Fully relativistic predictions in Horndeski gravity from standard Newtonian N-body simulations

Guilherme Brando

Based on 2105.04491

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PPGCosmo

Outline

- Introduction
- N-Body gauge
- Modified Gravity
- Results
- Conclusions

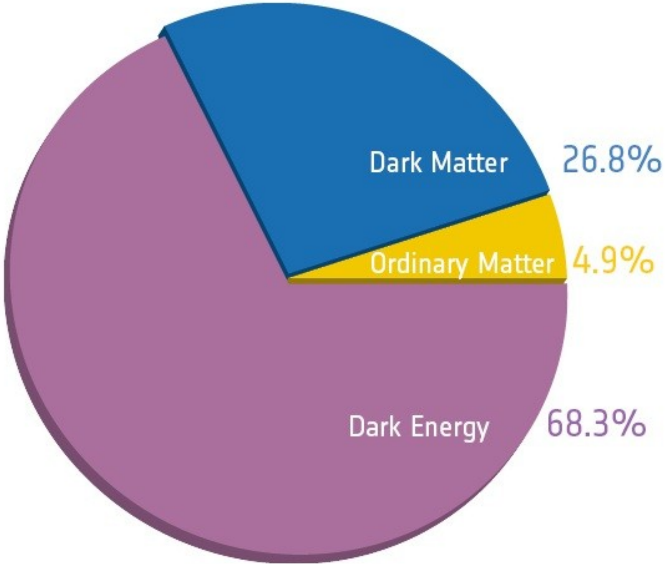
Introduction

- Standard Model of Cosmology GR is assumed in all scales

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu},$$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) u_\mu u_\nu + p g_{\mu\nu}$$



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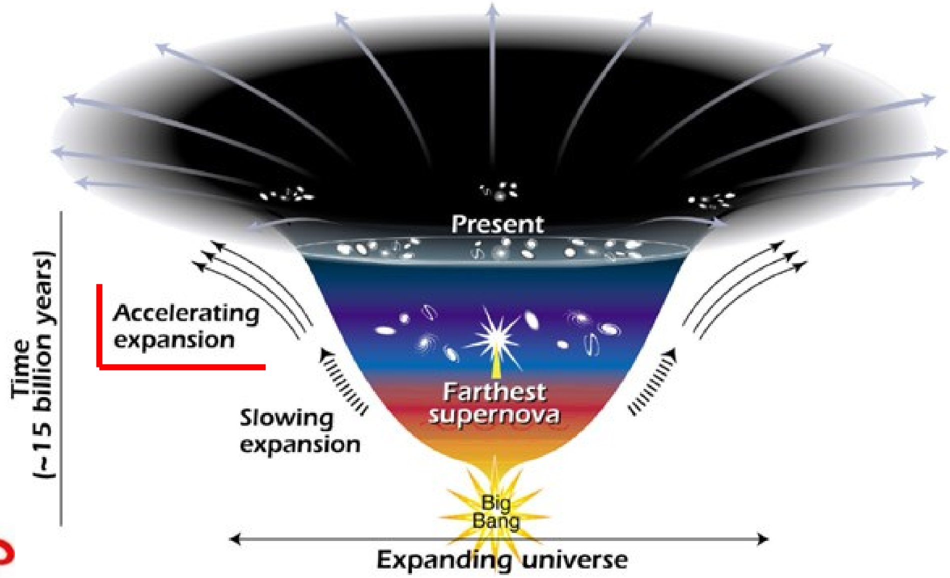
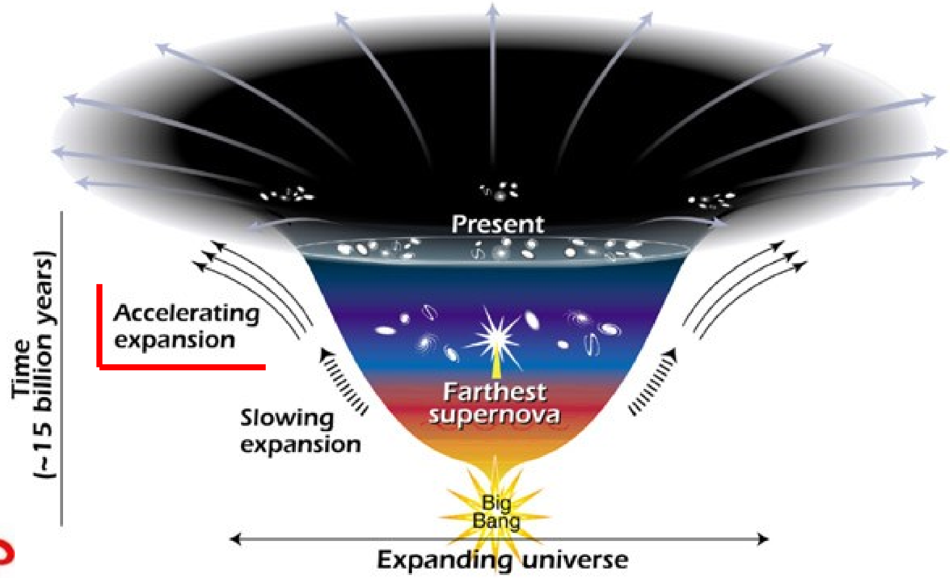


Introduction

Cosmological Constant?

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



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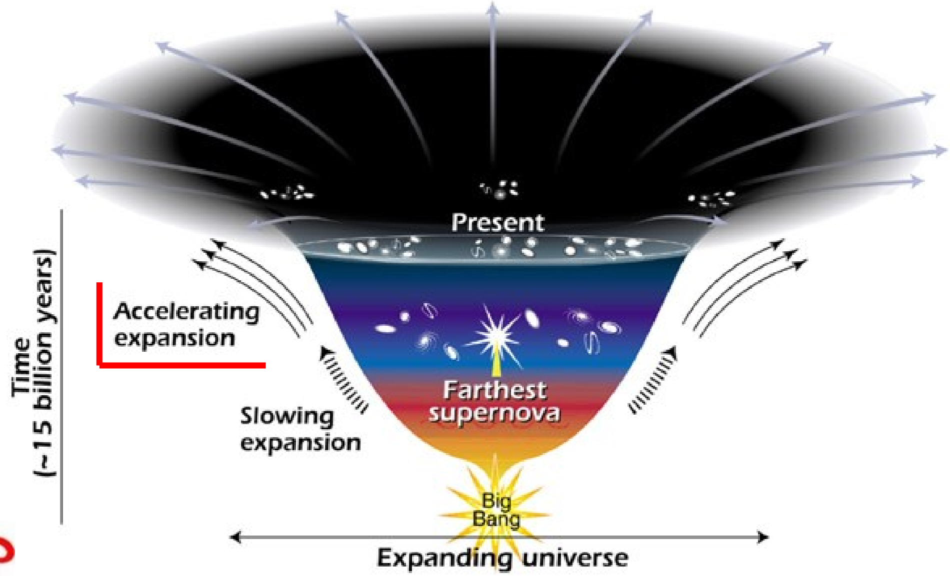
Exotic Fluid?

Modified gravity?



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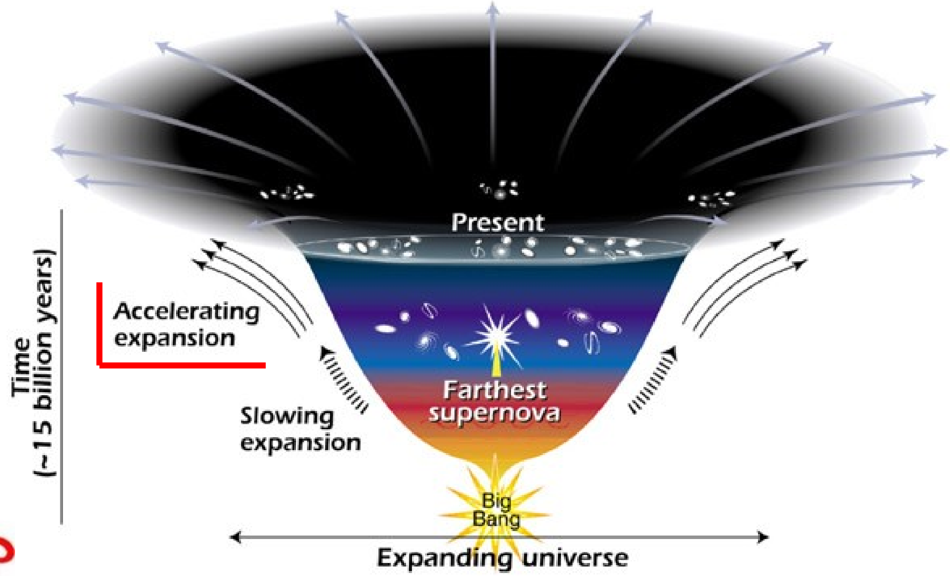

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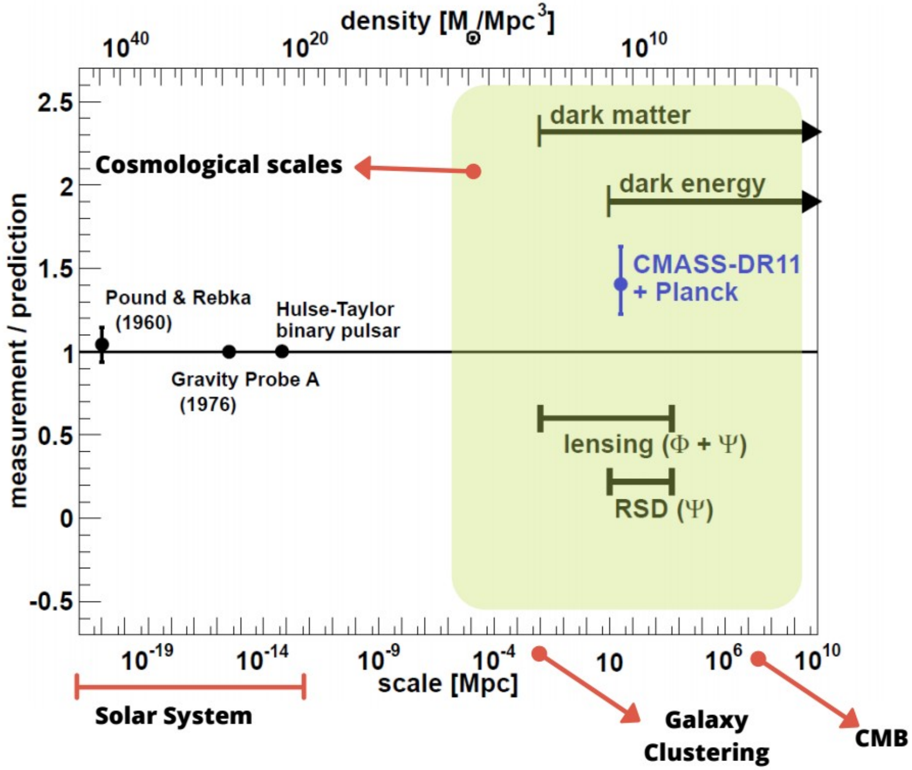
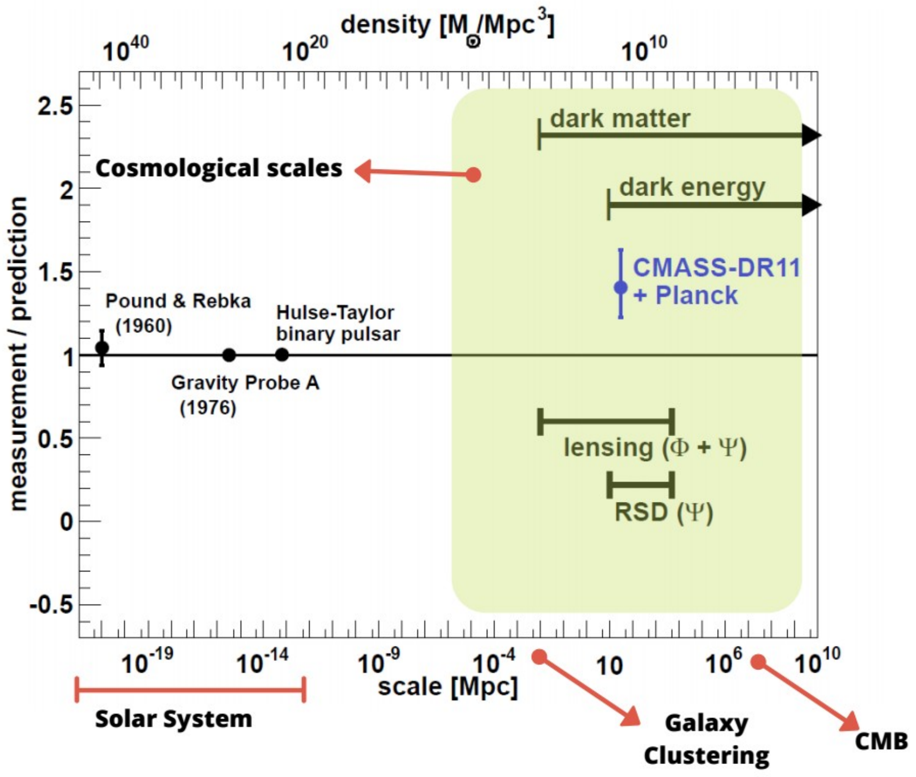


Adapted from 1312.4611

Introduction

- We already have multiple constraints on gravity from different probes

- Astrophysical and solar system are the tighest



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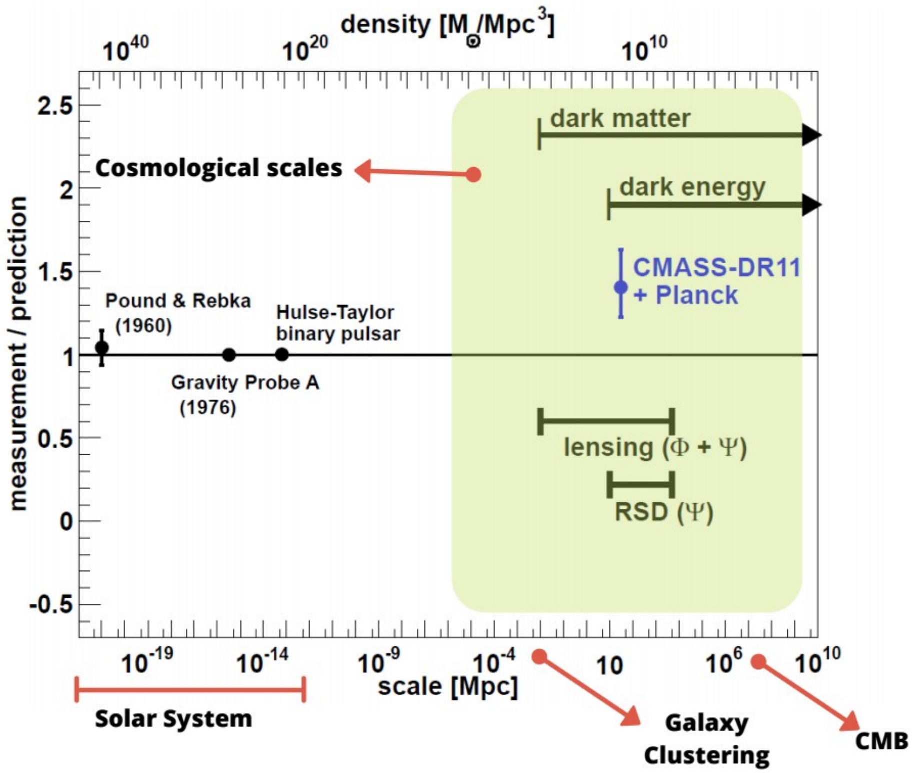
Introduction

- We already have multiple constraints on gravity from different probes
- Astrophysical and solar system are the tighest
- Cosmological are still not competitive when compared to local, how can we improve?



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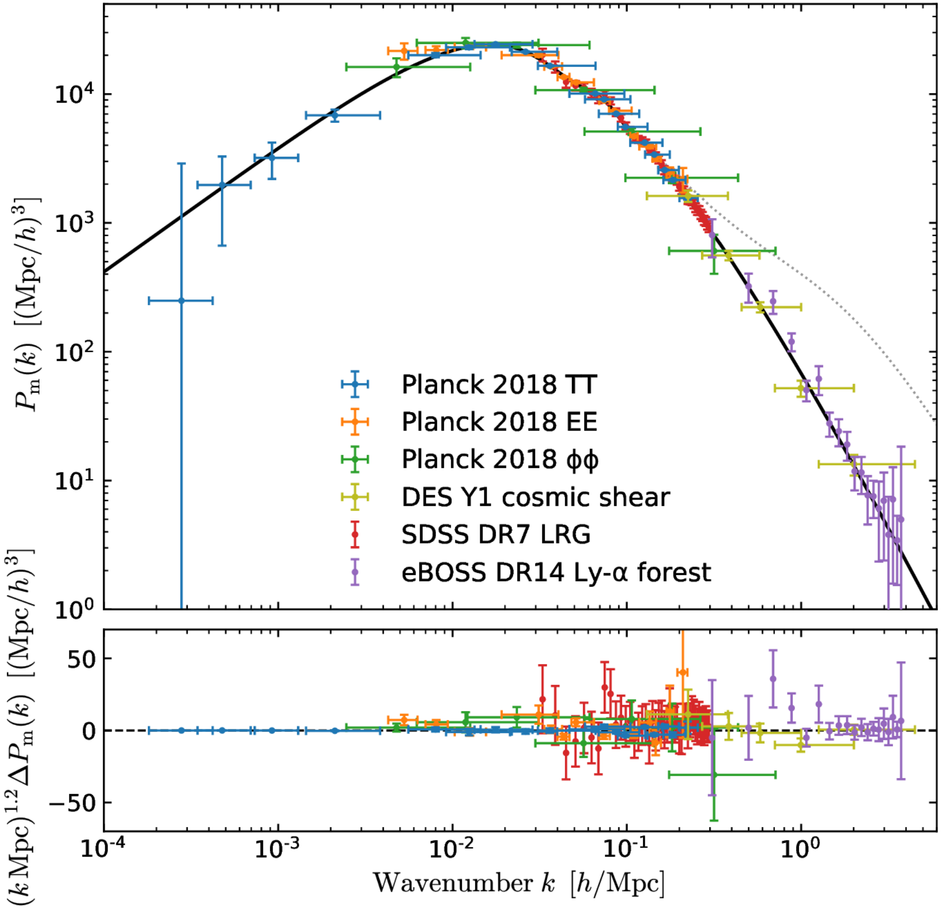

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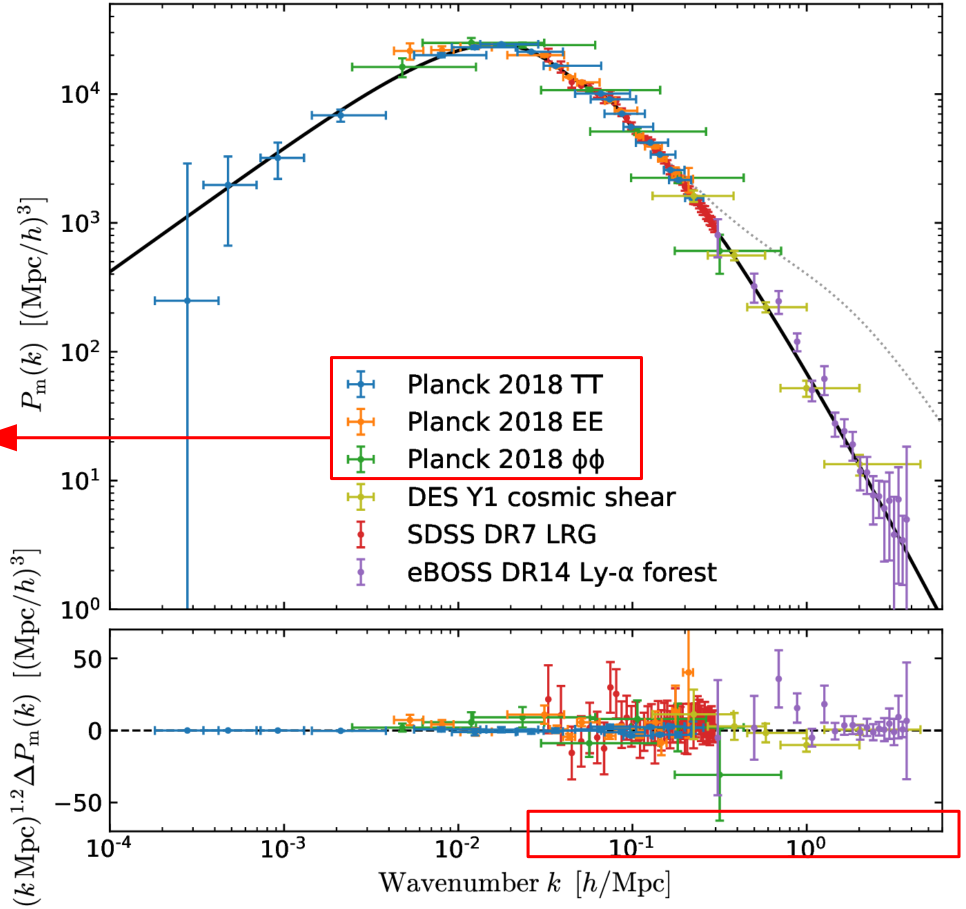
Points inferred (assume model)



Scales measured

Introduction

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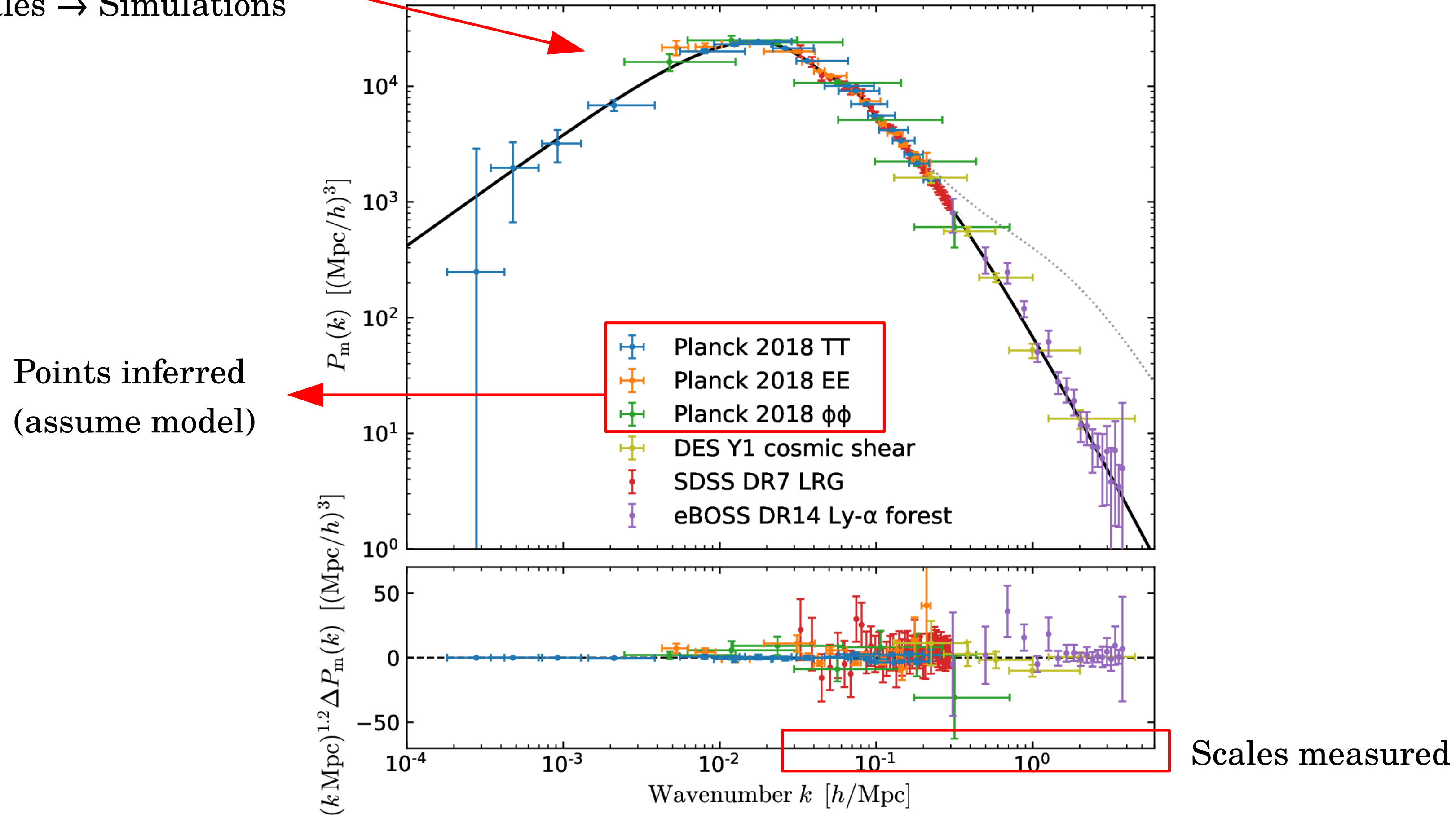

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- Euclid, DESI, LSST → bulk of information will be on non-linear scales → Simulations

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$$\nabla^2\Phi \propto G_{eff}(\tau, k)\rho_m\delta_m$$

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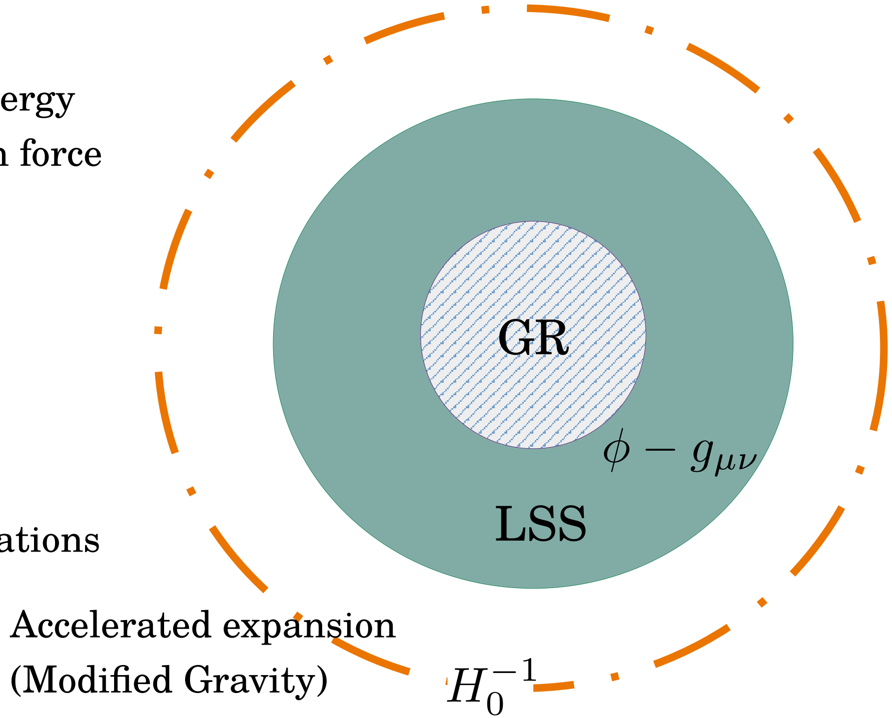

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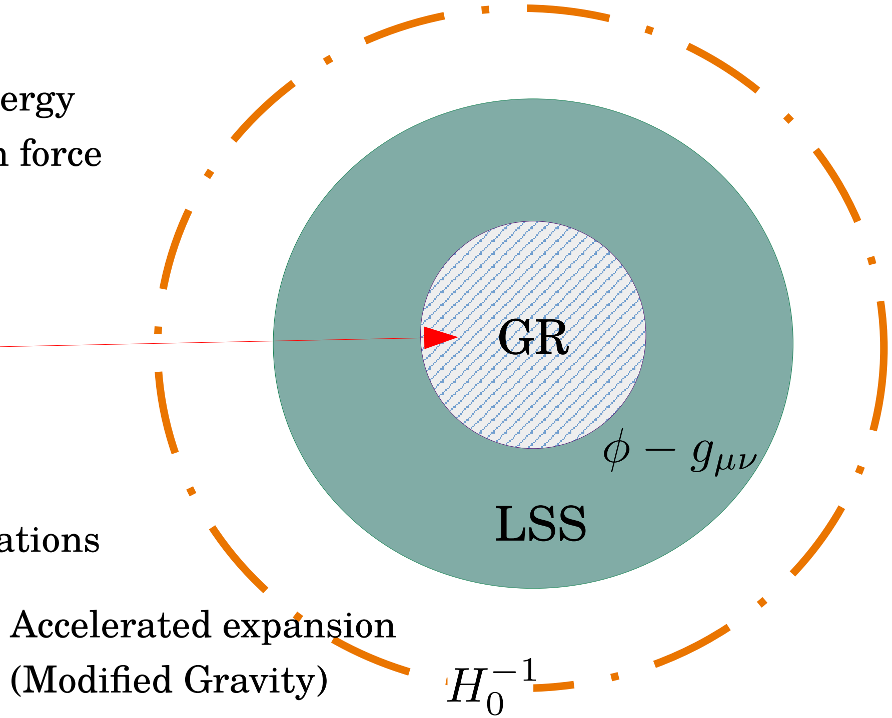


Introduction

- A note on screening:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[G_4(\phi)R + K(\phi,\nabla\phi) + G_3(\phi,\nabla\phi,\nabla^2\phi)\right]$$

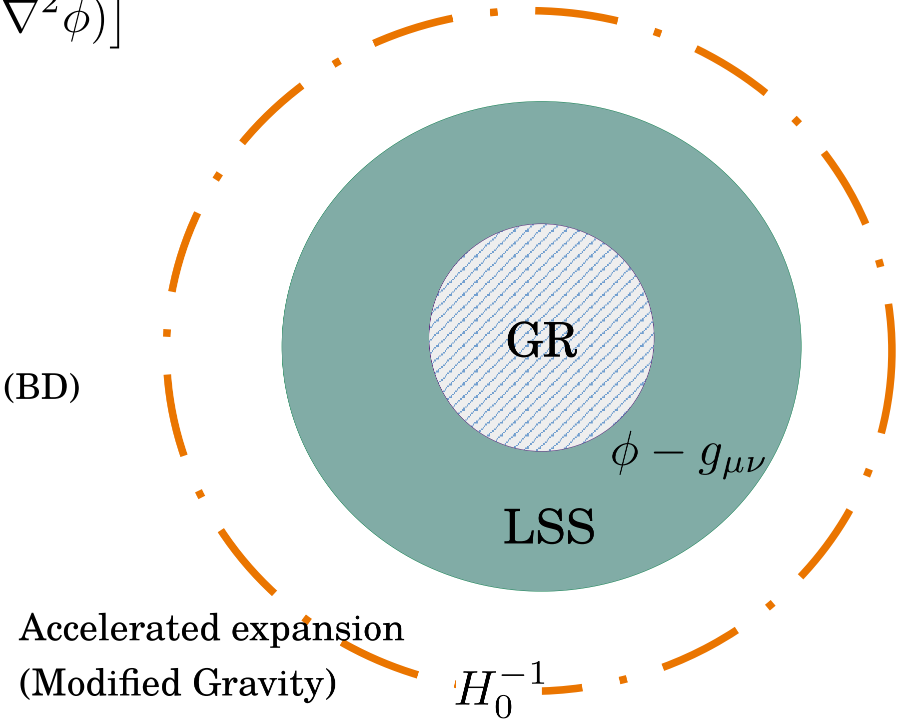
- $K = X + V(\phi) \rightarrow$ make the scalar short-ranged (chameleon)

- $K = \frac{\omega_{\mathrm{BD}}}{\phi}(\nabla\phi)^2 \rightarrow$ kinetic term large $\rightarrow$ suppress matter coupling (BD)

- $K + G_3 = N(\nabla\phi, \nabla^2\phi) \rightarrow$ non-linearities in EoM $\rightarrow$

  derivatives suppress scalar field charge

  (k-mouflage, Vainshitein)



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  I. Use specific gauge (N. Chisari and M. Zaldarriaga 1101.3555) :

     - N-Body gauge (C. Fidler et al: 1505.04756, 1606.05588, …)

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  II. Poisson gauge weak-field approximation → K-evolution (F. Hassani, et al: 1910.01104, 1910.01105)

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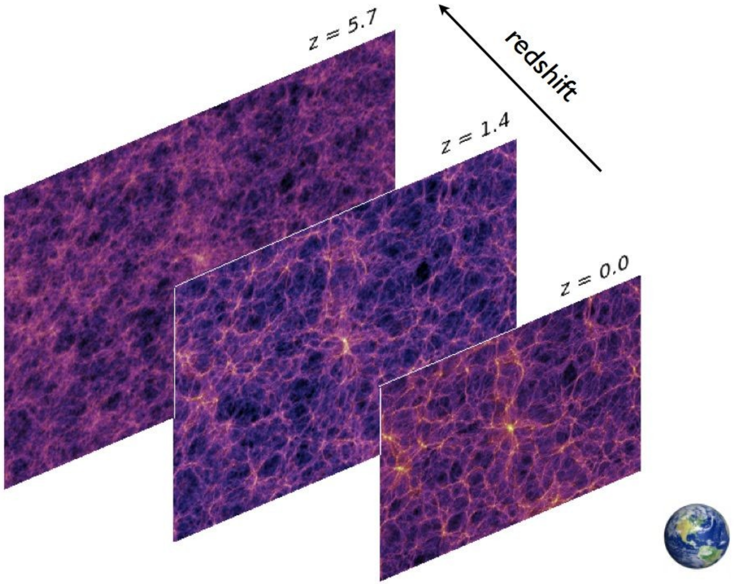

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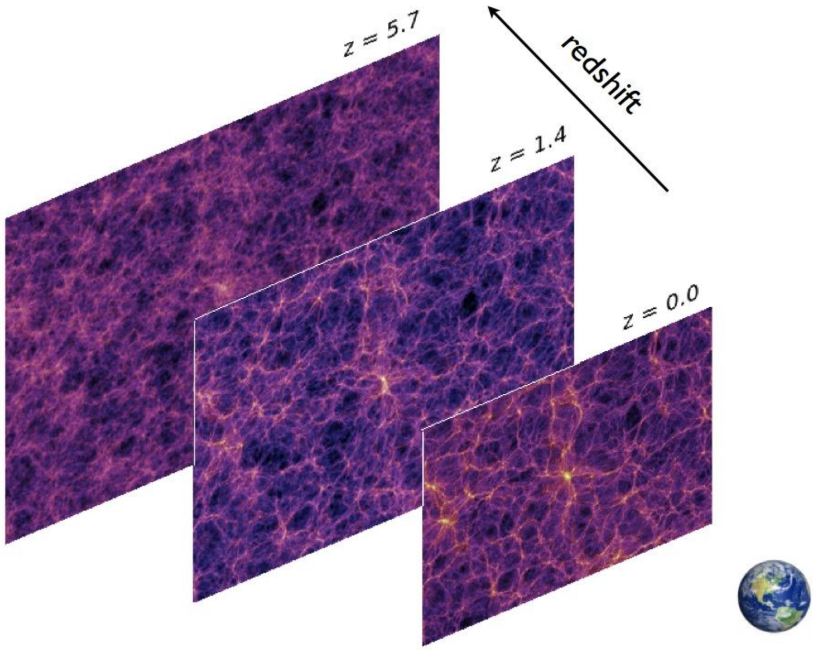

- Ray tracing (build light cone)
- Galaxy number count (what surveys “observe”)
- LoS relativistic corrections (RSD, WL, ISW, …)
- Mock galaxy catalogs (emulators)
- Covariance matrices (modelling for future surveys)

The N-Body Gauge

N-Body gauge

- CDM dynamics:

$$
\begin{aligned}
k^2\Phi^{\mathrm{N}} &= 4\pi G a^2 \bar{\rho}^{\mathrm{N}}_{\mathrm{cdm}}\delta^{\mathrm{N}}_{\mathrm{cdm}} \,, \\
\dot{\delta}^{\mathrm{N}}_{\mathrm{cdm}} + k v^{\mathrm{N}}_{\mathrm{cdm}} &= 0 \,, \\
\left[\partial_\tau + \mathcal{H}\right] v^{\mathrm{N}}_{\mathrm{cdm}} &= -k\Phi^{\mathrm{N}}
\end{aligned}
$$

$$
\begin{aligned}
k^2\Phi &= 4\pi G a^2 \left[\bar{\rho}\delta + 3\mathcal{H}\left(\bar{\rho}+\bar{p}\right)(v-B)/k\right] \,, \\
\dot{\delta}_{\mathrm{cdm}} + k v_{\mathrm{cdm}} &= -3\dot{H}_{\mathrm{L}} \,, \\
\left[\partial_\tau + \mathcal{H}\right] v_{\mathrm{cdm}} &= -k\left[\Phi + (\partial_\tau + \mathcal{H})\dot{H}_{\mathrm{T}} + 12\pi G a^2 \left(\rho + p\right)\sigma\right]
\end{aligned}
$$

Newton

GR

$$
\begin{aligned}
g_{00} &= -a^2\left(1+2A\right) \,, \\
g_{0i} &= a^2\, \mathrm{i}\hat{k}_i B \,, \\
g_{ij} &= a^2\left[\delta_{ij}\left(1+2H_{\mathrm{L}}\right) + 2\left(\delta_{ij}/3 - \hat{k}_i\hat{k}_j\right) H_{\mathrm{T}}\right]
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N-Body gauge

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Newton

GR

$$H_{\mathrm{L}} = \Phi^{\mathrm{P}},$$
$$H_{\mathrm{T}} = B = 0,$$ Newtonian (Poisson) gauge
$$A = \Psi^{\mathrm{P}}$$

$$g_{00} = -a^2\left(1+2A\right)\,,$$
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N-Body gauge

- Setting: $H_{\mathrm{L}} = 0, \quad B = v_m$

$$\begin{aligned}
k^2\Phi^{\mathrm{Nb}} &= 4\pi G a^2 \bar{\rho}_{\mathrm{cdm}} \delta^{\mathrm{Nb}}_{\mathrm{cdm}}\,, \\
\dot{\delta}^{\mathrm{Nb}}_{\mathrm{cdm}} + k v^{\mathrm{Nb}}_{\mathrm{cdm}} &= 0\,, \\
\left[\partial_\tau + \mathcal{H}\right] v^{\mathrm{Nb}}_{\mathrm{cdm}} &= -k\left(\Phi^{\mathrm{Nb}} + \gamma^{\mathrm{Nb}}\right)\,, \\
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\end{aligned}$$

- Comoving curvature perturbation:

$$\zeta = H_{\mathrm{L}} + \frac{H_{\mathrm{T}}}{3} - \frac{\dot{a}}{a}\frac{v_m - B}{k}$$

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$$
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$$

- Comoving gauge

$$
\dot{\zeta} = \frac{\mathcal{H}}{\rho + p}\left[\left(\rho + p\right)\sigma - \delta p^{\rm com.}\right]
$$

N-Body gauge

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$$
\dot H^{\rm Nb}_{\rm T} = 3\frac{\mathcal{H}}{\rho + p}\left[(\rho + p)\,\sigma - \delta p^{\rm S/P} + \dot p\frac{\theta^{\rm S/P}_{\rm tot}}{k^2}\right],
$$

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$$

- But CDM has no pressure and no anisotropic stress, then at linear order... $H_{\rm T} = 0$

N-Body gauge

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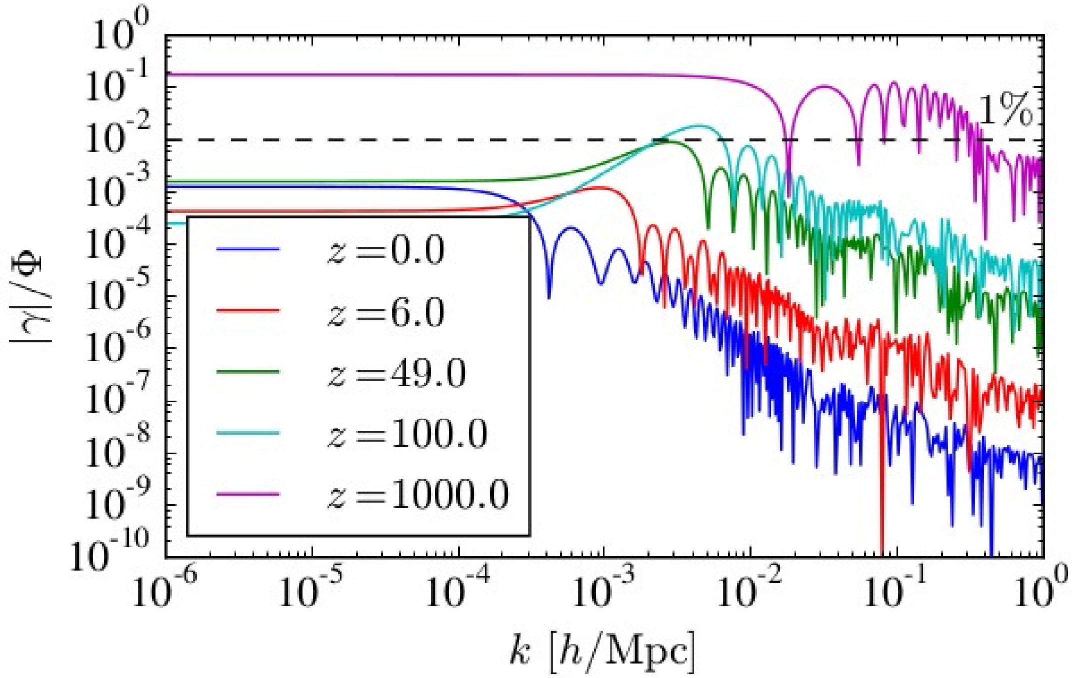
$$\begin{aligned} &k^2\Phi^{\rm N} = 4\pi G a^2 \bar{\rho}^{\rm N}_{\rm cdm}\delta^{\rm N}_{\rm cdm}\,, \\ &\dot{\delta}^{\rm N}_{\rm cdm} + k v^{\rm N}_{\rm cdm} = 0\,, \\ &\left[\partial_\tau + \mathcal{H}\right] v^{\rm N}_{\rm cdm} = -k\Phi^{\rm N} \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} &k^2\Phi^{\rm Nb} = 4\pi G a^2 \bar{\rho}_{\rm cdm}\delta^{\rm Nb}_{\rm cdm}\,, \\ &\dot{\delta}^{\rm Nb}_{\rm cdm} + k v^{\rm Nb}_{\rm cdm} = 0\,, \\ &\left[\partial_\tau + \mathcal{H}\right] v^{\rm Nb}_{\rm cdm} = -k\Phi^{\rm Nb} \end{aligned}$$

Newton GR

N-Body gauge



Only photons and neutrinos.

N-Body gauge

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  iii. We can introduce effects of photons, neutrinos and dark energy at large sacales:

  $$\begin{aligned}
  &\ddot{\delta}_m^{\mathrm{Nb}} + \mathcal{H}\dot{\delta}_m^{\mathrm{Nb}} - 4\pi G a^2 \rho_m \delta_m^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{GR}}, \\
  &\delta\rho_{\mathrm{GR}} = \delta\rho_\gamma^{\mathrm{Nb}} + \delta\rho_\nu^{\mathrm{Nb}} + \delta\rho_{\mathrm{DE}}^{\mathrm{Nb}} + \delta\rho_{\mathrm{metric}}^{\mathrm{Nb}}, \\
  &k^2\gamma^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{metric}}
  \end{aligned}$$

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$$\underbrace{\ddot{\delta}_m^{\mathrm{Nb}} + \mathcal{H}\dot{\delta}_m^{\mathrm{Nb}} - 4\pi G a^2 \rho_m \delta_m^{\mathrm{Nb}}}_{\text{Same as Newtonian at small scales}} = 4\pi G a^2 \delta\rho_{\mathrm{GR}},$$

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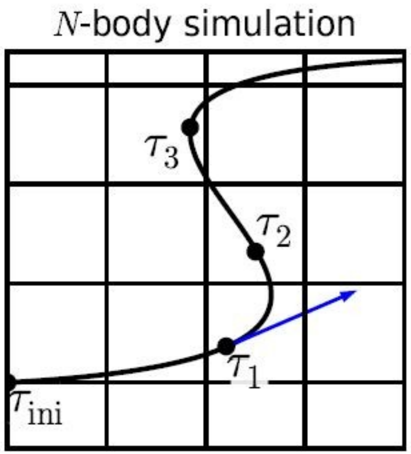

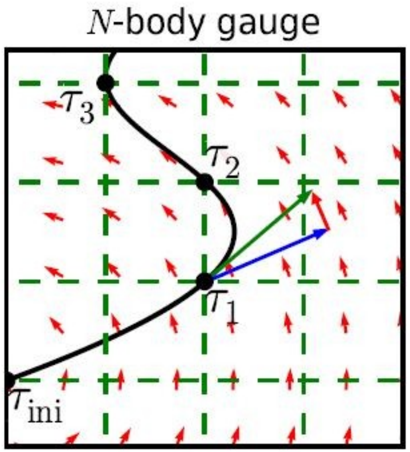

$$\underbrace{\ddot{\delta}_m^{\rm Nb} + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2 \rho_m \delta_m^{\rm Nb}}_{\text{Same as Newtonian at small scales}} = \underbrace{4\pi G a^2 \delta\rho_{\rm GR}}_{\text{Relevant at large scales}} \Rightarrow$$

Output of the simulation already in the N-Body gauge

N-Body gauge

- Large scales → linear, use an Einstein-Boltzmann solver

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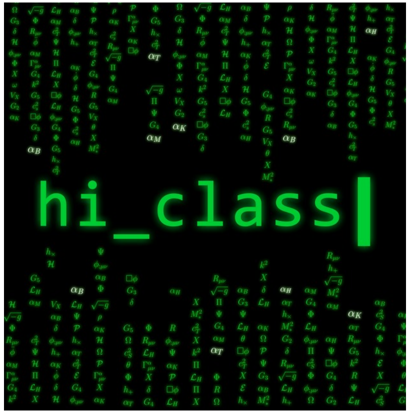
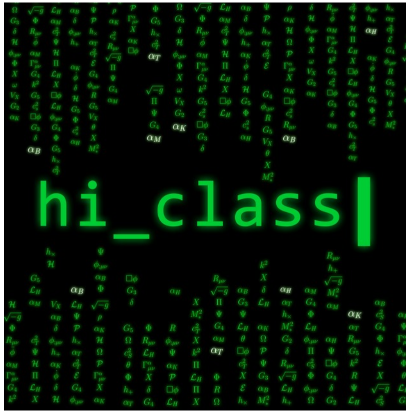


N-Body gauge

- Large scales → linear, use an Einstein-Boltzmann solver

$$\delta\rho_{\mathrm{GR}} = \delta\rho_{\gamma}^{\mathrm{Nb}} + \delta\rho_{\nu}^{\mathrm{Nb}} + \delta\rho_{\mathrm{DE}}^{\mathrm{Nb}} + \delta\rho_{\mathrm{metric}}^{\mathrm{Nb}}$$



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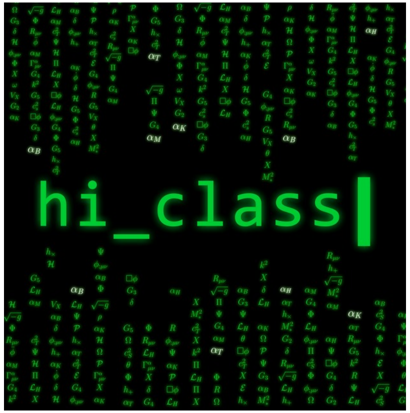

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- For the last term we need

$$k^2\gamma^{\mathrm{Nb}} = 4\pi G a^2 \delta\rho_{\mathrm{metric}}$$

N-Body gauge

- Compute inside hi_class: $k^2\gamma^{\mathrm{Nb}} = -(\partial_\tau + \mathcal{H})\dot{H}_{\mathrm{T}}^{\mathrm{Nb}} + 12\pi G a^2\,(\rho + p)\,\sigma$

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- Use Boltzmann equations for photons and neutrinos (massive and massless)

What about DE?

Modified Gravity

- Horndeski theory: most general scalar tensor theory with 2nd order equations in both fields.

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$$S[g_{\mu\nu},\phi] = \int \mathrm{d}^4x\,\sqrt{-g}\left[\sum_{i=2}^{5}\frac{1}{8\pi G}\mathcal{L}_i[g_{\mu\nu},\phi] + \mathcal{L}_\mathrm{m}[g_{\mu\nu},\psi_M]\right]$$

- But can be made simpler:

$H(z)$	Fixes background
$\alpha_\mathrm{K}(z)$	Kineticity
$\alpha_\mathrm{B}(z)$	Braiding
$\alpha_\mathrm{M}(z)$	Running Planck Mass
$\alpha_\mathrm{T}(z)$	Tensor excess

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Modified Gravity as a fluid

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$$\underbrace{G_{\mu\nu}}_{\text{GR}} = T^{(m)}_{\mu\nu} + \underbrace{T^{MG}_{\mu\nu}}_{\text{Horndeski}}$$

Modified Gravity as a fluid

- So EoM at linear order:

$$k^2\eta - \frac{1}{2}\mathcal{H}h' = 4\pi G_N a^2 \sum_\alpha \delta\rho_\alpha,$$

$$k^2\eta = 4\pi G_N a^2 \sum_\alpha (\rho_\alpha + p_\alpha)\,\theta_\alpha,$$

$$h'' + 2\mathcal{H}h' - 2k^2\eta = 8\pi G_N a^2 \sum_\alpha \delta p_\alpha,$$

$$h'' + 6\eta'' + 2\mathcal{H}\,(h' + 6\eta') - 2k^2\eta = -24\pi G_N a^2 \sum_\alpha (\rho_\alpha + p_\alpha)\,\sigma_\alpha,$$

$$\alpha : \mathrm{cdm}, \mathrm{b}, \gamma, \nu, \mathrm{DE}$$

$$V_X = a\frac{\delta\dot{\phi}}{\phi}$$

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α : cdm, b, γ, ν, DE

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GR

Modified Gravity as a fluid

- Where

$$
\begin{aligned}
\delta\rho_{\rm DE} &= \delta\rho_{\rm DE}\left(\tau, k^2\eta, V_X, V_X', \delta\rho_m\right)\,, \\
\theta_{\rm DE} &= \theta_{\rm DE}\left(\tau, V_X, V_X', \theta_m\right)\,, \\
\delta p_{\rm DE} &= \delta p_{\rm DE}\left(\tau, k^2\eta, h', V_X, \delta p_m\right)\,, \\
\left(\rho_{\rm DE}+p_{\rm DE}\right)\sigma_{\rm DE} &= \left(\rho_{\rm DE}+p_{\rm DE}\right)\sigma_{\rm DE}\left(\tau, k^2\eta, \eta', h', V_X, \sigma_m\right)
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\boxed{
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$$\delta\rho_{\rm DE} = \delta\rho_{\rm DE}\left(\tau, k^2\eta, V_X, V_X', \delta\rho_m\right),$$

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- These depend on background functions (time dependent only) and synchronous gauge metric potentials (already computed in hi_class)

- Can compute $\delta\rho_{\rm GR}$! And feed it into N-Body codes!

Results

Results

- Effects at extremely large scales in matter power spectra

$$H(z) = \Lambda\mathrm{CDM},$$
$$\alpha_i = c_i \Omega_{\mathrm{DE}}, \qquad i = \mathrm{B}, \mathrm{M}, \mathrm{T},$$
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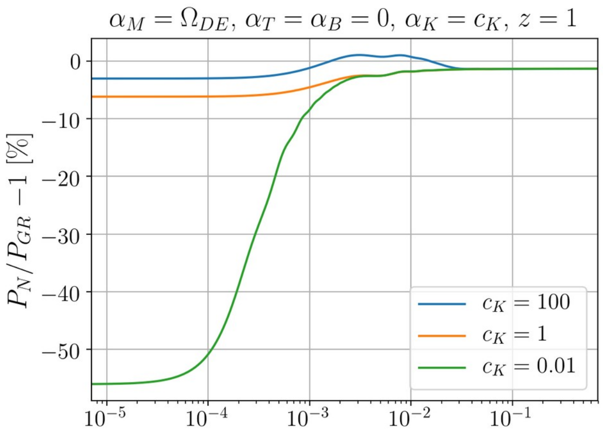

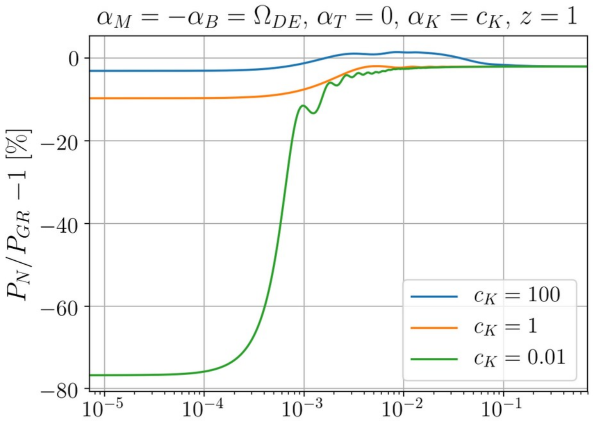

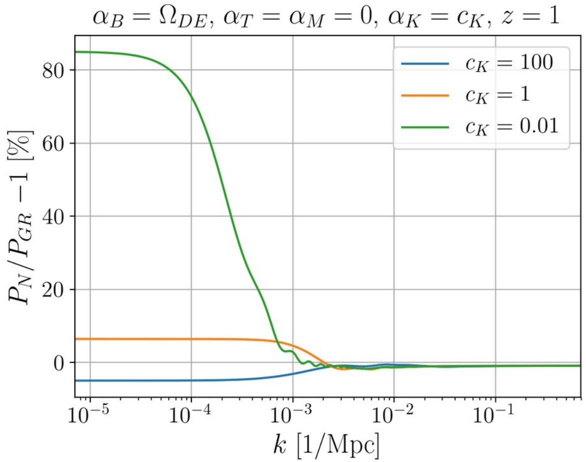

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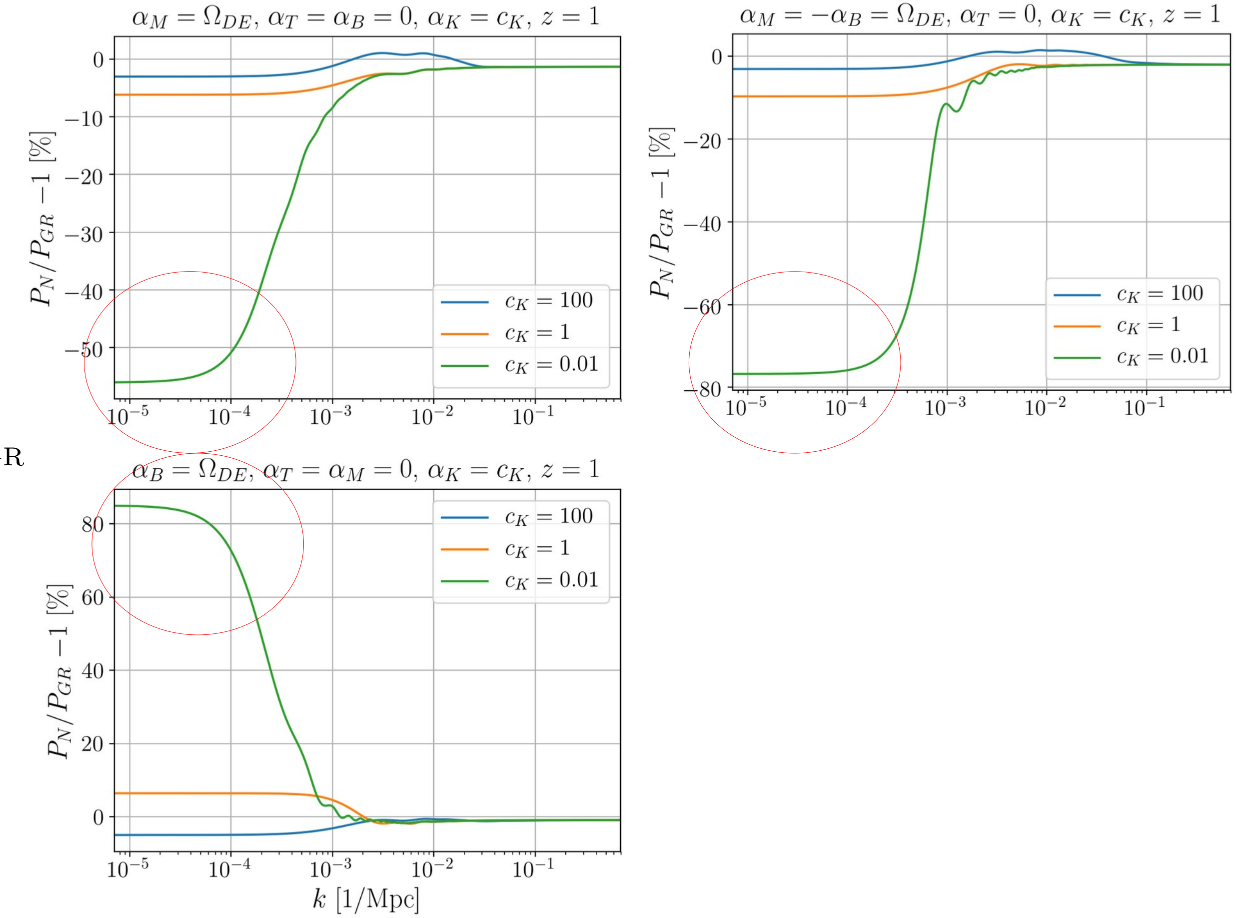

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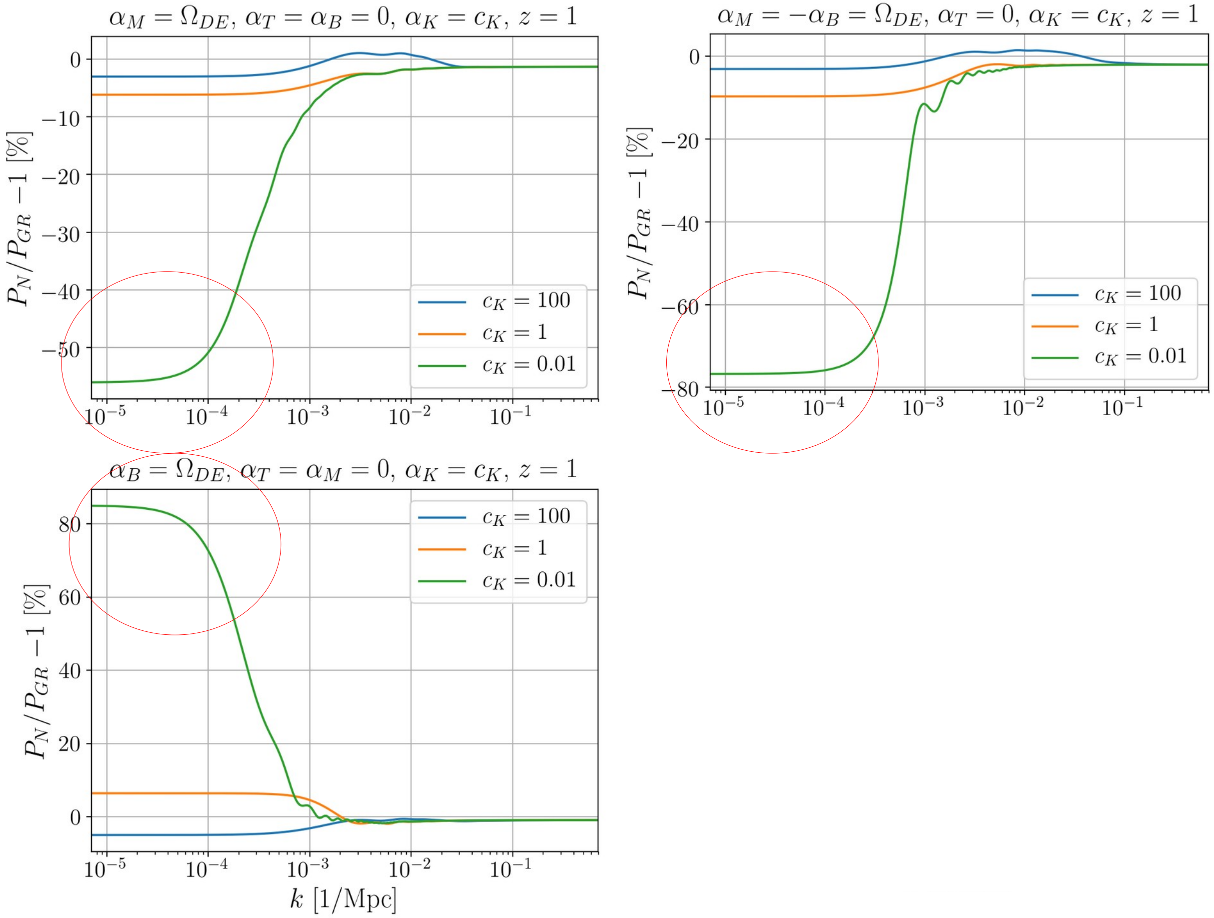
$$V_X'' + aHV_X' + a^2H^2V_X =$$
$$A(\tau)\delta\rho_{\mathrm{m}} + B(\tau)\delta p_{\mathrm{m}} + \frac{2c_s^2}{aH(2-\alpha_{\mathrm{B}})}k^2\eta$$

$$c_{\mathrm{s}}^2 \propto \frac{1}{\alpha_{\mathrm{K}}}$$



What about the other scales?

Results

- Separation small from large → Quasi-Static Aprroximation (QSA), valid deep inside hor.

$$\left.\begin{aligned} \delta\rho_{\rm DE}^{\rm QSA} &\propto \delta\rho_m \\ \sigma_{\rm DE}^{\rm QSA} &\propto \delta\rho_m \end{aligned}\right\} \quad G_{\rm eff} \qquad \nabla^2\Phi \propto G_{\rm eff}(\tau, k)\rho_m\delta_m$$

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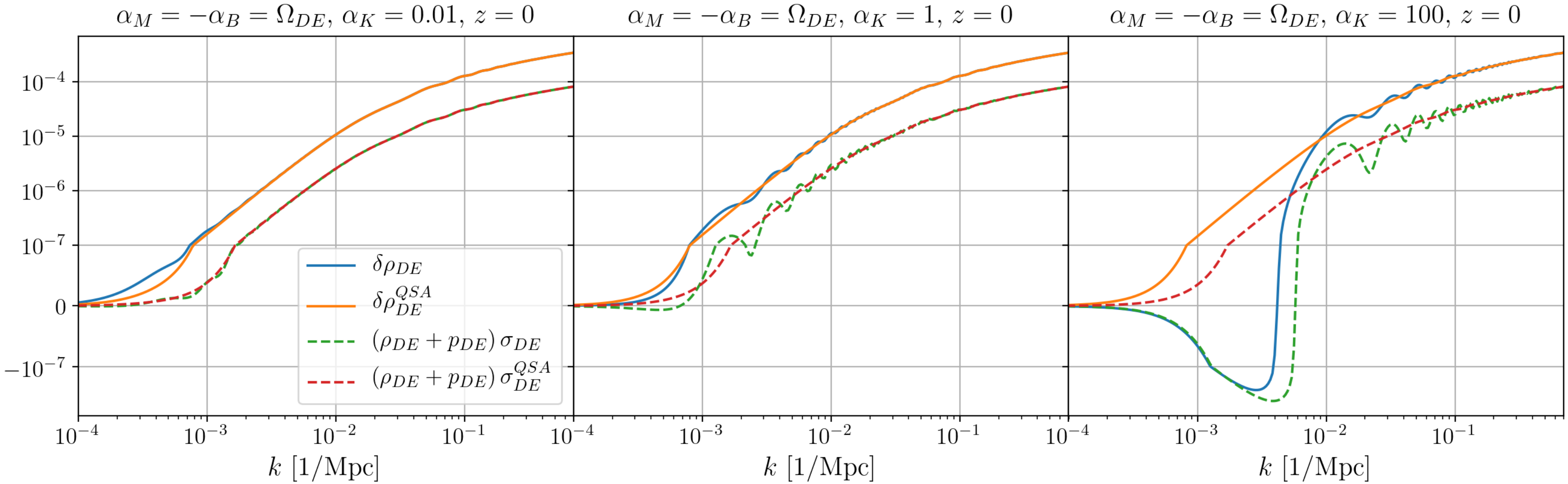
- QSA:
  - Write the small scale contributions of the dark energy density perturbations and anisotropic stress (diff. between Newtonian Potentials)

$$\delta\rho_{\rm DE} = \delta\rho_{\rm DE}^{\rm QSA} + \delta\rho_{\rm DE,\ rel.}$$
$$\sigma_{\rm DE} = \sigma_{\rm DE}^{\rm QSA} + \sigma_{\rm DE,\ rel.}$$

Results

- Bigger kineticity → Smaller sound speed → Agreement pushed to larger k



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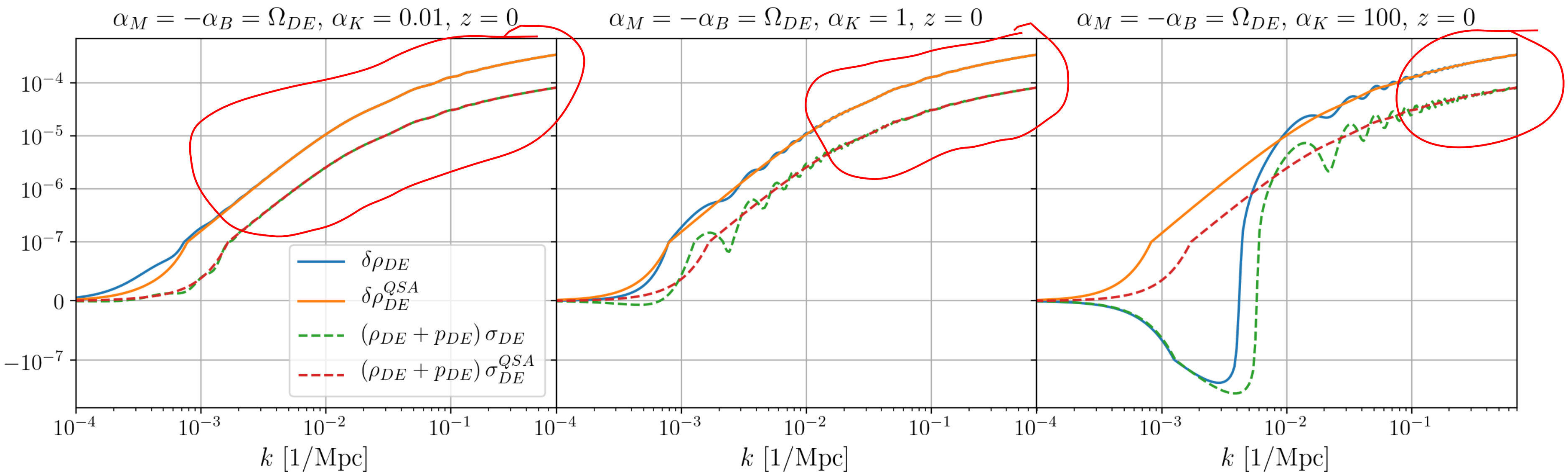

Results

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  1) Modified gravity Newtonian simulations: 5th force

$$\nabla^2\Phi \propto G_{\rm eff}(\tau,k)\rho_m\delta_m \qquad\qquad \ddot{\delta}_m^{\rm N} + \mathcal{H}\dot{\delta}_m^{\rm N} - 4\pi G_{\rm eff}a^2\rho_m\delta_m^{\rm N} = 0.$$

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2) N-Body gauge:

$$\delta\rho_{\rm GR} = \delta\rho_\gamma^{\rm Nb} + \delta\rho_\nu^{\rm Nb} + \delta\rho_{\rm DE}^{\rm Nb} + \delta\rho_{\rm metric}^{\rm Nb} \qquad \ddot{\delta}_m^{\rm Nb} + \mathcal{H}\dot{\delta}_m^{\rm Nb} - 4\pi G a^2\rho_m\delta_m^{\rm Nb} = 4\pi G a^2\delta\rho_{\rm GR}$$

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3) QSA:

$$\begin{aligned}
\delta\rho_{\rm GR,\ rel.} &= \delta\rho_\gamma^{\rm Nb} + \delta\rho_\nu^{\rm Nb} + \delta\rho_{\rm DE,\ rel.}^{\rm Nb} + \delta\rho_{\rm metric,\ rel.},\\
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$$P_{\rm N}^{\rm Geff}$$

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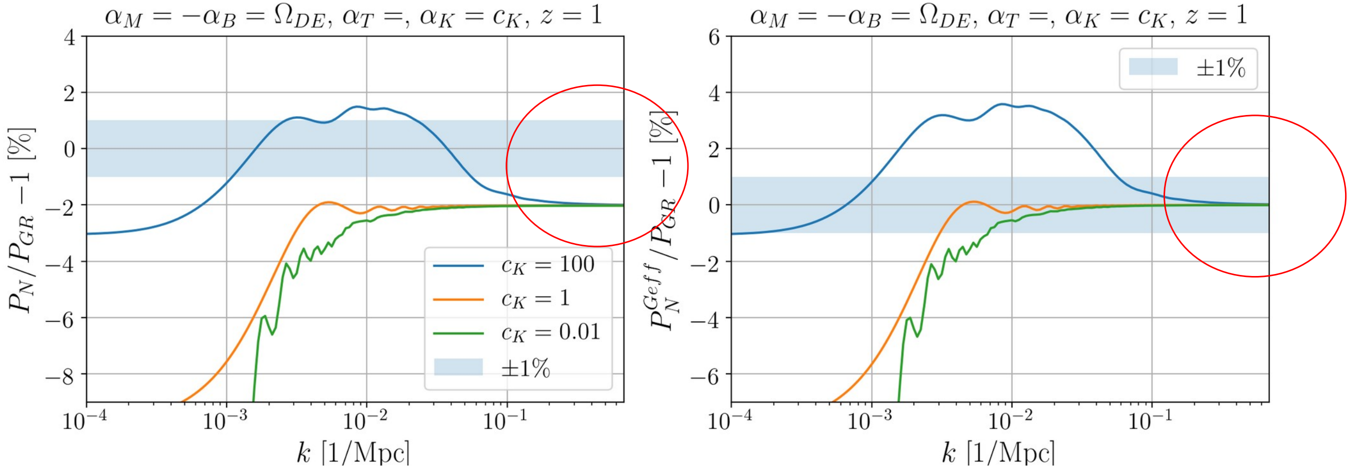
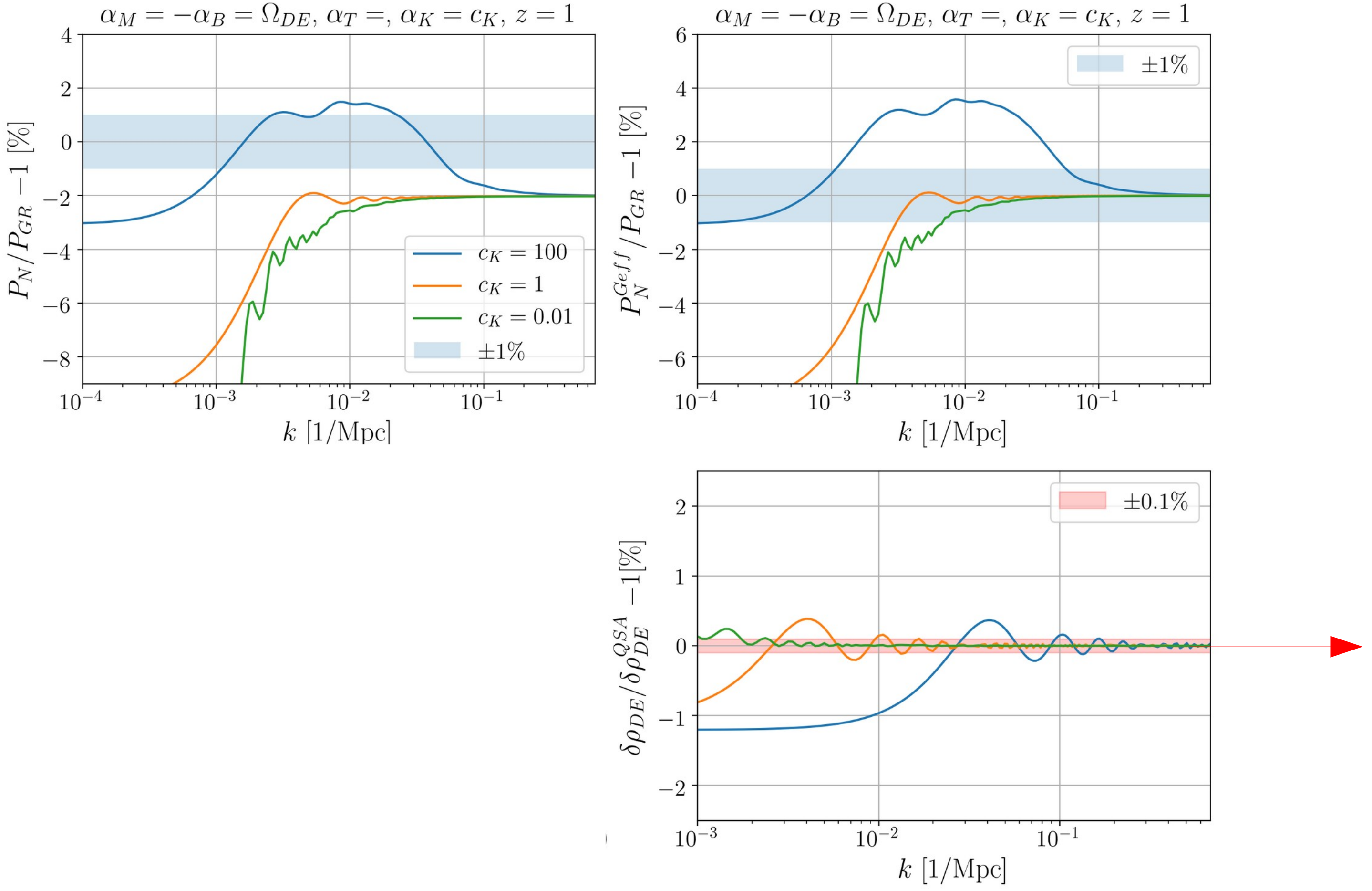
$$P_{\rm GR}$$

Results



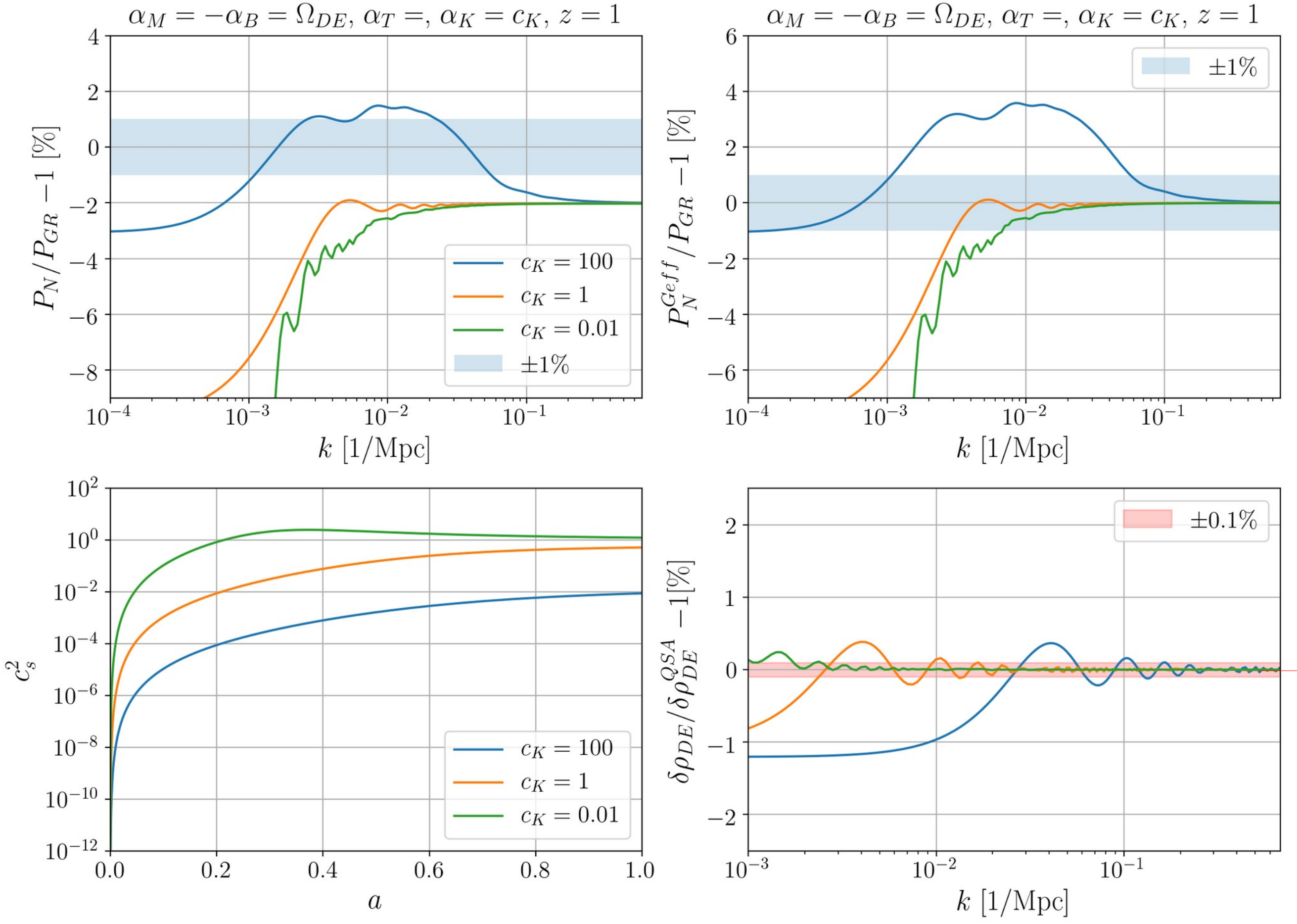
Mitigate the effect of: G_{eff}

Results



Smooth transition to Newtonian gravity (QSA)

Results



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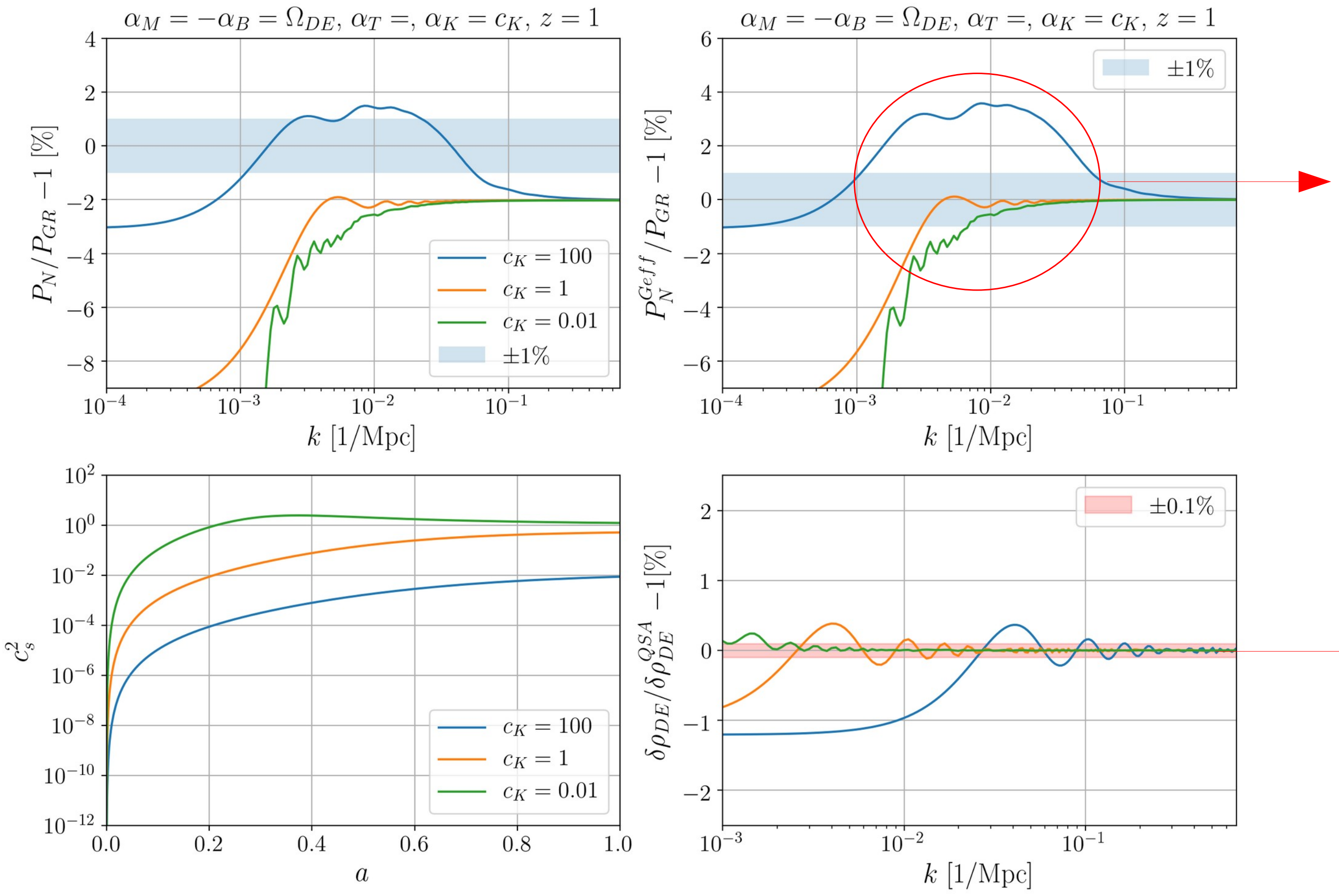
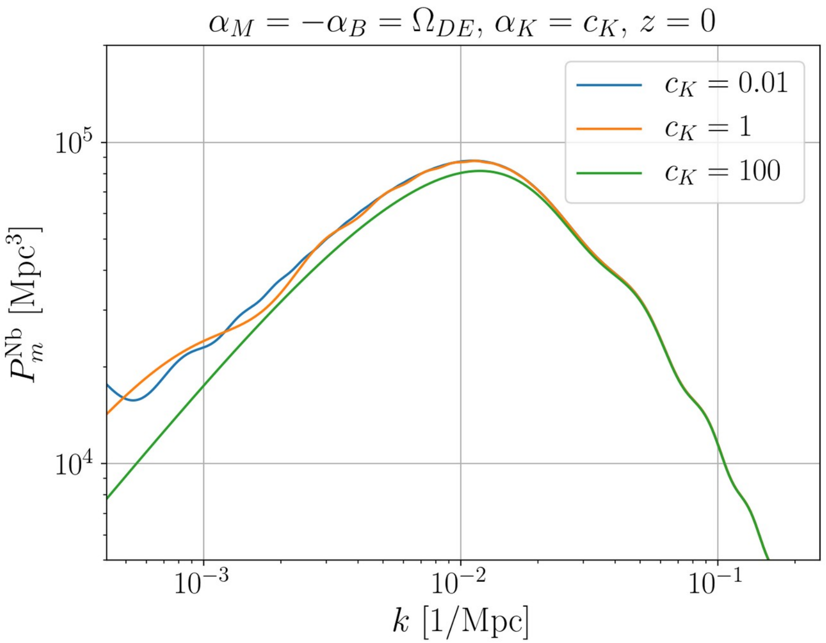


Relativistic effects
Not captured by QSA

Smooth transition to
Newtonian gravity (QSA)

Results

- Gravity acoustic oscillations (GAOs) from 10^-3 – 10^-2 1/Mpc



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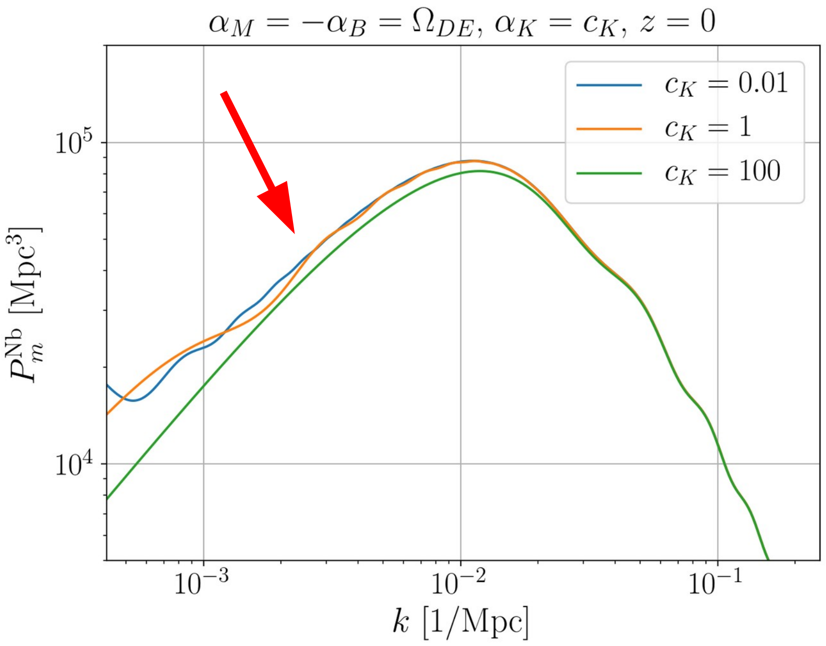

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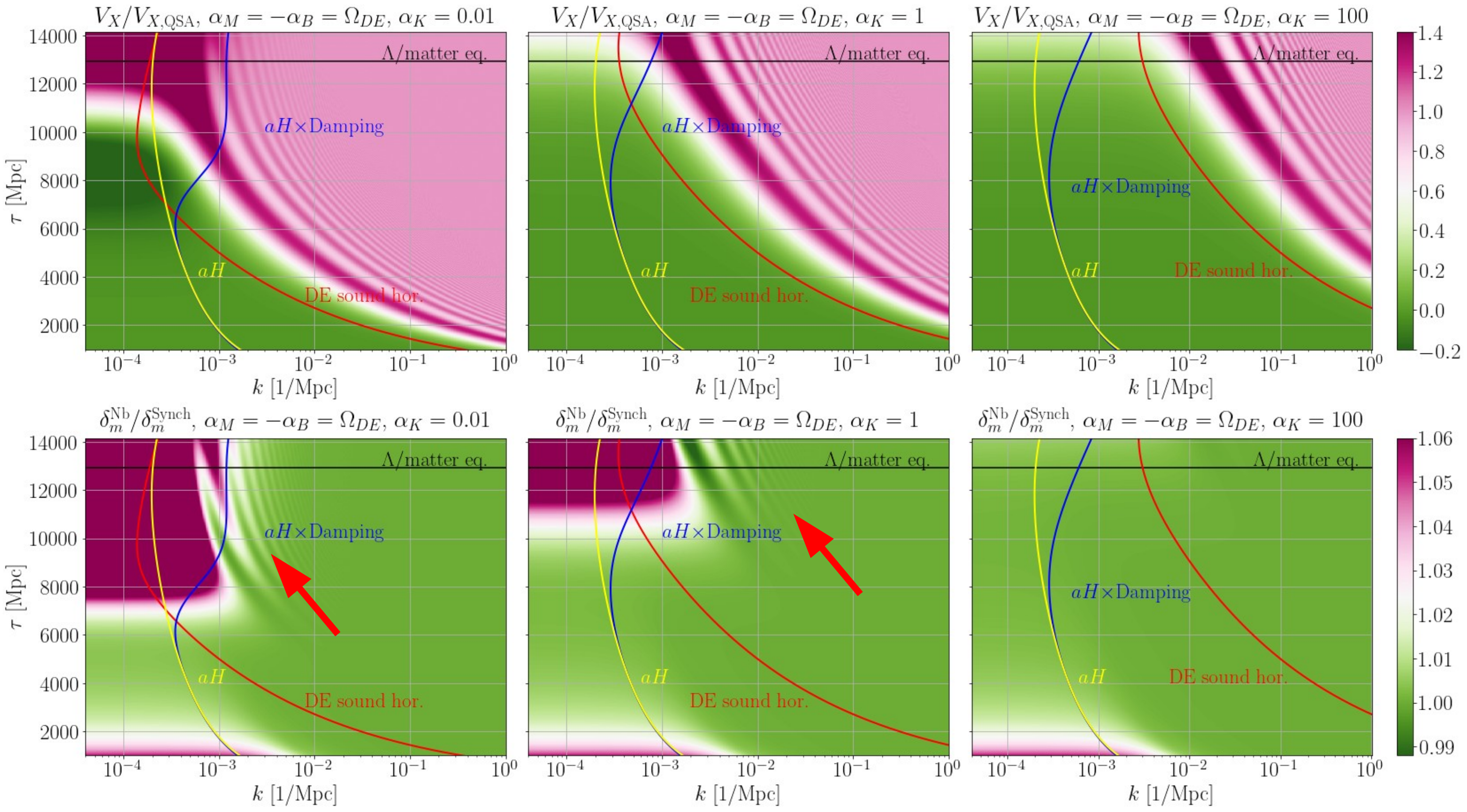

$V_X/V_{X,\mathrm{QSA}}$, $\alpha_M = -\alpha_B = \Omega_{DE}$, $\alpha_K = 0.01$
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Λ/matter eq.
$aH\times$Damping
aH
DE sound hor.
τ [Mpc]
k [1/Mpc]

Conclusions/Future

- Relativistic effects of DE not accounted for in MG simulations can now be readily implemented with $\delta\rho_{\mathrm{GR}}$

- Kineticity enhances the signal at large scales

- Emergence of GAOs at scales 10^-3 – 10^-2 1/Mpc in the matter power spectrum,

  amplified to (possibly) detecteable levels in models with rapid DE sound horizon evolution

Conclusions/Future

- Move to non-linear scales→ Implement in Newtonian MG simulation (MG-COLA, gevolution)

- Other example in the literature of relativistic MG simulation: k-evolution,

  so it would be interesting to introduce in k-essence (clustering DE) model to cross-check and validate the results.

Thank you!