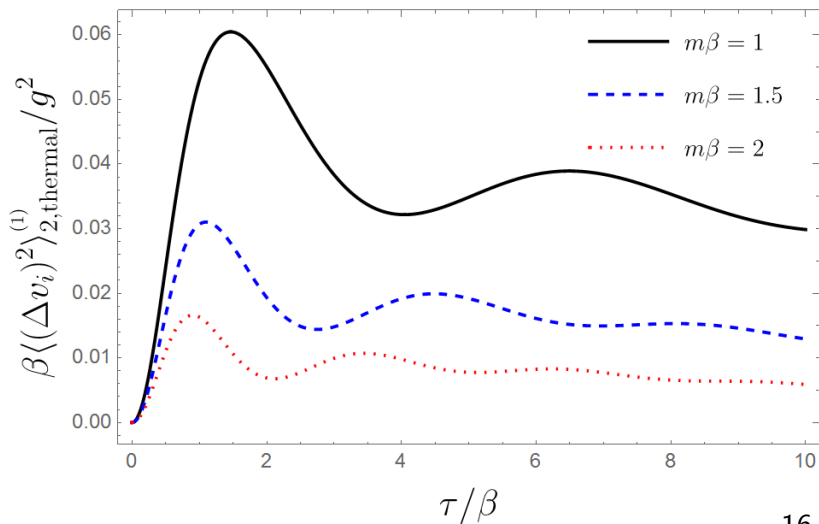
where $a_l = (\tau/2\beta)(\psi_{n,p} - \psi_{n,q}^*) - il$. Note that, when $\beta \rightarrow \infty$ or $m \rightarrow \infty$ the dispersions are exponentially suppressed.

For $D = 1$, and $m\beta = 1$

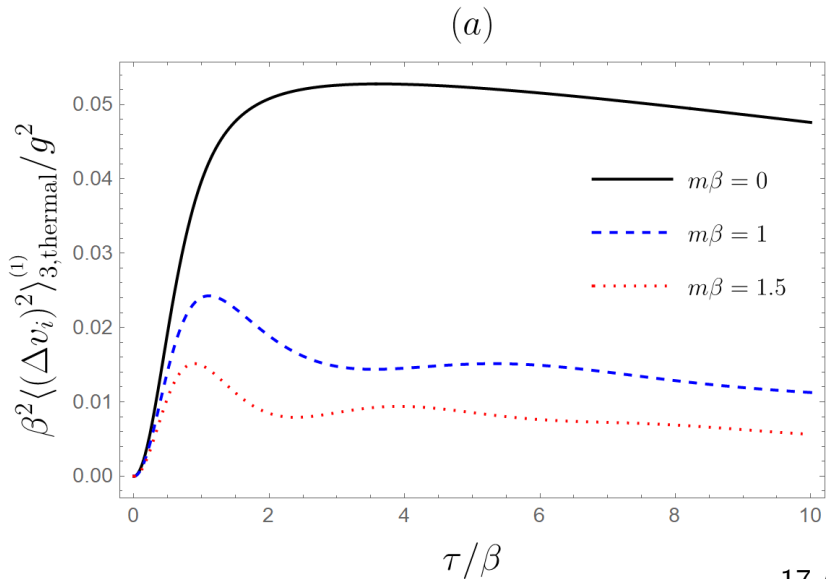


Here $D = 2$, and $n = 20$.

(b)



Here $D = 3$, and $n = 20$.



Dispersions due to a boundary at finite temperature

In a physical setup, the thermal bath can be detached from the insertion of the boundary. Thence, we investigate the boundary contributions to the fluctuations, given by

$$\langle (\Delta v_i)^2 \rangle_{D,\text{boundary}} = \langle (\Delta v_i)^2 \rangle_{D,\text{vacuum}} + \langle (\Delta v_i)^2 \rangle_{D,\text{mixed}}. \quad (12)$$

The pure thermal contribution will be only the residual dispersions, which can be zero for τ_s long enough.

First, using the switching function $F_n^{(1)}(t)$, the mixed contribution to the fluctuations is

$$\begin{aligned} \langle (\Delta v_{\parallel})^2 \rangle_{D,\text{mixed}}^{(1)} &= -\frac{2g^2}{x^{D-1}} \left(\frac{(\tau/x)\pi c_n}{2n} \right)^2 \\ &\times \sum_{p,q=n}^{2n-1} \sum_{l=1}^{\infty} \psi_{n,p} \psi_{n,q}^* \left(\frac{mx}{4\pi\sqrt{1-\gamma_l^2}} \right)^{\frac{D+1}{2}} K_{\frac{D+1}{2}} \left(2mx\sqrt{1-\gamma_l^2} \right), \quad (13) \end{aligned}$$

$$\langle (\Delta v_{\perp})^2 \rangle_{D,\text{mixed}}^{(1)} = 8\pi x^2 \langle (\Delta v_{\parallel})^2 \rangle_{D+2,\text{mixed}}^{(1)} - \langle (\Delta v_{\parallel})^2 \rangle_{D,\text{mixed}}^{(1)} \quad (14)$$

where we have defined $\gamma_l = (\tau/4x)(\psi_{n,p} - \psi_{n,q}^*) - il(\beta/2x)$.

This contributions vanishes as $\beta \rightarrow \infty$ or $m \rightarrow \infty$.

The vacuum contribution is just half the mixed term with $l = 0$. Thence:

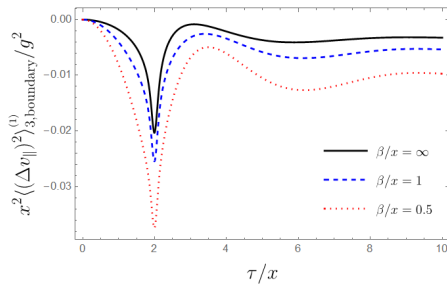
$$\begin{aligned} \langle (\Delta v_{\parallel})^2 \rangle_{D,\text{vacuum}}^{(1)} &= -\frac{g^2}{x^{D-1}} \left(\frac{(\tau/x)\pi c_n}{2n} \right)^2 \\ &\times \sum_{p,q=n}^{2n-1} \psi_{n,p} \psi_{n,q}^* \left(\frac{mx}{4\pi\sqrt{1-\gamma_0^2}} \right)^{\frac{D+1}{2}} K_{\frac{D+1}{2}} \left(2mx\sqrt{1-\gamma_0^2} \right) \quad (15) \end{aligned}$$

$$\langle (\Delta v_{\perp})^2 \rangle_{D,\text{vacuum}}^{(1)} = 8\pi x^2 \langle (\Delta v_{\parallel})^2 \rangle_{D+2,\text{vacuum}}^{(1)} - \langle (\Delta v_{\parallel})^2 \rangle_{D,\text{vacuum}}^{(1)}. \quad (16)$$

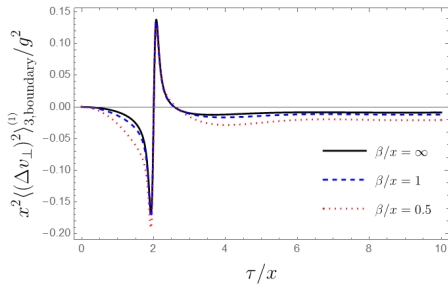
As $m \rightarrow \infty$ the vacuum term is also suppressed. However, as it is the zeroth order term of the summation, the temperature contributions disappear earlier.

Here $D = 3$, $m\beta = 1$, and $n = 20$

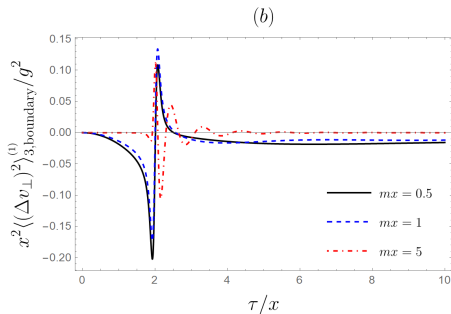
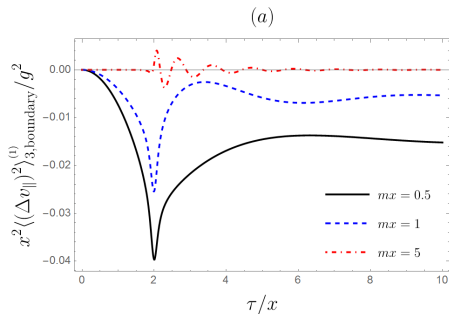
(a)



(b)



For $D = 3$, $\beta/x = 1$, and $n = 20$



Late-time dispersions due to a pure thermal bath

We find for the pure thermal bath at the late-time regime:

$$\lim_{\tau \rightarrow \infty} \langle (\Delta v_i)^2 \rangle_{\text{thermal}}^{(2)} = \frac{2g^2}{\pi \beta^{D-1}} \sum_{l=1}^{\infty} \left\{ \left[\frac{m\beta}{2\pi(2\tau_s/\beta + l)} \right]^{\frac{D-1}{2}} \right. \\ \left. \times K_{\frac{D-1}{2}}(m\beta(2\tau_s/\beta + l)) - \frac{(2\tau_s/\beta + l)(m\beta)^D}{2^D \pi^{\frac{D}{2}-1} \Gamma(\frac{D}{2} + 1)} I(D, m\beta(2\tau_s/\beta + l)) \right\} \quad (17)$$

with

$$I(D, \alpha) = \int_1^{\infty} du \frac{(u^2 - 1)^{\frac{D}{2}}}{u} e^{-\alpha u} = -\frac{\pi}{2} \operatorname{cosec} \left(\frac{\pi}{2} D \right) \\ - \frac{\alpha}{2\sqrt{\pi}} \Gamma \left(-\frac{D}{2} - 1 \right) \Gamma \left(\frac{D}{2} + 1 \right) {}_1F_2 \left[1/2; 3/2, (D+3)/2; \alpha^2/4 \right] \\ + \frac{1}{\alpha^D} \Gamma(D) {}_1F_2 \left[-D/2; (1-D)/2, 1-D/2; \alpha^2/4 \right] \quad (18)$$

Late-time dispersions due to a boundary at finite temperature

For the mixed contribution we find:

$$\lim_{\tau \rightarrow \infty} \langle (\Delta v_{\parallel})^2 \rangle_{D,\text{mixed}}^{(2)} = -\frac{2g^2}{\pi x^{D-1}} \times \sum_{l=1}^{\infty} \left\{ \left[\frac{mx}{4\pi \sqrt{1+\alpha_l^2}} \right]^{\frac{D-1}{2}} K_{\frac{D-1}{2}} \left(2mx \sqrt{1+\alpha_l^2} \right) - \frac{\alpha_l (mx)^{\frac{D}{2}}}{2^{D-1} \pi^{\frac{D}{2}-1}} \int_1^{\infty} du \frac{(\sqrt{u^2-1})^{\frac{D}{2}}}{u} e^{-2mx \alpha_l} J_{\frac{D}{2}}(2mx \sqrt{u^2-1}) \right\}, \quad (19)$$

$$\lim_{\tau \rightarrow \infty} \langle (\Delta v_{\perp})^2 \rangle_{D,\text{mixed}}^{(2)} = 8\pi x^2 \lim_{\tau \rightarrow \infty} \langle (\Delta v_{\parallel})^2 \rangle_{D+2,\text{mixed}}^{(2)} - \lim_{\tau \rightarrow \infty} \langle (\Delta v_{\parallel})^2 \rangle_{D,\text{mixed}}^{(2)}, \quad (20)$$

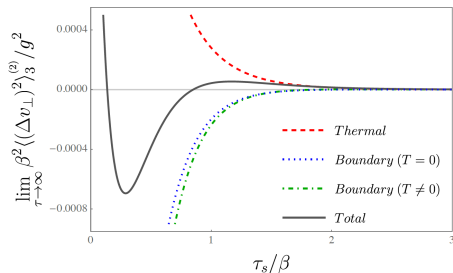
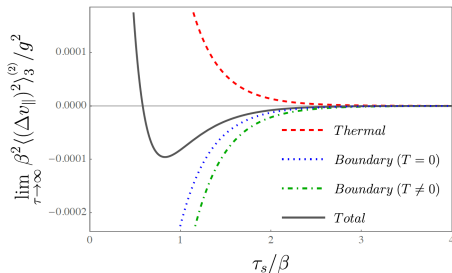
with $\alpha_l = \tau_s/x + l\beta/2x$.

The vacuum contribution is then:

$$\lim_{\tau \rightarrow \infty} \langle (\Delta v_{\parallel})^2 \rangle_{D, \text{vacuum}}^{(2)} = -\frac{g^2}{\pi x^{D-1}} \times \left\{ \left[\frac{mx}{4\pi \sqrt{1 + \alpha_0^2}} \right]^{\frac{D-1}{2}} K_{\frac{D-1}{2}} \left(2mx \sqrt{1 + \alpha_0^2} \right) - \frac{\alpha_0 (mx)^{\frac{D}{2}}}{2^{D-1} \pi^{\frac{D}{2}-1}} \int_1^{\infty} du \frac{(\sqrt{u^2 - 1})^{\frac{D}{2}}}{u} e^{-2mx \alpha_0} J_{\frac{D}{2}}(2mx \sqrt{u^2 - 1}) \right\}, \quad (21)$$

$$\lim_{\tau \rightarrow \infty} \langle (\Delta v_{\perp})^2 \rangle_{D, \text{vacuum}}^{(2)} = 8\pi x^2 \lim_{\tau \rightarrow \infty} \langle (\Delta v_{\parallel})^2 \rangle_{D+2, \text{vacuum}}^{(2)} - \lim_{\tau \rightarrow \infty} \langle (\Delta v_{\parallel})^2 \rangle_{D, \text{vacuum}}^{(2)}, \quad (22)$$

Residual dispersions for $D = 3$, and $m\beta = 1$



Distance behavior of the dispersions

To investigate how the dispersions behaves with the distance to the plate, note that, when $x/\beta \ll 1$, we have

$$\begin{aligned}\sqrt{1 - \gamma_l^2} &= \frac{\beta}{2x} \sqrt{\frac{4x^2}{\beta^2} + \left(l + i \frac{\tau}{2\beta} (\psi_{n,p} - \psi_{n,q}^*) \right)} \\ &\simeq \frac{\beta}{2x} \sqrt{l + i \frac{\tau}{2\beta} (\psi_{n,p} - \psi_{n,q}^*)} = \sqrt{-\gamma_l^2}. \quad (23)\end{aligned}$$

So that

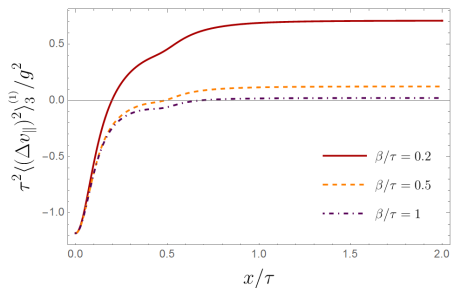
$$\langle (\Delta v_{\parallel})^2 \rangle_{D,\text{mixed}}^{(1)} \simeq -\langle (\Delta v_i)^2 \rangle_{D,\text{thermal}}^{(1)},$$

and

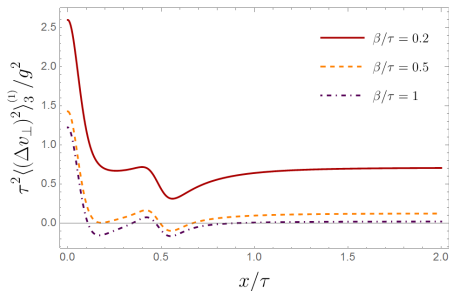
$$\langle (\Delta v_{\perp})^2 \rangle_{D,\text{mixed}}^{(1)} \simeq \langle (\Delta v_i)^2 \rangle_{D,\text{thermal}}^{(1)} - 8\pi x^2 \langle (\Delta v_i)^2 \rangle_{D+2,\text{thermal}}^{(1)}.$$

For $D = 3$, $n = 5$ and $m\beta = 1$

(a)



(b)



Vacuum vs thermal dominance

To investigate the vacuum versus thermal dominance as the boundary is approached, we study the mean squared velocity:

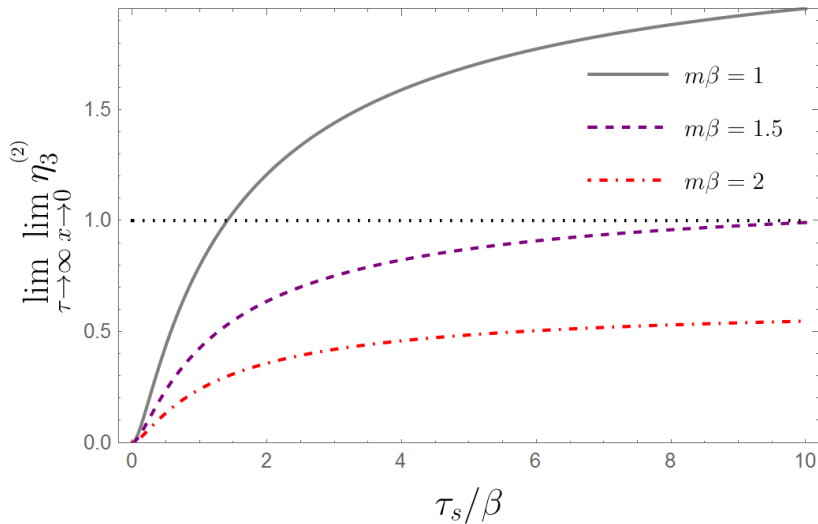
$$\langle v^2 \rangle_{D,\beta} = \sum_i \langle v_i^2 \rangle_{D,\beta} = (D-1) \langle (\Delta v)_{\parallel}^2 \rangle_{D,\beta} + \langle (\Delta v)_{\perp}^2 \rangle_{D,\beta}. \quad (24)$$

Then, defining the quantity

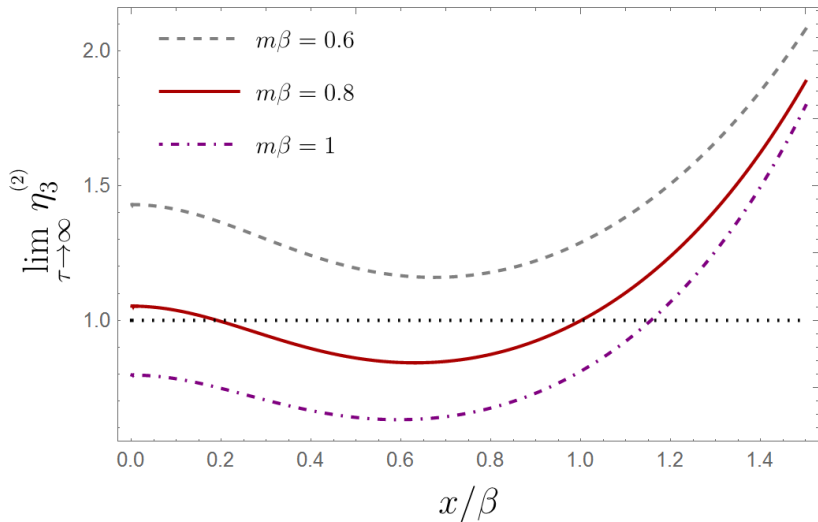
$$\eta_D = \left| \frac{\langle v^2 \rangle_{D,\beta} - \langle v^2 \rangle_D}{\langle v^2 \rangle_D} \right|. \quad (25)$$

If $\eta_D > 1$ thermal effects dominate, if $\eta_D < 1$, the modified vacuum contribution is higher.

For $D = 3$



Here $D = 3$, and $\tau_s/\beta = 1$



Remarks on the assumptions of the model

Regarding the assumptions made in the model:

- Idealized boundary: Infinite potential barrier.
- Dirichlet's over Neumann's boundary conditions.
- $\langle (\Delta x_i)^2 \rangle = \int_0^\tau dt \int_0^\tau dt' \langle v_i(t) v_i(t') \rangle \ll x_i^2$ [Yu & Ford (2004) [arXiv:0406122], De Lorenci et al. (2016) [arXiv:1606.09134]].
- Backreaction due to particle's emitted radiation [Yu & Ford (2004) [arXiv:0406122], De Lorenci et al. (2016) [arXiv:1606.09134]].

In Summary

- Subvacuum effects are present even at finite temperatures and at late times.
- In the absence of the thermal bath, temperature increases subvacuum effects, facilitating its detection.
- Thermal effects can dominated over the vacuum near the boundary, for switching times long enough.
- Mass creates an interesting pattern of domination as the wall is approached.

References



H. Yu and L. H. Ford, Vacuum fluctuations and Brownian motion of a charged test particle near a reflecting boundary, Phys. Rev. **D 70**, 065009 (2004).



M. Seriu and C.H. Wu, Switching effect on the quantum Brownian motion near a reflecting boundary, Phys. Rev. **A 77**, 022107 (2008).



M. Seriu and C.H. Wu, Smearing effect due to the spread of a probe particle on the Brownian motion near a perfectly reflecting boundary, Phys. Rev. **A 80**, 052101 (2009).



V.A. De Lorenci, E.S. Moreira Jr. and M.M. Silva, Quantum Brownian motion near a point-like reflecting boundary, Phys. Rev. **D 90**, 027702 (2014).



V.A. De Lorenci, C.C.H. Ribeiro and M.M. Silva, Probing quantum vacuum fluctuations over a charged particle near a reflecting wall, Phys. Rev. **D 94**, 105017 (2016).



C.H.G. Bessa, V.B. Bezerra and L.H. Ford, Brownian motion in Robertson-Walker spacetimes from electromagnetic vacuum fluctuations, J. Math. Phys. **50**, 062501 (2009).

References



G.H.S. Camargo, V.A. De Lorenci, C.C.H. Ribeiro, F.F. Rodrigues and M.M. Silva, Vacuum fluctuations of a scalar field near a reflecting boundary and their effects on the motion of a test particle, *JHEP* **07**, 173 (2018).



V.A. De Lorenci and L.H. Ford, Subvacuum effects on light propagation, *Phys. Rev. A* **99**, 023852 (2019).



L. H. Ford, Stochastic spacetime and Brownian motion of test particles, *Int. J. Theor. Phys.* **70**, 1753 (2005).



H. Yu, J. Chen and P. Wu, Brownian motion of a charged test particle near a reflecting boundary at finite temperature, *JHEP* **02**, 058 (2006).



V.A. De Lorenci and C.C.H. Ribeiro, Remarks on the influence of quantum vacuum fluctuations over a charged test particle near a conducting wall, *JHEP* **04**, 072 (2019).



G.H.S. Camargo, V.A. De Lorenci, C.C.H. Ribeiro, and F.F. Rodrigues, Vacuum induced dispersions of the motion of test particles in $D + 1$ dimensions, *Phys. Rev. D* **100**, 065014 (2019).

Thank you!